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FROM JANUARY 1931 TO SEPTEMBER 1931

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PROCEEDINGS AT THE MEETINGS OF THE PHYSICAL SOCIETY

SESSION 1930–31

Except where the contrary is stated, the meetings were held at the Imperial College of Science and Technology, South Kensington.

October 17, 1930.

Mr R. W. PAUL in the Chair.

1. The President announced that the Council had elected R. A. Newing, G. D. Pegler, Wilfred Pagley, Donovan Chilton, Hilda E. Carpenter, Elsie A. Simmons, Edith Dora Vedy, Charles John Birkett Clews to be Student Members of the Society.

2. A paper entitled "A simple approximate theory of the pressure between two bodies in contact," was read by J. P. ANDREWS, M.Sc., F.Inst.P.

3. A paper entitled "Experiments on impact," was read by J. P. ANDREWS, M.Sc., F.Inst.P.

4. A paper entitled "Observations on percussion figures," was read by J. P. ANDREWS, M.Sc., F.Inst.P.

5. A paper entitled "Some physical radiometric investigations of technical interest," was read by Dr RUDOLF HASE.

November 7, 1930.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Roger Daniel Hewart Jones, Robert Stephen Dadson, George Paget Thomson, Syed Mohamad Ali Khan, Herbert John Gough, Thomas Musgrave Pyke, R. F. Hanstock, R. S. Mani, P. V. Kuruvila, H. R. Robinson, H. Amorim Ferreira, Louis George Melio, Donald Thomas Jones, H. Harrison Macey were elected Fellows of the Physical Society.

The President announced that the Council had elected E. R. F. P. Mendis to be a Fellow of the Physical Society under Article 14, and Allen James Lewis to be a Student Member of the Society.

2. A paper entitled "The absorption and dissociative or ionizing effect of monochromatic radiation in an atmosphere on a rotating earth," was read by Prof. S. CHAPMAN.

3. A paper entitled "Turbulent flow through tubes," was read by W. N. BOND, M.A., D.Sc., F.Inst.P., Lecturer in Physics in the University of Reading.

4. A paper entitled "The spectrum of trebly-ionized cerium (Ce IV)," was read by J. S. BADAMI, Imperial College of Science and Technology.

5. A paper entitled "The photographic effects of gamma-rays," by J. S. ROGERS, B.A., M.Sc., F.Inst.P., Senior Lecturer in Natural Philosophy, University of Melbourne, was read.

November 21, 1930.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Thomas Eric Banks, Frank Arthur Long, Mark Thompson, Robert C. G. Williams, Charles Seymour Wright, and William Rees Williams were elected Fellows of the Physical Society.

The President announced that the Council had elected F. B. Levetus and James Alfred Conway Student Members of the Society.

2. A paper entitled "The determination of the acoustical characteristics of singly-resonant hot-wire microphones," was read by E. T. PARIS, D.Sc., F.Inst.P.

3. A paper entitled "The spectrum of doubly-ionized arsenic," by K. R. RAO, D.Sc., Madras Government Research Scholar, University of Upsala, was read.

4. A paper entitled "The effect of temperature on spark-potential," was read by H. C. BOWKER, B.Sc. (Eng.), Ph.D., A.K.C.

5. A paper entitled "The Curie points," was read by L. F. BATES, B.Sc., Ph.D., University of London, University College.

6. A demonstration was given of an instrument for compounding curves, designed by J. L. HAUGHTON, D.Sc., The National Physical Laboratory.

December 5, 1930.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Alan Julian Maddock, John Francis Robertson, Thomas Lidyard Rosgyll Ayres, William Lethersich, Karl Manne Georg Siegbahn, Mary John, Homer L. Dodge, Wilfred Basil Mann, and Leonard Francis Osmond were elected Fellows of the Physical Society.

2. A paper entitled "A point of analogy between the equations of the quantum theory and Maxwell's equations," by M. FAHMY, The Egyptian University, Cairo, was read.

3. A paper entitled "Sources of illumination for ultra-violet microscopy," was read by B. K. JOHNSON, F.R.M.S.

4. A paper entitled "The influence of the crystal-orientation of the cathode on that of an electro-deposited layer," was read by W. A. WOOD, M.Sc., National Physical Laboratory.

5. A paper entitled "Relations between the fundamental physical constants," was read by C. A. KLOSS, B.Sc., A.I.C., British Electrical and Allied Industries Research Association.

6. A demonstration of some stroboscopic effects was given by Prof. G. B. BRYAN, D.Sc., Royal Naval College, Greenwich.

January 6, 7 *and* 8, 1931.

The twenty-first annual exhibition of the Physical Society and the Optical Society was held in the Imperial College of Science.

Discourses were delivered as follows:

January 6th: Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., opening address.

January 7th: E. LANCASTER-JONES, B.A., "Searching for minerals with scientific instruments."

January 8th: Prof. Sir GILBERT WALKER, F.R.S., "The physics of sport."

January 16, 1931.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Henry C. Calvert, Claude L. Lyons, Luang Brata, Robert Donaldson, Thomas Arthur Chalmers, William Edward Thomas Perry, Charles Edward Robinson and Clifford Wainwright were elected Fellows of the Physical Society.

The President announced that Robert Welham Haward, Aileen Mary Prior and Walter Wilson had been elected to Student Membership of the Society.

2. A paper entitled "The influence of low temperatures on the thermal diffusion effect," by T. L. IBBS, M.C., Ph.D. and K. E. GREW, Ph.D., was read by Dr Ibbs.

3. A paper entitled "Further experiments on magnetostriction oscillators at radio frequencies," was read by J. H. VINCENT, M.A., D.Sc., F.Inst.P.

4. A paper entitled "The equivalent circuit of the magnetostriction oscillator," by S. BUTTERWORTH, M.Sc. and F. D. SMITH, M.Sc., A.M.I.E.E., was read by Mr Butterworth.

5. A paper entitled "The theory of the microscope," was read by L. C. MARTIN, D.Sc., A.R.C.S., D.I.C.

6. A demonstration of plug and ring gauges was given by F. H. ROLT, National Physical Laboratory.

February 6, 1931.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Robert William Corkling, S. Rama Swamy, Basil Gordon Dickins, and Ernest Gordon Cox were elected Fellows of the Physical Society.

The President announced that Eric George Longhurst, Arthur Joseph Woodall, Ailsa W. Ikin, David Arthur Bell, David Evered Harrell Jones, and Charles Henry Kemp had been admitted to Student Membership of the Society.

2. A paper entitled "The radiation-reflecting powers of rough surfaces," was read by H. E. BECKETT, B.Sc.

3. A paper entitled "A ballistic recorder for small electric currents," was read by E. B. Moss, B.Sc.

4. A paper entitled "The instrumental phase-difference of seismograph records; an illustration of the properties of damped oscillatory systems," was read by F. J. SCRASE, M.A., B.Sc., Kew Observatory.

February 20, 1931.

Mr J. GUILD in the Chair.

1. Mrs Freda Stilwell, James Ronald Clarkson, Raymond George Wood, and Cyril Alfred Sinfield were elected Fellows of the Physical Society.

2. A paper entitled "On the velocity of sound waves in a tube," by G. G. SHERRATT, B.A. and J. H. AWBERY, B.A., B.Sc., F.Inst.P., Physics Department, National Physical Laboratory, Teddington, Middlesex, was read by Mr Sherratt.

3. A paper entitled "A note on the elimination of the β wave-length from the characteristic radiation of iron," was read by W. A. WOOD, M.Sc., Physics Department, National Physical Laboratory.

4. A paper entitled "The tube effect in sound-velocity measurements," was read by P. S. H. HENRY, Coutts Trotter Student, the Laboratory of Physical Chemistry, Cambridge.

March 6, 1931.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Guy Burniston Brown, Thomas Eran Leyshon, Frank Ernest Hoare, Alastair Watson Gillies, and Reginald Frederick Clark were elected Fellows of the Physical Society.

2. A paper entitled "Practical investigations of the earth resistivity method of geophysical surveying," was read by G. F. TAGG, B.Sc., A.M.I.E.E.

3. A paper entitled "A photoelectric spectrophotometer for measuring the amount of atmospheric ozone," was read by G. M. B. DOBSON, D.Sc., F.R.S.

4. A paper entitled "Displacements of certain lines of the spectra of ionized oxygen (O II, O III), neon (Ne II) and argon (A II)," was read by W. E. PRETTY, B.Sc., A.R.C.S., D.I.C., Assistant Lecturer in Physics, Imperial College, South Kensington.

5. The following experiments and slides were shewn by ERIC J. IRONS, Ph.D.:

(1) Mechanical lantern slide to demonstrate the formation and properties of stationary waves. The slide consists of two chemically fixed photographic plates upon each of which a sine curve is drawn. Means are provided whereby the curves move in opposite directions and the resultant displacements for various positions of the waves are registered on the board upon which the slide is focussed.

(2) Five lantern slides* illustrating the formation of dust figures in a Kundt's tube excited by a rod.

(3) Determination of the end-correction of a tube. Dust figure gives an ocular demonstration of the fact that an antinode is not formed exactly at the open end of a Kundt's tube and enable an estimate of the end-correction to be made.†

(4) Demonstration of the effect of a Quincke filter on the sound emitted from a valve-maintained tube. If a side branch tuned to a note of particular frequency is fixed in a conduit down which sound waves are passing, the energy associated with that frequency is absorbed in the branch.

Annual General Meeting, March 20, 1931.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. The Minutes of the preceding Annual Meeting were read and confirmed.

2. The Reports of the Council and of the Hon. Treasurer were presented and adopted.

3. Mr J. E. Calthrop and Major W. S. Tucker having been appointed scrutineers, the following officers and members of Council were elected for the year 1931–32:

President: Prof. Sir A. S. Eddington, M.A., D.Sc., F.R.S.

Vice-Presidents (who have filled the office of President): Sir Oliver J. Lodge, D.Sc., LL.D., F.R.S.; Sir Richard Glazebrook, K.C.B., Sc.D., F.R.S.; Sir Arthur Schuster, Ph.D., Sc.D., F.R.S.; Sir J. J. Thomson, O.M., D.Sc., F.R.S.; Prof. C. Vernon Boys, F.R.S.; Prof. C. H. Lees, D.Sc., F.R.S.; Prof. Sir W. H. Bragg, K.B.E., M.A., F.R.S.; Alexander Russell, M.A., D.Sc., F.R.S.; F. E. Smith, C.B., D.Sc., F.R.S.; Prof. O. W. Richardson, M.A., D.Sc., F.R.S.; W. H. Eccles, D.Sc., F.R.S.

* *Phil. Mag.* **7**, 523 (1929).

† *Phil. Mag.* **6**, 580 (1928).

Vice-Presidents: A. B. Wood, D.Sc.; Prof. A. O. Rankine, O.B.E., D.Sc.; J. S. G. Thomas, D.Sc.; J. Guild, A.R.C.S., D.I.C.

Hon. Secretaries: Ezer Griffiths, D.Sc., F.R.S.; Allan Ferguson, M.A., D.Sc.

Hon. Foreign Secretary: Prof. O. W. Richardson, M.A., D.Sc., F.R.S.

Hon. Treasurer: R. S. Whipple, M.I.E.E.

Hon. Librarian: J. H. Brinkworth, M.Sc., A.R.C.S.

Ordinary Members of Council: Lewis Simons, D.Sc.; J. H. Awbery, B.Sc.; T. Smith, M.A.; W. Jevons, D.Sc., D.I.C.; Prof. W. Wilson, Ph.D., D.Sc., F.R.S.; D. Owen, B.A., D.Sc.; B. P. Dudding, M.B.E., A.R.C.S.; Major I. O. Griffith, M.A.; D. W. Dye, D.Sc., F.R.S.; Prof. G. P. Thomson, F.R.S.

4. Votes of thanks to the auditors (proposed by Dr D. Owen and seconded by Dr J. H. Brinkworth), to the retiring officers and Council (proposed by Prof. F. L. Hopwood and seconded by Prof. L. C. Martin), to the Governing Body of the Imperial College of Science and Prof. G. P. Thomson for permission to meet at the College (proposed by Mr T. Smith and seconded by Mr J. Guild) and to the scrutineers (proposed from the Chair) were carried unanimously.

5. The Duddell Medal, 1930, was presented to Prof. Sir J. Ambrose Fleming, M.A., D.Sc., F.R.S., a founder Fellow of the Society who read the first paper at the opening meeting of the Society in March 1874. Sir Ambrose Fleming exhibited a collection of apparatus of historical interest.

Prof. H. E. Armstrong, also a founder Fellow, gave a brief address of reminiscence.

April 17, 1931.

Dr J. S. G. Thomas in the Chair.

1. Ralph Jessel, Sidney Zeidenfeld and Leslie Charles Bailey were elected Fellows of the Physical Society.

The Chairman announced that Miss T. M. Lloyd had been admitted to Student Membership of the Society.

2. A paper entitled "The generation of current-pulses of rectangular waveform," by A. J. Maddock, M.Sc., A.Inst.P., was taken as read.

3. A paper entitled "An improved method for the comparison of small magnetic susceptibilities," was read by R. A. Fereday, B.Sc., F.Inst.P.

4. A paper entitled "Edge tones," was read (with demonstration) by E. G. Richardson, B.A., Ph.D., D.Sc., University College, London.

May 1, 1931.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Richard Leonard Ascough Borrow, M. Farrell, R. W. Sutton, Godfrey Henry Barker and Frank Cecil Connelly were elected Fellows of the Physical Society.

2. A lecture was delivered by Prof. J. E. LENNARD-JONES, D.Sc., Ph.D., who took as his subject "Cohesion."

May 15, 1931.

Meeting held at the Science Museum, South Kensington.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. Frank Gill, John Rankine, G. A. Wedgwood, and W. Ewart Williams were elected Fellows of the Physical Society.

The President announced that Denis Raymond Wilson had been admitted to Student Membership of the Society.

2. The Sixteenth Guthrie Lecture was delivered by Sir RICHARD T. GLAZEBROOK, K.C.B., M.A., Sc.D., F.R.S., who took as his subject "Standards of measurement, their history and development."

A vote of thanks to the lecturer was proposed by Sir Frank Smith and seconded by Sir Henry Lyons.

June 5, 1931.

Prof. A. O. RANKINE, O.B.E., D.Sc., in the Chair.

1. Francis Alan Burnett Ward, Sidney J. Kennedy and Sidney Jefferson were elected Fellows of the Physical Society.

2. A paper entitled "Electro-osmosis and electrolytic water-transport," Part 2, was read by H. C. HEPBURN, Ph.D., Birkbeck College.

3. A paper entitled "Spectra of trebly and quadruply ionized antimony, Sb IV and Sb V," by J. S. BADAMI, Imperial College of Science, was read in title.

4. A paper entitled "A time base for the cathode-ray oscillography of irregularly recurring phenomena," by G. I. FINCH, R. W. SUTTON and A. E. TOOKE, Imperial College of Science, was read (with demonstration) by Prof. Finch.

5. A paper entitled "The attenuation of ultra-short radio waves due to the resistance of the earth," by R. L. SMITH-ROSE, D.Sc., Ph.D., A.M.I.E.E. and J. S. MCPETRIE, B.Sc., National Physical Laboratory, was read by Mr McPetrie.

6. A paper entitled "The absorption and dissociative or ionizing effect of monochromatic radiation in an atmosphere on a rotating earth. Part II—Grazing incidence," by S. CHAPMAN, M.A., D.Sc., F.R.S. was read in title.

7. A ripple-tank demonstration of beats as moving interference fringes was given by Mr M. O. CLARKE.

June 19, 1931.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., and subsequently Prof. A. O. RANKINE, O.B.E., D.Sc., in the Chair.

A discussion on audition was held.*

The discussion was opened by Dr C. S. MYERS, F.R.S. The following also contributed papers:

Dr E. D. ADRIAN, "The microphonic action of the cochlea in relation to theories of hearing." Dr R. T. BEATTY, "Auditory mechanisms." Dr A. W. G. EWING, "High-frequency deafness." Dr F. ALLEN, "The perception of intensity of sound in normal, depressed and enhanced states of aural sensitivity." Dr E. G. RICHARDSON, "The dynamical theory of the ear." Sir RICHARD A. S. PAGET, "Audition in relation to speech, and the production of speech sounds by the human vocal apparatus, by acoustic or electrical resonators and by musical instruments." Dr E. W. SCRIPTURE, "The nature of the vowels." Dr ERWIN MEYER, "The analysis of noises and musical sounds." Prof. E. N. DA C. ANDRADE, "Absolute measurement of sound-amplitudes and intensities." Dr C. V. DRYSDALE, "Acoustic measuring instruments." Dr H. BANISTER, "The basis of sound-localization." Dr A. H. DAVIS, "The measurement of noise." Dr F. TRENDELENBERG, "Objective measurement and subjective perception of sound." Dr G. WAETZMANN and Herr H. HEISIG, "The measurement by resonance telephone of the threshold sensitivity of the ear." Major W. S. TUCKER, "The localization of sound derived from observations of intensity." Prof. E. M. VON HORNBOSTEL, "The time-theory of sound-localization. A re-statement." Mr F. C. BARTLETT, "On certain general conditions of auditory measurements."

June 20, 1931.

Meeting held at University College, Reading.

Prof. Sir ARTHUR EDDINGTON, M.A., D.Sc., F.R.S., in the Chair.

1. A paper entitled "The absorption of X-rays" was read by Prof. J. A. CROWTHER, M.A., Sc.D.

2. A paper entitled "Magnetostriction and hysteresis" was read by Dr W. N. BOND, M.A.

3. A paper entitled "Radiation from a point source" was read by Mr L. G. VEDY, B.A., B.Sc.

4. After tea a series of seventeen exhibits were shown in the laboratories.

5. The meeting concluded with a trip on the river.

* Published in a separate volume.

REPORT OF COUNCIL FOR THE PERIOD ENDING FEBRUARY 28TH, 1931

MEETINGS

DURING the period covered by the report 12 ordinary science meetings were held at the Imperial College of Science. At these meetings 45 papers were presented and 7 demonstrations given.

On June 4 and 5, 1930, a joint discussion with the Optical Society was held on "Photo-electric cells and their applications," and on June 14, 1930, members of the Society and their friends visited Rugby. In the forenoon they were shown round the research laboratories and works of the British Thomson-Houston Company and were their guests at lunch. In the afternoon the Rugby Radio Station was visited and the party then proceeded to Rugby School where, after tea with the Headmaster and staff, the class-rooms and laboratories were inspected under the guidance of Mr F. A. Meier, the Senior Science Master.

A discussion on "Magnetism" was held on May 23, 1930, whilst on October 17, 1930, a lecture was delivered by Dr Rudolf Hase, entitled "Some physical radiometric investigations of technical interest."

EXHIBITION

The twenty-first Annual Exhibition, arranged jointly by the Physical and Optical Societies, was held on January 6, 7 and 8, 1931, through the courtesy of the Governing Body, at the Imperial College of Science. The exhibition was opened by the President of the Physical Society, Sir Arthur S. Eddington, F.R.S., who reviewed the salient points in connexion with the history of the exhibition for the past twenty years. The Research and Experimental Section contained exhibits from 29 sources. Trade exhibits were arranged by 85 firms. Discourses were given by Mr E. Lancaster-Jones on "Searching for minerals with scientific instruments," and by Professor Sir Gilbert Walker, F.R.S., on "Physics of sport."

REPRESENTATIVES ON OTHER BODIES

Dr Ezer Griffiths and Dr A. B. Wood have been nominated as representatives of the Society on the National Committees of the Royal Society, on Physics and Radio-Telegraphy respectively, of the International Unions founded under the International Research Council. Dr Ferguson and Dr D. Owen were appointed as representatives of the Society on the Board of the Institute of Physics, and also on the Science Abstracts Committee. Sir Richard Threlfall represented the Society on the occasion of the jubilee celebrations of the foundation of the Mason Science College and the thirtieth anniversary of the granting of the charter to the University of Birmingham.

THE DUDDELL MEDAL

At the Annual Meeting on March 28, 1930, the seventh (1929) Duddell Medal was presented to Professor A. A. Michelson through the intermediary of the United States Embassy. The Council has awarded the eighth (1930) Duddell Medal to Professor Sir Ambrose Fleming, F.R.S.

GUTHRIE LECTURE

Professor P. Debye delivered the fifteenth Guthrie Lecture on April 11, 1930, the subject being "The scattering of X-rays in gases, in relation to molecular structure."

OBITUARY

The Council records with deep regret the deaths of Professor W. G. Duffield, Mr H. Chester Bell, Mr T. C. Lewis, Mr M. J. Jackson, Mr H. S. S. Harvey, Professor Sir L. Ray Lankester, Mr V. Lough, Dr H. Borns and Lt.-Col. J. W. Gifford.

MEMBERSHIP ROLL

The number of Honorary Fellows on the roll on December 31, 1930, was 11. At the same date ordinary Fellows numbered 754 and students 51.

The changes in the membership of the Society are shown in the appended table:

	Total Dec. 31, 1929	Changes during 1930			Total Dec. 31, 1930
Honorary Fellows	11				11
Ordinary Fellows	724	Elected		54	
		Student transfers		10	
				64	
		Deceased	15		
		Resigned or lapsed	19		
			—	34	
		Net increase		30	754
Students ...	51	Elected		12	
		Trans. to Fellow ...	10		
		Resigned	2		
			—	12	
		Net increase		0	51
Total membership	786	Net increase		30	816

REPORT OF THE TREASURER

THE accounts for the year ended December 31st, 1930, show an adverse balance of £22. 2*s*. 10*d*.

This is largely due to the increase in the number of pages of the *Proceedings* published during the financial year. The publication, in conjunction with the Optical Society, of the discussion on "Photo-electric cells and their applications," was also a heavy expense, but it is hoped that the Society will more than recover this expenditure by sales.

The Society desires to thank the Council of the Royal Society for a grant of £200 towards the cost of publications.

The decrease in the amount charged to the Physical Society for *Science Abstracts* is due to the larger number of copies now taken by members of the Institution of Electrical Engineers.

£350 of 5 % War Loan was purchased in August at a cost of £364. 18*s*. 10*d*. The Society's investments have been valued at market prices on December 31st, 1930, through the courtesy of the Manager of the Charing Cross Branch of the Westminster Bank. Their value was approximately 7 % less at this date than on December 31st, 1929, owing to the continued depression of the stock market.

(Signed) ROBERT S. WHIPPLE
Honorary Treasurer

March 16th, 1931

INCOME AND EXPENDITURE ACCOUNT FOR THE YEAR ENDED 31ST DECEMBER, 1930

1929 £ s. d.	Expenditure	£ s. d.	£ s. d.
	To		
500 0 0	Stock of Publications at 1st January, 1930		600 0 0
647 0 0	"Science Abstracts"		566 0 0
	Ordinary Publications:		
1126 1 11	"Proceedings"	1374 12 10	
65 13 1	"Bulletin" and Notices	80 7 7	
56 6 3	General	72 19 6	
			1527 19 11
120 0 7	Postage on Publications		116 8 5
	Share of cost of printing and publishing "Photo-Electric Cells and their Applications"		378 10 2
72 17 1	Reporting at Meetings		66 4 7
34 13 5	Expenses at Meetings		33 11 2
610 15 8	Exhibition Expenses	744 10 2	
56 2 3	Optical Society Exhibition Account	52 17 9	
			797 7 11
6 4 7	Periodicals and Library		5 19 2
	Administration Expenses:		
178 18 8	Institute of Physics	169 11 8	
110 7 9	Office Expenses	112 18 9	
			282 10 5
	Printing List of Members		36 0 0
20 0 0	Guthrie Lecture (Honorarium)		20 0 0
28 15 4	Expenses incurred at Rugby Meeting		40 12 6
182 18 7	*Balance, being excess of Income over Expenditure, carried forward*		
£3816 15 2			£4471 4 3
	To		
	Balance brought forward		£22 2 10

1929 £ s. d.	Income	£ s. d.	£ s. d.
	By		
600 0 0	Stock of Publications at 31st December, 1930		650 0 0
	Stock of "Photo-Electric Cells and their Applications"		129 0 0
	Subscriptions:		
1198 11 10	Fellows*	1242 1 3	
8 8 0	(Voluntary†)	8 3 0	
26 4 6	Students	28 17 6	
74 17 0	For "Science Abstracts" and Advance Proofs	102 4 0	
			1381 5 9
663 17 11	Sale of Publications		819 17 8
141 8 0	Advertisements in "Proceedings"		165 5 4
	Dividends from Investments and Bank Interest	245 1 5	
	Add: Income Tax refunded Jan.–April 4, 1930	16 17 11	
	Income Tax claimed April 5–Dec. 31, 1930	15 10 9	
		277 10 1	
250 14 11	*Less:* Transferred to Duddell Memorial Fund	20 0 0	
			257 10 1
	Exhibitors' Payments on account of Exhibition		805 10 1
673 17 8			
150 0 0	Royal Society Grant for Publications		200 0 0
	Provincial Meeting (Rugby): Receipts from Sale of Railway and Coach Tickets		40 12 6
28 15 4	Balance, being excess of Expenditure over Income, carried forward		22 2 10
£3816 15 2			£4471 4 3
	By		
	Balance carried to Accumulated Fund		£22 2 10

* Eighty-seven Fellows paid reduced subscriptions by the arrangement with the Institute of Physics, the total rebate being £30. 5*s*. 9*d*.

† Voluntary subscriptions are subscriptions paid by Fellows who compounded for the low sum of £10.

BALANCE SHEET AT 31ST DECEMBER, 1930

LIABILITIES	£	s.	d.	£	s.	d.
Accumulated Fund:						
As per last Balance Sheet				2161	19	7
Entrance Fees, 1930				38	17	0
				2200	16	7
Less: Balance brought forward from Income and Expenditure Account	22	2	10			
Decreased value of Investments	282	18	10			
				305	1	8
				1895	14	11
Life Compositions:						
As per last Balance Sheet				2321	0	0
Received during year				63	0	0
				2384	0	0
Duddell Memorial Trust Fund:						
As per last Balance Sheet				426	12	9
Add Increased value of Investments				8	0	0
				434	12	9
W. F. Stanley Trust Fund (For the "Bulletin"):						
As per last Balance Sheet				344	0	0
Less Decreased value of Investments				79	0	0
				265	0	0
A. W. Scott Bequest				250	0	0
Sundry Creditors				1801	15	0
Subscriptions paid in advance				59	12	10
				£7090	15	6

ASSETS	£	s.	d.	£	s.	d.
Investments at Market Value on date:						
£600 Consolidated Stock 2½ %	336	0	0			
£1000 War Loan 5 % 1929/47 Inscribed "A" Account	1020	0	0			
£400 War Loan 5 % 1929/47 Inscribed "B" Account	408	0	0			
£650 Funding Loan 4 % 1960/90	579	0	0			
£500 India 3½ % Stock	280	0	0			
£254. 2*s.* 9*d.* New South Wales 5 % Stock 1935/55	128	0	0			
£211 London County Consolidated 4½ % Stock	206	0	0			
£400 Lancaster Corporation 3 % Redeemable Stock	268	0	0			
£399 London Midland and Scottish Railway 4 % Debenture Stock	292	0	0			
£1000 London Midland and Scottish Railway 4 % Preference Stock	510	0	0			
£500 London and North Eastern Railway 4 % Debenture Stock	355	0	0			
£150 Southern Railway 5 % Debenture Stock	144	0	0			
£300 Southern Railway Preferred Ordinary Stock	180	0	0			
£442 Southern Railway Deferred Ordinary Stock	85	0	0			
				4791	0	0
Stock of Publications (as per Treasurer's Valuation)				779	0	0
Subscriptions due				32	8	6
Sundry Debtors				75	11	0
Inland Revenue—Income Tax reclaimed for 1930				15	10	9
Cash at Bank:						
On Deposit Account	1200	0	0			
On Current Account	172	6	10			
Cash in hand	24	18	5			
				1397	5	3
				£7090	15	6

ROBERT S. WHIPPLE, *Honorary Treasurer.*

We have audited the above Balance Sheet and have obtained all the information and explanations we have required. We have verified the Bank Balances and the Investments. In our opinion such Balance Sheet is properly drawn up so as to exhibit a true and correct view of the state of the Society's affairs according to the best of our information and the explanations given to us and as shown by the books of the Society.

SPENCER HOUSE, E.C. 2
16th March 1931.

KNOX, CROPPER & CO.,
Chartered Accountants

Examined and approved on behalf of the Society { (Signed) E. H. RAYNER. (Signed) W. S. TUCKER.

LIFE COMPOSITION FUND AT DECEMBER 31ST, 1930

	£	s.	d.
128 Fellows paid £10	1280	0	0
3 Fellows paid £15	45	0	0
1 Fellow paid £20	20	0	0
1 Fellow paid £20. 10s.	20	10	0
14 Fellows paid £21	294	0	0
23 Fellows paid £31. 10s.	724	10	0
	£2384	0	0

W. F. STANLEY TRUST FUND (FOR THE "BULLETIN")

	£	s.	d.		£	s.	d.
£300 Southern Railway Preferred Ordinary Stock	180	0	0	Carried to Balance Sheet	265	0	0
£442 Southern Railway Deferred Ordinary Stock	85	0	0				
	£265	0	0		£265	0	0

DUDDELL MEMORIAL TRUST FUND

CAPITAL

	£	s.	d.		£	s.	d.
£400 War Loan 5 % 1929/47 Inscribed Stock	408	0	0	Carried to Balance Sheet	408	0	0

REVENUE

	£	s.	d.		£	s.	d.
Balance at 31st December, 1929	26	12	9	Honorarium to Medallist	20	0	0
Dividends	20	0	0	Balance carried to Balance Sheet	26	12	9
	£46	12	9		£46	12	9

THE PROCEEDINGS OF THE PHYSICAL SOCIETY

VOL. 43, PART 1 *January* 1, 1931 No. 236

A SIMPLE APPROXIMATE THEORY OF THE PRESSURE BETWEEN TWO BODIES IN CONTACT*

BY J. P. ANDREWS, M.SC., F.INST.P.

Received May 8, 1930. *Read and discussed October* 17, 1930.

ABSTRACT. When two solid bodies are pressed together without at any point exceeding the elastic limit, their common area of contact is frequently circular, and in such cases the normal stress at each point of this area may be calculated by a simple approximate method giving results correct, as a rule, to within 1 or 2 per cent. The approximation makes use of two principles: (*a*) The displacement at the centre of the circle of contact is twice that at its edge, and (*b*) for the purpose of calculating the stresses we may replace the two bodies by a single sphere of which the circle of contact is a diametral section, and write the strain at any point as the ratio of the displacement of that point to the length of the line drawn from the point to the sphere in the direction of displacement. When the elastic modulus with which this is multiplied is taken as that appropriate to a rod with sides fixed, the agreement with accurate theory is close. It is shown that the principle (*a*) remains nearly true for elliptical areas of contact.

THE problem of the stresses called forth when two bodies are pressed together, a problem of considerable practical interest, has been thoroughly worked out by Hertz†, and it is possible to obtain from his theory all that is required in practice, provided that the elastic limit is not surpassed. It is nevertheless a fact that the mathematical processes involved in the analysis are far from elementary, and it is not a simple matter to deduce the formulae required in simple instances. This complexity arises in part from the thoroughness of the solution, which includes the distribution of stresses in the interior of the substances. Now in practice we rarely need to know anything beyond the pressure at each point of the surface of contact, and the dimensions of that area, which is generally circular or approximately so. In cases where this information will suffice, a much easier approximate calculation is possible.

This simpler method rests on the fact that in either of a pair of bodies in contact the normal displacement at the centre of a circular area of contact is twice that at its

* Portion of thesis approved for the degree of Doctor of Science in the University of London.

† See A. E. H. Love, *Elasticity*, p. 194 (Cambridge, 1927).

edge. This may be deduced from Hertz's theory, or may be regarded as an experimental fact true for perfectly elastic bodies. The following experiment is a verification of the rule for the case of a solid rubber sphere in contact with a glass plane.

A solid rubber ball (Croydex type), about 6·2 cm. in diameter, was compressed between two parallel planes in such a way that the total force employed and the corresponding diminution of the diameter of the rubber ball could both be measured. If the ball behaves in a perfectly elastic fashion in the sense employed by Hertz, and

H, *k* *H* is the total diminution of the diameter under a thrust *F*, $F = kH^{\frac{3}{2}}$, where *k* is a constant.

Figure 1 *a* shows that, for small compressions, the ball may be considered perfectly elastic.

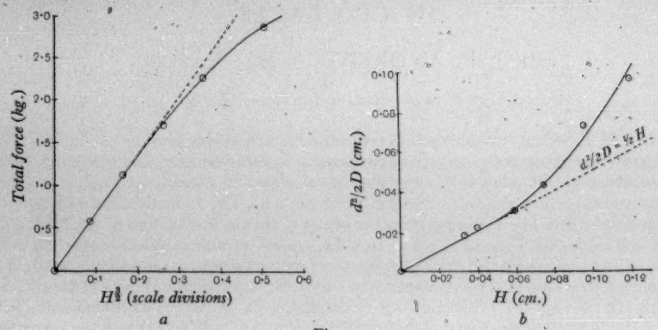


Fig. 1.

Next, the ball was inserted in the apparatus shown in figure 2, in which it is compressed between two blocks of glass. In order to avoid the errors due to its weight, the ball was floated in water, so that it just touched the bottom glass. The levels of the two glass surfaces were read through a microscope, with the aid of stiff glass pointers projecting from them, the upper glass being kept stationary and the ball gradually pressed on to it by means of a screw which raised the bottom block. The bottom of the upper block had been smeared very thinly with a minute quantity of grease, just sufficient to alter the reflecting power of the surface; and when the compressed ball was flattened against this surface, the circle of contact was clearly visible through the glass and was measured microscopically.

D, *d* If the diameter of the ball is *D* and that of the area of contact *d* the height of the spherical cap whose base is a circle of diameter *d* is $d^2/4D$. Hence, if the compression merely takes place where contact is made, the total diminution *H* of the diameter, considering both top and bottom, should be $d^2/2D$. If, on the other hand, this only accounts for half the diminution, as we have assumed, $H = d^2/D$. Figure 1 *b* shows the actual experimental results. Evidently for small compressions, up to 1 per cent. of the radius of the ball, our assumption is justified; and this is the order of strain to which Hertz's theory is supposed to apply.

Consider therefore two spheres, of radii R_1, R_2 respectively, just touching as in figure 3 *a*, so that the distance between their centres is $(R_1 + R_2)$. Press them together, figure 3 *b*, so that their centres, originally at B_0, B_1, are now at B_0, B_2 respectively. If with B_0, B_2 as centres we draw two spheres of radii R_1, R_2, they intersect in the circle whose diameter is CD and cut off GH on the line B_0B_2. The actual shapes of the surfaces are shown by unbroken lines, that of the first sphere having become the surface $AEOFJ$, and that of the second $KEOFL$, EOF being the surface of contact. Draw through E a line parallel to B_0B_2, cutting the dotted spheres in M, N. Then the principle quoted at the beginning of the paper indicates that $MN = \frac{1}{2}GH$.

R_1 R_2

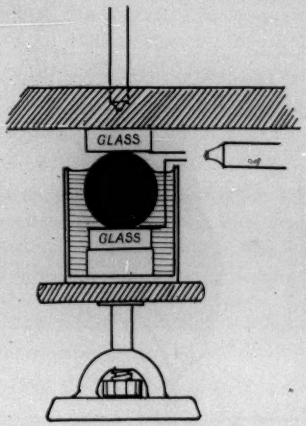


Fig. 2.

In the approximation to be given we assume that the surfaces AE, KE do not differ appreciably from spheres of the radii R_1, R_2, at any rate near the point E, so that we account for the intersection of the actual surfaces at E instead of at C by assuming the spherical masses surrounding the area of contact to have been simply pressed back, without appreciable deformation, by amounts ME, EN respectively, where $ME = \frac{1}{2}GO$ and $EN = \frac{1}{2}OH$. It follows then that if through E and F we continue the spherical surfaces AE, KE, etc., these will cut off a length PQ on B_0B_2, where $PQ = \frac{1}{2}GH = \frac{1}{2}B_1B_2$, and the circle of contact is the circle of intersection of these two spheres. The process of compression may then be thought of as the production of (*a*) a mass displacement of the material surrounding the area of contact, and (*b*) an additional displacement of points within the area of contact which increases from the edge to the centre and makes the total displacement at the centre just double that at the edge.

Turning now to an actual example of the approximate method, consider the pressing together of two different bodies whose surfaces in the neighbourhood of

the points of contact are parts of spheres of radii R_1, R_2 respectively. Since we are only concerned with displacements in this neighbourhood we may replace the bodies by complete spheres of radii R_1, R_2. In order, now, to find the radius of the circle
α of contact when the centres of the spheres have approached by B_1B_2 or α after the surfaces have touched, we draw two intersecting spheres representing the bodies in

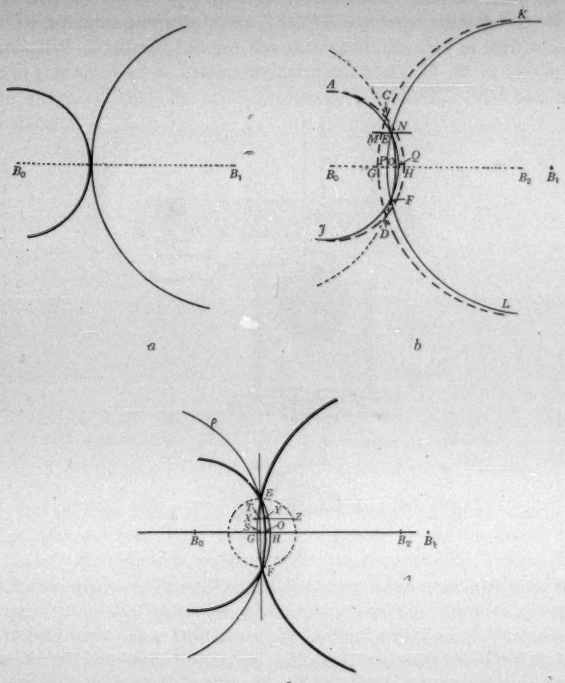


Fig. 3.

contact, figure 3 *c*, and if EF is small compared with the radius of curvature of
a either sphere

$$SG \times 2R_1 = SE^2 = a^2, \text{ where } 2a = EF,$$

$$SH \times 2R_2 = SE^2 = a^2,$$

$$SG + SH = \alpha/2.$$

Hence

$$a^2 = \alpha R_1 R_2/(R_1 + R_2).$$

This is the expression given by Love.

The pressure at each point of the area of contact may be calculated thus. In the figure the unbroken lines represent the two spheres and the surface of contact (part of a sphere of radius ρ, say). The displacement of the point X on the surface of one of the spheres is then XY. To calculate the pressure at Y, we first require the strain at that point, and the second part of the approximation enters here. On EF as diameter construct the sphere shown by the dotted line. We will suppose that at the surface of contact the normal stresses in, say, the larger sphere of figure 3 *c* are the same as those at the upper surface of a body bounded originally by the spherical surfaces EGF, EZF, when EGF is depressed to EYF, and EZF is held rigidly fixed. The appropriate strain of an element such as XZ is XY/XZ. Draw the straight line EF cutting XZ in T. Let EF be equal to $2a$ and ST to r, where S is the mid-point of EF. Then from the figure we have, if the displacements are all small,

ρ

a, r

$$2R_1XT = a^2 - r^2 = 2\rho \,.\, TY = (TZ)^2.$$

Remembering that $YZ = TZ$ very nearly, in all cases where Hertz's theory applies, there is no difficulty in showing that

$$XY = (1 - R_1/\rho)\,(a^2 - r^2)/2R_1,$$

and the strain, to the same approximation, is then $(a^2 - r^2)^{\frac{1}{2}}\,(1 - R_1/\rho)/2R_1$, while the stress at Y is E_1 times this quantity, where E_1 is the appropriate elastic modulus. But since the normal stresses must be the same in both bodies

E_1

$$\tfrac{1}{2}E_1\,(a^2 - r^2)^{\frac{1}{2}}\,(1/R_1 - 1/\rho) = \tfrac{1}{2}E_2\,(a^2 - r^2)^{\frac{1}{2}}\,(1/R_2 + 1/\rho).$$

E_2

From which we derive

$$1/\rho = (E_1/R_1 - E_2/R_2)/(E_1 + E_2).$$

The stress may finally be written

$$\tfrac{1}{2}\,(1/R_1 + 1/R_2)\,(a^2 - r^2)^{\frac{1}{2}}/(1/E_1 + 1/E_2).$$

The proper elastic modulus appears from general considerations to be that appropriate to the longitudinal compression of a rod whose sides are fixed, or one with a value between this and Young's modulus q. For a rod with sides fixed, E would have the value $E = q/[1 - 2\sigma^2/(1 - \sigma)]$, where σ is Poisson's ratio.

q

σ

We may now compare these results with those of accurate theory. The stress at Y is, on Hertz's theory,

$$[2/\pi]\,[1/R_1 + 1/R_2]\,(a^2 - r^2)^{\frac{1}{2}}\,[(1 - \sigma_1^2)/q_1 + (1 - \sigma_2^2)/q_2].$$

The coefficient of $(1/R_1 + 1/R_2)\,(a^2 - r^2)^{\frac{1}{2}}$ in the accurate case is

$$[2/\pi]\,\{1/[(1 - \sigma_1^2)/q_1 + (1 - \sigma_2^2)/q_2]\},$$

where σ_1, σ_2 are the values of Poisson's ratio for the two materials, and in the simple theory

$$\tfrac{1}{2}\,\{1/[\{1 - 2\sigma_1^2/(1 - \sigma_1)\}/q_1 + \{1 - 2\sigma_2^2/(1 - \sigma_2)\}/q_2]\}.$$

To illustrate the extent of the agreement, consider the two spheres to be made of the same material, and Poisson's ratio to be 0·3. Then the values of the coefficients in the two cases are,

by the accurate theory, $0{\cdot}349q$,

by the approximate theory, $0{\cdot}336q$.

This agreement is quite good enough for most purposes. The following example relates to a practical case. A steel ball of radius 1 inch is pressed into a flat copper plate. The values of Young's modulus for the steel and the copper are respectively 2×10^{12} dynes/cm.^{2} and $1{\cdot}2 \times 10^{12}$ dynes/cm.^{2}, while the values of Poisson's ratio are 0·30 and 0·26 respectively. We suppose the centre of the ball to have been pressed down, after the ball rested on the copper, by 0·001 cm. It is required to find the pressure at the centre of the area of contact.

By the method already explained, we find that the radius of this area is 0·0316 cm. Employing this value in the necessary calculation, and remembering that $R_2 = \infty$, we have, for the pressure at the centre of the area of contact, by the accurate method $6{\cdot}46 \times 10^9$ dynes/cm.^{2} or $9{\cdot}38 \times 10^5$ lb./in.^{2}, and by the approximate method $6{\cdot}56 \times 10^9$ dynes/cm.^{2} or $9{\cdot}51 \times 10^5$ lb./ in.^{2}. The total force on the ball could be calculated by the usual simple integration, and the degree of approximation is the same.

As a further example let us consider two equal cylinders of the same substance, with their axes perpendicular, to be pressed into one another. Now two cylindrical surfaces of radius R, when allowed to intersect, cut in a curve which is approximately a circle if its area is small compared with R^2. Simple calculation gives the
a radius of this circle as $(2R\alpha)^{\frac{1}{2}}$, where $(2R - \alpha)$ is the perpendicular distance between the axes. In the case of the solid cylinders, however, the surrounding surface is pressed back, and the flattened area of contact is that obtained when the approach is considered as $\alpha/2$. This is merely the application of the first principle of approximation laid down in this paper. The radius a of the area of contact is therefore $(R\alpha)^{\frac{1}{2}}$.

The actual area of contact will be an anticlastic surface, the two radii of curva-
ρ ture in this instance being equal to ρ, say. If we take the origin of coordinates to be at the centre of this area, and the coordinate axes to be parallel to the axes of the cylinders, elementary geometry shows that the displacement of a point on the surface of either cylinder, reckoned as in the case of the spheres, is

$$(a^2 - x^2)\,(1/2R - 1/2\rho) - y^2/2\rho.$$

In order to find the strain at the point in question, we construct as before a sphere with the circle of contact as its diametral section. If we divide this into longitudinal prisms supposed to be uniformly compressed in the direction of their length, and proceed exactly as before, we find the strain to be

$$\{(a^2 - x^2)\,(1/2R - 1/2\rho) - y^2/2\rho\}/(a^2 - r^2)^{\frac{1}{2}}.$$

The condition that the stress in the direction of pressure must be the same at the surface in each cylinder leads to the relation

$$\rho = 2R,$$

and the stress, finally, may be written

$$[E/4R]\,(a^2 - r^2)^{\frac{1}{2}},$$

where, as before, E is the appropriate elastic modulus, which we may take to be that for the compression of a rod with sides fixed. The full expression then becomes

$$q\,(a^2 - r^2)^{\frac{1}{2}}/4R\,\{1 - 2\sigma^2/(1 - \sigma)\}.$$

The stress calculated from Hertz's theory is

$$q\,(a^2 - r^2)^{\frac{1}{2}}/\pi R\,(1 - \sigma^2),$$

and this is of the same order of agreement as in the case of spheres.

The validity of these methods depends upon the fact that when the area of contact has a circular perimeter the ratio of two definite integrals which occur in Hertz's theory is equal to 2·0*. These integrals are

$$\int_0^\infty du/\{(1+u)^{\frac{3}{2}}\,[u\,(1-e^2+u)]^{\frac{1}{2}}\}, \text{ which} = \pi/2 \text{ when } e = 0,$$

and

$$\int_0^\infty du/\{u\,(1+u)\,(1-e^2+u)\}^{\frac{1}{2}}, \text{ which} = \pi \text{ when } e = 0.$$

In these expressions, e is the eccentricity of the perimeter of the area of contact, proved by Hertz to be always elliptical, within the limits laid down in his theory. e

It is owing to this simple relation that what is called in this paper the first principle is accurately true for circular areas of contact. In so far however as the ratio mentioned approximates to 2 over a wide range of values of e no serious inaccuracy would be caused by using the approximate method for elliptical areas of contact. The list below indicates how the ratio varies for different values of e, the numbers having been calculated from the approximations

$$\int_0^\infty du/(1+u)^{\frac{3}{2}}\,\{u\,(1-e^2+u)\}^{\frac{1}{2}} = \pi/2\,(1+\tfrac{3}{8}e^2),$$

$$\int_0^\infty du/\{u\,(1+u)\,(1-e^2+u)\}^{\frac{1}{2}} = \pi\,(1+e^2/4),$$

which values apply when e is not too near to unity.

The test thus provides a guide to the accuracy of the approximation in any unusual case where the area of contact is elliptical. In such a case the method of reckoning the strain is similar; but it now becomes necessary to construct an

e	0	0·2	0·4	0·6	0·8
Ratio of integrals	2·0	1·990	1·960	1·910	1·840

ellipsoid on the elliptical area of contact. Simple comparison with accurate theory suggests that the length of the third semi-diameter of this ellipsoid should be taken as the harmonic mean of the lengths of the other two.

In conclusion I have pleasure in thanking Prof. C. H. Lees for his interest and advice in the preparation of this paper.

DISCUSSION

For discussion see p. 25.

* A. E. H. Love, *loc. cit.* p. 197, equations (56) and (57).

EXPERIMENTS ON IMPACT*

By J. P. ANDREWS, M.Sc., F.Inst.P.

Received May 8, 1930. *Read and discussed October* 17, 1930.

ABSTRACT. This paper, which describes the continuation of a research upon the impact of soft metallic bodies, contains the results of four investigations, as follows: (1) Impact of equal spheres for low values of the velocity of approach v. The duration of contact is found to vary inversely as $v^{\frac{1}{5}}$, while the coefficients of restitution e in the same cases are found to be unity for all speeds below a value characteristic of each material, as though the metals were perfectly elastic for smaller speeds. (2) The effect of duration of contact upon the size of the permanent deformations. No effect was observed. (3) The variation of duration of contact t with mass of sphere at high speeds of approach. Within the limits of experimental error, t varies as the square root of the mass, as theory predicts. (4) The impact of crossed cylinders. No important new phenomena are found.

§ 1. INTRODUCTION

The experiments to be described in this paper continue a previously published series of investigations† into the impact of soft metallic bodies. The ultimate goal of the research was the elucidation of the phenomenon of the flow of solids under pressure; the method employed, that of impact of pairs of similar spheres of soft metal, for in this method the phenomena are all of such brief duration that it is possible to neglect cumulative effects such as the elastic after-effect, etc. The duration of contact during collision, and the size of the flattened parts generally produced, afforded the data; and the chief results recorded in these previous investigations are summarised below and in figures 3 and 4.

t — t_0, a, n — $d,$ — v, b, v_0, m — p_0

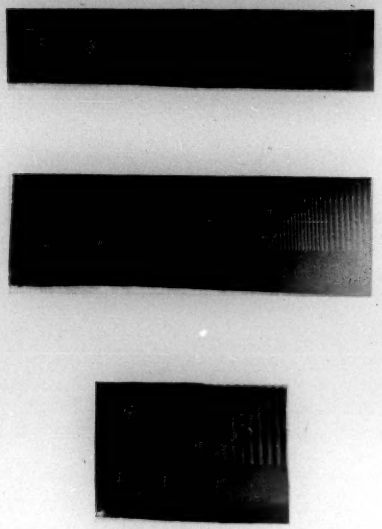
(*a*) The duration of contact is related to the velocity of approach in the manner shown in figure 3. The empirical equation $t = t_0 + a/v^n$, where t is the duration of contact and t_0, a, n are constants, fits these results fairly well, but smooths over the characteristic inward bend at α. (*b*) The circular permanent deformations are bounded by a rim. (*c*) The diameters d of these rims are related to the velocity of impact v by the law $d = b(v - v_0)^m$ (see figure 4), where b, v_0, and m are constants. When $v < v_0$, no deformations are produced.

These results, together with the discovery by C. V. Raman that the coefficients of restitution of solid spheres approach unity at very slow speeds of approach, were all explained in detail by a theory of impact‡ based on the following two fundamental principles: (1) That all the metals employed are perfectly elastic until the pressure exerted between them attains a value p_0 characteristic of the material; (2) that this pressure p_0 is never exceeded, any further mutual approach of the spheres producing plastic flow of the material.

* Portion of thesis approved for the degree of Doctor of Science in the University of London.

† J. P. Andrews, *Phil. Mag.* **8**, 781 (1929); **9**, 593 (1930).

‡ *Ibid.*



While the theory very satisfactorily explained all the results obtained, it was felt that the evidence for it was still deficient in one particular, namely the direct experimental verification of perfect elasticity for small stresses. To supply this deficiency, therefore, a more accurate method of investigating the variation of duration of contact with velocity of approach at very small speeds, is described in this paper. An additional assumption, implicit in the theory, that "time-effects" have no influence on the results owing to the extreme brevity of the impact, which is of the order 10^{-4} sec., had no experimental support: and this point also is dealt with. It was felt desirable also to test the consequences of the theory in some particular case, and for this purpose, experiments were performed at comparatively high speeds of approach (about 150 cm./sec.), when the duration of contact is primarily determined by the plastic properties of the metal, and defects due to the elastic reaction may be considered as secondary. The variation of the duration of contact with the mass of the spheres was investigated, though with some difficulty. Finally, experiments on the impact of crossed cylinders were tried, to find out whether the shapes of the bodies in contact had any important significance not taken into account in the theory.

§ 2. EXPERIMENTS AT LOW SPEEDS

For these experiments it was essential that the velocity of approach should not exceed 1 or 2 cm./sec. and in any case should be less than that required to initiate permanent deformation. The method of measuring this speed consequently became very important. The method finally adopted was suggested by Raman's device for measuring coefficients of restitution at low speeds. The chief improvement adopted in the present case consisted in an accurate timing device, without which absolute measurement of velocity is impossible.

The image of a fine slit S, figure 2 (a), strongly illuminated from a pointolite P is focussed at M above the line joining the centres of the spheres. The further progress of the light is prevented, when the balls are touching, by two small pieces of mica M attached to the little brass blocks to which the balls also are attached, figure 2 (b). When the balls are drawn apart, light passes through the lens L, figure 2 (a), and a magnified image of the gap at M is produced on a photographic plate F. (In the diagram the rays are drawn for one point only in the gap at M.) A cylindrical lens—a thick glass rod—at C concentrates the light into a fine line on the plate, and an electrically maintained tuning-fork T cuts off the light 64 times a second. If the balls are withdrawn through a fixed small distance and the plate is pushed down by hand, it will show on development a series of fine lines of equal length. If, however, the spheres are slowly approaching, the lengths of the lines will slowly diminish, and after impact slowly increase, giving a negative such as those from which the prints of plate 1 were taken. To obtain the speed of approach, the negative is measured under low magnification; and the lengths of the lines are plotted as the ordinates of a graph whose abscissae are times measured in periods of the tuning-fork. Figure 5 shows a series of such graphs.

The slope of a line at the moment of impact determines the required velocity, since the magnification and the frequency of the fork are known; in the same way

the speed of separation may be measured. The prints in plate 1 are examples for three different speeds of approach. In practice, measurements were always made on the negative itself; by this means the velocities could be measured to an accuracy of 2 or 3 per cent.

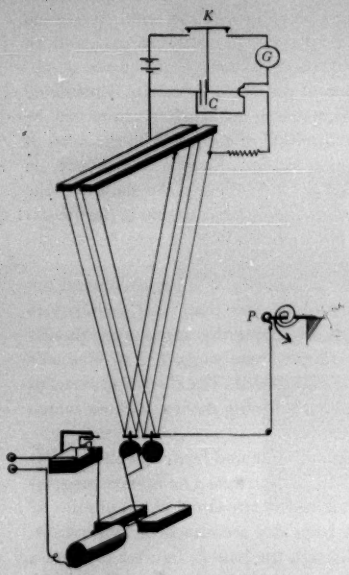


Fig. 1. Diagram of apparatus, apart from the timing arrangement of Fig. 2 (*a*).

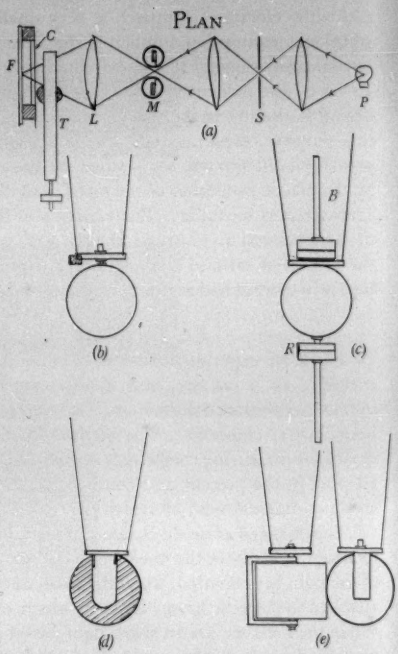


Fig. 2.

Connected to the spheres, figure 1, was an electrical circuit for measuring the duration of contact. When the spheres touched they partially discharged condenser C through a resistance; and when the key K was subsequently depressed to the right, the charge remaining passed through the ballistic galvanometer G whose throw afforded a means of calculating the time of contact.

The results are rather more scattered than the method seems to justify. In order to test the reliability of the electrical circuit, the balls were separated and the condenser charged and immediately discharged. This process was repeated in one experiment twelve times, care being taken that all other conditions were those which would obtain in an actual observation. The average deflection was 33·767, the mean deviation from the average was 0·019, and the maximum deviation 0·033.

This indicates that the electrical circuit is not to blame for the scattering. The optical arrangement suggests one possible source of chance variation. The distance between the balls and the lens L may vary slightly. The change in the magnification produced by an alteration of 1 mm. in this distance is about 0·01, or about 1 in 600; this cannot therefore account for the effect. The conclusion must be that the spheres differ slightly at different parts of their surfaces in regard to either shape or elastic properties. This has been fairly obvious throughout the work on impact; in the case of aluminium the surface was pitted with minute cavities, while with Babbitt's metal a lens revealed small crystals of dark substance (possibly antimony) scattered

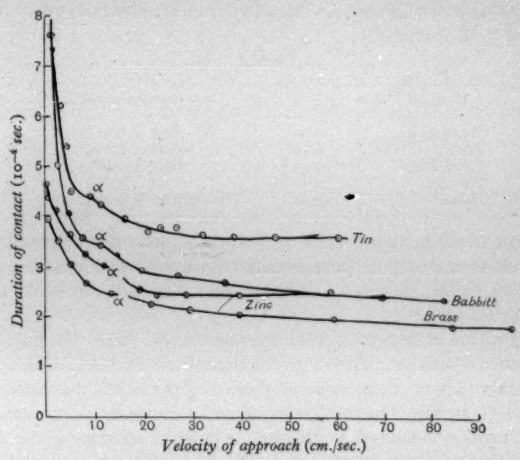


Fig. 3.

through the material. The speeds of approach are so small in the experiments that the maximum area of contact is small, and the method becomes one which deals with minute samples of the material in which small variations do not average out.

The results were treated as follows. From the electrical circuit we know that

$$t = RC \log_e (\theta_0/\theta),$$

where R is the resistance in series with the spheres, C the capacitance of the condenser, θ_0 the deflection of the galvanometer on immediate discharge, θ after the spheres have collided. If R remains constant throughout a series of experiments on a pair of spheres,

R, C
θ_0, θ

$$t = k_1 \log_{10} (\theta_0/\theta),$$

k_1

where k_1 is a constant. Now, according to Hertz's theory,

$$t = k_2 v^{-\frac{1}{5}},$$

v where v is the velocity of approach. Let us suppose that in these experiments $t = k_2 v^{-n}$. Then we have the relation

n
$$\log \log (\theta_0/\theta) = -n \log v + \text{const.}$$

The value of the constant depends upon the radius of the spheres, their mass and elastic properties, and the constants of the electric circuit. We are not at present concerned with its value.

If then we plot log log (θ_0/θ) as ordinate and log v as abscissa, the slope of the graph gives n. Typical graphs are shown in figure 6.

The results for the materials investigated are tabulated in the second column of table 1, the value of n having been calculated from the observations by the method described by J. H. Awbery*.

Table 1

Material	$-n$	e and mean deviation
Aluminium	0·21	1·00 ± 0·013
Zinc	0·20	0·99 ± 0·005
Lead-Tin	0·21	0·97 ± 0·06
Babbitt	0·20	1·00 ± 0·018
Brass	0·21	1·00 ± 0·024

e In the case of tin the values of the coefficients of restitution e showed that small permanent deformations were being produced down to about 0·3 cm./sec., although these were not visible. Figure 6 *a* illustrates this. The variation is like that found previously by Raman.

The coefficients of restitution were investigated and these, like the results just recorded, varied somewhat. Table 2 gives a typical list of results, while in the third column of table 1 are the mean values of these coefficients and the mean deviations for the various materials. The coefficients cannot of course actually exceed 1·00 but they are recorded as measured, to indicate the order of accuracy of the method.

Table 2. Coefficients of restitution for brass

Speed of approach	0·39	0·57	0·55	1·08	1·18	1·65	1·49	1·72	2·04	2·22
e	1·02	1·02	1·00	0·95	1·02	0·93	1·00	1·04	1·00	1·02

The evidence is decidedly in favour of perfect elasticity at very low speeds, the deviations of n from $-\frac{1}{5}$ and of e from 1·00 being of the same order as the experimental error.

§ 3. VARIATION OF DURATION OF CONTACT WITH MASS OF SPHERE, AND INVESTIGATION OF TIME EFFECTS

The best manner of increasing the mass of the spheres, while keeping other variables constant, was considered to consist in the loading of the spheres with riders. Rods B were screwed into the spheres, figure 2 (*c*), and brass weights R (50 gm. each) were fixed thereon. When the experiments were performed, however,

* J. H. Awbery, *Proc. Phys. Soc.* **41**, 384 (1929).

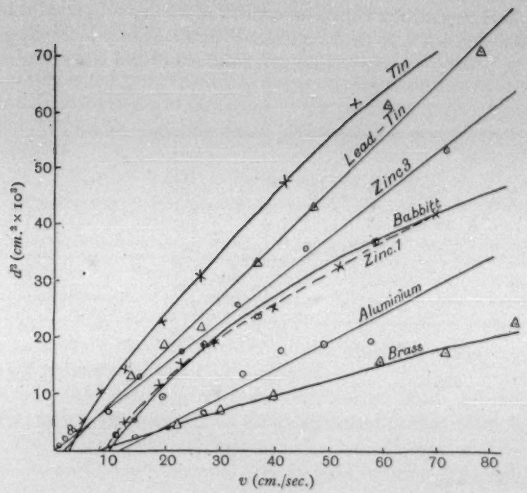

Tin
Lead - Tin
Zinc 3
Babbitt
Zinc 1
Aluminium
Brass
d^2 (cm.^{2} × 10^2)
v (cm./sec.)

Fig. 4.

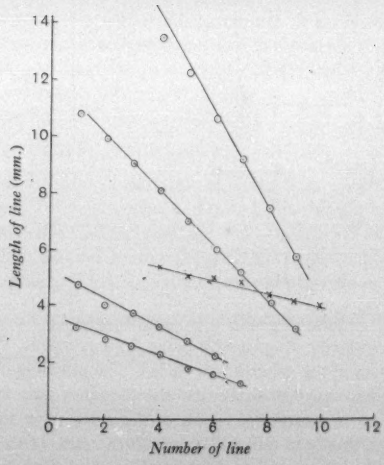

Length of line (mm.)
Number of line

Fig. 5.

it was soon discovered that a vitiating effect had been introduced. The impact appears to have generated a transverse oscillation whose half period controlled the duration of contact, just as the longitudinal vibrations in a long rod govern that duration. In order to test this matter, one rider above and one below each sphere

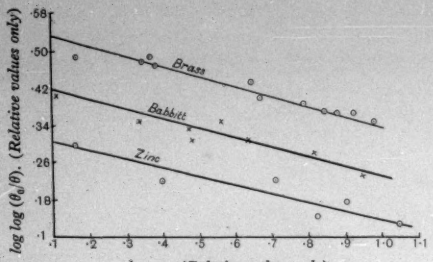


Fig. 6.

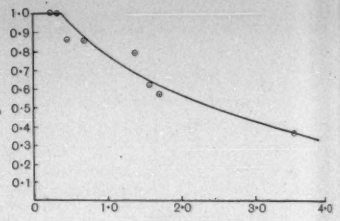


Fig. 6 *a*.

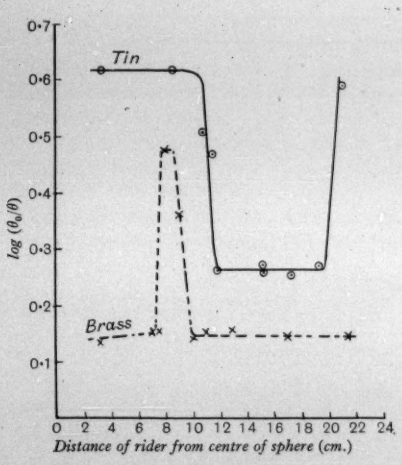


Fig. 7.

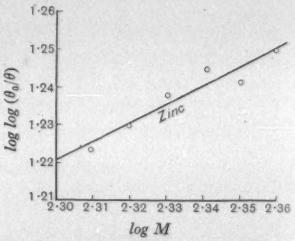


Fig. 8.

were used, and the velocity of approach being kept constant, the distance of the riders from the centres of the spheres was varied. The duration of contact was then observed. The graphs, figure 7, show that the tendency was for the duration to remain unchanged up to a certain position of the rider, then to pass suddenly to another position, and finally to return. If we call the ratio of the two durations T, and let L be the ratio of the total length of rod R to the length between sphere and

rider when the first sudden change occurs, then for brass $T = 3{\cdot}2$, $L = 3{\cdot}3$, and for tin, $T = 2{\cdot}4$, $L = 2{\cdot}2$. This suggests that the rod tends to vibrate so that the rider is at a node, the upper and lower ends of the rod being antinodes; and the duration of the contact is determined by the period of this vibration. This effect was employed to investigate the influence of longer or shorter contact on the size of the deformations. Times varying in the ratio 3 : 1 were thus obtainable. Upon measurement of the deformations, no significant difference was observable in their size. Table 3 is typical.

Table 3. Deformation of tin

Duration of contact $\times 10^{-4}$ (sec.)	7·0	3·17	3·13	3·02	3·22	2·96
Diameter of deformation (cm.)	0·397	0·356	0·388	0·385	0·377	0·395
Duration of contact $\times 10^{-4}$ (sec.)	7·32	7·35	3·09	6·02	5·59	—
Diameter of deformation (cm.)	0·387	0·352	0·355	0·365	0·357	—

We may therefore conclude that only negligible differences are made to the results by longer or shorter duration of contact.

The problem of variation with mass now becomes more difficult, since any arrangement for carrying riders involves a risk similar to that which has just been discussed. The only safe method appears to be the hollowing of the spheres and subsequent filling with heavy material. The range of variation of the mass is thereby made very restricted, however; for if the hollowing is carried too far, the sphere will vibrate as an elastic shell. Figure 2 (*d*) shows the dimensions finally adopted for the boring. An experiment on duration of contact was tried first with the spheres empty: then with an increasing quantity of lead shot, fixed with paraffin wax poured in hot. (After the wax had been inserted, 40 minutes were allowed to elapse before an observation was taken, so that temperatures should recover.) The speed of approach, kept as constant as possible, was made large—of the order 150 cm./sec.—so that plastic properties were most important. Under these circumstances, the duration of contact t_0 is given by t_0

$$t_0 = \pi/2\,(M/\pi R p_0)^{\frac{1}{2}} \text{ approximately} \qquad \text{.....(1).}$$

Experiment has already shown that at these high speeds the duration t is t
practically a constant. All metals were known to behave similarly, so only three were investigated, namely aluminium, zinc and tin. In figure 8 is a typical graph, in which $\log \log (\theta_0/\theta)$ is plotted against $\log M$. The slopes calculated for the three cases are given in table 4. They would all be 0·50 if equation (1) held good.

Table 4. Values of x where $t = kM^x$ x

Substance	Aluminium	Zinc	Tin
x	0·52	0·49	0·53

The restricted range of variation of the masses accounts for the deviation from 0·50.

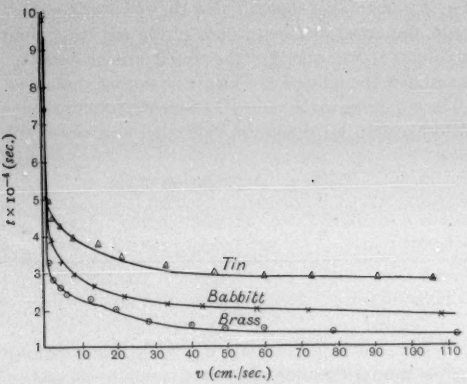


Fig. 9. Duration of contact and velocity of approach (cylinders).

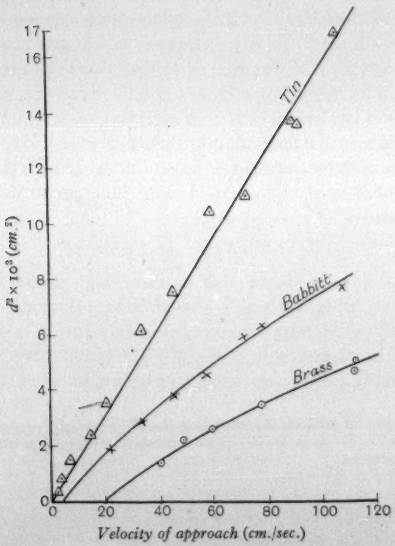


Fig. 10. (Diameter)² and velocity of approach (cylinders).

§ 4. EXPERIMENTS ON CROSSED CYLINDERS

The theory was based upon experiments on spheres only, and there remained a possibility that some essential part of the variation of duration of contact and of size of deformation may have been due to shape of surface. Two equal cylinders of radius 2 cm. were therefore taken, one suspended with its axis vertical the other with its axis horizontal as in figure 2 (*e*), and the time of contact, etc. were investigated. The same precautions were taken to present a fresh surface at every collision. Nothing new emerged from these experiments, and their results are represented graphically in figures 9 and 10.

The conclusion may be drawn that while the value of the duration of contact may depend on the shapes of the surfaces, its variation with velocity does not change in any fundamental manner when the shapes of the surfaces are altered. The variation of the size of the permanent deformations also follows no new law.

§ 5. CONCLUSION

The experiments justify the assumption of perfect elasticity at low speeds and the neglect of any cumulative effect due to the differences in the times of contact. The variation in this time for different colliding masses is in accordance with the theoretical results, and when the shape of the colliding surfaces is changed from spherical to cylindrical no new factor is introduced.

§ 6. ACKNOWLEDGMENT

My thanks are due to Prof. C. H. Lees both for the facilities given me for the work and for advice during its progress.

DISCUSSION

For discussion see p. 25.

OBSERVATIONS ON PERCUSSION FIGURES*

By J. P. ANDREWS, M.Sc., F.Inst.P.

Received May 8, 1930. *Read and discussed October* 17, 1930

ABSTRACT. The formation of percussion figures by the pressure of a steel ball on glass blocks is here traced out, and an investigation suggested by this exploration elicits the following facts: (1) The diameter of the innermost circular or part-circular crack remains constant for one specimen of glass, and is independent of the maximum pressure exerted by the ball on the glass. (2) The diameter of the outermost circular or part-circular crack varies with the maximum pressure in a manner which suggests that the crack tends to keep to the outer edge of the area of contact. (3) No crack is formed until the pressure exceeds a value characteristic of the glass. A method of testing the fragility of glass is developed: and finally some observations on percussion figures upon spherical surfaces are mentioned.

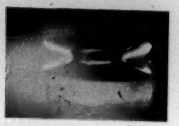
§ 1. INTRODUCTION

The principal names associated with the study of percussion figures—the ring-formed cracks produced when a steel ball is pressed into, or is allowed to fall upon a flat glass block—are those of C. V. Raman†, K. Banerji‡, and J. W. French§. The characteristics of such figures, as described by these observers, may be summarized thus. A plane circular area is surrounded by a double ring-crack, the annulus enclosed between the rings often being broken up by cross cracks. The diameters of the internal and external rings increase with the final static pressure upon the steel ball, or the height from which it is dropped (Banerji), the external ring being roughly coincident with the circumference of the area of contact as calculated by Hertz's theory for perfectly elastic bodies. The plane area within the rings is at a slightly lower level than the surrounding glass, which is heaped up into a minute rim just outside the annular region (Raman). The two cracks extend into the interior of the glass, forming circular truncated cones with bases below the glass surface.

§ 2. PROCESS OF FORMATION OF PERCUSSION FIGURES

During some work on the collision of spheres of soft metals‖ I had met with percussion figures of a different kind, namely the flattened patches formed when two spheres of such metal collide. An outstanding characteristic of these circular patches was the thread-like rim surrounding each of them; and a less prominent feature, not invariably visible, was the convex curvature of the surface within this rim. Comparison with the glass percussion figures of Raman was suggested, and experiments

* Portion of thesis approved for the degree of Doctor of Science in the University of London.
† C. V. Raman, *Nature*, **104**, 113 (1919); *J.O.S.A.* **12**, 387 (1926).
‡ K. Banerji, *Ind. Ass. for Cultiv. of Science*, **10**, 59 (1926).
§ J. W. French, *Nature*, **104**, 312 (1919).
‖ *Phil. Mag.* **8**, 781 (1929).

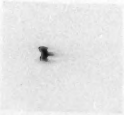


(a)

(b)

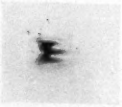
(c)

(d)

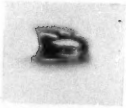


(e)

(f)



(g)



(h)

(i)

(j)

were made with two objects in view, namely (1) to investigate the somewhat surprising observation of Banerji that the diameter of the internal crack varies with the final static pressure, or the height of fall, and (2) to find how far the character of percussion figures on glass throws light upon the formation of similar figures on metal spheres.

In figure 1 is plotted an example of Banerji's observations, showing how the two diameters were found to increase with the height of fall of a steel ball on a glass block. Such a result seems to imply that the crack was formed at the moment of recoil of the sphere. Similarly, for static pressure, if Banerji is right, the crack must have been formed as the pressure was released. An attempt to demonstrate this with

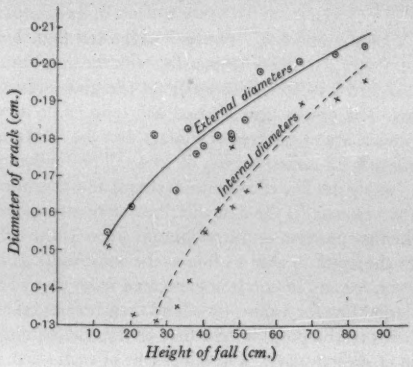


Fig. 1.

static pressure threw light on the problem. A crack does not form all at once, but starts at one point or simultaneously at several points on the surface. If a block of glass is carefully pressed in a parallel-jawed vice against a steel ball, the genesis and development of the crack may be watched through the side of the block. Figure (*a*) of plate 2 shows what is usually seen for a moderate pressure*. A click is sometimes heard when the glass cracks, and one cannot always see where the breakdown has occurred. Thus, in the case of (*a*), which promised a perfectly circular crack, when the pressure was released only two or three short cracks were found, of which (*b*) is the photograph from the top of the plate. This view from above may, however, tell only part of the story. The cracks which appear are often remarkably black, a fact which probably indicates an actual gap in the glass: and it is conceivable that the fracture extends further than appears. I have indeed succeeded in one or two instances in reflecting light from a surface inside the glass which did not end in a visible crack on the surface: while in one case, while the fracture was being examined between crossed nicols, a short crack appeared to continue in a thread of

* The upper half is the reflected image of the lower half.

light through a much larger arc of a circle. In many instances, however, the development of the crack is easier to follow, and in the set of magnified photographs from (*c*) to (*i*) is recorded a particularly marked case, much more pronounced than is usual. The series is intended to show how a horn-shaped crack, starting on the left, extends as the pressure is increased, finally finishing in a roughly-shaped complete ring-crack. The view of this crack from above, after removal of the ball, is figure (*j*). There follows, in plate 3, a number of magnified photographs of the surface cracks seen from above, varying from simple to highly complex, and formed, some by static pressure, some by the dropping of the ball from a height. No essential difference was observed in the two cases: but all the photographs show evidence of the development of the rings from one or more points, from which curved cracks have set out.

Inspection of a large number of "figures" suggested that their formation proceeded as follows. When the stress responsible for the fracture (almost certainly shear stress) has reached a value characteristic of the glass, a breakdown occurs at one or more points. The cracks thus formed will tend to be arcs of circles, since stresses are symmetrical about the area of contact, but the relaxation brought about by the fracture will mean a redistribution of stress. The tendency will now be for the crack to "run" along the line of maximum stress, and this line will depart from the circular from two causes: (i) the new distribution of stress due to the existence of the crack; (ii) the new position of the maximum stress as the ball presses into the glass. If therefore the crack is able to follow the maximum stress, it will not be circular. If, however, for any reason it is prevented from doing so, it may follow a more nearly circular outline for a short time, and then branch off so as to bridge the distance between its present position and that of the line of maximum stress. If fracture has begun at several places, a development of each crack in this manner is to be expected, giving rise to the complicated figures found in practice. The importance of this view derives from the fact that for one piece of glass fracture may be expected when and where the maximum stress reaches a definite characteristic value. If the process of development of the crack is that just described, the first cracks should approximate to arcs of circles with the same radius, no matter what the height of fall of the ball or the final static pressure applied to it.

On the other hand, the largest external diameter of the crack system would vary: and if the glass behaved in a perfectly elastic manner up to the time of fracture, we should anticipate—as others have anticipated—that this diameter would approximate to the diameter of the circle of contact calculable from Hertz's theory, for at the circumference of this circle the shear stresses are greatest. There is, however, little point in calculating these diameters of contact, since the elastic constants required are generally not known and are not easy to find. A more reliable test is obtained by plotting a graph to find whether the diameter varies as the $\frac{2}{5}$-power of the height of fall, as Hertz's theory predicts.

§ 3. EXPERIMENTAL RESULTS

A series of observations was obtained where a 1-inch steel sphere was dropped on to a glass block from heights varying from 5 cm. to 45 cm. These observations are shown in figure 2, where the crosses represent the smallest diameters of the innermost cracks and the circles the largest external diameters. The line through the circles represents the variation required by Hertz's theory, on the assumption that the shear stress is greatest at the edge of the circle of contact.

The important fact emerges, that the smallest internal diameter is constant, and this no doubt measures the least stress required to produce fracture. The external cracks do roughly follow the law required by Hertz's theory.

The interest attaching to the constancy of the internal diameter is twofold. In the first place, we might use this method to test the "fragility" of specimens of glass. In the second place it affords a further means of investigating the process of compression. As a simple deduction from geometrical similarity, one would expect,

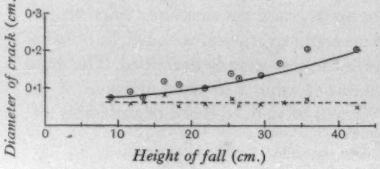


Fig. 2.

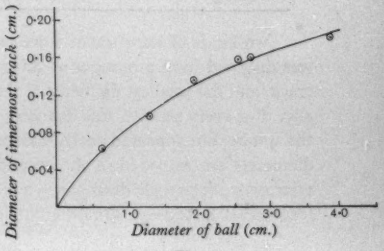


Fig. 3.

for example, the diameter of the smallest crack to be proportional to the diameter of the sphere used in producing it. Figure 3 shows the results of an experiment on these lines. For the larger balls, the crack is smaller than would be expected. The explanation probably depends on two factors: (1) since the first break-down probably occurs at a point where the glass is slightly weaker, the chance of the occurrence of the necessary flaw will be more nearly proportional to the area of contact, and hence to the square of the radius of the ball; (2) these cracks are associated with a rim formed just beyond, and due in an indirect way to, the volume of glass displaced inwards by the pressure of the ball. This volume is proportional to R^3, where R is the radius of the ball. The circumference of the rim will be roughly proportional to R, so that we may consider the tendency to form a rim to depend on R^2. Large shear stresses are formed in the neighbourhood of the rim. We are thus led to suppose that the diameter of the crack should vary as some power of R, less than unity. A logarithmic plot shows the actual power to be 0·63 in this instance.

We return now to the other aspect, "fragility" or "brittleness"—terms which are preferable to "hardness," on account of the rather different phenomenon described. Five samples of optical glass were obtained from Messrs Chance. The

names of these glasses and their approximate chemical compositions, for which I am indebted to Messrs Chance, are given in table 1.

Table 1. List of chemical compositions of glass specimens

	Name of glass				
	HC.	DBC. 5804	DF. 5374	EDF. 5909	EDF. 5570
	(%)	(%)	(%)	(%)	(%)
SiO_2	72·5	37·6	46·4	42·0	40·8
K_2O	10·7	5·9	3·7	5·3	5·3
Na_2O	6·0	—	2·5	—	—
CaO	10·8	—	—	—	—
BaO	—	44·7	—	—	—
Al_2O_3	—	3·7	—	0·2	0·2
ZnO	—	6·8	—	—	—
Sb_2O_3	—	1·3	—	—	—
PbO	—	—	47·4	52·3	53·5
As_2O_3	—	—	—	0·2	0·2

Two kinds of experiment were performed on each block. Firstly, a $\frac{3}{4}$-in. ball was dropped from a number of different heights, each sufficiently great to cause a crack, and the smallest diameter of the innermost crack was measured microscopically. For every sample, this diameter had no regular variation with height of fall of the sphere, but appeared nearly constant as in the case already described. The mean diameters are recorded in the second column of table 2 in scale divisions of the microscope (570 scale divisions = 1 cm.). It will be noticed that a considerable difference exists between the different samples.

Table 2

Glass	Diameter of innermost crack	Least height of fall to form crack
	(divs.)	(cm.)
DBC.	89·7	21·1
EDF. 5570	80·3	9·0
HC.	78·3	7·4
EDF. 5909	77·3	6·58
DF.	75·3	5·3

Secondly, a steel ball was dropped from varying heights above the glass surface, until the height just sufficient to form a crack was found. In order to obtain greater distances to measure, a smaller ball, half an inch in diameter, was used. The adjustment was found to be surprisingly sharp, and a critical distance of, say, 15 cm. could be estimated to within 2 mm. The cracks formed by a fall from a height just greater than this were naturally very small, and could rarely be detected from the upper surface of the specimen. The smallest cracks become visible as bright specks, however, when the block is held up to the light and the observer looks through one of the sides. Total reflection then occurs at the fissure, and the crack is observable. In the last column of table 2 these minimum heights are recorded. The table

emphasizes two points, namely (1) that both methods arrange the samples in the same order; (2) that the second method is much more sensitive.

We should anticipate this last fact from general considerations: for consider the cognate case where a number of spheres of different diameters are dropped upon a single sample of glass, the two experiments just described being performed for each sphere. Then, on the supposition that Hertz's theory may be applied, it may be shown that the innermost diameters vary as the $\frac{2}{5}$-power of the least height. We cannot predict what exact relation ought to hold in the case of the actual experiment, because the elastic constants will vary in an unknown manner. In figure 4 the two

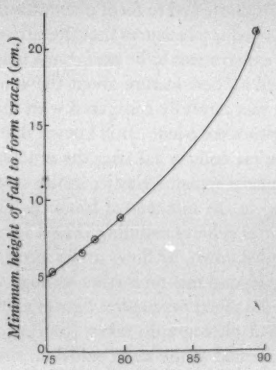


Fig. 4.

measurements are plotted one against the other, and are found to fall on a regular curve. If F is the least height of fall and d is the innermost diameter, in cm., the results are fairly well represented by the formula

$$F = 6{\cdot}83 \times 10^{7} . d^{8{\cdot}1}$$

which brings out the greater sensitivity of the minimum height method. The formula also gives an interesting item of new information about glasses in general. The maximum depth of penetration of the steel ball when falling from a given height is governed chiefly by Young's modulus for the glass. The critical depth of penetration at which the first crack appears probably depends on the shear strength of the glass: and there is no *a priori* reason for supposing that these two should vary together, at least in any simple fashion, in passing from one kind of glass to another. The formula indicates that they do so, nevertheless.

The reciprocal of F or of d might be employed as a measure of the "fragility" or "brittleness" of the glass. This is of practical interest, for according to J. W. French the action of abrasives is of the same character as that exerted by spheres when forming percussion figures. In optical glassworks measurements of the "hard-

ness" of the glass are sometimes made by measurement of the quantity of abrasive required to remove a measured quantity of glass. This operation could be profitably replaced by the much simpler measurement of F.

When the second object of the research was entered upon, close observation did not encourage much hope that percussion figures in glass would throw light on the formation of similar figures in metals. The following experiment was tried, nevertheless. A glass sphere about one inch in diameter was dropped, or thrown with force, on to a hardened steel anvil, in order to form percussion figures. The object was to simulate the case of two exactly similar glass spheres colliding, in which the surfaces of contact must be depressed to form planes during the collision. The very small yielding of the hardened steel ensures that the surface of contact shall be very nearly plane, and enables experiments to be performed much more easily. Examination of the figures revealed no new feature about the cracks. The curvature of the surface within the cracks was carefully compared with that outside in a number of cases: in each case these two were equal. It is known that the surface of a flat glass plate depressed by a spherical body is flat after the process. These two observations tend to show that the material remains elastic, in the sense that no permanent deformation is produced, up to the moment of fracture. This does afford a point of contact with the case of metal spheres colliding. There is reason to believe that up to the moment of definite breakdown, or flow, under pressure, the metal behaves as though perfectly elastic. Beyond this no further analogy was found. It is true that in a number of instances the metal percussion figures exhibited a rim and a central convex region. A magnified photograph, taken from an oblique direction, of two such deformations, both formed at the same collision, shows this convexity by the disposition of the shadows: see plate 3 (*g*) and (*h*). But whereas mainly the shear stresses are operative in the case of glass, it is almost certain that longitudinal or volume compressions are at the root of the phenomenon in the case of metals.

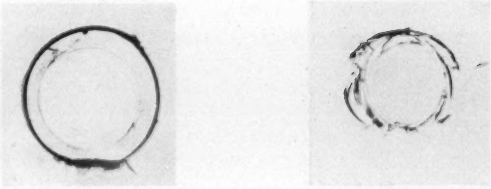
§ 4. CONCLUSIONS

The results attained may now be summarized. We have shown that the diameter of the innermost crack remains constant for the same specimen of glass and the same ball. This constant diameter varies, however, according to the power 0·63 of the diameter of the ball. The distance between extreme external cracks, a measure of the diameter of the outer ring, justifies their association with the circumference of the greatest circle of contact of ball and block. When different specimens of glass are tested by this method and by the measurement of the minimum height of fall necessary to cause fracture the samples are graded in the same order of fragility or brittleness. Percussion figures upon glass spheres were unable to throw important new light on the problem of impact of metal spheres.

§ 5. ACKNOWLEDGMENTS

Finally, I would like to thank Prof. C. H. Lees for affording facilities for this work, and both Prof. Lees and Dr Allan Ferguson for their continued interest in its progress.

(a) (b) (c) (d)



(e) (f)

(g) (h)

DISCUSSION ON THE PRECEDING THREE PAPERS

Dr J. H. VINCENT. I congratulate the author on the great experimental skill which he has brought to bear on this subject, especially as regards the remarkably neat arrangement for determining the circumstances of impact for very small velocities of approach.

Mr T. B. VINYCOMB said that circular markings found in museum flints had at first been ascribed to organisms, in view of their definite minimum size. Some of them had, however, been found to be of recent origin and to have been caused by the impact of waves, the minimum diameter of the markings having different values for different materials. The fracture is conical, the section of the cone decreasing from the surface inwards down to a certain size, whereupon the angle of the cone undergoes an abrupt change.

Mr A. F. DUFTON. On reading the proofs of the first and third papers I was curious to know what Hertz wrote on the subject and found that the percussion figures were described by him in 1882. Hertz measured the hardness of a body by the normal pressure per unit area which must act at the centre of a circular surface of pressure in order that in some point of the body the stress may just reach the limit consistent with perfect elasticity. From the pressure necessary to produce the first crack and from the size of the crack he determined the hardness of glass, obtaining for mirror glass the value 130 to 140 kg./mm.^{2}.

In the first paper Mr Andrews determines for a practical case the pressure at the centre of the area of contact and states that the total force could be calculated by integration. As Hertz showed, the average pressure is two-thirds of the pressure at the centre; the integration need not be performed for individual cases.

Mr B. P. DUDDING. I wish to associate myself with Dr Vincent in congratulating Mr Andrews on the simplicity and accuracy of the method employed to determine the velocity of approach. I am sure that glass technologists will be interested to note the suggested relationship between the data given in the paper and the hardness tests usually employed in the glass industry. I suggest that the author should state whether the glass specimens were annealed or not, as that would be expected to affect profoundly any comparison of glasses having similar compositions. (See tables 1 and 2, p. 22.)

Mr T. SMITH suggested that the author might care to refer to some interesting photographs published by J. W. French in the *Journal of the Optical Society*.

AUTHOR's reply. I should like to thank Dr Vincent and Mr Dudding for their appreciative remarks. The curious markings on pebbles mentioned by Mr Vinycomb are, from his description, percussion figures of the same kind as those I have dealt with. I am unable to state whether or not the blocks of glass were anealed; but since they were in a condition suitable for final polishing it seems probable that they were. I am grateful for Mr Dufton's reminder of Hertz's original work. Hertz says that he made a few experiments, but scarcely any detail is provided. Except therefore for the mention of that work, the present paper is an account of additional information about a phenomenon first brought prominently into notice by Sir C. V. Raman.

THE ABSORPTION AND DISSOCIATIVE OR IONIZING EFFECT OF MONOCHROMATIC RADIATION IN AN ATMOSPHERE ON A ROTATING EARTH

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ABSTRACT. The absorption of monochromatic radiation from the sun in an atmosphere of which the density varies exponentially with height is considered; the energy of the radiation, or a definite fraction of it, is supposed to dissociate or ionize the air, and the dissociation products are supposed to recombine with one another only, and not to diffuse away from the element in which they were formed. The resulting distribution of density of the dissociation products is determined, a constant recombination coefficient being assumed, while account is taken of the variation in rate of dissociation due to the earth's rotation. The results are illustrated by numerous diagrams, showing the density of the dissociation-products as a function of height, time, latitude and season.

§ 1. INTRODUCTION

THE main object of this paper is to consider the following idealized problem: (*a*) A uniform beam of monochromatic radiation from a sun falls upon a rotating earth, the rate of rotation of the earth and the changing declination of the sun being the same as for the actual earth and sun; the radiation is absorbed (before reaching the ground) in an atmosphere of uniform composition, in which the density varies exponentially with height. (*b*) The absorption of radiation at each point is proportional to the air-density and the intensity of the radiation reaching that point. (*c*) The energy of the absorbed radiation, or a constant fraction of it, is expended in dissociating some constituent A of the air into two components A_1, A_2 which may be electrically neutral or not. (*d*) The two components recombine with one another (alone) to re-form A, at a rate αn^2, where α is a constant independent of height and time, and n is the number of A_1 or A_2 particles per unit volume. (*e*) The particles A_1, A_2 do not move from the element of volume in which they were formed.

The problem is to determine (*f*) the rate of absorption and dissociation or ionization at each point, as a function of height, time of day, season of year, and latitude, and (*g*) the value of n at each point, as a function of the same variables.

This problem is easy to formulate in mathematical terms, and the differential equations can be partly, but not fully, integrated; at a certain point it is necessary to resort to numerical calculation. A feature of the present treatment is that, by suitable choice of units, the analysis and discussion are expressed in general terms, without the assumption of any special values for the numerical quantities involved, until the

latest possible stage; when this can no longer be avoided, particular numerical values are considered, but, even so, the differential equation is written so that only a single constant (σ_0, § 9) comes in question. This procedure makes it possible to apply the present analysis and results to different types of radiation (which may be absorbed at different levels in the atmosphere, and have different dissociative or ionizing effects) when the particular numerical data for each such case become available.

On the other hand, the problem is an ideal one which will scarcely represent adequately all the factors of importance in any actual case. It is thought likely, however, to be of value as an approximation, and as a starting point for further investigation into the influence of factors here neglected.

The number of independent variables involved is four, namely height, time of day, season of year, and latitude; this renders it difficult to gain a comprehensive view of the results. It has seemed most convenient to represent them graphically, rather than by numerical tables, though a considerable number of diagrams are required, each of which includes several curves. Figures 1–5 relate to the absorption and rate of dissociation or ionization; Figures 6–11 represent n as a function of the time of day, for various heights, seasons, and latitudes (for the three adopted values of σ_0). The seasons considered are midsummer, midwinter, and the equinoxes; the latitudes are 0° (the equator) and 60°.

In § 14 the results are briefly considered in relation to the ionization of the upper atmosphere, though as regards the conditions (*d*) and (*e*) the present problem only approximately illustrates the ionization changes. But as this is the most interesting application of the results at the present time, the whole paper has been written as though the dissociated components A_1 and A_2 were ions and electrons; by merely verbal changes the discussion could be expressed relative to the corresponding dissociation phenomena in which A_1 and A_2 are neutral atoms or molecules.

§ 2. THE RATE OF ION-PRODUCTION

Consider a point P at height h above the ground at an angular distance θ (the colatitude) from the north pole. Let t, or ϕ, denote the local time reckoned from noon, that is, the longitude of P measured eastward from the noon meridian at the instant, ϕ being in angular measure (radians), and t in time-measure (seconds). Clearly h, θ t, ϕ

$$t = 86400\ \phi/2\pi = 1{\cdot}37 . 10^4\ \phi, \qquad \ldots\ldots(1),$$

there being 86400 seconds in a day.

Over the range of level considered, the density of the atmosphere will be supposed to vary exponentially, i.e. $\rho = \rho_0 \exp(-h/H)$, where ρ_0 is the density that would exist at ground level if the formula were valid at all heights. In the actual atmosphere this formula is valid only if H itself is a slowly varying function of height, but the probable variation over the range of level that is important in this investigation is not great, and may in a first approximation be neglected. Constancy of H would result if the atmosphere were of uniform composition and temperature, but these conditions are only sufficient, not necessary. When they are fulfilled, $H = kT/mg$, ρ_0, H

k, T, m, g where k, T, m and g denote respectively Boltzmann's constant $1{\cdot}37.10^{-16}$, the absolute temperature, the mean molecular mass, and the acceleration of gravity; for example, if $T = 300°$ and the composition is the same as in the lower atmosphere (mainly nitrogen N_2 and oxygen O_2), $H = 8{\cdot}4$ km. This figure may be borne in mind in considering the following results, which, however, are independent of any particular numerical value of H, because heights will in general be expressed in terms of H as unit. It is quite possible that, above 100 km., the value of H differs from 8·4 km., but the value, when found, can at once be applied to the results of the general theory here given. The theory may also be relevant to different strata in the atmosphere, for which H has different values.

h' Thus, when H is taken as the unit of height, $\rho = \rho_0 \exp(-h')$, where h' $(= h/H)$ denotes height measured in this unit.

§ 3. THE INCIDENCE OF THE RADIATION

χ Since the sun is on the noon meridian, its zenith distance χ at P is given by

$$\cos\chi = \sin\delta\cos\theta + \cos\delta\sin\theta\cos\phi \qquad \text{.....(2),}$$

δ where δ denotes the north declination of the sun.

At the equinoxes $\delta = 0$ and

$$\cos\chi = \sin\theta\cos\phi \text{ (equinox)} \qquad \text{.....(3).}$$

At noon ($\phi = 0$), denoting χ by χ_0, we have

$$\chi_0 = \tfrac{1}{2}\pi - (\theta + \delta) \text{ (noon)} \qquad \text{.....(4).}$$

When it is desired to indicate the variables on which χ depends, it will be denoted by $\chi(\delta, \theta, \phi)$. Then, by (2),

$$\chi(\delta, \tfrac{1}{2}\pi, \phi) = \chi(0, \tfrac{1}{2}\pi - \delta, \phi) \qquad \text{.....(5),}$$

i.e. the sun's zenith distance, at any local time ϕ, is the same for the equator at the season when the sun's declination is δ, as at a point in latitude δ (or colatitude $\frac{1}{2}\pi - \delta$) at the equinoxes ($\delta = 0$).

§ 4. THE ABSORPTION OF RADIATION

Consider a beam of monochromatic ionizing solar radiation, of unit cross-section and of intensity S_∞ outside the atmosphere, passing through the layer between h and $h - dh$, above the point θ, ϕ, and therefore at the inclination χ to the vertical. Subject to the usual assumption that the absorption is proportional to the intensity S at that height, and to the mass $\rho_0 \sec\chi \exp(-h/H).dh$ of air traversed, the change of intensity dS after crossing the layer will be given by

$$dS = AS\rho_0 \sec\chi \exp(-h/H)\, dh \qquad \text{.....(6),}$$

where A is the coefficient of absorption. The solution of this differential equation is*

$$S = S_\infty \exp\{-A\rho_0 H \sec\chi . \exp(-h/H)\} \qquad \text{.....(7).}$$

* P. Lenard. *Sitz. d. Heidelberger Ak. d. W.*, 12 Abh. (1911).

The absorption of radiation per cc. of atmosphere is $dS/(\sec\chi . dh)$, and if the number of ions produced by the absorption of unit quantity of the radiation is β, the rate of production of ions per cc. will be $\beta \cos\chi . dS/dh$; this, being a function of h and χ, will be denoted by $I(\chi, h)$. By (7),

$$I(\chi, h) = \beta \cos\chi \frac{dS}{dh} = \beta A S_\infty \rho_0 \exp\left\{-\frac{h}{H} - A\rho_0 H \sec\chi \exp\left(-\frac{h}{H}\right)\right\} \quad \ldots\ldots(8).$$

The total number of ions produced in a square cm. column of air by the complete absorption of the radiation (S_∞) concerned is $\beta S_\infty \cos\chi$.

This rate of production of ions per c.c. has a maximum at the height $h(\chi)$ given by

$$\exp\frac{h(\chi)}{H} = A\rho_0 H \sec\chi \quad \ldots\ldots(9),$$

and the corresponding maximum value $I(\chi)$ of $I(\chi, h)$ is given by

$$I(\chi) = \beta S_\infty \cos\chi / H \exp 1 \quad (10),$$

where exp 1 is written instead of e (or 2·718 ...) to avoid confusion with the usual symbol for the electronic charge. Let the values of $h(\chi)$ and $I(\chi)$ for $\chi = 0$ be denoted by h_0 and I_0. Then

$$\exp(h_0/H) = A\rho_0 H, \quad I_0 = \beta S_\infty / H \exp 1 \quad (11);$$

consequently

$$h(\chi) = h_0 + H \ln \sec\chi \quad \ldots\ldots(12),$$

where ln denotes the Napierian logarithm, and

$$I(\chi) = I_0 \cos\chi \quad \ldots\ldots(13).$$

In terms of I_0 and h_0, (8) may be written

$$I(\chi, h) = I_0 \exp\left\{\frac{h_0 + H - h}{H} - \sec\chi \exp\frac{h_0 - h}{H}\right\} \quad \ldots\ldots(14).$$

It should be noted that these equations are not valid for very large values of $\sec\chi$ (corresponding to grazing incidence of the beam of radiation), because then the level surfaces traversed by the beam can no longer be treated as parallel planes, as they were when the distance along the beam between h and $h - dh$ was taken as $\sec\chi . dh$. The approximation is probably sufficiently accurate up to $\sec\chi = 12$, or $\chi = 85°$, which along the equator corresponds to about 20 minutes after sunrise or before sunset. In a succeeding paper the special conditions existing near sunrise will be examined more accurately; it will be shown that at and near the level of maximum ion-content the results of the present paper are very nearly true except near dawn, at the equator; but that in higher latitudes, *in winter*, the necessary corrections are appreciable up to noon.

§ 5. THE DATUM UNIT FOR DIFFERENCES OF LEVEL

It is convenient (cf. § 2) to measure heights in terms of H as unit, and reckoned from h_0 as datum; H will have the value appropriate to the height h_0, and its change with height over the ionized layer, at and near the level h_0, will be assumed small and neglected. Thus, let

$$z = (h - h_0)/H \qquad \text{.....(15).}$$

Then if $z(\chi)$ corresponds to the level $h(\chi)$, we have, by (12),

$$z(\chi) = \ln \sec \chi \qquad \text{.....(16),}$$

and by (14),

$$I(\chi, h) = I_0 \exp\{1 - z - \sec\chi . \exp(-z)\} \equiv I(\delta, \theta, z, \phi) \qquad \text{.....(17),}$$

where the notation $I(\delta, \theta, z, \phi)$ is used to indicate the independent variables of which $I(\chi, h)$ is a function.

§ 6. THE NOON DISTRIBUTION OF ABSORPTION

At any point P the minimum value of χ and $\sec\chi$, and consequently the maximum value of $I(\chi, h)$ or $I(\delta, \theta, z, \phi)$, occur at noon ($\phi = 0$), when, by (4), $\chi = \frac{1}{2}\pi - (\theta + \delta)$. Hence, by (17),

$$I(\delta, \theta, z, 0) = I_0 \exp\{1 - z - \operatorname{cosec}(\theta + \delta) \exp(-z)\} \text{ (noon)} \qquad \text{.....(18).}$$

This has its maximum value, with respect to height, of amount

$$I(\delta, \theta) = I_0 \sin(\theta + \delta) \qquad \text{.....(19),}$$

by (13), at the level

$$z(\delta, \theta) = \ln \operatorname{cosec}(\theta + \delta) \qquad \text{.....(20),}$$

by (16).

These formulae are exemplified by the graphs in figure 1, which represent $I(\delta, \theta, z, 0)/I_0$ as a function of z; that is, they indicate, for a number of values of $\theta + \delta$, the ratio of the noon rate of ion-production at various heights, to the noon-rate at the equator at height h_0. The curves relate to the values (1) 6·5°, (2) 15°, (3) 30°, (4) 45°, (5) 53·5°, (6) 66·5° and (7) 90°.

At the equinoxes ($\delta = 0$) they refer to points having these colatitudes, or to latitudes varying from 83·5° (curve 1) to zero (the equator, curve 7). They show how the ion-production at noon decreases as we recede from the equator, while at the same time the level of maximum production rises, at first slowly, then more rapidly as the pole is approached.

The curve (6) shows the noon-rate of ion-production at the equator ($\theta = \frac{1}{2}\pi$) at the solstices ($\delta = \pm 23{\cdot}5°$), and by comparison with the equinoctial curve (7) for the equator indicates how slight is the seasonal change in the height distribution of ion-production above the equator.

At latitude 60° ($\theta = 30°$), however, the seasonal change is very considerable, as is shown by curves (1), (3) and (5), which refer to this latitude at midwinter ($\delta = -23{\cdot}5°$, $\theta + \delta = 6{\cdot}5°$), the equinoxes ($\delta = 0$, $\theta + \delta = 30°$), and midsummer ($\delta = 23{\cdot}5$, $\theta + \delta = 53{\cdot}5°$) respectively. The level of maximum production varies

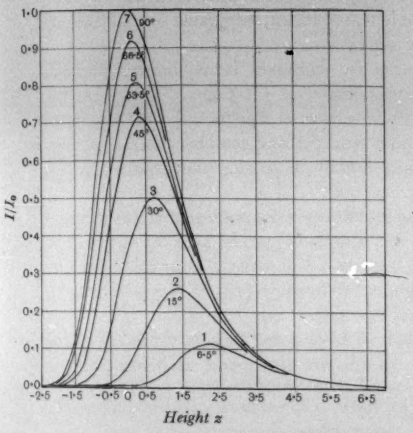

Fig. 1. Ion-production at noon for various values of $(\theta \pm \delta)$

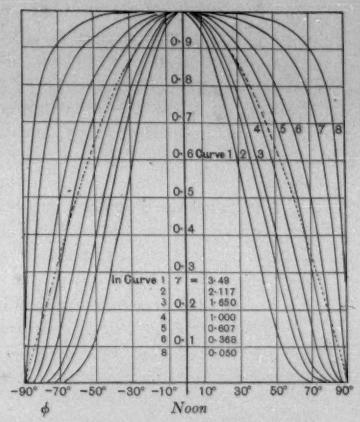

Fig. 2. Ion-production at the equinoxes

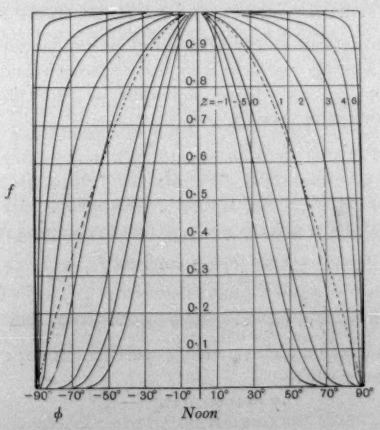

Fig. 3. Ion-production at the equinoxes in latitude 60°

from $z = 2{\cdot}2$ in midwinter to $z = 0{\cdot}2$ in midsummer (a change of 17 km. if $H = 8{\cdot}4$ km.), and the maximum rate of production is increased in the ratio $\sin 53{\cdot}5^\circ / \sin 6{\cdot}5^\circ$ or $7{\cdot}1$; the total rate of ion-production, integrated over all heights (i.e. $\beta S \cos \chi_0$, cf. § 4), is altered in the same ratio.

Considering curve (7) for the equator at the equinoxes, it may be seen that I/I_0 falls from 1 at height h_0 ($z = 0$) to $\frac{1}{8}$ at approximately $z = 1{\cdot}5$ and $z = 3{\cdot}0$; thus the greater part of the ion-production occurs in a layer of thickness $4{\cdot}5\ H$* (or about 39 km. if $H = 8{\cdot}4$ km.). Here, and at other latitudes and seasons, the rate of ion-production at noon decreases much more rapidly in passing downwards than in *ascending* from the level of maximum production.

The higher the level above h_0, the more uniform is the noon rate of ion-production at nearly all latitudes and seasons; this is already marked at $z = 3$, where I/I_0 is approximately $\frac{1}{8}$ for all values of $\theta + \delta$ between 15° and 165°. Over this range the term $\operatorname{cosec}(\theta + \delta) \exp(-z)$ in the exponential formula (18) is very small when $z > 3$, and I then becomes approximately $I_0 \exp(1 - z)$.

The variation of $z\,(\delta, \theta)$ is also shown in table 1 for various values of χ_0 or $\frac{1}{2}\pi - (\theta + \delta)$; the corresponding values in km. are also given, H being taken as 8·4 km.

Table 1.

$\chi_0 = \frac{1}{2}\pi - (\theta + \delta)$	0°	30°	45°	60°	75°	80°	82·5°	85°
$z\,(\delta, \theta)$	0	0·062	0·151	0·301	0·587	0·760	0·884	1·060
z (km.)	0	1·2	2·9	5·8	11·3	14·7	17·1	20·5

§ 7. THE PROPORTIONATE DAILY VARIATION OF ABSORPTION

It is of interest to indicate also how the rate of ion-production varies with respect to ϕ, that is, throughout the day at different heights, seasons and latitudes. This can conveniently be done by representing graphically, as a function of ϕ, the ratio of the rate at time ϕ to that at noon ($\phi = 0$), i.e.

f
$$\begin{aligned} f(\delta, \theta, z, \phi) &= I(\delta, \theta, z, \phi)/I(\delta, \theta, z, 0) \\ &= \exp\{(\sec \chi_0 - \sec \chi).\exp(-z)\} \\ &= \exp[\{\operatorname{cosec}(\theta + \delta) - \sec \chi\}.\exp(-z)] \end{aligned} \qquad \text{.....(21).}$$

The range of ϕ is that for which $\cos \chi > 0$, so that the limiting values of ϕ are given by
$$\cos \phi = -\tan \delta \cot \theta \qquad \text{.....(22).}$$

At the equinoxes $\delta = 0$ and ϕ ranges between $\pm \frac{1}{2}\pi$, while (21) takes the form
$$f(0, \theta, z, \phi) = \exp\{-\operatorname{cosec} \theta \exp(-z)(\sec \phi - 1)\} \qquad \text{.....(23),}$$
in which the dependence on each variable is specially simple; z and θ are involved together in the factor γ where

γ
$$\gamma = \operatorname{cosec} \theta \exp(-z) \qquad \text{.....(24),}$$
in terms of which
$$f(0, \theta, z, \phi) = \exp\{-\gamma(\sec \phi - 1)\} \qquad \text{.....(25).}$$

* H itself is assumed to have a negligible variation throughout a layer of thickness about $5H$.

The values of γ for various values of z and θ are given in table 2.

Table 2

$\gamma = \operatorname{cosec}\theta . \exp\left(-z\right)$

z	θ							
	6·5°	15°	30°	45°	53·5°	60°	66·5°	90° (eqr.)
4·0	0·162	0·071	0·037	0·026	0·023	0·021	0·020	0·018
3·0	0·440	0·192	0·100	0·070	0·062	0·057	0·054	0·050
2·0	1·196	0·523	0·271	0·191	0·168	0·156	0·148	0·135
1·5	1·971	0·862	0·446	0·316	0·278	0·258	0·243	0·223
1·0	3·250	1·421	0·736	0·520	0·458	0·425	0·401	0·368
0·5	5·358	2·343	1·213	0·858	0·755	0·700	0·661	0·607
0	8·834	3·864	2·000	1·414	1·244	1·155	1·090	1·000
−0·5	14·56	6·37	3·297	2·332	2·051	1·904	1·798	1·649
−0·75	18·70	8·18	4·23	2·99	2·63	2·445	2·308	2·117
−1·0	24·01	10·50	5·44	3·84	3·38	3·14	2·96	2·72
−1·25	30·83	13·49	6·98	4·94	4·34	4·03	3·81	3·49
−1·5	39·59	17·32	8·96	6·34	5·58	5·18	4·89	4·48

In figure 2, curves representing $f(0, \theta, z, \phi)$ as a function of ϕ, equation (25), are drawn for various values of γ. They indicate the proportionate diurnal variation in the rate of ion-production, *at the equinoxes* ($\delta = 0$), for any point at height z (above or below h_0) in any colatitude θ; the value of γ corresponding to θ and z may be found by interpolation from table 2, and the corresponding curve can then be obtained from figure 2 by interpolation.

At the level h_0 ($z = 0$), at the equator ($\theta = \frac{1}{2}\pi$), $\gamma = 1$; the curve $\gamma = 1$ in figure 2 resembles, though except at $\phi = 0$ it is below, the curve of $\cos\phi$ (shown by a dotted line for comparison). For $\gamma > 1$, corresponding, *inter alia*, to levels below h_0 at the equator, or to levels extending somewhat above h_0 at higher latitudes (cf. table 2), the excess of $\cos\phi$ over $f(0, \theta, z, \phi)$ is greater, indicating that the part of the day during which these levels absorb the radiation is more and more concentrated round the hour of noon. Conversely, for $\gamma < 1$, $f(0, \theta, z, \phi)$ approaches unity the more rapidly and over a wider range of ϕ, the smaller the value of γ, indicating that in the higher levels of the atmosphere the rate of ion-production is near its maximum and varies little, from near sunrise to near sunset, the rise and fall near these epochs being rapid.

§ 8. THE SEASONAL CHANGES IN THE PROPORTIONATE DAILY VARIATION

At times other than the equinoxes the curves showing $f(\delta, \theta, z, \phi)$ as a function of ϕ differ from those of figure 2, except for the equator ($\theta = \frac{1}{2}\pi$). There, by virtue of (5),

$$f(\delta, \tfrac{1}{2}\pi, z, \phi) = f(0, \tfrac{1}{2}\pi - \delta, z, \phi),$$

indicating that the equatorial curve for the season δ and for any height z is the same as the *equinoctial* curve for the same height in latitude δ. The slightness of the seasonal change in the proportionate diurnal variation of ion-production at any

height above the equator is indicated by the small difference between the values of γ in the two last columns of table 2, which refer respectively, for any height z, to the solstices and equinoxes.

At latitudes far from the equator the proportionate daily variation in the rate of ion-production at any height changes considerably with the season, as also does the duration of ion-production. This is illustrated by the curves in figures 3, 4, 5, which refer to latitude 60° ($\theta = 30°$) at the equinoxes, the summer solstice, and the winter solstice ($\delta = 23{\cdot}5.0, -23{\cdot}5$), for a series of heights between $z = -2$ and $z = 7$; the curve representing $\cos\chi$ is also shown on each diagram by a dotted line.

The different base-lengths in the three diagrams refer, of course, to the different periods of daylight at the three seasons. In summer, when the sun's rays are most direct, the rate of ion-production at the level $z = 0$ exceeds half its maximum (noon) value for about half the period of daylight; this is nearly true at the same level at the equinoxes also, but in winter the absorption of radiation at this level increases more slowly, and it exceeds half the noon value only for about a quarter of the period of daylight. At $z = 6$ at all seasons the absorption is nearly at its full value throughout almost the whole day.

To obtain a comprehensive idea of the variation of ion-production with respect to season, height, latitude and local time the curves in figure 1 should be considered in conjunction with those of figures 2–5.

§ 9. THE DISTRIBUTION AND VARIATIONS OF ION-DENSITY

n, n_e, n_- Let n, n_e, n_- denote the number per c.c. of positive ions, electrons, and negative ions, at a point ϕ, θ, z. The ions of either kind are, for simplicity, supposed to be all alike and simply charged; hence, the air being electrically neutral,

$$n = n_e + n_- \qquad \text{.....(26).}$$

The number of positive ions produced per c.c. per second is denoted, as in the preceding sections, by $I(\delta, \theta, z, \phi)$. The equation of variation for n, due to new formation of positive ions, and their disappearance by recombination, is

$$dn/dt = I(\delta, \theta, z, \phi) - \alpha_e n n_e - \alpha_- n n_- \qquad \text{.....(27),}$$

α_e, α_- where α_e, α_- denote coefficients of recombination. Their values are not known with any certainty, at the low pressures which obtain in the atmospheric region under consideration; for simplicity it will be assumed that they are equal, though this is a pure assumption. Thus we take

α $$\alpha_e = \alpha_- = \alpha \qquad \text{.....(28),}$$

so that, by virtue also of (26), (27) reduces to

$$dn/dt = I - \alpha n^2 \qquad \text{.....(29).}$$

This is the equation that expresses the condition (*d*) of § 1.

Here, as has been shown in §§ 2–8, I is a function of δ, θ, z (or h) and ϕ, so that n also will depend on these variables; α may be a function of z, but will be supposed independent of δ, θ, and ϕ; the solutions of (29) which are obtained here will, however, only be discussed for values of α independent also of z.

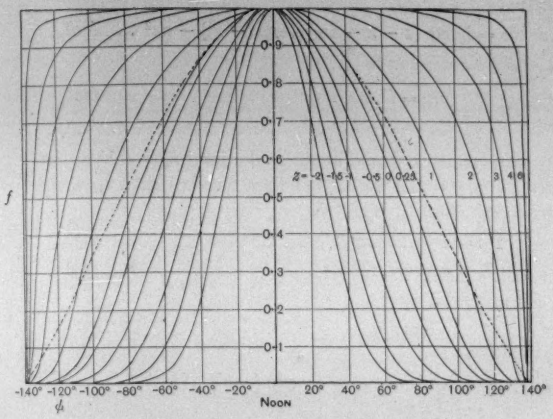

Fig. 4. Ion-production at the summer solstice in latitude 60°.

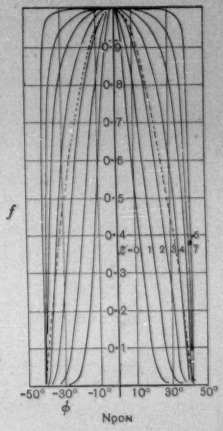

Fig. 5. Ion-production at the winter solstice in latitude 60°.

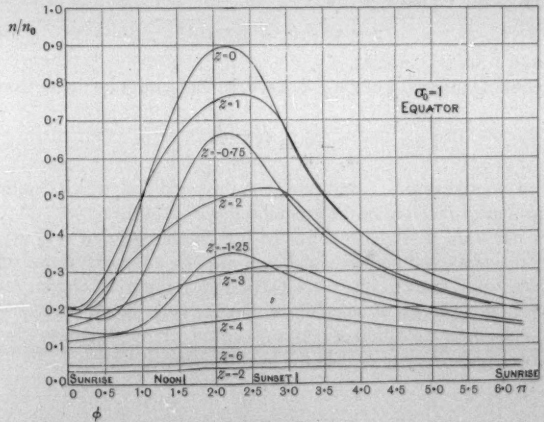

Fig. 6.

t It is convenient to express the local time t in (29) in terms of ϕ, by (1). Then (29) becomes, using (17):

$$(1/1{\cdot}37.10^4).dn/d\phi = I_0 \exp\{1 - z - \sec\chi.\exp(-z)\} - \alpha n^2 \quad \text{.....(30)},$$

where χ depends on ϕ, θ and δ according to (2).

n_0 Let $$n_0 = (I_0/\alpha)^{\frac{1}{2}} \quad \text{.....(31)},$$

σ_0 $$1/\sigma_0 = 1{\cdot}37.10^4\,(I_0\alpha)^{\frac{1}{2}} \quad \text{.....(32)},$$

so that n_0 is the steady value which n would attain at the level h_0 ($z = 0$) at the equator at midday (when $z = 0$, $\chi = 0$) if the earth did not revolve. Then n_0, σ_0 are alternative modes of specifying α and I_0, i.e.

$$1/\alpha = 1{\cdot}37.10^4\,\sigma_0 n_0 \quad \text{.....(33)},$$

$$I_0 = n_0/(1{\cdot}37.10^4\,\sigma_0) \quad \text{.....(34)}.$$

In terms of n_0, σ_0, and ν, defined by

$$\nu \equiv n/n_0 \quad \text{.....(35)},$$

(30) may be written in the form

$$\sigma_0.d\nu/d\phi + \nu^2 = \exp\{1 - z - \sec\chi\exp(-z)\} \quad \text{(day)}$$

F $$= F(z, \chi) \quad \text{.....(36)}.$$

This represents the variation of ν, or n/n_0, during the hours of daylight; during the hours of darkness the right-hand side must be replaced by zero, i.e.

$$\sigma_0.d\nu/d\phi + \nu^2 = 0 \quad \text{(night)} \quad \text{.....(37)}.$$

The solution of this equation is

$$\nu = \sigma_0/(\phi + C) \quad \text{(night)} \quad \text{.....(38)},$$

C where C is an arbitrary constant.

r, s Distinguishing the values of ν and ϕ at sunrise and sunset by the suffixes r and s, we have

$$\frac{1}{\nu_r} - \frac{1}{\nu_s} = \frac{\phi_r - \phi_s}{\sigma_0} \quad \text{.....(39)}.$$

The solution of (36) has to be found subject to this condition, which determines the arbitrary parameter involved in the general solution of (36).

It does not seem possible to solve the non-linear equation (36) in terms of elementary functions, and numerical methods must be adopted. If one solution of (36) has been found numerically, not satisfying (39), the true solution can be obtained from it by a process of quadratures, but the latter is not appreciably easier than finding the correct solution of (36), subject to (39), by successive direct trials; it is therefore unnecessary to explain the method referred to.

§ 10. SOLUTIONS FOR THREE VALUES OF σ_0

The equation (36) has been solved, subject to the condition (39), for three values of σ_0, namely 1, $\frac{1}{5}$ and $\frac{1}{25}$, for various heights (at distances z above and below h_0) at the equator ($\theta = \frac{1}{2}\pi$) at the equinoxes ($\delta = 0$); and also, for $\sigma_0 = \frac{1}{25}$, at various

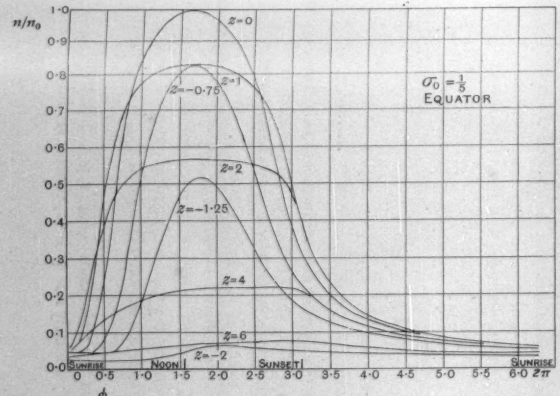

Fig. 7.

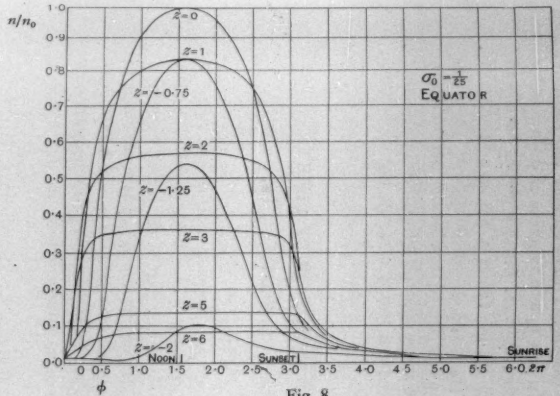

Fig. 8.

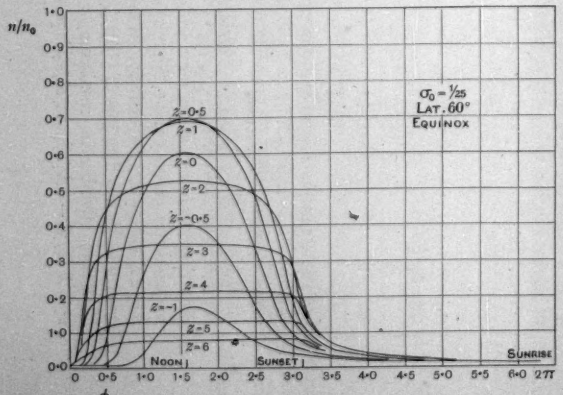

Fig. 9.

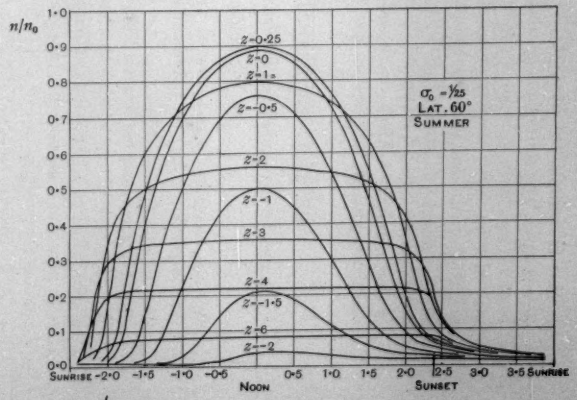

Fig. 10.

heights in latitude 60° ($\theta = 30°$), at the equinoxes ($\delta = 0$) and the solstices ($\delta = \pm 23{\cdot}5$). The corresponding values of ν or n/n_0 as a function of ϕ (in circular measure) are shown by series of graphs in figures 6–11. Further graphs have been derived from these, showing, in figures 12–17, the values of n/n_0 as a function of height at various local times, for the equator ($\sigma_0 = 1, \frac{1}{5}, \frac{1}{25}$) and latitude 60° ($\sigma_0 = \frac{1}{25}$). In the "equinoctial" figures 6–9 and 12–15, ϕ is reckoned from *sunrise*, not from *noon* as in figures 2–5, 10, 11, 16, 17, and elsewhere in this paper. These curves, and further general conclusions which can be inferred from them, or from the differential equations (36), (37), will now be discussed.

§ 11. THE ION-DENSITY AS A FUNCTION OF TIME

At and between sunset and sunrise ν varies according to (37), i.e.

$$d\nu/d\phi = -\nu^2/\sigma_0;$$

thus ν is decreasing at and between these epochs. Between sunrise and sunset ν varies according to (36), the right-hand side of which is essentially positive, and has its maximum at noon (when $\chi = \frac{1}{2}\pi - \theta - \delta$); during part of the hours of sunlight must increase, and the reversal from the decreasing rate ${\nu_r}^2/\sigma_0$ at sunrise occurs after an interval of time which is shorter, the smaller the value of σ_0. In many of the graphs in figures 6–10 this interval is too small to be shown, though for heights *below* h_0 (z negative) a considerable time elapses after sunrise before the increase in ν becomes noteworthy.

At the equator, for $\sigma_0 = 1$ the maximum value of n/n_0 occurs distinctly after noon at heights adjacent to h_0, and at greater heights the maximum is deferred till near sunset. But for $\sigma_0 = \frac{1}{5}$ and $\sigma_0 = \frac{1}{25}$ the value of n/n_0 at noon is very near the maximum value; this is true likewise for $\sigma_0 = \frac{1}{25}$, at 60° latitude, and therefore also at intermediate latitudes. For such values of σ_0 it is possible to deduce a close approximation to $\nu_{\max}$; for if, at noon, when $\sec\chi = \operatorname{cosec}(\theta + \delta)$, $d\nu/d\phi$ is small or zero, then by (36)

$$\nu^2 = F(z, \chi) = \exp\{1 - z - \operatorname{cosec}(\theta + \delta)\exp(-z)\},$$

or

$$n_{\max}/n_0 = \exp\tfrac{1}{2}\{1 - z - \operatorname{cosec}(\theta + \delta).\exp(-z)\}$$
$$= \sqrt{(I/I_0)} \quad \text{(noon)} \qquad \ldots\ldots(40)$$

by (18). At height h_0 ($z = 0$) at the equator ($\theta = \frac{1}{2}\pi$) at the equinoxes ($\delta = 0$), the value of I/I_0 is unity (cf. figure 1), so that $n_{\max} = n_0$ almost exactly. At the solstices at the equator $(I/I_0)_{\text{noon, max}} = 0{\cdot}938$ (curve 6, figure 1), so that $n_{\max} = 0{\cdot}968\, n_0$; the maximum value occurs very shortly after noon, at the height $z = 0{\cdot}1$. At the equinox in 60° latitude $(I/I_0)_{\text{noon, max}} = 0{\cdot}5$ (curve 3, figure 1) so that $n_{\max}/n_0 = 0{\cdot}707$, occurring at $z = 0{\cdot}7$; at the same latitude the values of $n_{\max}/n_0$ at midsummer and midwinter (cf. curves 5 and 1, figure 1) are 0·90 and 0·33, occurring nearly at noon at $z = 0{\cdot}2$ and $z = 2{\cdot}2$ respectively. At this latitude the seasonal change in $n_{\max}$ is very great, though less than that of $(I/I_0)_{\text{noon, max}}$.

Before attaining its maximum, ν must be less than $\sqrt{F(x, \chi_0)}$, because $\nu^2 = F(z, \chi) - \sigma d\nu/d\phi$, and $d\nu/d\phi$ is positive; similarly $\nu > \sqrt{F(z, \chi)}$ after attaining

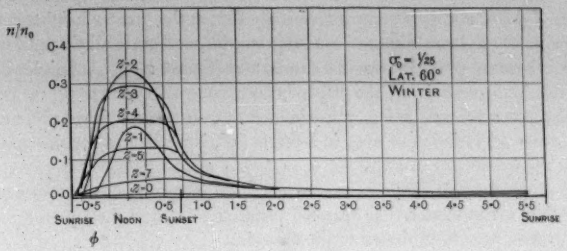

Fig. 11.

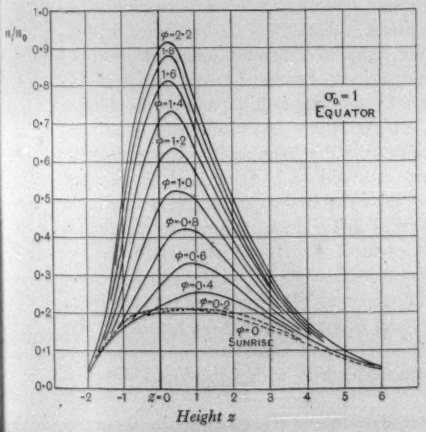

Fig. 12*a*.

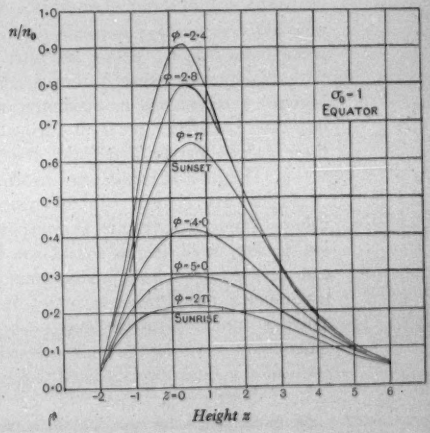

Fig. 12*b*.

its maximum. But there appears to be no easy general way of finding the amount of the difference.

At heights above the level of maximum noon ionization the graphs of n/n_0 as a function of ϕ (for $\sigma \leqslant \frac{1}{5}$) become increasingly flat; at the greater heights considered n/n_0 rises nearly to its maximum soon after sunrise, and varies little until near sunset; this is because $\sqrt{(I/I_0)}$ varies in a similar way (figures 2–5). At heights below the level of maximum noon (n/n_0), the graphs become increasingly narrow, the maximum ion density at such levels being approximated to over a short time only, near noon.

The seasonal variation of n/n_0, in latitude 60°, is much less for the upper levels than for the lower; for example for $\sigma_0 = \frac{1}{25}$ at $z = 6$, n_{max}/n_0 lies between 0·8 and 0·9 all through the year, while for $z = -1$ it is 0·6 in summer, 0·5 at the equinoxes, and less than 0·1 in winter. The seasonal variation of n/n_0 at the equator is so small that no graphs have been drawn to illustrate it.

§ 12. THE ION-DENSITY AS A FUNCTION OF HEIGHT

The curves in figures 12–17, giving n/n_0 as a function of height at various local times, will next be considered.

Figures 12*a*, *b*, for $\sigma_0 = 1$ at the equator, show that in this case n has its maximum, at all heights, at about $2\frac{1}{2}$ hours after noon ($\phi = 2{\cdot}2$); the maximum value of n occurs at about $z = \frac{1}{4}$, or about 2 km. (if $H = 8{\cdot}4$ km.) above h_0, the level of maximum ion-production at midday; moreover the maximum value of n is distinctly less than n_0, being about 0·92 n_0. Before and after the epoch of maximum ionization, n varies fairly rapidly with respect both to height and time; at sunset ($\phi = \pi$) the ion-distribution has fallen from its maximum only by about one-third, and a considerable decrease of ionization proceeds during the night. But even at sunrise ($\phi = 0$ or 2π) there is a well-marked distribution of ionization, with maximum ion-density at about $z = \frac{1}{2}$. The level of maximum ion-density falls while n is rising, and *vice versa*.

Figures 13*a*, *b*, for $\sigma_0 = \frac{1}{5}$ at the equator, show that in this case n/n_0 has a maximum value of almost exactly unity at or very near to the height h_0, and that the maximum ion-density, at all heights considered, is attained very shortly after noon. Before and after noon the level of maximum ion-density is above h_0, the value of z for maximum n, near sunrise or sunset, being about 2. The maximum ion-density at sunset occurs at about the level $z = 1{\cdot}5$, and is approximately 0·4 n_0; the further reduction in n during the night is considerable, and at sunrise the ion-distribution consists of a thick layer of nearly uniform ion-density with a maximum of about $\frac{1}{20}$ n_0 at about $z = 2$.

Figures 14*a*, *b*, for $\sigma_0 = \frac{1}{25}$ at the equator, show that in this case n attains the maximum value n_0 at the level h_0 at noon, almost exactly. At sunset the maximum ion-density is reduced to about 0·23 n_0 and occurs at the level $z = 2{\cdot}5$ (about 21 km. above h_0, if $H = 8{\cdot}4$ km.). During the night the ion-density decreases still further to about $\frac{1}{100}$ n_0 at sunrise, when n is nearly uniform throughout a thick layer extending from about $z = -2{\cdot}5$ to $z > 6$. These remarks apply to the equinoxes, but the reduction, and changes of distribution, of the ionization at the solstices are very slight.

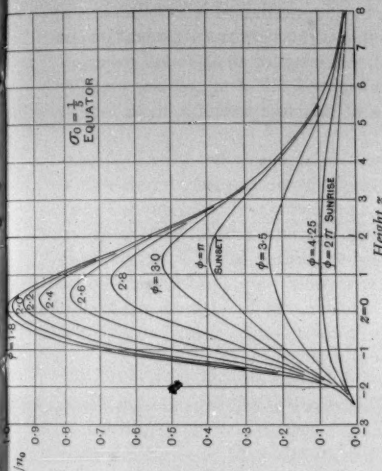

Fig. 13*b*.

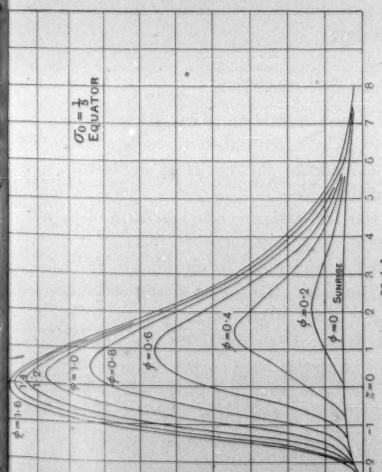

Fig. 13*a*.

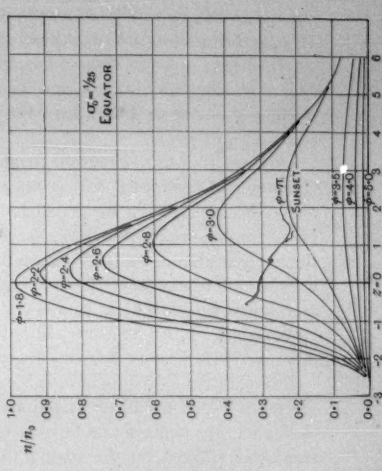

Fig. 14*b*.

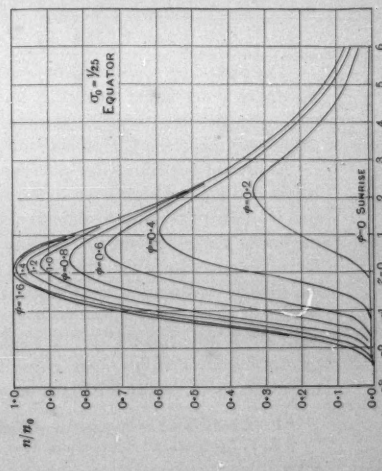

Fig. 14*a*.

Figures 17–18 for $\sigma_0 = \frac{1}{25}$ at latitude 60° show that at the equinox here, as at the equator, n has its maximum nearly at noon, but the maximum is less than at the equator, being about 0·71 n_0 instead of n_0; moreover it occurs at a somewhat higher level, where $z = 0{\cdot}7$ or about 6 km. above h_0. Allowing for an all-round reduction in n at all times and heights, and for a rather higher level of n_{max} at any time (by about 0·5 to 0·7, or 4 to 6 km.) the features of this case resemble those for the equator ($\sigma_0 = \frac{1}{25}$).

Figures 16 and 17 show the height-distribution of n/n_0 at 60° latitude at midsummer and midwinter, for $\sigma_0 = \frac{1}{25}$; at midsummer the sunrise curve is a little above the sunrise curve for the equator (figure 14*a*), owing to the long day and short night in summer at 60° latitude; in midwinter the converse is true. In midsummer the noon maximum value of n is nearly as great at latitude 60° as at the equator; the sunset ionization at 60° latitude is greatest at about $z = 3{\cdot}3$, and is about 0·16 n_0, as compared with 0·23 n_0 at $z = 2{\cdot}5$ at the equator. At 60° latitude the sunset ionization in midwinter is very little less than in midsummer, though at noon the difference of ionization between the two seasons is large.

For still smaller values of σ_0 it is evident that the main changes from the curves of figures 14–17 would be as follows: the ion-density at sunrise and sunset, and throughout the night, would be reduced; the noon distribution of ion-density would scarcely be affected, but the rise to it would be initially less rapid, mainly taking place in a shorter interval before noon; similarly most of the fall from the noon maximum would be completed in a shorter interval, the period of high ion-density thus being concentrated more towards the middle of the day.

§ 13. UPPER ATMOSPHERIC IONIZATION

The preceding results will be briefly considered in relation to the ionization in the upper atmosphere; for further information, as regards both observational data and more detailed theory, reference may be made to the works of Pedersen*, Appleton†, Eckersley‡, Hulburt§ and other writers.

At present our knowledge of the actual values and variation of n as a function of height and time, at high levels in the atmosphere, is uncertain, though there is hope that it will later become possible to obtain detailed information of the kind by radio methods. There can scarcely be any doubt, however, that at least one strongly ionized layer exists in the atmosphere, at a height of the order 100 km., in which the ion-density undergoes a considerable daily variation; the evidence for this consists of various kinds of radio measurements, together with the daily, especially the lunar-diurnal, variations of the earth's magnetism. These suggest that the ion-density is greater by day than by night, rising from sunrise to about noon, and decreasing towards sunset, but still leaving at sunset a considerable distribution of ion-density,

* P. O. Pedersen, *The Propagation of Radio Waves* (Copenhagen, 1927).

† E. V. Appleton and collaborators, *Proc. R.S.* A, **128**, 133, 159 (1930) and earlier papers there cited.

‡ T. L. Eckersley, *Proc. Inst. Rad. Eng.* **18**, 106 (1930), and earlier references.

§ E. O. Hulburt, *Phys. Rev.* **31**, 1028 (1928); **34**, 1167 (1929); **35**, 240 (1930).

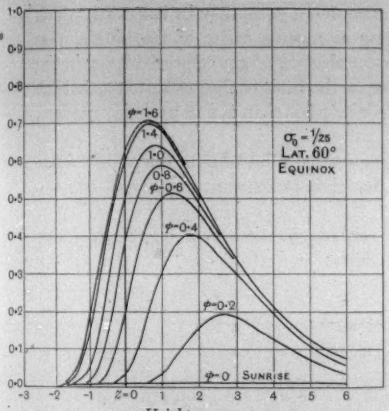

Fig. 15*a*.

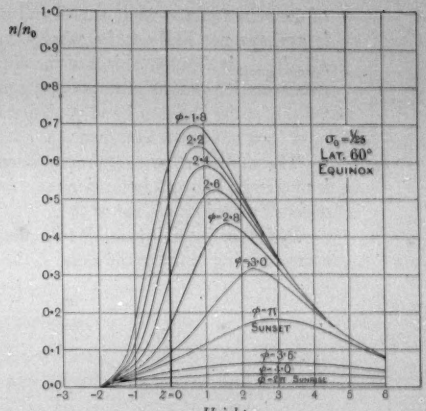

Fig. 15*b*.

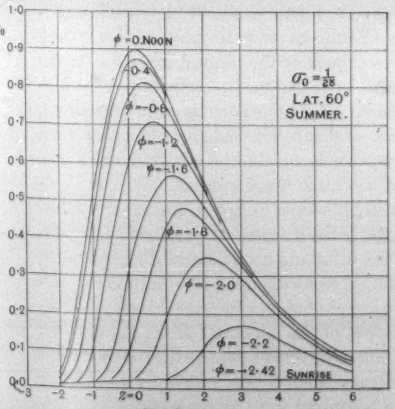

Fig. 16*a*.

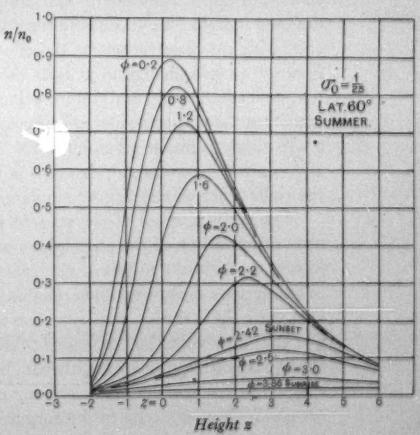

Fig. 16*b*.

which slowly decreases throughout the night. The theory of the lunar daily magnetic variations is not sufficiently well established to justify any safe estimate of the ratio of the values of n at noon, sunset and sunrise, but a guess may be hazarded, which is perhaps not likely to be wholly wrong as regards order of magnitude, that $n_{\text{noon}}/n_{\text{sunset}}$ is about 5. This, and the apparent absence of any appreciable lag in the maximum ion-density after noon, suggests, on the basis of figures 12 to 17, that the value of σ_0 lies between $\frac{1}{5}$ and $\frac{1}{25}$, being definitely less than 1, and probably not less, or at least not much less, than $\frac{1}{25}$.

n_0 The value of n_0, the maximum (noon) equatorial value of n for the positive ions, is uncertain, but such indications as exist (based on radio measures) suggest that the order of magnitude is 10^6 or 10^7.

By (33) or (34) it is possible from these very rough estimates of n_0 and σ_0 to derive corresponding estimates of α and I_0, as follows:

σ_0	n_0	α	I_0
$\frac{1}{25}$	10^6	$2\cdot10^{-9}$	$2\cdot10^3$
$\frac{1}{25}$	10^7	$2\cdot10^{-10}$	$2\cdot10^4$

The corresponding total rate of production of ions in a cm.^{2} column of air at the equator at noon is (cf. § 3) βS_∞ or, by (8), $HI_0 \exp 1$; if $H = 8{\cdot}4$ km., the values of βS_∞ corresponding to the above two values of I_0 are about $5\cdot10^9$ and $5\cdot10^{10}$.

These estimates of α, I_0 and βS_∞ are in general accordance with those of the same quantities (there denoted by α, q, and $\int qdh$) which I made in an earlier discussion of upper atmospheric ionization*; this discussion was based on data rather different in kind from those here considered. They seem also to be in general accordance with the observations of the effects of eclipses on radio transmission, which indicate a rapid change in at least the lower part of the ionized layer due to the temporary (total or partial) interception of solar radiation; if, at time $t = 0$, the ion-density at the point considered being n, the ion-producing agent is entirely removed, n will at later time be given by $n/(1 + \alpha nt)$, so that it will be halved in a time $1/\alpha n$. If $\alpha = 2\cdot10^{-9}$ and $n = 10^6$, this time is 500 seconds, or about 8 minutes, which is of the right order of magnitude; similarly if $\alpha = 2\cdot10^{-10}$ and $n = 10^7$.

The main purpose of the present paper, however, is not to discuss the actual state of ionization of the atmosphere on the basis of the scanty available data; it is intended to afford a means of discussing the value and variations of the ion-content of the upper atmosphere when reliable data become available. Its results are applicable not only to the ionized layer near 100 km., but also to the higher layer, at about 250 km., discovered by Appleton†; the values of the constants H, β, S_∞, α, ... for the two layers may, and probably will, be different. The present analysis is applicable also to the absorption of non-ionizing radiation, such as that which, by dissociating oxygen molecules, leads to the formation of ozone; but the work of §§ 9 *et seq.* is valid for dissociating-radiation only if the products of dissociation recombine according to the simple law (29), which may not be the case for ozone.

* *Q.J.R. Met. Soc.* **52**, 229 (1926). † *Loc. cit.*

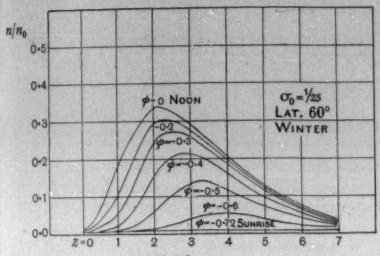

Fig. 17*a*.

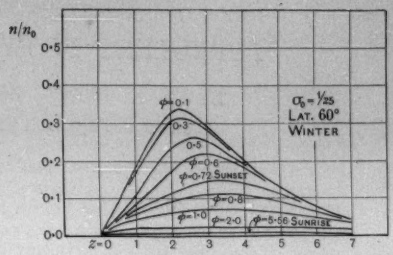

Fig. 17*b*.

§ 14. ACKNOWLEDGMENTS

In conclusion I have to acknowledge assistance received from Miss M. C. Gray, who made the detailed numerical calculations involved in this paper, and to Mr W. Reeve, Miss V. Hatcher and Miss R. Rossiter, who assisted in the preparation of the numerous diagrams.

TURBULENT FLOW THROUGH TUBES

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Received May 27, 1930. *Read and discussed November* 7, 1930.

ABSTRACT. The flow of fluid through long straight tubes of circular section was experimented on, at speeds in the neighbourhood of the lower critical velocity, in an attempt to detect any trace of periodicity in the turbulent motion. The experimental methods included (*a*) an aural method; (*b*) photography of the motion of a deflected vane; and (*c*) injection of colour-streamers about halfway along the tube. Intermittent turbulence was investigated at speeds near the critical speed, and measurements of the critical speed were made. It was shown that the velocity parallel to the tube-axis is sometimes almost uniform momentarily over the transverse section.

No trace was found of a simple frequency, but evidence was obtained of a predominant wave-length in the turbulent motion at the critical speed. Both these observations seem to agree with the approximate theory for flow between a pair of parallel planes given by Heisenberg.

§ 1. INTRODUCTION

The primary aim of this investigation was to search for any evidence of regular periodicity in the motion of fluid that is flowing turbulently in a long straight tube.

The subject of turbulent flow through tubes was first treated systematically by Osborne Reynolds*; extensive accurate experiments were subsequently performed by Stanton and Pannell†; and a theoretical basis was finally given by Heisenberg‡. Heisenberg's paper contains extensive references to prior papers on the subject.

Heisenberg gives a detailed approximate theory for the flow between a pair of *parallel planes*. He concludes that all turbulence will ultimately get suppressed when the "Reynolds' number" $vd\rho/\mu$ is less than a critical value of the order 10^3; here v, d, ρ, μ, denote the mean velocity, distance between planes or tube diameter, fluid density and viscosity respectively. At greater speeds turbulence will not be suppressed if its "wave-length" is between maximum and minimum values, which are functions of $vd\rho/\mu$. When $vd\rho/\mu$ has the critical value, only turbulence of a certain critical wave-length will persist. Heisenberg suggests a critical wave-length equal to about π times the distance between the planes; this is only an approximate estimate, and he says a more exact calculation is not feasible. Though Heisenberg suggests a single critical wave-length at the critical rate of flow, yet there is not supposed to be a single frequency; the velocity, and consequently the frequency, vary across the transverse section of the system.

v, d, ρ, μ

* *Phil. Trans. R.S.* **174**, 935 (1883).
† *Proc. R.S.* A, **85**, 366 (1911); *Phil. Trans. R.S.* A, **214**, 199 (1914).
‡ *Ann. der Physik*, **74**, 7, 577 (1924).

§ 2. AURAL EXPERIMENTS

As the ear is known to be an excellent harmonic analyser, the first experiments were made by an aural method. A small hole was drilled in the side of a long tube and a stethoscope was connected to the hole. When air was blown through the long tube, the only sound heard in the stethoscope was like that of a wind. Next the stethoscope was connected to a small glass Pitot-tube, which was inserted in the exit end of the long tube. It was then found that, as the air speed was increased, a speed was reached at which pronounced crackling, like pistol shots, was heard. At still higher speeds a continuous roar was heard, but no trace of a musical note, except for æolian tones due to the presence of the Pitot-tube itself. The result of the experiment was the same if the Pitot-tube was placed in or near the jet of air emerging from the end of the long tube, except that in the latter position æolian tones were avoided. The failure to detect any trace of a musical note by this very sensitive method is in agreement with Heisenberg's theory.

In the tube, near its entrance, the flow was turbulent over the whole range of speeds considered above: the crackles heard near the exit therefore represent the first turbulence that has persisted throughout the length of the tube, which is from 500 to 550 diameters. It is thus the *lower* critical velocity that is detected. For tubes shorter than about 25 diameters no abrupt transition to turbulence is found: the tube is then so short that the eddies have not time to get damped down or to develop fully.

The transition to turbulence could also be detected by ear or eye by letting the emergent jet impinge on a long luminous gas flame. Or even the unaided ear placed near the air jet could detect the effects.

Table 1

Fluid and method	Tube diameter (cm.)	Distance from entrance (in tube-diameters)	Lower critical velocity. Values of Reynolds' number, $vd\rho/\mu$		
			(i)	(ii)	(iii)
Air, stethoscope	0·199$_8$	500	1950	2060	2130
Air, stethoscope	0·178$_5$	550	1890	1990	2060
Water, deflected vane	0·178$_5$	550	—	2260	—
Water, deflected vane	0·74	150	1980	—	2200
Water, colour-band	0·77$_9$	130	—	2250	—

Quantitative experiments were next performed, the results of which are given in the upper part of table 1. Though the crackling was very precise, yet the velocity

for which it just started was not sharply defined. The crackles appeared to be distributed at random in time, becoming much more frequent as the rate of air flow was slightly increased. For convenience of measurement three stages were distinguished: (i) crackles every few seconds, (ii) 8 or 10 crackles a second, (iii) a roar with momentary lulls every few seconds.

§ 3. DEFLECTED VANE EXPERIMENTS

In order to find in more detail the nature of the turbulence and the transition, the air jet emerging from the long tube was allowed to impinge on a mica flake clamped at one edge, and measuring 1·5 × 1·5 × 0·005 cm. The mica vane carried a very small mirror, from which a beam of light could be reflected to a moving photographic film: and the vane was placed a few tube diameters away from the end of the tube, so as neither to affect the flow in the tube appreciably nor to be so far along the jet that the instability of the long jet itself should complicate the results.

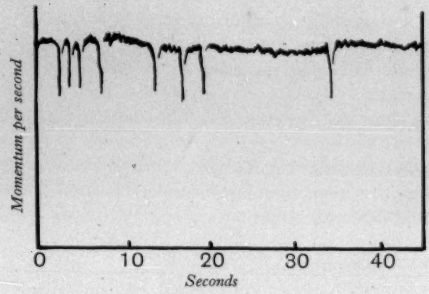

Fig. 1. Results for water (d=0·74 cm., $vd\rho/\mu$=1980).

In the case of air flow through the 0·178 cm.-diameter tube, the photographic records showed the intermittent turbulence at the transition stage (the crackling in the aural method), and indicated that each such burst of turbulence had occupied a length of tube of between 12 and 25 diameters.

The same mica vane was next used with water flowing through wider tubes. The change to wider tubes results in smaller speeds, and the change to water leads to smaller speeds, larger forces and more damping of the vane's free oscillations, all of which are desirable; the only disadvantage is the increase in the vane's free period. Below the critical speed the reflection was found to be proportional to the square of the rate of flow, as was to be expected. The first traces of turbulence are shown in figure 1, where the ordinates represent the force exerted by the jet on the vane, or the momentum per second of the emergent liquid. In figure 2 various results are collected together with a larger time-scale so as to show the transition to turbulence in more detail. The natural period and damping of the vane are also shown at the bottom of the diagram.

In the curves of figure 2, and in others obtained with yet wider tubes, no regular periodicity could be detected, apart from that due to the vane itself. This is in agreement with Heisenberg's theory which suggests* a single wave-length at the critical stage, but velocities and frequencies that vary across the transverse section of the tube†.

The noticeable features of the transition stage are the alternate periods of stable and turbulent flow, and the gradual commencement but rapid termination of each period of turbulence. The ratio of the extreme values of the ordinates (corresponding to maximum turbulence and stable flow) was estimated for each curve, allowance being made for errors due to the free oscillation of the vane. The mean of the values so obtained was $0{\cdot}73 \pm 0{\cdot}006$. This number may have been affected by constant errors. But it is significant to note that for a transition from parabolic velocity-distribution to uniform velocity across the whole transverse

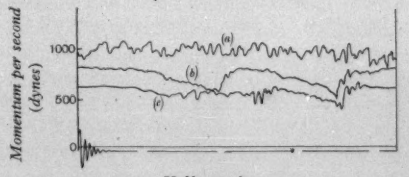

Fig. 2. Results for water (tube diameter 0·74 cm.); $vd\rho/\mu = (a)$ 2900; (b) 2150; (c) 2250.

section, the ratio should be 0·75: and a change from parabolic to a velocity distribution typical of that supposed to exist when there is turbulence should correspond to a ratio of ordinates of about 0·79. We are driven to the conclusion that during the transition stage and probably also during full turbulence, there are times when the velocity parallel to the tube-axis is appreciably more uniform over the transverse section than is indicated by the curves of *average* velocity distribution‡ as determined by Pitot-tubes.

Finally we may note the results of some measurements of the critical speeds made by this vane method (table 1). All the values fall within the range obtained by other observers, but the different lines in the table cannot be compared simply. The frequency of the eddy groups varies as $vd\rho/\mu$ varies: but for a fixed value of $vd\rho/\mu$ the frequency is probably proportional to $\mu/\rho d^2$, and changes with the fluid and tube-diameter.

§ 4. COLOUR-BAND EXPERIMENTS

The colour-band method has (by some workers) been considered inapplicable to the determination of the *lower* critical velocity, as the motion near the entrance of the tube is then turbulent, and any colour injected there will have become more or less uniformly distributed across the tube before it reaches the later parts of the

* *Loc. cit.* 605 and 608.

† *Loc. cit.* 609 and 616.

‡ T. E. Stanton, *Proc. R.S.* A, 85, 366 (1911).

tube where the motion may be in stable stream-lines. This difficulty was avoided by injection of the colour through holes in the wall of the tube, at a considerable distance from the tube entrance. At the transition stage intermittent turbulence was again observed, and each turbulent portion could be watched as it moved rapidly along the tube. Each was more abrupt and regular at its end than at its front, the appearance being approximately as indicated in figure 3.

The tubes for the above experiments were arranged almost horizontally, and water was passed through them. Small air-bubbles, that moved along the upper side of the tube, gave some further indication as to the effect of the turbulence. They

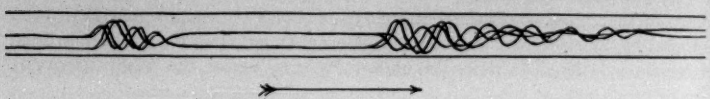

Fig. 3. Intermittent turbulence. (Colour-bands observed visually.)

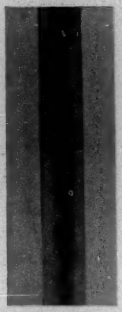

Fig. 4. Distortion of colour-band due to abrupt stoppage of flow (the speed being initially slightly less than the critical).

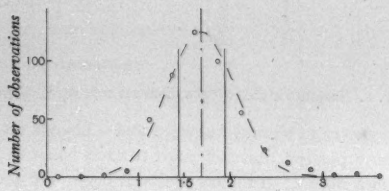

Fig. 5. Values of λ expressed as multiples of the tube diameter.

no longer moved uniformly, but jumped forward rapidly, with a tendency to move sideways also, when any group of eddies passed, and then resumed their uniform motion. Even for quite small and slowly moving air-bubbles near the wall, the ratio of the two velocities was considerably greater than 2 : 1. This momentary increase in the velocity near the walls, and decrease near the axis, are more marked than we might have deduced by taking the ratio of the pressure-gradients for speeds just greater than and just less than the critical, as given by Stanton and Pannell*. The apparent discrepancy is explained (as we saw in the previous section of this paper) when we remember that only time-average values of the pressure-gradient were measured. The observations of the motion of air-bubbles show that, though the flow near the walls may remain in stable stream-lines whilst the flow in the

central parts of the tube is turbulent, yet the stream-lines will be distorted and the velocities will vary when the eddies are passing.

Let water be flowing through a long straight tube with abrupt entrance, so that in the earlier part of the tube the flow is turbulent but in the later part stable, as indicated by a colour-band injected part-way along. Then if the flow of liquid be abruptly stopped, it is found that the colour-band along the bottom side will remain unchanged if the speed has been appreciably below the critical: but if the speed has been nearly equal to the critical, the colour-band develops a sinuous form or becomes broken up into a series of approximately equal lengths, λ, as illustrated in figure 4. The details of the distortion of the colour-band vary with the initial speed and rapidity of stoppage, and the spacing is apt to be rather irregular: but the phenomenon proved sufficiently distinct to enable measurements to be made.

460 measurements of this "wave-length," λ, from 30 experiments with each of two tubes, are collected statistically in figure 5. I conclude that this "wave-length," of about 1·7 times the diameter of the tube, corresponds to the critical value suggested by Heisenberg, and estimated by him to be of the order π times the distance between a pair of parallel planes. (We may compare the ratio $1{\cdot}7 : \pi$ with the ratio of the "hydraulic mean depths," which is 1 : 2.)

Thus, no regular frequency was detected in the turbulent motion in the tube: but at the critical speed a more or less definite wave-length λ equal to $1{\cdot}7 \times d$ was found. These two results appear to be in agreement with the theoretical work of Heisenberg, who suggests a critical wave-length of the order stated above, but a wave-velocity that varies across the transverse section of the system.

§ 5. ACKNOWLEDGMENTS

In conclusion, I should like to thank Prof. J. A. Crowther, in whose laboratory the work was carried out, for his continued interest in it. I have also to thank Mr J. S. Burgess, the laboratory steward, for his help in experimental matters.

DISCUSSION

Dr E. G. Richardson. I should like to know whether Dr Bond traversed the Pitot tube across the end of the pipe from side to side. At lower velocities the turbulence will persist to the end of the pipe at the axis though not at the edges, so that the radial distance of the Pitot tube from the axis seems to me to be an important factor in measurements of critical velocity by the aural method. It is interesting to compare these results with the attempt made by Prof. Burgers in 1926 to detect, by means of a couple of hot wires, a definite wave-length in disturbances propagated along the floor of his large wind channel at Delft. As far as I know, a negative result was obtained.

* *Loc. cit.*

AUTHOR'S reply. I did try the effect of traversing the Pitot tube across the end of the pipe, but merely observed that the aeolian tones were most noticeable when there was only a small gap between the Pitot tube and the wall of the pipe. The crackling was heard for all positions of the Pitot tube, even when it was not actually in the air-jet. Hence this experiment gave no evidence as to whether the turbulence was near the axis only of the pipe or was distributed more or less over the whole of the transverse section. The results given in the first two lines of table 1 were obtained with the stethoscope connected to a tube near to, but not in, the air-jet.

On the other hand, the experiments, especially the measurements of the decrease in momentum per second (see figures 1 and 2), indicated that during intermittent turbulence the end of each body of turbulence consists of a disturbance extending almost completely over the transverse section of the tube.

THE SPECTRUM OF TREBLY-IONIZED CERIUM (Ce IV)

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Read November 7, 1930

ABSTRACT. The spectrum of the condensed spark of cerium has been investigated in the ultra-violet, and doublet combinations in Ce IV, in addition to those given by Gibbs and White, have been found. The $6^2P - 6^2D$ combination of La III also has been identified. Term values and ionization potentials of La III and Ce IV are calculated.

THE spectrum of Ce IV is very simple, as is evident from the term scheme given in table 1. It is similar in electronic structure to the spectra of Cs I, Ba II and La III. Some of its combinations can be identified by extrapolation from Cs I, etc. by means of the irregular and regular doublet laws, and of the

Table 1. Predicted terms of Ce IV.

Orbits of outer electrons										Terms
4_1	4_2	4_3	4_4	5_1	5_2	5_3	6_1	6_2	6_3	7_1	
2	6	10		2	6	1					5^2D
2	6	10		2	6		1				6^2S
2	6	10		2	6			1			6^2P
2	6	10	1	2	6						4^2F
2	6	10		2	6					1	7^2S
2	6	10		2	6				1		6^2D

irregular doublet law modified so as to be applicable to transitions between two orbits of different total quantum numbers. The various transitions are considered below.

$6^2S_{\frac{1}{2}} - 6^2P_{\frac{1}{2},\frac{3}{2}}$: For the identification of this pair of lines good guidance is provided by comparison with the behaviour of NaI-, KI- and RbI-like spectra, the details for which are shown in table 2.

These give a good idea of how the irregular doublet law, according to which the second differences should be very small, will be obeyed in CsI-like spectra, and roughly extrapolated values for $6S_{\frac{1}{2}} - 6P_{\frac{1}{2}}$ are shown below in brackets.

$6^2S_{\frac{1}{2}} - 6^2P_{\frac{3}{2}}$

	ν	$\Delta\nu$	$\Delta^2\nu$
Cs I	11178		
		9084	
Ba II	20262		[970]
		[8114]	
La III	[28376]*		[430]
		[7684]	
Ce IV	[36060]		

Table 2. $^2S - {}^2P$ lines of Na-, K- and Rb-like spectra.

$3^2S_{\frac{1}{2}} - 3^2P_{\frac{1}{2}}$			$4^2S_{\frac{1}{2}} - 4^2P_{\frac{1}{2}}$			$5^2S_{\frac{1}{2}} - 5^2P_{\frac{1}{2}}$		
	ν	$\Delta\nu$ $\Delta^2\nu$		ν	$\Delta\nu$ $\Delta^2\nu$		ν	$\Delta\nu$ $\Delta^2\nu$
Na I	16956		K I	12985		Rb I	12579	
		18713			12207			11136
Mg II	35669	702	Ca II	25192	835	Sr II	23715	917
		18011			11372			10219
Al III	53680	411	Sc III	36564	402	Y III	33934	435
		17600			10970			9784
Si IV	71280		Ti IV	47534		Zr IV	43718	

By application of Sommerfeld's† regular doublet law

$$\Delta\nu = K\,(Z - S)^4,$$

$\Delta\nu$ the $(6^2P_{\frac{1}{2}} - 6^2P_{\frac{3}{2}})$ separation can be approximately calculated. $\Delta\nu$ is the required
Z, S separation, Z is the atomic number and S is the screening constant. The constant
K $K = 0{\cdot}0135$ for $6P$. Table 3 gives the values of the screening constant S for NaI-,

Table 3. Screening constants for $^2P_{\frac{1}{2}} - {}^2P_{\frac{3}{2}}$ separations in Na-, K-, Rb- and Cs-like spectra.

$3^2P_{\frac{1}{2}} - 3^2P_{\frac{3}{2}}$			$4^2P_{\frac{1}{2}} - 4^2P_{\frac{3}{2}}$			$5^2P_{\frac{1}{2}} - 5^2P_{\frac{3}{2}}$			$6^2P_{\frac{1}{2}} - 6^2P_{\frac{3}{2}}$		
	S	ΔS		S	ΔS		S	ΔS		S	ΔS
Na I	7·450		K I	13·036		Rb I	26·96		Cs I	40·77	
		0·844			1·396			2·56			3·58
Mg II	6·606		Ca II	11·640		Sr II	24·40		Ba II	37·19	
		0·455			0·73			1·45			[1·90]
Al III	6·151		Sc III	10·91		Y III	22·95		La III	[35·29]	
		0·227			0·48			1·00			[1·30]
Si IV	5·924		Ti IV	10·43		Zr IV	21·95		Ce IV	[33·99]	

KI- and RbI-like spectra, and the extrapolated values for La III and Ce IV, for purposes of comparison. Taking the extrapolated value of S for Ce IV we get $(6^2P_{\frac{1}{2}} - 6^2P_{\frac{3}{2}}) \fallingdotseq$ (approx. =) 4500. The line $6^2S_{\frac{1}{2}} - 6^2P_{\frac{3}{2}}$ would thus be located near $\nu = (36060 + 4500) = 40560$, i.e. at about $\lambda 2465$, and this line should be the stronger one of the pair. A line at $\lambda 2456{\cdot}81$ even by its appearance on one of the plates was marked out from other lines. It was intense only at the tips and could

* The observed value as given by Gibbs and White is 28424·3. This was not known to the author when this extrapolation was made. See footnote † on p. 55.

† A. Sommerfeld, *Atomic Structure and Spectral Lines*, Engl. Trans. p. 496 (Methuen and Co. 1923).

be unmistakably identified as $6^2S_{\frac{1}{2}} - 6^2P_{\frac{3}{2}}$. Simultaneously Gibbs and White* corrected their previous† identifications for this doublet of Ce IV and gave:

λ vac.	ν	Δν	
2779·07	35983·3		$6^2S_{\frac{1}{2}} - 6^2P_{\frac{1}{2}}$
		4706·9	
2457·59	40690·2		$6^2S_{\frac{1}{2}} - 6^2P_{\frac{3}{2}}$

$6^2P_{\frac{1}{2},\frac{3}{2}} - 7^2S_{\frac{1}{2}}$: This combination involves a change in the total quantum number and the irregular doublet law cannot be applied directly. Millikan and Bowen‡ give the following formula:

$$\nu' = \nu - \{R\,(Z-A)^2\,(n_2^2 - n_1^2)\}/n_1^2\,n_2^2, \qquad \nu'$$

where ν is the observed frequency, R the Rydberg constant, Z the atomic number, n_2 and n_1 the total quantum numbers involved, and A any suitable constant. To this ν', or modified ν, the irregular doublet law can be applied. For Cs the atomic number is 55, so we choose A equal to 54. We obtain the following values for $\nu - \nu'$: 808·3 for Cs I, 3233·1 for Ba II, 7274·5 for La III and 12932·5 for Ce IV. For the $6^2P_{\frac{3}{2}} - 7^2S_{\frac{1}{2}}$ transition the values of ν and ν' for Cs I and Ba II are as follows:

ν, R, Z

n_2, n_1, A

$$\text{Cs I, } \nu = 6803{\cdot}3,\ \nu' = 5995{\cdot}0;\ \text{Ba II, } \nu = 20402{\cdot}6,\ \nu' = 17169{\cdot}5.$$

$$\Delta\nu' = 17169{\cdot}5 - 5995{\cdot}0 = 11174{\cdot}5.$$

Taking a progressive variation with atomic number in the value of ν' we predict for La III $\nu' = 17169{\cdot}5 + 11174{\cdot}5 = [28344]$ and $\nu = 28344 + 7274 = [35618]$ and similarly for Ce IV $\nu' = [39518]$ and $\nu = [52451]$. For Ce IV the difference $6^2P_{\frac{1}{2}} - 6^2P_{\frac{3}{2}}$ being already known to be $\Delta\nu = 4707{\cdot}0$ we can locate the other line $6^2P_{\frac{1}{2}} - 7^2S_{\frac{1}{2}}$ at about $\nu = 52451 + 4707 = [57158]$.

There are two lines of cerium observed in the condensed spark at λ2009·98 and λ1836·19 with $\Delta\nu = 4706{\cdot}5$, λ2009·98 being the stronger of the two. From their behaviour and appearance, especially of λ2009·98, we can classify them as:

λ2009·98; $\nu = 49735{\cdot}7$; $6^2P_{\frac{3}{2}} - 7^2S_{\frac{1}{2}}$;

λ1836·19; $\nu = 54442{\cdot}2$; $6^2P_{\frac{1}{2}} - 7^2S_{\frac{1}{2}}$.

From the available data for La it was not possible to pick out the corresponding doublet.

$6^2P_{\frac{1}{2},\frac{3}{2}} - 6^2D_{\frac{3}{2},\frac{5}{2}}$: From data for Cs I and Ba II, by application of the irregular and regular doublet laws as before, we obtain the following approximate values for this combination:

	$6^2P_{\frac{1}{2}} - 6^2D_{\frac{3}{2}}$	$6^2P_{\frac{3}{2}} - 6^2D_{\frac{5}{2}}$	$6^2P_{\frac{3}{2}} - 6^2D_{\frac{3}{2}}$
La III	[40170]	[37505]	[37075]
Ce IV	[55287]	[51300]	[50580]

* R. C. Gibbs and H. E. White, *Phys. Rev.* **33**, 157 (1929).

† R. C. Gibbs and H. E. White, *Proc. Nat. Acad. Sc.* **12**, 551 (1926). This paper had escaped notice. Both La III and Ce IV, $6^2S - 6^2P$, doublets were identified; for Ce IV the values given were λ vac. 2769·2 and λ vac. 2455·1 with $\Delta\nu = 4619{\cdot}8$.

‡ R. A. Millikan and I. S. Bowen, *Phys. Rev.* **26**, 313 (1925).

The following three lines (La III) of the lanthanum spark spectrum satisfy the conditions regarding the differences and intensities:

λ	ν		
2476·71	40363·9		$6^2P_{\frac{1}{2}} - 6^2D_{\frac{3}{2}}$
2651·68	37700·7	3095·0	$6^2P_{\frac{3}{2}} - 6^2D_{\frac{5}{2}}$
		431·8	
2682·4	37268·9		$6^2P_{\frac{3}{2}} - 6^2D_{\frac{3}{2}}$

The value obtained for the difference $6^2D_{\frac{3}{2}} - 6^2D_{\frac{5}{2}}$ is very near that predicted.

In the cerium spectrum there is a faint line at $\lambda 1949{\cdot}85$, $\nu = 51269{\cdot}2$, which, from its appearance at tips on some plates and its position, may be classified as the $6^2P_{\frac{3}{2}} - 6^2D_{\frac{5}{2}}$ combination of Ce IV; but the other two lines are not observed, being perhaps very faint, so this finding cannot be confirmed.

Knowing the values of $6S - 6P$ and $6P - 7S$ for Ce IV, we can calculate approximately the term values for $6P$ and $6S$; and if the tentative location of $6P - 6D$ is correct the term value of $6D$ also can be calculated. Knowing $6D$, we can get an idea of the $5D$ term from Rydberg's tables. Thus we find

$$6^2P_{\frac{1}{2}} = 167810, \quad 6^2S_{\frac{1}{2}} = 203794, \quad 6^2D_{\frac{3}{2}} = 11834 \text{ and } 5^2D_{\frac{3}{2}} = 20012.$$

The $5D$ term in La III is considerably deeper than $6S$. The above estimated values show however that in Ce IV the $5D$ if not less than $6S$ cannot be very far above it.

$5^2D_{\frac{3}{2},\frac{5}{2}} - 6^2P_{\frac{1}{2},\frac{3}{2}}$: From the values of $\Delta\nu = 5^2D_{\frac{3}{2}} - 5^2D_{\frac{5}{2}}$ for Cs I, etc. the value of $\Delta\nu$ for Ce IV can be predicted, by means of the regular doublet law, to be about 3340. The following three lines of Ce IV, owing to their frequency differences and positions, are classified as the $5D - 6P$ multiplet:

λ	ν		
2430·25	41135·5		$5^2D_{\frac{3}{2}} - 6^2P_{\frac{1}{2}}$
2350·16	42537·3	4706·7	$5^2D_{\frac{5}{2}} - 6^2P_{\frac{3}{2}}$
		3304·9	
2180·71	45842·2		$5^2D_{\frac{3}{2}} - 6^2P_{\frac{3}{2}}$

The appearance and intensities of these lines are entirely appropriate for this combination.

From $5D - 6P$ and $6P - 6D$ by means of the Rydberg formula the term value of $6P$ can be calculated. Table 4 gives the term values for La III and Ce IV. For Ce IV a mean of the two values of $6P$, calculated from the S series and the D series respectively, is given. All the other terms are based on $6P$.

The behaviour of the $5D$ term is somewhat anomalous. For Cs I, Ba II, La III and Ce IV the term values for $5^2D_{\frac{3}{2}}$, $6^2S_{\frac{1}{2}}$ and $6^2P_{\frac{1}{2}}$ are given in table 5. They show that for Cs I, $6S$ is the deepest term and $6P$ is deeper than $5D$; for Ba II, $6S$ is still the deepest term but $5D$ is deeper than $6P$; for La III, $5D$ is the deepest term, $6S$ being of course deeper than $6P$; while for Ce IV also $5D$ is the deepest term

Table 4. Term values for La III and Ce IV.

Terms	La III		Ce IV	
$5^2D_{\frac{3}{2}}$	165574·3	1603·8	210895·3	3304·9
$5^2D_{\frac{5}{2}}$	163970·5		207590·4	
$6^2S_{\frac{1}{2}}$	151984·3		205743·7	
$6^2P_{\frac{1}{2}}$	123560·0	3095·4	169760·0	4706·9
$6^2P_{\frac{3}{2}}$	120464·6		165053·1	
$7^2S_{\frac{1}{2}}$	—		115317·6	
$6^2D_{\frac{3}{2}}$	83195·7	431·8	[114504]	[720]
$6^2D_{\frac{5}{2}}$	82763·9		113783·9	

but not so much deeper than $6S$ as in La III. Table 5 also gives the ionization potentials calculated from the deepest levels*, which are underlined.

Table 5. Deep terms and ionization potentials in Cs-like spectra.

Terms	$5^2D_{\frac{3}{2}}$	$6^2S_{\frac{1}{2}}$	$6^2P_{\frac{1}{2}}$	Ionization potential
Cs I	16905·0	31404·6	20226·3	3·88
Ba II	68674·3	80664·9	60403·4	9·96
La III	165574·3	151984·3	123560·0	20·4
Ce IV	210895·3	205743·7	169760·0	26·0

Tables 6 and 7 give lists of the classified lines of La III and Ce IV respectively. The wave-length measures for lanthanum are taken from Eder and Valenta's *Atlas Typischer Spektren*, corrected to I.A.; for cerium they are by the author.

Table 6. La III; classified lines.

λ air (Int.)	ν	Classification	
3517·11 (20)	28424·3	$6^2S_{\frac{1}{2}} - 6^2P_{\frac{1}{2}}$	Gibbs and White
3171·66 (15)	31520·2	$6^2S_{\frac{1}{2}} - 6^2P_{\frac{3}{2}}$	Gibbs and White
2682·4 (2)	37268·9	$6^2P_{\frac{3}{2}} - 6^2D_{\frac{3}{2}}$	Present investigation
2651·68 (10)	37700·7	$6^2P_{\frac{3}{2}} - 6^2D_{\frac{5}{2}}$	Present investigation
2476·71 (8)	40363·9	$6^2P_{\frac{1}{2}} - 6^2D_{\frac{3}{2}}$	Present investigation
2379·41 (10)	42014·3	$5^2D_{\frac{3}{2}} - 6^2P_{\frac{3}{2}}$	Gibbs and White
2297·83 (6)	43505·9	$5^2D_{\frac{5}{2}} - 6^2P_{\frac{3}{2}}$	Gibbs and White
2216·13 (2)	45109·7	$5^2D_{\frac{3}{2}} - 6^2P_{\frac{3}{2}}$	Gibbs and White

* It is assumed here, as suggested by Gibbs and White, *loc. cit.*, that the 4^2F term is not the deepest. The suggestion depends on the correctness of the assignment of term values of the $4^2F_{\frac{5}{2},\frac{7}{2}}$ levels for Ba II by Paschen and Götze.

Table 7. Ce IV; classified lines.

λ air	(Int.)	ν	Classification	
2778·23	(4)	35983·5	$6^2S_{\frac{1}{2}} - 6^2P_{\frac{1}{2}}$	Gibbs and
2456·81*	(6)	40690·9	$6^2S_{\frac{1}{2}} - 6^2P_{\frac{3}{2}}$	White
2430·25	(6)	41135·5	$5^2D_{\frac{3}{2}} - 6^2P_{\frac{1}{2}}$	
2350·16	(5)	42537·3	$5^2D_{\frac{5}{2}} - 6^2P_{\frac{3}{2}}$	Present
2180·71	(4)	45842·2	$5^2D_{\frac{3}{2}} - 6^2P_{\frac{3}{2}}$	investigation
2009·98	(3)	49735·7	$6^2P_{\frac{3}{2}} - 7^2S_{\frac{1}{2}}$	Present
1836·19	(2)	54442·2	$6^2P_{\frac{1}{2}} - 7^2S_{\frac{1}{2}}$	investigation
1949·85	(1)	51269·2	$6^2P_{\frac{3}{2}} - 6^2D_{\frac{5}{2}}$	Ditto

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* This line was classified independently in the present investigation.

THE PHOTOGRAPHIC EFFECTS OF GAMMA-RAYS

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ABSTRACT. The photographic action of γ-rays from radium and radon in equilibrium with their short-lived products has been investigated, the rays being filtered with lead screens of thicknesses 0, 0·19, 1·42, 2·61 and 4·03 cm. The variations of photographic density both with time and with intensity have been examined. The shape of the characteristic (C) curves ($D - \log I$) have been found to be independent of the filtering, and the same holds for the Hurter-Driffield (H.D.) curves ($D - \log t$), except when no lead filter is used. The value of p in the Schwarzschild relation, $D = f(It^p)$, has been found to be unity for all the lead filters. The absorption coefficient of lead for γ-rays from the sources used has been found to be 0·533 cm^{-1} for thicknesses of lead from 1 to 7 cm.

§ 1. INTRODUCTION

Although there have been several systematic investigations of the photographic effects of X-rays*, no experiments appear to have been carried out on the photographic action of γ-rays. This is no doubt due to the smallness of the action of these rays, which suffer very little absorption in the photographic emulsion. Generally, therefore, investigations of γ-rays have been carried out with ionization chambers designed to absorb an appreciable fraction of the rays. The photographic method has, however, one advantage, for although the instantaneous effect may be small this effect is additive with time, and with sufficiently long exposures even the smallest γ-ray intensities can be measured. Experiments in this laboratory have shown the convenience of the photographic method of measuring X-ray intensities†.

Experiments have been undertaken to examine the variation of photographic density with time of exposure (constant intensity) and the variation of density with intensity (constant time) for γ-rays filtered through various thicknesses of lead. From the curves so obtained the absorption-coefficient of γ-rays in lead has been obtained. A method has been developed for determining absorption-coefficients by a photographic method in which these curves need not be drawn.

* E.g. W. Friederich and P. P. Koch, *Ann. der Phys.* **45**, 399 (1914); R. Glocker and W. Traub, *Phys. Zeit.* **22**, 345 (1921); A. Bouwers, *Dissertation* (Utrecht 1924); *Zeit. für. Phys.* **14**, 374 (1923).

† Miss Allen and T. H. Laby, *Proc. R.S. Vict.* **31**, 421 (1919); H. C. Webster, *Proc. Phys. Soc.* **41**, 181 (1929); C. E. Eddy and T. H. Laby, *Proc. R.S.* **127**, 20 (1930).

§ 2. APPARATUS

Gamma-rays from a source R, figure 1, were confined to a narrow vertical channel by two rectangular lead blocks B, 15·3 cm. long, 2·3 cm. wide, and 7·5 cm. high, standing on a wooden base. The distance apart of these at the source end was 4 mm. and they opened to a distance of 8 mm. at the film end. The film, Agfa duplitized*, was held in a special holder H constructed as follows. A groove was milled in a piece of aluminium sheet 10 cm. by 6 cm. and approximately 4 mm. thick so that a piece of film 6 cm. by 1·8 cm. could just be accommodated, the depth of the groove being equal to the thickness of the film. The piece of film was backed with a thin sheet of aluminium the inside face of which was covered with velvet, and the latter sheet was clamped to the former by two screws, so that the film-holder was perfectly light tight. The film was exposed so that the long edge was

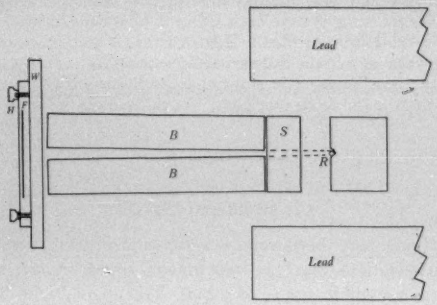

Fig. 1. Sketch of apparatus.

horizontal. During exposures the film-holder was carried by a vertical soft wood frame W the centre of which was cut away. This frame carried two aluminium bolts and two holes in H permitted it to be slipped over these bolts. The film-holder was then clamped by two aluminium nuts the ends of which fitted geometrically into the countersunk holes in H. By this means the film was always placed in the same position relative to the source. The γ-rays after passing through the absorbing screens passed through 17 cm. of air and 0·38 cm. of aluminium before striking the film.

The sources of γ-radiation used were radium and radon contained in equilibrium with their short-lived products, in glass tubes 4·8 cm. long and 0·35 mm. in diameter. The source was supported vertically in a shallow recess in a wooden block lined with cotton wool and was held in position with the minimum quantity of fine copper wire. At the sides of the source, and distant approximately 5 cm. from it,

* Duplitized film was used to reduce the time of exposure. The double layer of emulsion produced no uncertainty as the same value of the photographic density was obtained whichever side of the film was facing the photometer lamp.

were lead blocks 5 cm. wide. Over the top of the source were 10 cm. of lead. For the Hurter-Driffield curves (*HD* curves)—i.e. those in which density is plotted against the logarithm of time—a tube of radium carbonate was used, the radium content of this being 103·6 mg. For the characteristic curves (*C* curves), in which density is plotted against the logarithm of the intensity, a tube containing radon was used which had a γ-ray activity equal to 400 mg. of radium three hours after sealing off.

In some of the later absorption experiments, four tubes containing radium carbonate were employed and these were kept in the brass cases supplied by the Radium Belge. These tubes of radium content 94·8, 99·0, 90·6, and 89·8 mg. respectively were placed as closely together as possible in vertical holes in a wooden block so that their axes were in line with the centre of the channel formed by the lead blocks *BB*. The tubes were always placed in the same order in the wooden block. The distance of the source *R* was 21·6 cm. from the film when radium in glass alone was used, but was several centimetres more when the four tubes were used together.

The lead absorbing screens were placed at *S* and were sufficiently wide to cover the whole width of the blocks *B*. These were rectangular and were carefully machined. They were cut from a pig of Broken Hill Associated Smelters lead which is very nearly pure metal, being 99·9915 per cent. pure as estimated by difference; the most prevalent impurity is antimony (0·004 per cent.).

§ 3. EXPERIMENTAL

Development of films. The films were developed with a metol-hydroquinone developer for 5 minutes at 18° C. In order to ensure uniformity of development the films of any one series were all developed together. A holder was constructed capable of holding twelve pieces of film vertically and for development a litre of developer was used and was placed in a two litre beaker. This volume of developer was brought to 18° C. and the temperature remained sensibly constant during the period of development. During the whole five minutes of development the film holder was moved up and down, so that the liquid surfaces at the film were continually being renewed. During fixing the same vertical agitation of the film was maintained.

Density measurement. The density was measured with a Moll microphotometer. During photometry the current through the heating-lamp was maintained constant at 3·9 amp. As photographic density is given by $\log_{10}$ [(galvanometer deflection produced by the heat energy transmitted by the unexposed portion of the film)/(that for energy transmitted by the exposed portion)], two readings would be necessary for each film, provided that the readings in the exposed and unexposed portions were constant among themselves. Owing, however, to irregularities in the photographic emulsion and in the thickness of the gelatine, and to non-uniformity of development, readings in any one portion were never quite constant, though, with the film used and the precautions taken during development, variations in the

readings were small. Figure 2, a photometer curve obtained by movement of the film through a definite distance between consecutive readings, shows the magnitude of the variations. In obtaining the actual densities, however, curves similar to figure 2 were not drawn for each film, but the mean galvanometer deflections were obtained. For the unexposed portion this averaging was done over a distance of approximately 0·7 to 1·7 cm. from the apparent edge of the exposed portion on one side, this side being determined by a mark on the film placed there before exposure.

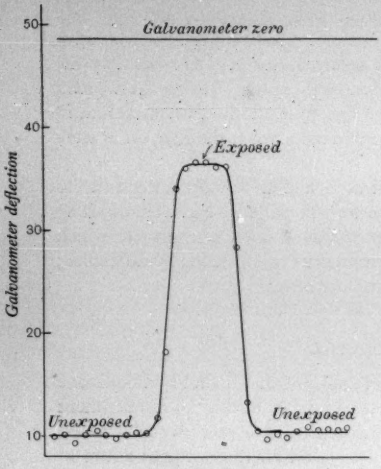

Fig. 2. Photometry of film exposed for 60 min.

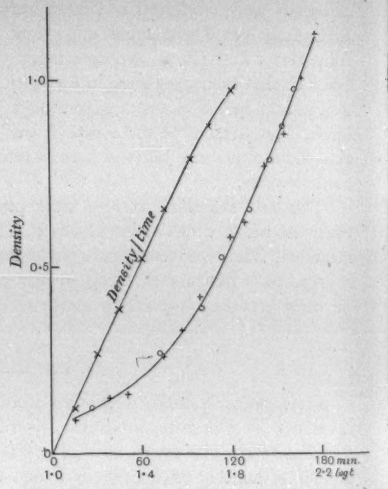

Fig. 3. Curves obtained with γ-rays filtered through 1·42 cm. of lead. o, *HD* curve; +, *C* curve.

The accuracy of the whole method—i.e. uniformity of conditions of exposure, uniformity of film, uniformity of development and accuracy in photometry—can be checked by exposure of a number of films in turn under exactly the same conditions; they are developed together and their densities are then measured. Table 1 represents a series of such determinations. It will be seen that the greatest accuracy is obtained with densities in the vicinity of 0·7, where the average departure from the mean is just over 1 per cent.

Table 1. Reproducibility of density determinations.

Individual readings	Mean
1·06, 1·10, 1·11	1·09 ± 0·02
1·16, 1·17, 1·22	1·18 ± 0·02
0·741, 0·760, 0·778, 0·769	0·76 ± 0·01
0·31, 0·35, 0·34, 0·31, 0·30, 0·34, 0·35	0·33 ± 0·02

In general, densities greater than 1 were not employed, for, with the method used for photometering, the accuracy with which densities greater than 1 can be measured decreases with density. Thus, for density 2 the galvanometer deflection for the exposed portion would be 0·01 of that for the unexposed portion, and it is questionable whether a 10 per cent. accuracy could be attained.

The curves. *HD* and *C* curves were obtained for γ-rays after they had passed through the following thicknesses of lead: 0, 0·19, 1·42, 2·61, 4·03 cm. The number of points for each curve varied between 8 and 12.

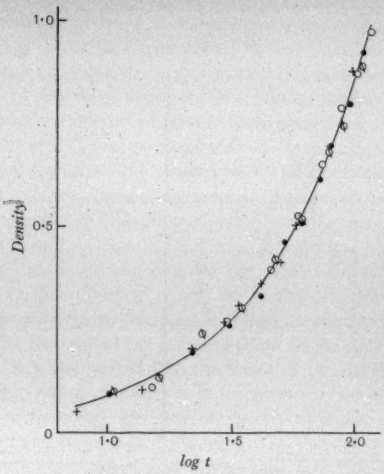

Fig. 4. *HD* curves obtained for γ-rays filtered through various thicknesses of lead. *HD* curves: •, 0·19 cm. lead; o, 1·42 cm. lead; +, 2·61 cm. lead; ⊘, 4·03 cm. lead.

(*a*) *Variation of density with time.* Figure 3 shows the results of readings obtained with 1·42 cm. of lead, the density D has been plotted both against the time of exposure t and against log t. It will be seen that D is proportional to t in the region extending from $D = 0$ to $D = 0{\cdot}8$, while D is proportional to log t at densities greater than 0·8. The same remarks apply to the curves obtained with the other four thicknesses of lead. Figure 4 shows a combined curve formed from the four *HD* curves corresponding to γ-rays filtered through 0·19, 1·42, 2·61, 4·03 cm. of lead; the scale for log t is the same for all four, but those for the first, third and fourth have been laterally displaced*. The single curve represents the four sets of readings equally well, so that the variation of density with log t is unaffected by the

$\frac{D}{t}$

* To produce density 0·9 exposures of approximately 60, 110, 210 and 420 minutes were required with the different filters.

range of filters used. This statement does not hold for the curve obtained without lead, for which the slope of that part of the curve where D is proportional to $\log t$ is less than that for the common curve. The numerical value of the slope γ for the common curve is 2·0, while that for no lead-filtering is 1·5.

D, I (*b*) *Variation of density D with intensity I.* Twelve exposures for each of the five filterings were obtained over a period of a fortnight, the times of exposure being 10, 15, 45, 60 and 120 minutes for 0, 0·19, 1·42, 2·61 and 4·03 cm. of lead respectively. For the calculation of the intensities the half-value-period of radon was taken as 3·85 days. As for the *HD* curves, D is proportional to I for values of D up to

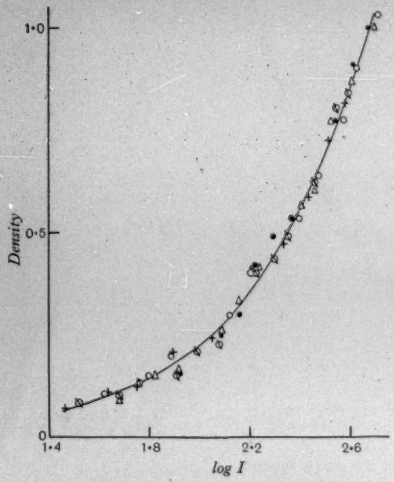

Fig. 5. Characteristic curves obtained with γ-rays with different filtering. *C* curves: o, no lead; •, 0·19 cm. lead; △, 1·42 cm. lead; ⊘, 2·61 cm. lead; +, 4·03 cm. lead.

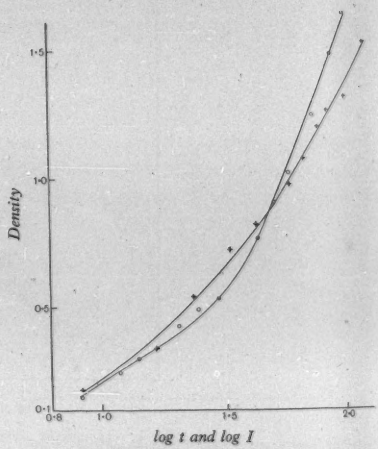

Fig. 6. Curves obtained with no lead filtering. o, *HD* curve; +, *C* curve.

0·8; and for higher densities D is proportional to $\log I$. In figure 5 all the readings for the *C* curves with the five different filterings are represented on one graph, the scale for $\log I$ being the same throughout, though the numerical values differ for the various sets of points. The conclusion is therefore drawn that the variation of density with $\log I$ is independent of the filtering of the γ-rays.

The readings for the *C* curve for γ-rays filtered by 1·42 cm. of lead are represented on figure 3, the points being indicated by +. It will be seen that the *HD* and *C* curves are identical. This statement holds also for the other three filterings of lead but does not hold when no lead filter was used. Figure 6 shows both the *HD* and *C* curves obtained with no lead filter. These have been taken to higher densities than the remainder of the curves in the attempt to separate them with greater certainty.

§ 4. DISCUSSION OF THE DENSITY CURVES

(i) As the *HD* and *C* curves for all filterings (except those obtained with no lead screen) coincide, the conclusion is drawn that, for γ-rays, the variation of density with time is of exactly the same nature as the variation of density with intensity, and hence the Bunsen-Roscoe reciprocity law holds, and the value of the exponent p in Schwarzschild's law ($D \propto It^p$) is unity. Various experiments have determined values of p for X-rays, and the values found have been in the vicinity of unity*. Bouwers has stated that this value may be due to the emission of X-rays being an intermittent process, as intermittent sources of ordinary light produce high values of p. In these experiments, however, the sources either have been constant—radium produces the most constant radiation that could affect a photographic plate—or else have decreased in a regular manner, so that the resulting value of p cannot be attributed to any intermittence of the source. The curves obtained without lead cannot be discussed in this connexion, as, in the absence of a magnetic field, β-rays from the sources were able to affect the photographic film, whereas with the other filterings these were all absorbed before reaching the film. The question whether or not the value of 0·7 of p, given by the curves obtained without lead, is due to β-rays has not been investigated, as the experiments have been performed in the strong room of the Commonwealth Radium Laboratory, where there is no direct current supply for an electromagnet.

(ii) As the curves for all thicknesses of lead are identical there are two possible conclusions. The first is that the γ-rays incident on the film are of the same quality for each filter, and the second that the shape of the density-curves representing the photographic action of γ-rays is independent of the wave-length of the rays. It will be shown later that, although the γ-rays emerging from the three thickest filters have the same absorption-coefficient, those through 0·19 cm. have a different value, and hence, if constancy of absorption-coefficient indicates constancy of wave-length, the effective wave-length of the rays from the thinnest filter differs from that from the others. Hence even if the quality of the γ-rays from the three thickest filters is the same, the second conclusion appears to be the correct one, and it is in agreement with the results found for X-rays by Glocker and Traub, who showed that this independence holds for wave-lengths from 1·1 to 0·4 A.U., and by Bouwers who obtained similar results in the range 1·54 to 0·18 A.U.

(iii) The direct proportionality between density and time or intensity up to $D = 0{\cdot}8$ is comparable with the result found by Glocker and Traub, who showed that the proportionality held for X-rays up to $D = 0{\cdot}6$ whatever development conditions (time and temperature) were used. These authors found that D varied as $\log t$ between densities 1·3 and 4. Such high densities have not been examined here, but the proportionality appears to begin at about $D = 0{\cdot}8$.

* See, for instance, R. Glocker and W. Traub, *loc. cit.* and A. Bouwers, *loc. cit.*

§ 5. DETERMINATION OF THE ABSORPTION COEFFICIENT OF LEAD FROM THE DENSITY CURVES

As the value of p has been shown to be unity, the photographic density is proportional to the product It when D has the value 0·8 or less. Suppose now that we consider within this range, some constant value of D, taken from each of the C curves obtained through 0·19, 1·42, 2·61 and 4·03 cm. of lead. To produce this density there must be emergent from each filter the same value of the product It,

I_1, t_1, d_1 but the value of $I_1 t_1$ incident on the filter will depend on the thickness d_1 of the

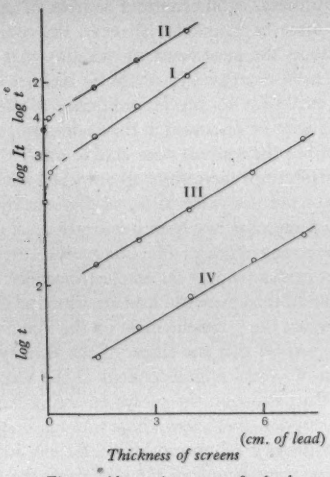

Fig. 7. Absorption curves for lead.

μ filter. The general equation, $J = J_0 e^{-\mu d}$ for μ, the absorption-coefficient, where

J, J_0 J_1, J_0 are the intensities of the emergent and incident beams respectively, can therefore be replaced by

$$It = I_1 t_1 e^{-\mu d_1},$$

and for a second filter of thickness d_2,

$$It = I_2 t_2 e^{-\mu d_2}.$$

Thus equating these two values of It we have

$$I_1 t_1 e^{-\mu d_1} = I_2 t_2 e^{-\mu d_2},$$

and hence $$\mu = (\log I_1 t_1 - \log I_2 t_2)/(d_1 - d_2) \quad \ldots\ldots(1).$$

If the source is of constant intensity,

$$I_1 = I_2,$$

and $$\mu = (\log t_1 - \log t_2)/(d_1 - d_2) \quad \ldots\ldots(2).$$

By use of (1) a series of values of μ for different densities can be obtained from the *C* curves, and by (2) a series of values of μ from the *HD* curves.

Curve I, figure 7, shows the values of log *It* plotted against *D* for $D = 0{\cdot}8$. Three such determinations for different densities gave the value of $\mu = 0{\cdot}560 \pm 0{\cdot}007$ cm^{-1}, for $d = 1{\cdot}4$ to $d = 4{\cdot}0$ cm. Between 0·19 cm. and 1·4 cm. the value of μ is higher.

Curve II of the same figure shows log *t* plotted against *d* for values obtained from the *HD* curves. Four determinations gave the value $\mu = 0{\cdot}532 \pm 0{\cdot}008$ cm^{-1}. Although the agreement between the two values is not very good, it is to be noted that values obtained from the curves depend on films which have been developed at quite different times; and although the greatest care has been exercised in reproducing the same conditions of development, there is, no doubt, some uncertainty introduced on this account.

§ 6. PHOTOGRAPHIC DETERMINATION OF ABSORPTION-COEFFICIENTS

The photographic method can however be used in a manner in which this uncertainty of development is eliminated. In this method a source of constant intensity (374 mg. of radium) was employed and exposures were obtained with a series of absorbers of different thicknesses. The time of exposure for each absorber was so chosen that approximately the same density was produced, the densities always being less than 0·8, i.e. in the range where density is proportional to time of exposure. All the films obtained in any one series were then developed and fixed together. From the densities so obtained the times of exposure to produce some standard density were calculated. Then log *t* could be plotted against *d*, and μ could be evaluated. The value of μ so obtained was 0·528 cm^{-1}. Another series obtained with densities of the order 0·3 is represented in Curve IV and gave the value 0·524 cm^{-1} for μ.

The mean value of μ from all the determinations is $0{\cdot}533 \pm 0{\cdot}008$ cm^{-1}, if half weight is given to the values obtained from the *C* and *HD* curves for reasons which have been previously discussed, and this value holds for thicknesses of lead ranging from 7 cm. down to about 1 cm. This is exactly the same value as that obtained by Kohlrausch* for the harder rays from RaC but is considerably less than the value 0·721 obtained by Ahmad† with a filter of 1 cm. of lead.

The absorption experiments are being extended to a large range of elements.

§ 7. ACKNOWLEDGMENTS

These experiments were carried out in the Commonwealth Radium Laboratory, University of Melbourne. The author wishes to express his thanks to Mr A. H. Turner, M.Sc., Physicist of that laboratory, for making available its facilities.

* K. W. F. Kohlrausch, *Wien. Ber.* **126**, 893 (1917).
† N. Ahmad, *Proc. R.S.* **109**, 206 (1925).

THE SPECTRUM OF DOUBLY-IONIZED ARSENIC

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ABSTRACT. The doublet system of As III published by R. J. Lang has been amended and extended in the light of fresh observations which have been made, and a scheme of terms has been evaluated.

§ 1. INTRODUCTORY

THE spectrum of arsenic has been under investigation by the writer for a considerable time. In a previous paper* some regularities among the arc lines of arsenic have been published. Evidence of a doublet system of As III was obtained very early in the course of the work and the chief separation 723 cm.$^{-1}$ of the $5p\ ^2P$ term and the doublets of the secondary series were obtained by extrapolation from Ge II by the writer and Prof. A. L. Narayan†. Our present knowledge of the series of doublets in this spectrum is due, however, chiefly to an investigation by R. J. Lang‡. Independently P. Pattabhiramiah and A. S. Rao§ have published an analysis of As III where the separation $4p\ ^2P_{\frac{1}{2}} - 4p_{\frac{1}{2}}\ ^2P_{1\frac{1}{2}} = 2940$ cm.$^{-1}$ is correctly identified.

In the present paper the doublet system published by Lang is considered in detail and, in the light of the present experimental observations, it has been altered in some respects and slightly extended. It is now possible to evaluate a reliable scheme of terms of As III by assuming a probable value for the term $5g\ ^2G$.

§ 2. EXPERIMENTAL

Several sources were employed for the excitation of the spectrum. From λ 8000 to λ 1300, they were the arc *in vacuo*, the spark between arsenic poles in air or hydrogen, and discharges through capillary tubes containing vapour of metallic arsenic. This part of the investigation was carried out in Prof. Fowler's laboratory at the Imperial College of Science, with various instruments. Below λ 1400 photographs were taken at Upsala, of the spark between aluminium electrodes containing arsenic, with a vacuum grating spectrograph designed by Prof. Siegbahn. The grating is of radius 150 cm. and is mounted at tangential incidence; the dispersion varies from about 3·5 A.U. per mm. at λ 1400 to about 2·0 A.U. per mm. at

* K. R. Rao, *Proc. R.S.* A, **125**, 238 (1929).
† K. R. Rao and A. L. Narayan, *Proc. R.S.* A, **119**, 611 (1928).
‡ R. J. Lang, *Phys. Rev.* **32**, 737 (1928).
§ P. Pattabhiramiah and A. S. Rao, *Ind. J. of Phys.* **3**, 437 (1929).

λ 400. The author is indebted to Dr J. E. Mack and Mr D. Borg for permitting him to see some plates taken by them of the spectrum of arsenic by the use of a highly condensed and violent vacuum spark in which lines of As VI are obtained with ease. The wave-lengths in the table which follows are from the author's measurements and the intensities are estimated on a scale having 10 for maximum.

§ 3. RESULTS

The analysis of As III published by Lang, while being quite correct in its essential features, appears to be defective in some details. The first member of the diffuse series, $4p\ ^2P - 4d\ ^2D$, is too weak and even fainter than the second member. Even if its correctness were assumed, no satisfactory pair could be found for the combination $4d\ ^2D - 4f\ ^2F$ which, at the same time, would give the other strong pair $4s\ 4p^2\ ^2D - 4f\ ^2F$. These latter two doublets are very prominent in spectra of this type, e.g. Ge II, Si II, etc. There are strong lines in the spectrum of arsenic which could be definitely ascribed to As III in the region where $4d\ ^2D - 4f\ ^2F$ may be expected, but no proper choice was found possible with Lang's classification.

Lang has drawn attention to the doubtful identification of the term cP of the configuration of three $4p$ electrons. Of the combinations of this term with $4s\ 4p^2\ ^2P$, while λ 2151 and λ 2053 are lines of As III, λ 2144 belongs to the arc spectrum and has been classified as such*. The group $bD - cP$ (in Lang's notation) shows abnormal intensities on Lang's plates. The line λ 1266 is certainly due to As II.

In view of the above considerations, the analysis has been slightly modified. The details of the new scheme are set forth in table 1, in which Lang's classification also is given in order to indicate clearly the modification that has been made in it by the writer. The wave-lengths above λ 2000 are in air; the others are *in vacuo*. The notation is that proposed by Russell, Shenstone and Turner†, the symbol ° which distinguishes the odd terms being omitted for convenience.

By alteration of the diffuse series member, the combinations of the $4f\ ^2F$ term with $4d\ ^2D$ and $4s\ 4p^2\ ^2D$ have been identified with certainty. The satellites λλ 2155·78 and 1274·13 are calculated and could not be observed. They are probably too faint and close to be resolved from the respective main lines, which are strong and diffuse. The pair at λ 4031, which is prominent and belongs to As III, has suggested the present assignment. Further, there would otherwise be a slight discrepancy in the agreement of the D separations, which is somewhat higher than the probable error in measurement. Two lines λ 7240 and λ 6923, photographed on a krypto-cyanine plate in the arsenic spark in hydrogen, are found in the calculated position of the inverted member $4d\ ^2D - 5p\ ^2P$.

Lang refers to a pair and satellite λλ 3319, 3122, 3091 which occur exactly in the calculated position of $bD - bP$ (according to his scheme). In the present classification, the last two lines correspond to the forbidden transition $4s^2\ 4d\ ^2D - 4s\ 4p^2\ ^2D$; the coincidence is believed to be only accidental and the origin of the lines also seems doubtful.

* *Loc. cit.* † Russell, Shenstone and Turner, *Phys. Rev.* **33**, 900 (1929).

Table 1

Classification				λ I.A. (int.)	ν (vac.)	△ν
Lang		Rao				
—		$4d\ ^2D_{1\frac{1}{2}}$ –	$5p\ ^2P_{\frac{1}{2}}$	7240 (3)	13808	—
—		$^2D_{2\frac{1}{2}}$ –	$^2P_{1\frac{1}{2}}$	6923 (1)	14441	—
—		$^2D_{1\frac{1}{2}}$ –	$^2P_{1\frac{1}{2}}$	—	—	—
$4s\ 4p^2\ ^2S_{\frac{1}{2}}$ –	$5p\ ^2P_{\frac{1}{2}}$	Same as Lang's		4226·74 (6)	23652·3	723·0
$^2S_{\frac{1}{2}}$ –	$^2P_{1\frac{1}{2}}$			4101·37 (7)	24375·3	
$5s\ ^2S_{\frac{1}{2}}$ –	$5p\ ^2P_{\frac{1}{2}}$	Same as Lang's		*4037·01 (9)	24763·8	723·2
$^2S_{\frac{1}{2}}$ –	$^2P_{1\frac{1}{2}}$			*3922·46 (10)	25487·0	
$5p\ ^2P_{1\frac{1}{2}}$ –	$6s\ ^2S_{\frac{1}{2}}$	Same as Lang's		*3255·55 (5)	30708·0	723·2
$^2P_{\frac{1}{2}}$ –	$^2S_{\frac{1}{2}}$			*3180·64 (4)	31431·2	
$5p\ ^2P_{1\frac{1}{2}}$ –	$5d\ ^2D_{1\frac{1}{2}}$	Same as Lang's		*2989·42 (3)	33441·6	84·5
$^2P_{1\frac{1}{2}}$ –	$^2D_{2\frac{1}{2}}$			*2981·88 (10)	33526·1	
$^2P_{\frac{1}{2}}$ –	$^2D_{1\frac{1}{2}}$			*2926·15 (10)	34164·6	723·0
$4s\ 4p^2\ ^2D_{1\frac{1}{2}}$ –	$5p\ ^2P_{1\frac{1}{2}}$	Same as Lang's		2132·76 (3)	46872·7	320·8
$^2D_{2\frac{1}{2}}$ –	$^2P_{1\frac{1}{2}}$			2147·46 (7)	46551·9	723·7
$^2D_{1\frac{1}{2}}$ –	$^2P_{\frac{1}{2}}$			2166·21 (5)	46149·0	
—		$4d\ ^2D_{2\frac{1}{2}}$ –	$4f\ ^2F_{2\frac{1}{2}}$	2156·20 (9)	46363·3	– 8·9
—		$^2D_{2\frac{1}{2}}$ –	$^2F_{2\frac{1}{2}}$	[2155·78]	46372·2	91·2
$4s\ 4p^2\ ^2P_{1\frac{1}{2}}$ –	$4p^3\ ^2P_{1\frac{1}{2}}$	$^2D_{1\frac{1}{2}}$ –	$^2F_{2\frac{1}{2}}$	2151·55 (8)	46463·4	
—		$4f\ ^2F_{2\frac{1}{2}}$ –	$5g\ ^2G$	4032·45 (10)	24791·8	– 8·9
—		$^2F_{3\frac{1}{2}}$ –	^2G	4031·01 (10)	24800·7	
$4s\ 4p^2\ ^2P_{\frac{1}{2}}$ –	$4p^3\ ^2P_{1\frac{1}{2}}$	$4s\ 4p^2\ ^2P_{1\frac{1}{2}}$ –	$4f\ ^2F_{2\frac{1}{2}}$	2053·37 (5)	48684·8	—
$4s\ 4p^2\ ^2D_{2\frac{1}{2}}$ –	$4p^3\ ^2P_{1\frac{1}{2}}$	$4s\ 4p^2\ ^2D_{2\frac{1}{2}}$ –	$4f\ ^2F_{2\frac{1}{2}}$	1274·27 (9)	78476	– 9
—		$^2D_{2\frac{1}{2}}$ –	$^2F_{2\frac{1}{2}}$	[1274·13]	78485	321
$^2D_{1\frac{1}{2}}$ –	$^2P_{1\frac{1}{2}}$	$^2D_{1\frac{1}{2}}$ –	$^2F_{2\frac{1}{2}}$	1268·95 (6)	78805	
$4p\ ^2P_{1\frac{1}{2}}$ –	$4s\ 4p^2\ ^2D_{1\frac{1}{2}}$	Same as Lang's		1214·00 (2)	82372	321
$^2P_{1\frac{1}{2}}$ –	$^2D_{2\frac{1}{2}}$			1209·29 (10)	82693	2941
$^2P_{\frac{1}{2}}$ –	$^2D_{1\frac{1}{2}}$			1172·16 (10)	85313	
$4p\ ^2P_{1\frac{1}{2}}$ –	$5s\ ^2S_{\frac{1}{2}}$	Same as Lang's		963·80 (8)	103756	2938
$^2P_{\frac{1}{2}}$ –	$^2S_{\frac{1}{2}}$			937·26 (6)	106694	
$4p\ ^2P_{1\frac{1}{2}}$ –	$4s\ 4p^2\ ^2S_{\frac{1}{2}}$	Same as Lang's		953·55 (4)	104871	2938
$^2P_{\frac{1}{2}}$ –	$^2S_{\frac{1}{2}}$			927·57 (4)	107809	
$4p\ ^2P_{1\frac{1}{2}}$ –	$4s\ 4p^2\ ^2P_{1\frac{1}{2}}$	$4p\ ^2P_{1\frac{1}{2}}$ –	$4d\ ^2D_{1\frac{1}{2}}$	†871·79 (8)	114707	94
—		$^2P_{1\frac{1}{2}}$ –	$^2D_{2\frac{1}{2}}$	871·07 (10)	114801	2941
$4p\ ^2P_{\frac{1}{2}}$ –	$4s\ 4p^2\ ^2P_{1\frac{1}{2}}$	$^2P_{\frac{1}{2}}$ –	$^2D_{1\frac{1}{2}}$	849·99 (9)	117648	
—		$4p\ ^2P_{1\frac{1}{2}}$ –	$4s\ 4p^2\ ^2P_{\frac{1}{2}}$	900·94 (6)	110995	1487
$4p\ ^2P_{1\frac{1}{2}}$ –	$4s\ 4p^2\ ^2P_{\frac{1}{2}}$	$^2P_{1\frac{1}{2}}$ –	$^2P_{1\frac{1}{2}}$	889·03 (8)	112482	2943
—		$^2P_{\frac{1}{2}}$ –	$^2P_{\frac{1}{2}}$	877·67 (7)	113938	
$4p\ ^2P_{\frac{1}{2}}$ –	$4s\ 4p^2\ ^2P_{\frac{1}{2}}$	$^2P_{\frac{1}{2}}$ –	$^2P_{1\frac{1}{2}}$	866·36 (7)	115425	
$4p\ ^2P_{1\frac{1}{2}}$ –	$5d\ ^2D_{1\frac{1}{2}}$	Same as Lang's		614·70 (1)	162681	85
$^2P_{1\frac{1}{2}}$ –	$^2D_{2\frac{1}{2}}$			614·38 (3)	162766	2939
$^2P_{\frac{1}{2}}$ –	$^2D_{1\frac{1}{2}}$			603·79 (2)	165620	

* These were identified independently by the writer.

† For this group Lang has λλ 849·34 (1), 845·86 (2), 828·65 (2). This and the lines λ 2144·13 (4) and λ 1266·39 (30), Lang's $bP_2 - cP_1$, and $bD_3 - cP_1$, have been omitted from the above table, as incorrectly identified.

§ 4. TERM VALUES

As some combinations are now available which involve the F and G levels, the terms can be evaluated as usual by assumption of a probable value for the term $5g\ ^2G$. In table 2 they are based on the value 39500 cm.$^{-1}$. The ionization potential corresponding to $4p\ ^2P_{\frac{1}{2}}$ is about 28·19 volts.

Table 2

Term	Term value	Term	Term value
$4p\ ^2P_{\frac{3}{2}}$	228406	$4f\ ^2F_{2\frac{1}{2}}$	64292
$^2P_{1\frac{1}{2}}$	225466	$^2F_{3\frac{1}{2}}$	64301
$5s\ ^2S_{\frac{1}{2}}$	121712	$5d\ ^2D_{1\frac{1}{2}}$	62783
$4d\ ^2D_{1\frac{1}{2}}$	110755	$^2D_{2\frac{1}{2}}$	62698
$^2D_{2\frac{1}{2}}$	110664	$5g\ ^2G$	[39500]
$5p\ ^2P_{\frac{3}{2}}$	96948	$4s\ 4p^2\ ^2D_{1\frac{1}{2}}$	143097
$^2P_{1\frac{1}{2}}$	96225	$^2D_{2\frac{1}{2}}$	142776
$6s\ ^2S_{\frac{1}{2}}$	65517	$^2S_{\frac{1}{2}}$	120600
		$^2P_{\frac{3}{2}}$	114468
		$^2P_{1\frac{1}{2}}$	112981

The results of the application of the regular and irregular doublet laws to Ga I-like spectra are set out by Lang. Table 3 shows the progressive variation of only the term values in these spectra—those of Ge II having been divided by 4 and those of As III by 9, as is usual in such comparisons.

Table 3

	$4p\ ^2P_{\frac{3}{2}}$	$5p\ ^2P_{\frac{3}{2}}$	$5s\ ^2S_{\frac{1}{2}}$	$6s\ ^2S_{\frac{1}{2}}$	$4d\ ^2D_{1\frac{1}{2}}$	$5d\ ^2D_{1\frac{1}{2}}$
Ga I	48380	[15326]	23592	10795	13598	7577
Ge II	32159	12408	16559	8464	11950	7138
As III	25378	10772	13524	7280	12306	6976

	$4f\ ^2F_{2\frac{1}{2}}$	$4s\ 4p^2\ ^2P_{\frac{3}{2}}$	$4s\ 4p^2\ ^2S_{\frac{1}{2}}$	$4s\ 4p^3\ _{\frac{3}{2}}D_{1\frac{1}{2}}$	$5g\ ^2G$
Ga I	—	—	8115	—	—
Ge II	[7080]	9406	13586	15906	—
As III	7144	12719	13400	15900	[4389]

§ 5. ACKNOWLEDGMENT

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THE DETERMINATION OF THE ACOUSTICAL CHARACTERISTICS OF SINGLY-RESONANT HOT-WIRE MICROPHONES

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ABSTRACT. A method is described for measuring the damping-coefficients and conductances of the orifices of the small resonators used in the construction of hot-wire microphones. Observations show that resonant microphones with cylindrical necks of diameter between 0·5 and 0·64 cm. and length between 1·2 and 2·25 cm. have damping-coefficients which are about 23 per cent. greater than the values indicated by theory. The observed conductances of the necks agree very closely with those calculated from a formula in which allowance is made for the added inertia due to viscosity.

§ 1. INTRODUCTORY REMARKS

A singly resonant hot-wire microphone consists of a Helmholtz resonator in the neck of which is mounted an electrically heated grid of fine platinum wire. The resistance of the hot-wire grid is a function of the magnitude of the aerial vibrations in the neck of the resonator.

The equation of motion of the air in the neck of the resonator is

$$\frac{\rho}{c}\frac{d^2q}{dt^2} + \frac{2h\rho}{c}\frac{dq}{dt} + \frac{a^2\rho}{Q}q = \rho\frac{d\phi}{dt} \qquad \ldots\ldots(1),$$

q where q is the volume of air in cc. which has entered the resonator at any instant,

t, ρ, c so that dq/dt is the current of air into the resonator; ρ is the density of air; c is a linear quantity, the 'conductance' of the neck, which is inversely proportional to the

h inertia of the air in the neck of the resonator; h is the damping coefficient (the re-

a, Q ciprocal of the modulus of decay); a is the velocity of sound in air; Q is the internal

ϕ volume of the resonator; and ϕ is the primary velocity-potential at the opening of the neck of the resonator due to an external source of sound: that is, ϕ is the velocity-potential which the source would produce at that position if the resonator were not there.

Second-order effects due to differences in ρ at positions just outside the resonator, in the neck, and inside the body of the resonator are ignored. The velocity of sound, a, is the velocity in the air inside the resonator.

The acoustical characteristics of the resonator are the three quantities c, h and Q. If these three quantities are known, then, by integration of (1) ϕ may be deduced from observed values of q, or conversely q may be forecasted for specified values of ϕ. The measurement of Q presents no difficulty and the remainder of this note is

devoted to a discussion of the experimental determination of c and h and a comparison of observed and calculated values.

A method of determining h has already been described in an appendix to a paper on the measurement of sound-absorption*. The method consists in exposing the microphone to sound of constant amplitude and frequency $\omega/2\pi$ and then varying the tuning by altering the volume of the resonator. A series of pairs of values of volume Q and resistance-change δR, in the hot-wire grid is recorded, and the relative amplitude of oscillation of the air in the neck of the resonator, corresponding to each observed resistance change, is found by a calibration with stationary wave apparatus. The relative amplitude appears as a circular function $\sin ky$ ($ky \neq \pi/2$), where $k = \omega/a$ and y is the distance which the microphone must be displaced from a loop position in a stationary wave in order that the grid may suffer the resistance-change δR. The process of calibration is fully described in the paper referred to.

ω

δR

k, y

The damping coefficient was calculated from the observations by means of the formula

$$h = \pm \tfrac{1}{2} \Delta \Big/ \left\{ \left(\frac{\sin ky_m}{\sin ky} \right)^2 - 1 \right\}^{\frac{1}{2}} \qquad \ldots\ldots(2),$$

where $\Delta = \omega_0 (\omega_0/\omega - \omega/\omega_0)$ and $\omega_0^2 = a^2 c/Q$†. $\sin ky_m$ is the relative amplitude when $\Delta = 0$.

Δ, ω_0, y_m

The use of the formula (2) is not satisfactory when the tuning of the resonator is such that its frequency is near to that of the sound, for then $\sin ky_m/\sin ky$ is near to unity, and both the numerator and denominator of the right-hand side of (2) tend to zero as the resonance frequency is approached.

The observations can be more satisfactorily dealt with by an application of the "circle and straight line" construction described by Mallett‡ for the analysis of resonance curves. The theory of the method as applied to the Helmholtz resonator is given in the next section.

§ 2. THEORY

The equation (1) can be written in the form

$$\frac{d^2q}{dt^2} + 2h\frac{dq}{dt} + \omega_0^2 q = c\frac{d\phi}{dt} \qquad \ldots\ldots(3).$$

If $\phi \propto \epsilon^{i\omega t}$ we may put

$$q = \Omega\phi \qquad \ldots\ldots(4),$$

where

$$\Omega = \frac{c}{2h - i\Delta} \qquad \ldots\ldots(5),$$

Ω

so that

$$|q| = \frac{c}{(4h^2 + \Delta^2)^{\frac{1}{2}}} |\phi| \qquad \ldots\ldots(6).$$

In the experiment ω remains constant and Δ is varied by making changes in ω_0.

* *Proc. Phys. Soc.* **39**, 287–288 (1927).

† The conductance c varies with frequency owing to the effects of viscosity. $\omega_0/2\pi$ is therefore only approximately the frequency of the resonator when its volume is Q since c is the conductance when the frequency is $\omega/2\pi$.

‡ E. Mallett, *J.I.E.E.* **62**, 517–525 (1924).

Let $\Delta/2h = \tan\alpha$. Then from (6)

$$|q| = \frac{c}{2h}\cos\alpha \,.\, |\phi| \qquad \text{.....(7).}$$

Now $(c/2h).|\phi|$ is the maximum value of $|q|$, that is, the value it attains when $\Delta = 0$ and the resonator is in unison with the sound. Denoting this value by $|q|_m$ we have

$$|q| = |q|_m \cos\alpha \qquad \text{.....(8).}$$

So that if a circle be drawn on a diameter $|q|_m$ and a chord of length $|q|$ be drawn from one end of a diameter the angle included between the chord and the diameter is the angle α. This is the circle construction for finding α from a resonance curve.

Suppose that in the experiment a series of corresponding values of $|q|$ and ϕ has been recorded, and let Q_m be the value of Q for which the greatest value of $|q|$ occurs. Since by Rayleigh's formula for the frequency of a Helmholtz resonator,

$$\omega^2 = a^2 c Q_m^{-1} \qquad \text{.....(9}a\text{),}$$

and since

$$\omega_0^2 = a^2 c Q^{-1} \qquad \text{.....(9}b\text{),}$$

we have

$$= (Q^{-1} - Q_m^{-1})\, a^2 c/\omega$$
$$= (Q^{-1} - Q_m^{-1})\, 2\pi n/Q_m^{-1} \qquad \text{.....(10),}$$

where n is the frequency of the sound. Now $\tan\alpha = \Delta/2h$ and therefore

$$\tan\alpha = (Q^{-1} - Q_m^{-1})\, \pi n/hQ_m^{-1} \qquad \text{.....(11),}$$

or

$$Q^{-1} = Q_m^{-1} + (hQ_m^{-1}/\pi n)\tan\alpha \qquad \text{.....(12),}$$

so that Q^{-1} and $\tan\alpha$ are linearly related, and if Q^{-1} is plotted against $\tan\alpha$ we obtain a straight line the slope of which is $dQ^{-1}/d\tan\alpha$ where

$$dQ^{-1}/d\tan\alpha = hQ_m^{-1}/\pi n \qquad \text{.....(13).}$$

If the slope is measured h may be calculated from the equation

$$h = \frac{\pi n}{Q_m^{-1}} \cdot \frac{dQ^{-1}}{d\tan\alpha} \qquad \text{.....(14).}$$

In the method as described by Mallett, $\tan\alpha$ is plotted against frequency and the linear relation is only approximate, but the relation given in (12) is exact.

Examples of the application of the method to the determination of the damping factors of hot-wire microphones are given in the following two sections.

§ 3. THE DETERMINATION OF THE DAMPING COEFFICIENT OF A MICROPHONE TUNED TO 30·66 ~

The microphone used in this experiment was one designed for recording the low-frequency sound emitted by an airscrew. The neck was cylindrical and made as shown in the section in figure 1. The dimensions of the cylindrical neck above the grid at M were: length 22·5 mm., diameter 6·4 mm. The grid was supported in the usual way on a porcelain bridge in a circular hole, 6·5 mm. in diameter, in a mica disc. The grid was about 1 mm. below the lower end of the neck.

For the determination of the damping coefficient the resonator was fitted with a side tube and tap so that water could be run in or out in order to vary the internal volume. A glass gauge was also fitted on one side of the resonator so that the level of the water could be read off, and the gauge was calibrated by the running of water into a measuring cylinder and observation of the gauge readings, allowance being made for the volume of water from the gauge. Sound of constant amplitude and frequency was provided by a moving-coil loud-speaker driven by a thermionic valve-oscillator, the microphone being placed about 2 ft. in front of the loud-speaker. The experiment was performed in a room, but the observer was at some distance from the microphone and was careful to make only small movements so as not to cause any sensible disturbance of the acoustic field round the microphone. The experiment was begun with the volume of the container small and the change in the resistance of the hot-wire grid also small, and the volume was increased step

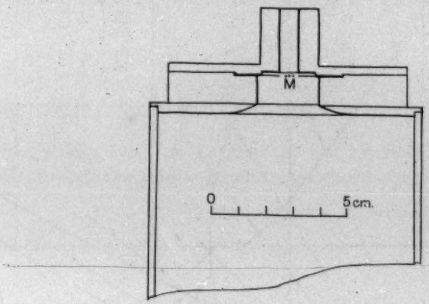

Fig. 1. Section through neck of resonator ($n = 30{\cdot}66 \sim$).

by step until the change in resistance had just passed through its maximum value. The microphone was then calibrated by the stationary-wave method. The apparatus employed for this purpose was the same in principle as that already described*; but in order to cope with the long wave-length the pipe in which the stationary wave was produced was 21 ft. long and 2 ft. 6 in. in diameter, made up of concrete sections cemented together. The pipe was sealed at one end with a heavy concrete disc in a circular iron frame held by bolts and nuts against a compressed cork washer. The other end of the pipe was covered with a three-ply baffle-board at the middle of which was mounted a moving-coil loud-speaker. This pipe was one that had been constructed at the Air Defence Experimental Establishment for the purpose of calibrating microphones at the low frequencies which occur in airscrew sound.

The stationary wave in the pipe could be made very strong at about 30 ~ and the loops were always very perfect. The change in resistance when the microphone was at a loop was only about 0·01 ohm when the sound was turned on or off (except

* *Proc. Phys. Soc.* **39**, 274–275 (1927).

for a transient effect which was of no importance so far as calibration was concerned) although the change in resistance at a node was about 50 ohms.

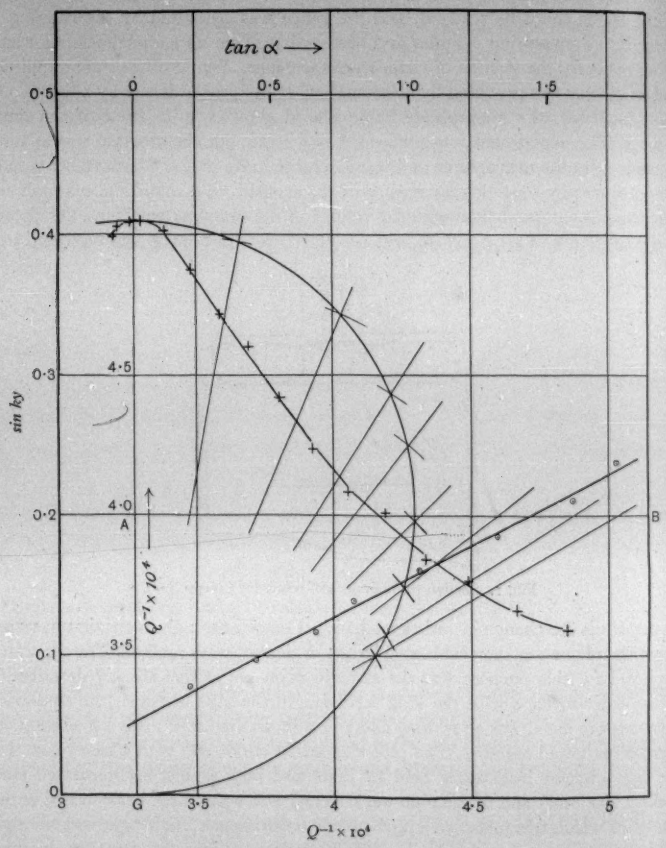

Fig. 2. Determination of damping coefficient and conductance by circle and straight line method ($n = 30{\cdot}66 \sim$).

In both parts of the experiment the frequency of the sound was found by a stroboscopic method, the primary of a transformer being placed in series with the moving-coil loud-speaker and the secondary connected to a neon lamp which illuminated intermittently (with the frequency of the sound) a rotating white disc

on which was drawn a black triangle. The disc was driven by a phonic motor at 10·17 rev./sec. The frequency was calculated from the observed slip of the triangle.

The set of figures shown in table 1 was obtained. The values of sin ky are of course proportional to $|q|$ in the theory given in § 2.

From these figures a resonance curve was drawn, sin ky being plotted against Q^{-1} (figure 2), and a set of values of tan α was determined from the curve by the circle construction. The intercept on AB between the chord drawn from G and the vertical diameter of the circle, GA being taken as unity, is, of course, equal to

Table 1. Determination of damping-coefficient at 30·66 ~ and 13° C.

$Q^{-1} \times 10^4$	sin ky	$Q^{-1} \times 10^4$	sin ky
c.c.		c.c.	
4·84	0·118	3·69	0·320
4·66	132	3·58	344
4·49	154	3·48	375
4·34	169	3·39	403
4·19	202	3·30	410
4·05	216	3·26	410
3·92	248	3·21	404
3·80	284	3·20	399

tan α. Finally tan α was plotted against Q^{-1} and a straight line was drawn as nearly as possible through the points. From measurement of the figure it was found that

$$dQ^{-1}/d \tan \alpha = 0{\cdot}88 \times 10^{-4}/1{\cdot}70,$$

and also that

$$Q_m^{-1} = 3{\cdot}26 \times 10^{-4},$$

so that by (13)

$$h = 15{\cdot}3 \text{ sec.}^{-1}.$$

The air-temperature being 13° C. we have $a = 33960$ cm./sec., and therefore

$$c = 0{\cdot}0989 \text{ cm.}$$

The accuracy with which c can be determined depends mainly upon the accuracy with which Q_m^{-1} can be found. The error in the value of c given above is probably not more than 4 parts in 1000—a much smaller error than can be claimed for any earlier measurements of conductance.

§ 4. THE DETERMINATION OF THE DAMPING-COEFFICIENT OF A MICROPHONE TUNED TO 512 ~

The observations used in this section are taken from an earlier paper*. The microphone is shown in figure 5 of that paper. The tuning was varied by alteration of the volume of the resonator by means of a sliding portion. The cylindrical neck above the hot-wire grid was 12 mm. long† and 5 mm. in diameter. A section through

* *Proc. Phys. Soc.* **39**, 288 (1927).

† Erroneously stated to be 1 cm. long in the earlier description (*loc. cit.* p. 276).

the neck and the upper part of the resonator is shown in figure 3. As before, the grid was about 1 mm. below the lower end of the neck.

The first two observations previously recorded must be discarded since the volume of the resonator given for the first observation is greater than that for the second whereas it should clearly be less. The error is probably due to a misprint.

The remaining observations yielded the figures given in table 2.

It was found from a figure that $(dQ^{-1}/d \tan \alpha) = (1{\cdot}40/3{\cdot}7) \times 10^{-2}$ and $Q_m^{-1} = 7{\cdot}665 \times 10^{-2}$ c.c.$^{-1}$ whence $h = 79{\cdot}4$ sec.$^{-1}$ (compared with the value of

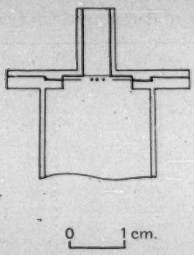

Fig. 3. Section through neck of resonator ($n = 512 \sim$).

81 sec.$^{-1}$ previously given). The temperature was not recorded, but assuming it to be 10° C. we have $a = 33760$ cm./sec. and $c = 0{\cdot}118$ cm., instead of $c = 0{\cdot}114$ cm. as previously given*.

Table 2. Determination of damping-coefficient at 512 ~

Q^{-1}	sin ky	Q^{-1}	sin ky
c.c.$^{-1}$		c.c.$^{-1}$	
$8{\cdot}64 \times 10^{-2}$	0·210	$7{\cdot}55 \times 10^{-2}$	0·531
8·38	260	7·37	459
8·11	344	7·22	353
7·94	465	6·90	241
7·72	542	6·52	182
7·61	549		

§ 5. SOME ADDITIONAL OBSERVATIONS

The following observations of damping-coefficients have been recorded during experiments with hot-wire microphones. The method of experiment was similar in all essential details to that described in § 3. The observations were not made with the object of obtaining specially accurate values for damping and conductance but were recorded during the ordinary course of experiments incidental to the design

* *Loc. cit.* p. 276.

and employment of microphones. All the orifices were cylindrical, and the letters l and r will be used to denote the length and radius respectively. l, r

(*a*) $l = 18{\cdot}4$ mm. $h = 20{\cdot}7$ sec.$^{-1}$.
$2r = 5{\cdot}8$ mm. $c = 0{\cdot}096$ cm.
$n = 56{\cdot}3 \sim$.
Temperature, 6° C.

(*b*) $l = 18{\cdot}4$ mm. $h = 21{\cdot}6$ sec.$^{-1}$.
$2r = 5{\cdot}8$ mm. $c = 0{\cdot}101$ cm.
$n = 61{\cdot}2 \sim$.
Temperature, 12° C.

(*c*) $l = 14{\cdot}95$ mm. $h = 26{\cdot}1$ sec.$^{-1}$.
$2r = 5{\cdot}74$ mm. $c = 0{\cdot}117$ cm.
$n = 90 \sim$.
Temperature, 7° C.

§6. POSSIBLE ERROR DUE TO REACTION BETWEEN MICROPHONE AND SOURCE OF SOUND

There is a possible source of error in the experiments which should be noted. In order to obtain effects of sufficient magnitude to ensure accurate measurement it is necessary to place the microphone near to the source of sound in the first part of the experiment. As the microphone is brought into tune with the source there may be acoustical reaction between resonator and source which will tend to enhance the output of the source when there is unison. In the case of microphones such as those described above for which the radiation damping is small, the effect will probably be quite unimportant. But with microphones which reradiate a larger proportion of sound an appreciable error might be introduced. The error could be avoided by means of an arrangement wherein the amplitude of the source remains constant during a set of observations, for example, by using a hot-wire grid in the throat of a loud-speaker*.

§7. THE CONTRIBUTION OF SOUND-RADIATION TO THE OBSERVED DAMPING-COEFFICIENTS

It was pointed out by Tucker and Paris† in 1921 that the contribution of radiation losses to the damping of the small resonators used in the construction of singly-resonant hot-wire microphones was relatively negligible. That this is the case is confirmed by the more accurate observations recorded in the preceding sections.

The damping coefficient, h, of a resonator can be regarded as the sum of two parts h_1 and h_2 representing radiation and viscosity effects respectively. The value of h_1 may be calculated from‡ h_1, h_2

$$h_1 = \pi n^2 c / 2a \qquad \text{.....(15).}$$

* Cf. *Proc. Phys. Soc.* **39**, 277 (1927). † *Phil. Trans.* A, **221**, 399 (1921).
‡ Cf. H. Lamb, *Dynamical Theory of Sound*, p. 265 (1910).

Thus in the case of the resonator used in the experiment described in § 3, $n = 30{\cdot}66 \sim$, $c = 0{\cdot}0989$ cm. and $a = 33960$ cm./sec. so that

$$h_1 = 0{\cdot}0043 \text{ sec.}^{-1},$$

which is to be compared with the observation that

$$h = 15{\cdot}3 \text{ sec.}^{-1}.$$

The radiation effect is thus quite insignificant, contributing only about 0·03 per cent. to the total damping.

The radiation effect is also negligible in the case of the observations recorded in § 5.

For the resonator used to obtain the observations given in § 4, $n = 512 \sim$, $c = 0{\cdot}118$ cm., and $a = 33760$ cm./sec., so that

$$h_1 = 1{\cdot}4 \text{ sec.}^{-1},$$

compared with the observation that

$$h = 79{\cdot}4 \text{ sec.}^{-1}.$$

Although the radiation effects in this case are relatively greater, they account for only 2 per cent. of the total damping. For viscosity alone

$$h_2 = 79{\cdot}4 - 1{\cdot}4 = 78 \text{ sec.}^{-1}.$$

§ 8. THEORETICAL EXPRESSIONS FOR THE DAMPING COEFFICIENT AND THE CONDUCTANCE OF AN ORIFICE

The theory of a resonator with a cylindrical neck, with allowance for the effects of viscosity, has been given by G. W. Stewart*. The effect is twofold; there is dissipation of acoustical energy and there is an increase in the effective inertia of the air in the neck of the resonator. Stewart deduced the acoustical impedance of a cylindrical neck from the equations of motion used by Helmholtz to compute the effect of viscosity on the velocity of propagation of sound in narrow pipes. Crandall† has given a detailed account of the solution of these equations and of the application of the solution to the calculation of the impedance of a cylindrical conduit. He obtained an expression for the resistance-coefficient of a conduit (the ratio of pressure-gradient to particle velocity) of which that part R depending on viscosity is given by

R

$$R = (2\mu\omega\rho)^{\frac{1}{2}}(1 + \iota)/r \qquad \text{.....(16),}$$

r, μ

where r is the radius of the conduit and μ is the coefficient of viscosity of air. By introducing this resistance coefficient into the equation of motion of the air in the neck of a resonator, and omitting the effects of radiation, with which we are not at present concerned, we obtain, instead of (1)

$$\frac{d^2q}{dt^2} + \frac{c}{\rho}\frac{L}{\pi r^2}\frac{1}{r}(2\mu\omega\rho)^{\frac{1}{2}}(1 + \iota)\frac{dq}{dt} + \frac{a^2c}{Q}q = c\frac{d\phi}{dt} \qquad \text{.....(17),}$$

* *Phys. Rev.* **27**, 488 (1926).

† *Vibrating Systems and Sound*, Appendix A, pp. 229–241 (1926).

where c is the conductance of the neck as it would be if viscous forces were non-existent and L is the effective length of the neck. When the length is great compared with the diameter, L will be very nearly the actual length, but in the case of short necks the addition of an end-correction will be necessary to allow for the fact that the viscous effects do not terminate abruptly at the ends of the neck. c L

Since ϕ and q vary as $\epsilon^{i\omega t}$, (17) becomes

$$\left[\left(\frac{a^2c}{Q} - \frac{\omega c}{\rho}\frac{L}{\pi r^3}(2\mu\omega\rho)^{\frac{1}{2}} - \omega^2\right) + \iota\frac{\omega c}{\rho}\frac{L}{\pi r^3}(2\mu\omega\rho)\right] q = \iota\omega c\phi \quad \text{.....(18).}$$

If we put

$$h_2 = \frac{cL}{\pi r^3}\left(\frac{\omega\mu}{2\rho}\right)^{\frac{1}{2}} \quad \text{.....(19·1),} \qquad h_2$$

and

$$c' = c\left(1 - \frac{2\omega Q}{a^2 c}h_2\right) \quad \text{.....(19·2),} \qquad c'$$

(18) may be written

$$\left[\left(\frac{a^2c'}{Q} - \omega^2\right) + 2\iota\omega h_2\right] q = \iota\omega c\phi \quad \text{.....(20).}$$

Of the two conductances which appear in this equation c' includes the effect of added inertia due to viscosity and is the conductance that is measured experimentally from observations of the volume and resonance frequency of a resonator. c is the conductance calculated on the assumption that there is no viscosity and can be found from the well-known formula

$$c = \pi r^2/(l + \alpha r) \quad \text{.....(21),}$$

where l is the length of the neck and αr is the sum of the end-corrections. l, α

Since $a^2c/Q = \omega_0^2$ (19·2) may be written

$$c' = c\,(1 - 2\omega h_2/\omega_0^2) \quad \text{.....(21·1),}$$

or

$$c' = c\,(1 - 2h_2/\omega) \quad \text{.....(21·2),}$$

for the limited variations of Q made in the experiments.

(19·1) gives the value of h_2 as it would appear from Stewart's expression for the impedance of a resonator.

In the case of necks with lengths greater than their diameters, it seems reasonable to identify L with the reduced length $(l + \alpha r)$ calculated for pure inertia effects, so that

$$h_2 = \frac{1}{r}(\omega\mu/2\rho)^{\frac{1}{2}} \quad \text{.....(22).}$$

This is the form in which the expression for the damping coefficient is given by Crandall*.

The resonator necks used in the experiments described in this paper were such that the ratio length/diameter always lay between 2·4 and 3·5, and it will be assumed that $L = l + \alpha r$ with sufficient approximation.

The acoustical impedance of an orifice, with allowance for viscosity, has also been calculated by E. G. Richardson†. His calculation proceeds on an assumption,

* *Vibrating Systems and Sound*, p. 56 (1926). † *Proc. Phys. Soc. Lond.* **40**, 214–215 (1928).

due to Prandtl, that viscosity effects are confined to a thin boundary layer of determinate thickness. Two expressions are given for the acoustical impedance of an orifice, from either of which h_2 can be found by use of the fact that the real part of the impedance is equal to $2h_2\rho/c$. Thus from Richardson's equation (10)* we find

$$h_2 = \frac{1}{r}\frac{\mu}{\rho x} \qquad \text{.....(23),}$$

x where x is the thickness of the boundary layer. The second expression† involves an
ν', μ' altered coefficient of viscosity, ν' or μ'/ρ, and is stated to be obtained from the first expression by means of a substitution equivalent to $x = \sqrt{(2\pi\mu'/\rho\omega)}$. This, however, appears to be an error. In order to obtain Richardson's equation (11) from (10) it is also necessary to substitute μ' for μ in (10)‡.

§ 9. COMPARISON OF THE OBSERVED DAMPING COEFFICIENTS WITH THEIR VALUES CALCULATED FROM (22)

A comparison of the experimental values of h_2 recorded in §§ 3, 4 and 5 with the values of h_2 calculated from (22) is contained in table 3.

Table 3. Observed and Calculated Values of h_2.

Frequency (n)	Diameter ($2r$)	Length/diameter ($l/2r$)	h_2 observed	h_2 calculated	Ratio of observed to calculated h_2
cycle/sec.	cm.		sec.$^{-1}$	sec.$^{-1}$	
30·66	0·64	3·51	15·3	12·4	1·23
56·3	0·58	3·17	20·7	17·0	1·22
61·2	0·58	3·17	21·6	18·1	1·19
90	0·574	2·50	26·1	21·8	1·20
512	0·5	2·4	78	60·3	1·29

The mean of the figures in the last column is 1·23, so that the conclusion to be drawn is that calculated values of h_2 must be increased by about 23 per cent. in order to procure agreement with observation.

The frequency of 30·66 ~ is near to that used in experiments by Richardson§
x to measure directly the thickness x of the boundary layer where viscosity effects occur. Measurements were made in an orifice 3 cm. in diameter and 2 mm. long and it was found that $x = 0{\cdot}075$ cm. at 25 ~ and $x = 0{\cdot}05$ cm. at 35 ~. By using (23) we can calculate x from the observed h_2 at 30·66 ~, and we find in this way

* *Loc. cit.* p. 214. † Equation (11), p. 215.

‡ The statement at the foot of p. 215 (Richardson, *loc. cit.*) to the effect that Stewart's form of the expression for impedance can be obtained from Richardson's formulae by the substitution $x = \sqrt{(\mu/\rho n)}$ also appears to be incorrect. To obtain Stewart's impedance from Richardson's (10) we must put $x = \sqrt{(2\mu/\rho\omega)} = \pi^{-\frac{1}{2}}\sqrt{(\mu/\rho n)}$, and to obtain it from (11) we must have $\nu' = \pi\nu$, whence

$$x = \sqrt{(\nu'/n)} = \pi^{\frac{1}{2}}\sqrt{(\mu/\rho n)}.$$

§ *Proc. Phys. Soc.* **40**, 217 (1928).

that $x = 0{\cdot}029$ cm., that is about half the value that would be expected from Richardson's observations. The discrepancy may be due to the relatively greater length of the orifices used in the present experiments.

§ 10. THE DISCREPANCY BETWEEN THE OBSERVED AND CALCULATED VALUES OF h_2

There seems little possibility of there being any experimental error which would account for the large difference between the observed and calculated values of h_2. The error mentioned in § 6 would tend to make the observed values of h_2 smaller than their true values.

Among the factors which might be suspected of being either wholly or partly the cause of the discrepancy are the following: (i) losses due to viscosity at the sharp edges of the circular hole in the mica disc supporting the hot-wire grid at the lower end of the neck; (ii) losses due to the motion of the air past the heated platinum wire (about 0·0006 cm. in diameter) constituting the grid of the microphone*; (iii) losses due to viscosity during the motion of the air past the porcelain bridge (1 mm. in diameter) which supports the heated wire in the circular hole in the hot-wire grid.

It may be noted, however, in regard to (i) that Stewart† found that viscosity effects were negligible in the case of a Helmholtz resonator with a circular orifice of diameter 4 mm. and length 0·15 mm. Regarded as a separate orifice, the hole in the mica disc has a diameter 6·5 mm. and length 0·13 mm. (the thickness of the mica), and Stewart's results indicate that there is no appreciable damping due to viscosity in an orifice of these dimensions.

Also, so far as I have been able to make out from the well-known work of Sewell‡, it appears that the losses under headings (ii) and (iii) are likely to be relatively insignificant. It must be admitted, however, that the question how much energy is dissipated during the motion of the air past the heated platinum wire requires more attention than I have yet given to it. The high temperature of the wire may be a factor of some importance.

It seems probable, however, that the discrepancy between the observed and calculated damping coefficients cannot be accounted for by any of the factors enumerated above but may be due to the fact that the theory of the vibrational motion of air in a narrow neck is not properly understood. In particular, in view of the experiments of E. G. Richardson§ on the velocity across circular orifices, it seems possible that the velocity distribution implicit in the theory from which formula (22) for the damping coefficient is deduced may not be correct. There is also some evidence that a jet effect exists in small cylindrical necks‖.

* The type of microphone used in the experiment had three loops. It is described in *Phil. Trans.* A, **221**, 391, figure 1 B (1921).

† *Phys. Rev.* **27**, 489 (1926).

‡ *Phil. Trans.* A, **210**, 239–270 (1910).

§ *Proc. Phys. Soc.* **40**, 215–218 (1928).

‖ W. S. Tucker and E. T. Paris, *Phil. Trans.* A, **221**, 422 (1921).

§ 11. COMPARISON OF OBSERVED AND CALCULATED VALUES OF CONDUCTANCE

Table 4 contains a comparison of the observed conductances recorded in §§ 3, 4 and 5 with values calculated from (21) and (21·2). The value of α in (21) has been taken to be 1·6 on the assumption that a fair value for the end-corrections will be obtained by means of the supposition that both ends of the neck are flanged*. It will be seen from figures 1 and 3 that the lower ends of the necks are much more restricted than by a plane flange while the upper ends are less restricted. The *observed* values of h_2 were used to calculate c' from (21·2).

Table 4. Observed and Calculated Values of Conductance.

Frequency (n)	Diameter ($2r$)	Length (l)	c calculated from (21)	c' calculated from (21·2)	Observed conductance
~	cm.	cm.	cm.	cm.	cm.
30·66	0·64	2·25	0·119	0·098	0·0989
56·3	0·58	1·84	0·115	0·102	0·096
61·2	0·58	1·84	0·115	0·102	0·101
90	0·574	1·495	0·132	0·120	0·117
512	0·5	1·2	0·123	0·117	0·118

The agreement between the figures in the fifth and sixth columns of this table verifies the correctness of the formula (21·2).

It is interesting to note that in the case of the microphone of frequency 30·66 the effect of viscosity was to reduce the conductance by more than 20 per cent.

§ 12. CONCLUSION

The observations on the damping coefficients of resonant hot-wire microphones with small cylindrical necks (table 3) show that the theoretical values are about 23 per cent. below those found by experiment. The observed values for conductance (table 4) agree very closely with those calculated from a formula which takes into account the increase in the effective inertia of the air in the neck due to viscosity.

§ 13. ACKNOWLEDGMENT

My acknowledgments are due to Mr A. Reading for the very considerable assistance he has given in the execution of the experiments described in this paper.

DISCUSSION

SIR RICHARD PAGET. A possible cause of the differences between observed and calculated conductance may lie in the shape of the resonator. I have found that in resonators such as those shown in figures 1 and 3 the effective resonating cavity is not stopped but is, as it were, smoothed out into a more or less stream-lined form—

* G. W. Stewart, *Phys. Rev.* **27**, 489 (1926), uses a value for end-correction which makes α equal to 1·57.

as if by a smooth surface (of rotation) touching the corners of the steps and merging into the full diameter of the neck and body of the resonator respectively. The effect of this stream-lining is to narrow the neck near its point of abrupt enlargement. The same type of effect occurs at the outer end of the neck where a rectangular cut-off behaves in fact as if the neck were extended outwards, enlarged, and rounded off. This phenomenon corresponds to the well known end-correction.

E. J. Irons. The present paper increases the debt which we owe to the author for his treatment of resonator problems. I should be pleased to know whether he considers that the method would retain its precision if used for the determination of the conductance of an orifice in a thin wall, i.e. an orifice for which, in terms of (21), l is very small.

Dr G. D. West asked whether the author had allowed for rise in temperature of the air in the neck of the resonator.

Dr A. B. Wood. It appears that the author is disposed to neglect altogether the damping due to the hot-wire grid. This seems tantamount to assuming that the hot-wire microphone is inefficient as a sound-receiver. Alternatively, assuming the receiver to be efficient, then the grid must absorb an appreciable proportion of the sound-energy falling on it, and this will appear as grid-damping. The latter quantity might conveniently be determined by the introduction of an auxiliary hot-wire grid (or grids) purely as a means of increasing the damping. In this manner also the damping due to the grid at different temperatures could be studied.

Dr E. G. Richardson (communicated). I agree with the author that there are at present several uncertain factors that will have to be properly accounted for in any theory which is to give values of the conductances of orifices in agreement with practice. With regard to measurements of the damping-coefficients for comparison with theory, I think that the impedance to the motion of the air in the neck offered by the mica insulator in the author's experiments, and by the fork held over the resonator in my own, is a factor of considerable magnitude. With regard to the difference in the thickness of the boundary layer, as calculated in § 9, the suggestion of the author that this is in part accounted for by the difference in the shape of the orifices used by us (tube and hole respectively) is supported by the corresponding differences of boundary-layer thickness in steady flow past obstacles of various shapes, and at different points along the surface of the same obstacle.

I am grateful to the author for pointing out the slip in the comparison-factor between equation (11) of my paper and the corresponding equation of Stewart, as given by me in the footnote on p. 215.

Author's reply. Sir Richard Paget's observations are of great interest and tend to show that the velocity-distribution assumed in the theory may not be that which actually exists in the neck of the resonator. In reply to Dr Irons: I think the experimental method described in the paper could be used to determine the conductance of a hole in a thin wall, but I would recommend the use of a single strand

of fine wire stretched across the hole. In answer to Dr West: I assumed the temperature of the air in the neck of the resonator to be that of the surrounding atmosphere. The true temperature is doubtless higher, but is very unlikely to be high enough to account for the discrepancy between the observed and calculated damping-factors. The matter is certainly one to which attention should be paid in future experiments. With regard to the point raised by Dr Wood, the opinion expressed in the paper is that any damping due to the presence of the grid is small compared with the damping due to viscosity in the neck of the resonator. The experiment which he suggests should however be performed to remove uncertainty. It is not clear to me that the sensitiveness of the hot-wire grid as a detector has any simple relation to its damping effect; a coarser and less sensitive grid would probably produce greater damping than a grid of the more sensitive fine wire.

THE CURIE POINTS

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ABSTRACT. Attention is directed to the fact that at least three temperatures may require to be specified in a description of the magnetic behaviour of a ferromagnetic substance. These are respectively termed the ferromagnetic critical point, the ferromagnetic Curie point, and the paramagnetic Curie point. The significance of the relative positions of the last two points is discussed, and it is shown that a slight extension of the view that ferromagnetism is due to a magnetic particle consisting of a group of associated atoms may account for the paramagnetic behaviour of iron, nickel and cobalt, and for that of more complicated substances such as the ferrocobalts, magnetite and manganese arsenide.

§ 1. STATEMENT OF PROBLEM

Following a recent discussion on magnetism*, some criticism was directed against the use of the term Curie point. It must be confessed that the term should not be used without discrimination, for in discussing the properties of a ferromagnetic material we may have to bear in mind at least three different temperatures, each of which may loosely be termed a Curie point. Suppose we consider the way in which the intrinsic magnetization of a ferromagnetic substance decreases with rise in temperature. In the first place we have a temperature at which the rate of change of the square of the intrinsic magnetic moment per unit volume with temperature is a maximum. This temperature, often referred to as the Curie point, is important in the discussion of the specific heat of the ferromagnetic substance, for, on the older Weiss theory†, or on the more recent theories of ferromagnetism‡, the specific heat of the substance should be a maximum at this temperature, and experiment has shown that it is so§. It has been suggested by the writer that this temperature should be termed the ferromagnetic critical point, as it is not sufficient to state that a change in specific heat occurs at the Curie point.

The ferromagnetic Curie point. A second temperature which is of interest is that at which the intrinsic magnetization may be considered to become zero. This point presents some difficulty, because it is not easy to understand how the intrinsic magnetization can become zero at a sharply defined temperature, as is so commonly supposed. At any rate, we may make our conception of this temperature more precise by supposing that it is obtained by extrapolation from the curve of intrinsic

* "Discussion on magnetism," *Proc. Phys. Soc.* **42**, 455 (1930).
† P. Weiss et P. N. Beck, *Journ. de Phys.* **7**, 279 (1908).
‡ R. H. Fowler and P. Kapitza, *Proc. R.S.* A, **29**, 1 (1929).
§ L. F. Bates, *Proc. Phys. Soc.* **42**, 441 (1930).

magnetic moment with temperature, that portion of the curve where the rate of change of the magnetization with temperature is a maximum being used for the purpose. This temperature is the one which is most frequently referred to as the Curie point. In accordance with the suggestion of Forrer*, we shall term it the ferromagnetic Curie point.

The paramagnetic Curie point. A third temperature which is of significance comes under consideration when we discuss the variation of the susceptibility with temperature, when the substance has passed into the paramagnetic state. It is frequently assumed that at temperatures considerably above the ferromagnetic Curie point the susceptibility χ is given by the Weiss law, viz. $\chi = C/(T - \theta)$, where C and θ are constants and T is the absolute temperature. Forrer terms the constant θ in the above equation the paramagnetic Curie point. He points out that on the Weiss theory of the internal field we should expect the ferromagnetic and the paramagnetic Curie points to coincide. Actually, however, various experimenters state that they are separated by a temperature range of some 15° to 20° C. Thus, for iron Preuss and Hegg† find a separation of 16° C. For nickel, separations of 15, 14 and 22° C. are given by Bloch, Alder and Peschard‡ respectively. For cobalt Preuss and Bloch‡ respectively give separations of 19 and 25° C. It should be remembered, however, that it is disputed whether the Weiss law holds for these substances at temperatures far above their ferromagnetic Curie points. For example, Terry§ finds that in all three cases a continuous curve is obtained when $1/\chi$ is plotted against the temperature. If tangents are drawn to his curves in the regions of the highest available temperatures, we obtain intersections on the axis of temperature which give separations between the ferromagnetic and paramagnetic Curie points of approximately 80, 30 and 40° C. for iron, nickel and cobalt respectively.

χ
C, θ, T

The reversal Curie point. Forrer‖ also shows that a fourth temperature may be of interest, for on taking a specially prepared specimen of nickel he obtained with it a series of hysteresis cycles up to, and, more particularly, in the near vicinity of, the ferromagnetic Curie point. The cycles were of distinctive appearance and characterized by very abrupt changes of magnetism when certain critical fields were attained. These critical fields were those at which the reversible portions of the magnetism changed abruptly; the coercive fields were the fields necessary for the irreversible reduction of the magnetism to zero. Incidentally, it was found that the residual magnetism was accurately proportional to the intrinsic magnetization of the specimen. Forrer plotted the above critical fields against the temperatures at which the hysteresis cycles were obtained, and by extrapolation he found the temperature at which the critical field could be taken to be equal to zero. It was also found by extrapolation that the coercive force became zero at the same temperature. This temperature Forrer terms the reversal Curie point. He found it to be some 20° C. above the ferromagnetic Curie point, and, in fact, he considers that it is

* R. Forrer, *Journ. de Phys.* **1**, 49 (1930).
† F. Hegg, *Arch. des Sci.* **31**, 4 (1911) and A. Preuss, *Thèse* (Zurich, 1912).
‡ These values are taken from the paper by R. Forrer mentioned above.
§ E. M. Terry, *Phys. Rev.* **9**, 39 (1917). ‖ R. Forrer, *loc. cit.*

identical with the paramagnetic Curie point, i.e. that the critical field ceases to exist at the paramagnetic Curie point, so that the simple conception of the Weiss internal molecular field is not adequate to explain the existence of intrinsic magnetization. Hence we need only consider, according to Forrer, the existence of the ferromagnetic and the paramagnetic Curie points, and we may represent the state of affairs diagrammatically as in figure 1. The curve on the left represents the variation of the intrinsic magnetization with temperature, whilst that on the right is supposed to represent the variation of $1/\chi$ with temperature well above the ferromagnetic Curie point. The ferromagnetic and paramagnetic Curie points are denoted by θ_f and θ_p respectively. This diagram Forrer considers to represent the behaviour of iron, nickel and cobalt.

$1/\chi$.
θ_f, θ_p

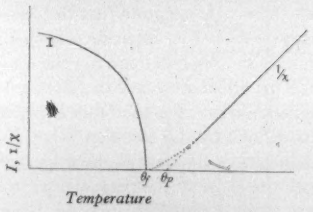

Fig. 1. Variation of I and $1/\chi$, with temperature

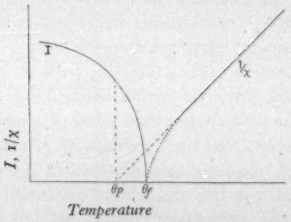

Fig. 2. Variation of I and $1/\chi$, with temperature

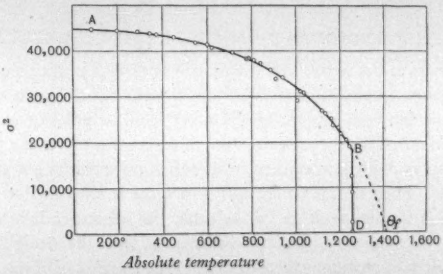

Fig. 3. Variation of σ^2 for alloy 40 % Fe, 60 % Co.

The Curie points of the ferrocobalts. There are, however, cases where the paramagnetic Curie point is lower than the ferromagnetic Curie point. Such cases occur with the ferrocobalts. Preuss* has found that alloys containing respectively 28·2, 38·1, 48·2, 59·8 and 69·0 parts of cobalt per 100 parts of metal all have very high ferromagnetic Curie points and that the paramagnetic Curie points are respectively 45, 75, 125, 140 and 74° C. lower. In these circumstances we might *a priori* expect that the state of affairs would be represented diagrammatically as in figure 2, but

* A. Preuss, *Thèse* (Zurich, 1912) and R. Forrer, *loc. cit.* Cf. also H. Masumoto, *Sci. Rep. Tokio*, 15, 449 (1926).

Forrer considers that the free movement which we associate with the magnetic particle in the paramagnetic state is incompatible with the existence of ferromagnetism, and that the latter must disappear abruptly in the neighbourhood of the paramagnetic Curie point, i.e. practically along the ordinate at θ_p. Actually, such cases are found in practice, for Preuss* has shown that all the ferrocobalts containing between 30 to 70 per cent. of iron exhibit this abrupt loss of ferromagnetism. A typical result is shown in figure 3. Let us note in passing that the deduction of the ferromagnetic Curie point now requires a somewhat doubtful extrapolation.

§ 2. FORRER'S EXPLANATION

In accepting the existence of the two Curie points, Forrer suggests that two different mechanisms are necessary to explain them. He suggests that the ferromagnetic Curie point corresponds to the disappearance of an intrinsic orientation of the elementary magnetic particle, in which only the direction and not the sense of the magnetic particle is taken into account. This orientation may therefore arise from forces other than those of a purely magnetic origin. On the other hand, he suggests that the paramagnetic Curie point is defined by the fact that below it a finite magnetic field is necessary to reverse the magnetic moment of the substance. Hence, in this case we are concerned only with the sense of the magnetic particle of the elementary particle without reference to its orientation. Forrer finally concludes that for the production of ferromagnetism intrinsic orientation and hysteresis are simultaneously necessary, one of these alone being insufficient.

§ 3. RECONSIDERATION OF PREVIOUS RESULTS

Now it appears to the writer that the matter may with profit be considered from another standpoint. Let us suppose that a long cylindrical specimen of ferromagnetic material is suspended vertically in a vacuum between the poles of an electromagnet, as in the Gouy method for the measurement of susceptibility. Let a uniform field H exist at the lower end of the specimen, whilst the upper end is not subjected to any appreciable field. Then the specimen will be acted upon by a force tending to pull it downwards. If the temperature be such that the substance is in a paramagnetic condition, and if it behaves as depicted in figure 1, then the straight portion of the curve may be taken to represent the variation of the reciprocal of the downward pull with temperature for a known value of H at high temperatures. What, then, will be the shape of the curve joining θ_f to the end of the straight portion? It will be some such curve as that represented by the dotted line, concave upwards, the actual shape of the curve depending on the strength of the field H. This is a point which does not appear to be sufficiently emphasized in the published investigations of the paramagnetic susceptibility of ferromagnetic substances. If now we erect an ordinate at θ_p in figure 1 we have to consider the significance of the portion of the dotted curve to the right of this ordinate. It is at once obvious that the pull recorded experimentally is less than it would be if the substance were actually in the para-

* A. Preuss, *loc. cit.*

magnetic state represented by the continuation of the straight line. The dotted curve in figure 1 appears to represent the behaviour of iron, nickel and cobalt, although systematic data, obtained by the method outlined above, appear to be lacking. There appears to be no doubt, however, that we have to explain why the pull is less than that theoretically possible over a considerable range of temperature. Turning now to figure 2, let us suppose that a specified ferromagnetic material gives the straight line shown when we plot the reciprocal of the pull against the temperature for a known value of H. The dotted curve running from the ferromagnetic

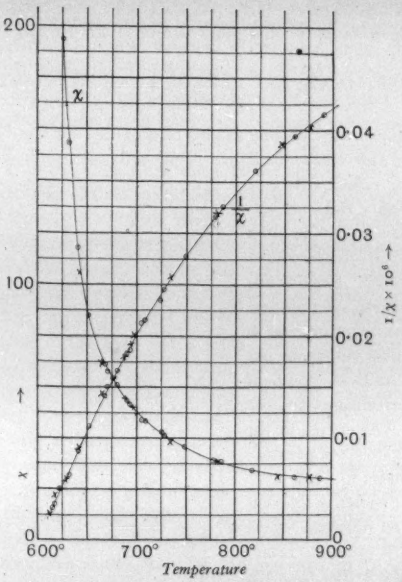

Fig. 4. Variation of χ and $1/\chi$.

Curie point to the straight portion must now be convex upwards. In this case the dotted curve always corresponds to a pull which is greater than that which would be observed if the Weiss law were obeyed throughout the whole range of temperature above the ferromagnetic Curie point. Magnetite, as far as the writer is aware, appears to be the only ferromagnetic substance which gives a curve similar to the complete curve of figure 2. In figure 4 the results obtained by Takagi* are reproduced. His work was carried out by the Faraday method for the determination of susceptibility apparently with a single value of $H.dH/dx$ equal to $2{\cdot}035 \times 10^6$ c.g.s. units. There does not appear to be a straight portion in the curve thus obtained. We must note, however, that Takagi does not discuss the relation between H and χ.

* H. Takagi, *Sci. Rep. Tokio*, **2**, 117 (1913).

In the case of the ferrocobalts which show the abrupt loss of ferromagnetism illustrated in figure 3, Preuss* has shown that the graphs of $1/\chi$ with temperature are straight lines. Just above the temperature at which the ferromagnetism disappears the alloy exists in a β state which persists only over a limited temperature range of some 50 to 100° C., depending on the relative proportions of iron and cobalt. It then passes, very suddenly, in some cases, into the γ state, which is characterised by a much lower susceptibility. The graph for the β state intersects the axis of temperature at the point where the ferromagnetism disappears, within the limits of experimental error. Here, too, the results were obtained by the Faraday method, in which, apparently, only one value of $H.dH/dx$ was used.

§4. EXPERIMENTS ON FERROMAGNETIC COMPOUNDS OF MANGANESE

A series of experiments on manganese phosphide† by the Gouy method gave a curve similar to that shown in figure 1, the difference between θ_f and θ_p being about 20° C. The values obtained for the susceptibility did not appreciably depend on the applied field, at any rate at temperatures over 10° C. above θ_f. Results obtained with manganese arsenide are, however, of much greater interest in this discussion. This substance has a ferromagnetic critical point at 42·2° C. and a ferromagnetic Curie point at 43·2° C. The curves of the reciprocal pulls against temperature for values of H equal to 3500, 2750 and 1950 gauss are shown in figure 5. The pulls obtained with the lowest field are naturally open to rather larger experimental errors than the others. It was found that the susceptibility varied with the applied field up to 92° C., and it is noteworthy that those portions of the curves which are convex upwards end approximately at 100° C. Above 92° C. the values of the susceptibility at different temperatures were quite definitely independent of the applied field. The curves eventually become straight lines at higher temperatures, and the value of the paramagnetic Curie point obtained by extrapolation is 1·5° C., i.e. the ferromagnetic Curie point, is considerably higher than the paramagnetic Curie point, as in the case of the γ states of iron and of the ferrocobalts.

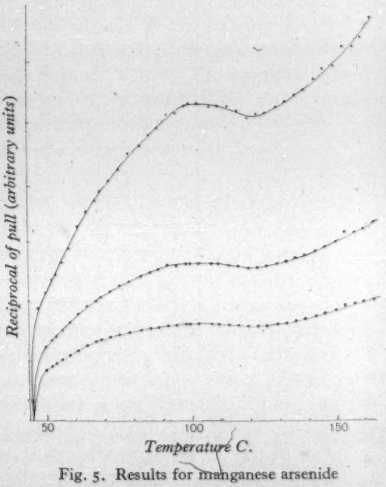

Fig. 5. Results for manganese arsenide

* Preuss, *loc. cit.*

† L. F. Bates, *Phil. Mag.* **8**, 726 (1929).

§ 5. SUGGESTED INTERPRETATION

It appears to the writer that the question whether or not the Weiss law is obeyed at high temperatures is of secondary importance, and that the explanation of the initial curvature of the graph of $1/\chi$ against the temperature is much more important. The explanation appears quite straightforward if we adopt a simple picture of the elementary magnetic particle. Experimental evidence indicates that the latter consists of a group of atoms, the interaction of particular electrons in the group being responsible for its ferromagnetic properties. As the temperature of a ferromagnetic substance rises the number of such groups may change owing to dissociation of the group, thus producing a change in the intrinsic magnetization. There is no reason to suppose that this dissociation is complete at θ_f. We know that the intrinsic magnetization is practically zero at the ferromagnetic Curie point, but that fact need not necessarily imply that the atoms have entirely ceased to be associated in groups. It is easy to see that if the groups still persist above the ferromagnetic Curie point —naturally, in a different state from those below this point—they may possibly produce less magnetic effect than would be produced if all the atoms were free from the forces holding them in the group. In this case the curve of $1/\chi$ with temperature would be concave upwards, and a very slight trace of residual ferromagnetism would be sufficient to cause the shape to change considerably with the applied magnetic field. A tangent drawn at any point on this curve would strike the temperature axis at a point above the ferromagnetic Curie point. As we have seen, iron, nickel and cobalt give curves of this type. If Terry's results* are accepted, then we must admit that the association of the atoms persists over large ranges of temperature above the ferromagnetic critical point. In any case, the work of all experimenters appears to show that the amount of residual ferromagnetism in all three cases must be very small, although a detailed study exists only in the case of nickel†. If, however, the ferromagnetism does not entirely disappear at the ferromagnetic Curie point, but persists in amounts sufficient to mask the paramagnetic effect of the individual atoms, then we should expect the curve of $1/\chi$ against the temperature to be convex upwards, owing to the continual decrease in ferromagnetism with rise in temperature. The curve would, however, depend markedly upon the strength of the applied field, and it is therefore unfortunate that in the case of magnetite, we possess no detailed investigation of the variation of the susceptibility with the applied field. Broadly speaking, then, if association of the atoms persists without ferromagnetism we get a curve concave upwards, whilst if ferromagnetism persists slightly we get a curve convex upwards. It is clear that if ferromagnetism persists slightly over a limited range of temperature above the ferromagnetic Curie point whilst association of the atoms in groups persists over a greater range, we may obtain curves which are at first convex upwards and then concave upwards, and possibly giving a straight portion when the association has entirely disappeared. In these cases we should expect the shape of the convex portion to vary with the strength of the applied field: This is precisely the type of curve which is found with manganese

* E. M. Terry, *loc. cit.* † P. Weiss and R. Forrer, *Ann. de Phys.* **5**, 153 (1926).

arsenide, and it is noteworthy that above 100° C., where the susceptibility is independent of the applied field, the curve approximates closely to those obtained for iron, nickel and cobalt. If further evidence in favour of the views advanced here to explain the behaviour of manganese arsenide is necessary, it is provided by the behaviour of the temperature-hysteresis of the substance as described in an earlier paper*. One further point is of interest. The value of the Weiss constant of the internal molecular field is calculated from the straight portions of such curves. This constant is used in calculations dealing with the properties of the substance below the ferromagnetic Curie point. It is clear from the above views, which we have adopted to explain the complete course of the curves, that its use should more correctly be restricted to regions above the ferromagnetic Curie point. The fact that the agreement between the experimental and calculated values, e.g. of the specific heat, below the ferromagnetic Curie point is so good, is only further evidence that the forces which cause the groups of atoms to associate and form magnetic particles are small compared with the forces which bind the atom in the crystal.

Let us now turn to the results obtained by Preuss with the ferrocobalts, such as those depicted in figure 3. It seems most reasonable to suppose that in these cases the groups of atoms suddenly and completely break up. Hence there is a complete loss of ferromagnetism and the individual atoms are also completely free of the forces which caused them to associate in groups. We should therefore expect the substance to be perfectly paramagnetic, and to give a graph of $1/\chi$ with temperature intersecting the temperature axis at the point at which the ferromagnetism completely disappears. There is some additional evidence for this view, for if the groups are so completely broken we should expect a temperature-hysteresis to occur. Such hysteresis in the ferrocobalts has been shown to exist by Masumoto†.

§ 6. CONCLUSION

It is therefore considered that a reasonable extension of the view that ferromagnetism is due to the association of atoms in groups may account for the paramagnetic behaviour of the simpler elements iron, nickel and cobalt and for that of more complicated substances such as the ferrocobalts, magnetite and manganese arsenide. The position of the paramagnetic Curie point is relatively unimportant. Our survey of existing knowledge indicates that more detailed investigation of the variation of the susceptibility of ferromagnetic bodies just above their ferromagnetic Curie points, particularly with respect to the applied magnetic field, is desirable. The existing experimental evidence in the case of iron, nickel and cobalt indicates that the existence of a true paramagnetic Curie point is doubtful and may only be assumed for the purpose of an approximate calculation of the Weiss constant of the internal molecular field, but it certainly appears desirable to employ the terms ferromagnetic Curie point and paramagnetic Curie point in our discussion of magnetic behaviour.

* L. F. Bates, *Phil. Mag.* **8**, 714 (1929).
† H. Masumoto, *Sci. Rep. Tokio*, **15**, 449 (1926).

§ 7. ACKNOWLEDGMENT

The writer has frequently availed himself of opportunities of discussing the problems raised in this paper with Prof. E. N. da C. Andrade, to whom he desires to express his best thanks.

DISCUSSION

The PRESIDENT referred to the diagram for the ferrocobalts, which showed a very abrupt loss of ferromagnetism, and asked whether the graph obtained by plotting the reciprocal of the susceptibility against the temperature in the region above the ferromagnetic Curie point was a straight line or a curved line.

Dr BATES, in reply, showed lantern slides of the graphs of the reciprocal of the susceptibility against temperature for various ferrocobalts, reproduced from Preuss's dissertation. The graphs for regions just above the ferromagnetic Curie points were straight lines. At higher temperatures more or less abrupt changes, corresponding to the β–γ change in the case of iron, were observed.

VARIATION OF SPARK-POTENTIAL WITH TEMPERATURE IN GASES*

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ABSTRACT. The effect of temperature on spark-potential in hydrogen and nitrogen at ordinary pressures has been studied up to 860° C., over 6000 voltmeter-readings having been taken. The spark-potential is found to depend on the density of the gas and to be independent of temperature and pressure for a given density. Additional ionization of limited amount does not lower the spark-potential at ordinary densities.

§ 1. OBJECT AND SCOPE

The object of the research described in this paper was to determine the effect on the spark-potential of the temperature of the gas in which sparking takes place. This research, which has been carried to much higher temperatures than any reached by previous experimenters in the same field, leads to the conclusion that, over the range of the experiments, the spark-potential is only affected by variation of temperature in so far as it alters the density of the gas. This is in agreement with Paschen's law which, in its general form, states that in a uniform field and in any particular gas the spark-potential is a function of the mass of gas between the electrodes.

Tests have been made at temperatures up to 860° C. at pressures ranging from about 0·25 to 2 atmospheres absolute in hydrogen and nitrogen, the electrodes used being spheres both of copper and of nickel, having a radius of 1 cm. Earlier investigators who have worked within the above range of pressure, have only carried out tests up to about 300° C.† in air, so that a considerable advance has been made. (For details see next section.) Higher temperatures could not be reached with the present apparatus, as the quartz vessel, being necessarily hotter than the spark-gap which it enclosed, showed signs of devitrification. In order to reach higher temperatures, apparatus of a different type must be designed.

* Abridged from a thesis approved for the degree of Doctor of Philosophy in the University of London.

† Since the experiments described here were completed, an account of similar work in Germany has been published by S. Franck, *Archiv f. Elekt.* **21**, 318 (1928). Although Franck's curves do not agree with standard calibrations, they lead to the same conclusions as are given here, except that he finds a drop in the spark-potential at temperatures above 700° C. for gaps greater than 4 mm. As he apparently used steel spheres in air, the surface may have been covered with oxide at that temperature, and some such effect as the roughness of the surface might account for this deviation from the usual law.

§ 2. INTRODUCTION

Theoretical considerations. It is well known that the spectrum of a spark corresponds to that of the gas through which the spark passes, whereas the spectrum of an arc contains that of the electrodes. Hence spark-over depends on the ionization of the gas. Usually a few ions diffuse into the gap from the surrounding space, and then, when the spark-potential is reached, the number of ions is multiplied enormously, and a spark takes place. Townsend's theory, which is to a large extent accepted, accounts for this intense ionization by collision of negative and positive ions. Ionization by collision is affected by the density of the gas, as well as the kind of gas, since the kinetic energy of the ions and their mean free path are concerned. The effect of the density of the gas is therefore of great importance, and many investigators have examined the relation between pressure and spark-length in uniform fields.

The problem on which this research is an attempt to shed light is whether the number of ions is increased as the temperature rises, to such an extent that the spark-potential is appreciably lowered.

Previous work on the effect on spark-potential of variation of pressure. A large number of experimenters have investigated the effect of variation of the pressure and gap-length at atmospheric temperature, chiefly between electrodes giving a uniform or nearly uniform field. Paschen [1] made a series of tests on a number of different gases between spheres of 1 cm. radius, using gap-lengths of 1 to 10 mm. at pressures from atmospheric down to a few cm. of mercury. As a result of these tests he formulated the law known as Paschen's law in the form: "Spark-potential is a function of (pressure × spark-length)." Similar tests had been made by Baille [2], Wolf [3], and de la Rue and Muller [4], and have since been confirmed by many other workers. Further references are not given here, as a long bibliography of earlier work is given at the end of a paper by Edler [5] in 1925, and the whole of this subject is treated by Whitehead [6], who gives a large number of useful references. An important work which should be mentioned here is Schumann's *Elektrische Durchbruchfeldstarke von Gasen* (Berlin, 1923). He also gives a large bibliography (pp. 94–96).

In 1903 Carr [7] made several series of tests with a gap of 3 mm. between parallel plates at low pressures down to 0·5 mm. His curves show a rise in spark-potential at the lowest pressures, but he still finds that Paschen's law holds. The work of Cardani [8], published about the same time as that of Paschen, is cited by Guye and C. Stancescu [9] in support of the use of density, instead of pressure, in Paschen's law. They made tests with a short gap, at pressures ranging from atmospheric up to about 10 atmospheres, using commercial carbon dioxide, and found that Paschen's law holds for the whole range if used in the more general form: "Spark-potential is a function of (mass of gas between electrodes) × (spark-length)."

Tests made by Hayashi [10] at higher pressures still (up to 70 atmospheres) indicate that the spark-potential falls off above 10 atmospheres, and tends towards a limiting value for a given gap-length.

Before leaving the work of the earlier investigators, it is well to draw attention to the fact that many of them made tests in a number of different gases. Very little appears to have been done recently in this direction. Generally speaking, their results show that high spark-potentials are associated with high molecular weights, but the spark-potential is by no means proportional to the molecular weight. This is brought out clearly by Carr's results[7].

Previous work on the effect on spark-potential of variation of temperature. The variation of spark-potential caused by changes in temperature has not attracted so much attention. In 1834, W. Snow Harris[11] performed some experiments on the potential required to cause spark-over between two spheres in a glass receiver. The volume was fixed, and the temperature was varied between 10° and 150° C., "but without in the least affecting the result." "The insulating power of the air was found to be quite independent of its temperature, and to depend only on the density."

An improvement on this method was made by Cardani[8], in 1888. The volume was again fixed, but the pressure of the air was measured with a manometer, the temperature being calculated from the change in pressure. Temperatures up to 300° C. were obtained, but above this the glass vessel became conducting. The temperature-distribution was far from uniform, so that great accuracy could not be expected, but the results were sufficiently consistent to enable Cardani to conclude that the spark-potential is a function of the mass of gas traversed by the spark, and that a temperature-effect is not to be expected until the temperature becomes high enough to reduce the stability of the molecule.

The only further tests* on the effect on the spark-potential of variation of temperature appear to have been made at very low pressures by Bouty[12] (in 1903), and Earhart[13] (in 1910), in air, hydrogen, and carbon dioxide. Bouty's tests were made at temperatures up to 190° C. with pressures ranging from 4 to 10 cm. at atmospheric temperature. Earhart's work does not apply to the region with which the present paper is concerned, as the air he used was only at a few mm. pressure. Under these conditions, up to 600° C.†, he found that the spark-potential in air and carbon dioxide, in this region, depends on the mass of gas between the electrodes and is otherwise independent of temperature. A more recent investigation on the effect of temperature was made by Peek[14] with concentric cylinders, but the electric discharge in this case was visible corona, and not the spark. The range of temperature was − 20° C. to 140° C., and the results show excellent agreement with curves obtained by variation of the pressure only.

From the preceding sections it is seen that Paschen's law holds in its most general form for a wide range of densities, from a pressure of a few cm. up to about 10 atmospheres. The effect of temperature (over the small range studied) appears to be confined to the alteration of the density of the gas, and is, therefore, auto-

* See footnote on p. 96.

† Earhart continued his tests up to 1000° C. in air, but, owing to the experimental difficulties, the results for this higher range do not exhibit the same close agreement, so that it is uncertain whether the above conclusion holds for the higher range.

matically included in the general form of Paschen's law. It remains to be shown whether this holds for temperatures above 300° C. at ordinary pressures. The agreement among Cardani's results is not close enough to remove all question of any deviation, even up to 300° C., and certainly warrants no extrapolation.

Experimental details. In order that the results might be comparable with those of other investigators and with standard calibrations, it was decided, at the beginning of this research, to use a spark-gap comprising spheres 1 cm. in radius. The spheres were made of copper and nickel, the latter being chosen on account of its high melting point, about 1450° C., and its slowness to oxidize. Hydrogen was the gas chiefly used, as it gives a definite spark-over voltage with spheres and keeps the electrodes clean and free from oxide. One series of tests was made in nitrogen as a check, this gas being selected on account of its inert nature. After a considerable time spent in overcoming experimental difficulties a suitable apparatus was evolved having the spheres enclosed in a quartz tube. A heater of the resistance-wire-wound type surrounded this tube, and was found adequate for the immediate requirements, although, in the light of the experience gained during the tests, many improvements might be made with advantage.

The particular object was to compare values of spark-potential obtained at low temperatures with those obtained at higher temperatures, with a view to detecting any decrease in spark-potential at constant density as the temperature rose.

On account of the difficulty of obtaining accuracy in measurements of high temperatures and spark-over voltages, a large number of tests was made, involving over 6000 voltmeter-readings, besides many other measurements. The frequency of supply in all these tests was approximately 51·5 ~. No special attempt was made to maintain a constant frequency or to check it often, as the spark-potential of the sphere-gap is independent of it at all ordinary frequencies. According to Peek[15] and Reukema[16], the same results are obtained for frequencies ranging from 10 ~ to 20,000 ~, after which the spark-potential falls until 60,000 ~ is reached, when the decrease ceases and a steady value is obtained.

§ 3. DESCRIPTION OF APPARATUS

General remarks. The apparatus consisted of a spark-gap chamber of quartz in the form of a tube arranged vertically, in which were enclosed the spheres forming the spark-gap, and the thermo-couple for measuring the temperature. Apparatus for supplying the gas and high voltage were connected to the spark-gap chamber, which was surrounded by a heater at the middle.

Spark-gap chamber. To enclose the spheres in an atmosphere of gas at the required conditions a spark-gap chamber was used, the main part of which was a vitreosil tube *A*, figure 1, $2\frac{1}{4}$ in. in bore and 16 in. long, fixed in a flanged brass casting *B* at the bottom with a sealing-wax joint. A tube $\frac{5}{8}$ in. in bore and 6 in. long was sealed on the top of the main tube, and terminated in a copper tube fixed in with a lead seal. The upper electrode rod, the top part of which was of nickel $\frac{5}{16}$ in. in diameter, passed through the copper tube, and was held by several brass nuts and

adaptors, a washer of special rubber and paint being used to make the joint gas-tight. It was possible to see that the spheres were aligned and that the thermo-couple was placed in its correct position as only the lower part of the large tube was opaque, the rest being transparent. The flanged brass end-piece *B* was bolted down to the base *C*, which also was made from a brass casting. A large washer of the same special rubber was used here, and both sides of the washer and both brass surfaces were carefully greased with tap grease before bolting-up to render the vessel gas-tight. The inlet for gases was through a copper tube, which was screwed and soldered into the base.

Arrangements for measuring spark-gap. One of the most important measurements was the length of the spark-gap. In order to obtain an accuracy of about 1 per cent. on a gap of 5 mm., measurements had to be made to hundredths of a millimetre. To provide for measurement and variation of the gap, the lower electrode rod projected downwards through a stuffing box in the centre of the base *C*. It was of nickel, $\frac{5}{16}$ in. in diameter and 22 in. long. At the top it was turned down on a taper and screwed to take the spheres, which were 20 mm. in diameter. A length of about 3 in. at the bottom also was turned down and screwed ($\frac{1}{4}$ in. Whitworth). The next 4 in. was carefully turned, so as to be smooth and straight for passing through the stuffing box.

To prevent the electrode rod turning round, and also to carry a small steel ball used in measuring the gap, a small brass plate *D* was screwed on the rod, and fixed with a lock-nut. Two rods, screwed into the base *C*, projected vertically downwards, and passed through two holes in this plate. Two strips, shown in section at *E*, were fixed on these rods at the bottom, and formed stops for the nut by which the electrode rod was raised or lowered. Another steel ball was fixed in the bottom of the base vertically above the one on the plate *D*, and an internal micrometer was used to measure the distance between them. This distance was first measured with the spheres just touching, and, when the electrode rod was lowered, the increase in the micrometer reading indicated the length of the spark-gap.

Arrangements for measurement of temperature. This is a measurement of the greatest importance. Owing to the uneven temperature-distribution, it was found necessary to measure the temperature actually in the spark-gap. For this purpose a thermo-couple of platinum and platino-rhodium was used and was mounted in two concentric vitreosil tubes, 22 in. long, which supported and insulated the thermo-couple wires, the lower 12 in. being surrounded by a brass tube $\frac{3}{8}$ in. in diameter, shown at *F*. All three tubes were closed up at the bottom with sealing-wax. The brass tube was smooth and strong for passing through the stuffing box, which was provided in the base *C* for this purpose. The thermo-couple wires terminated in an oil bath in which a thermometer was placed, and copper leads, heavily rubber-covered, connected the thermo-couple from this cold bath to the instrument on which the temperature readings were obtained.

The calibration used was obtained by the National Physical Laboratory for similar thermo-couples taken from the same ingots. The thermo-couple was also compared with another pyrometer, and was found to agree as closely as could be

expected. A curve was obtained giving the E.M.F. in micro-volts corresponding to the cold bath temperature, and this was added on when the temperature was estimated.

Heater. In order to heat the spheres and the gas in the spark-gap, a heater surrounded the main tube of the spark-gap chamber and was capable of raising it to

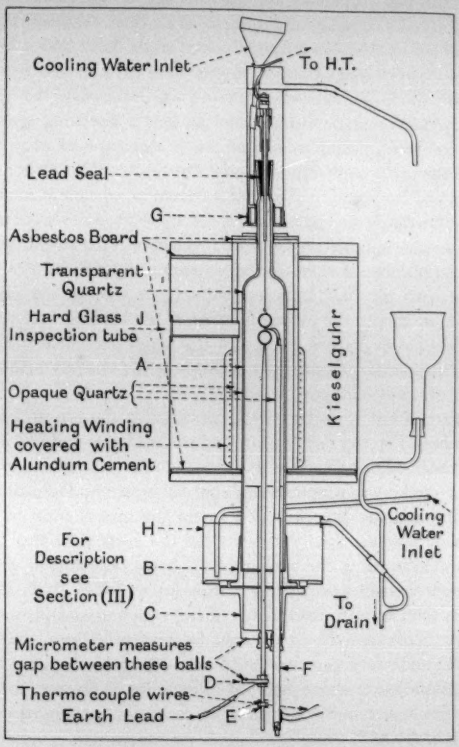

Fig. 1. Diagram of apparatus without supporting framework.

temperatures of about 1000° C. The heating-winding was wound non-inductively on an opaque vitreosil tube $3\frac{3}{4}$ in. in outside diameter, and was about $6\frac{1}{2}$ in. long, "brightray" wire, which is made from an alloy of 80 per cent. of nickel with 20 per cent. of chromium, being used. The winding was covered with alundum cement. To reduce the loss of heat by radiation the furnace tube was surrounded with kieselguhr, which formed an effective lagging material. A sheet iron drum 11 in.

in diameter and 13½ in. high had a hole cut in the bottom, and a sheet of asbestos board, having a hole 3¼ in. diameter in the centre, placed in it. The tube carrying the heating-winding was placed vertically over this hole, and two hard glass tubes for inspection fitted in position as shown at *J*, figure 1. A short length of similar quartz tube was placed above the other, and the space between the tubes and the outer drum was filled in with kieselguhr. Another sheet of asbestos board covered it, and the whole formed a compact unit quite separate from the rest of the apparatus. The base *C* was bolted down in a framework of angle-steel, and cross pieces were arranged for the heater to stand on. Uprights were fixed in this framework to act as guides, so that, when the spark-gap chamber was in position, the heater could be let down over it, into its place, without any danger of knocking against it. Pieces of asbestos board were placed on top of the heater tube to prevent a constant stream of air passing through the inside of the heater. The heater was supplied during the tests and for some time previously from a 100-volt storage-cell battery, but while the temperature was being raised the 100-volt mains were used. For this purpose a double-pole double-throw switch was connected to the heater, and the circuit included a number of rheostats, an ammeter and a fuse.

Interference with the field between the spheres due to the presence of the heating-winding, or due to the magnetic effect of the heating current, has been so reduced as to become negligible, except for the larger gaps of 8 and 10 mm. This has been brought about by placing the heating-winding entirely below the gap, and by employment of a non-inductive winding.

Cooling jackets. Cooling jackets were provided at the top and bottom of the spark-gap chamber. The top jacket protected the lead seal and the joint by which the upper electrode rod was fixed in. It consisted of a glass tube 2 in. in bore with an overflow tube, and it was supported on a special brass cup *G* pushed on to a layer of asbestos string and red lead wrapped round the quartz tube. A piece of thin rubber tube made a water-tight joint between the glass tube and the brass cup. The water was able to reach the bottom of the brass cup, and no trouble was experienced through the asbestos packing getting hot or loose. Both inlet and outlet from this jacket were by drip feed, and sufficient cooling and insulation were obtained in this way up to about 15 kV, when the flow sometimes had to be stopped, as it tended to form a continuous stream. A cylinder with a flange at the bottom on the inside formed the lower water-jacket *H*. It was bolted down to the base, keeping the sealing-wax joint cool and free from strain, as well as cooling the stuffing boxes in the base.

Gas supply. Gas was let into or exhausted from the spark-gap chamber through a copper tube, which was screwed and soldered into the base supporting it and was connected to a glass tube by a sealing-wax joint. This glass tube was several feet long and was connected through a stop-cock to one side of a phosphorus-pentoxide drying-tube, a branch between the stop-cock and the drying-tube being joined to a closed mercury manometer about 12 in. long and an open U-tube, also containing mercury, about 4 ft. long. The U-tube was used for measuring the pressure of the gas, in conjunction with a standard barometer, and the closed manometer indicated

the degree of exhaustion obtained before fresh gas was admitted, or when tests for leaks were being made. In order to connect the apparatus with the vacuum-pump, with a cylinder of gas or with the atmosphere, there were several branch tubes on the other side of the drying-tube, each provided with stop-cocks. The pump was a pulsometer 4-inch Geryk rotary vacuum pump, stated by the makers to be capable of reaching pressures as low as 0·02 mm. It dealt rapidly with the volumes of gas used.

The gases* used were hydrogen and nitrogen, and were obtained from cylinders without any subsequent purification, other than passage through the drying-tube.

High tension supply. The high voltage supply was obtained from an alternator and a small high-tension transformer having a ratio of 75 : 20,000. The field of the

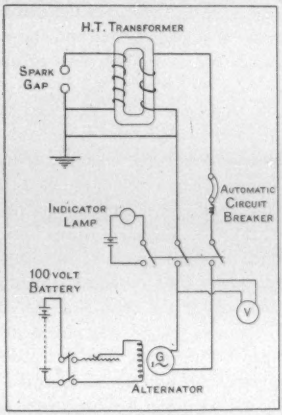

Fig. 2. Wiring diagram.

alternator was separately excited from a 100-volt storage-cell battery, the field-current being controlled by several sliding rheostats conveniently placed so that they could be adjusted while the voltmeter was being read. The voltmeter-readings recorded are those of primary voltage. The primary voltmeter was calibrated against a sphere-gap on the secondary, but an accurate knowledge of the actual secondary voltage is not important, since the primary object is to detect a decrease, if any, in the spark-potential at constant density, and not to compare different spark-potentials. The voltmeter on the primary side provided an accurate and convenient method of measuring the voltage, much quicker than other methods.

The frequency was approximately 51·5 ~ in all the tests. The wiring diagram is shown in figure 2. The primary circuit included a small automatic circuit-breaker

* The British Oxygen Co., Ltd., gave the following approximate analyses: *Hydrogen* 99·5 per cent. pure, the balance being oxygen with a trace of nitrogen; *nitrogen* 99·2–99·8 per cent. pure, the balance being oxygen with traces of helium and neon.

and a three-pole switch. Two poles of this switch connected the alternator to the transformer, and the third connected a small red indicator-lamp, for a danger-signal, to an accumulator. The automatic circuit-breaker was set so that it would come out as readily as possible. All sparks, except those formed at the lowest voltages, caused the breaker to operate very quickly. This was a distinct advantage, as it greatly reduced the heating of the spheres caused by continued sparking.

§4. PROCEDURE DURING TESTS

Four series of tests were made as follows: (i) nickel spheres in hydrogen (preliminary); (ii) copper spheres in hydrogen; (iii) nickel spheres in hydrogen; (iv) nickel spheres in nitrogen. The main series was the third. The chief object of the first series was to obtain data about the heater and the temperature distribution: see figure 8. The temperature was raised by steps of nearly 50° C. when nickel spheres in hydrogen were used, and by 100° C. in the second and fourth series. These two series were made to discover what effects, if any, are due to the use of a different metal in series (ii) and of a different gas in series (iv). The spheres were carefully polished with metal-polish before being put in the apparatus, and remained quite clean in hydrogen. They were not touched during each series of tests.

At each temperature the spark-gap chamber was filled with gas to about 2 atmospheres' pressure, which was a convenient upper limit to the range of pressure used. The spheres were then brought together, so that they were just touching. The micrometer zero-reading was then obtained, and the gap was set to 4 mm. The thermo-couple was replaced in the centre of the gap and the temperature was taken, after which the thermo-couple was pulled down several inches so as not to disturb the spark-gap during tests. The gas-pressure was then read and testing began. About five voltmeter-readings were recorded for each gap, the first one or two sparks being usually not counted. Readings were taken at gaps of 4, 5, 6, 8 and 10 mm. at each pressure, and the pressure was reduced by several steps—usually four or five pressures were used at each temperature—until the primary volts were as low as 8 or 10.

To confirm the temperature-reading and to find out if it had altered by the end of the test, the gap was set to 4 mm., the thermo-couple was again carefully adjusted to the centre of the gap and the temperature-reading was taken. After this the spheres were made to touch again, and the zero checked. As the measurement of the gap depended on the constancy of the zero-reading this was important. Usually it agreed to within 0·02 mm. On a few occasions, when the difference was fairly large, the whole test was repeated. A gradual change of temperature could be allowed for in the calculations by interpolation, but changes in the gap-length had to be corrected for by the drawing of preliminary curves, as mentioned in the next section. Changes not greater than 0·10 mm. were allowed for in this way.

§ 5. RESULTS

Graphical representation. About five voltmeter-readings were recorded for each set of conditions, and the average of these was taken. These average voltmeter-readings were first plotted against the spacing between the spheres for each density, for one temperature only, and in this way variation in the length of the gap could be cor-

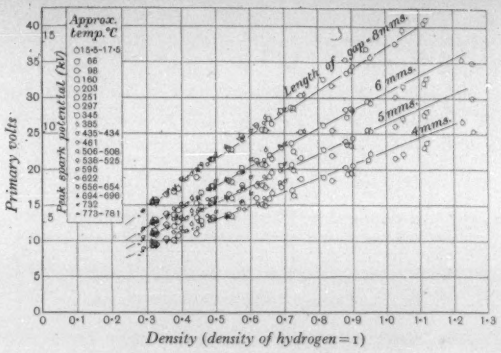

Fig. 3 (1st series). Nickel spheres in hydrogen.

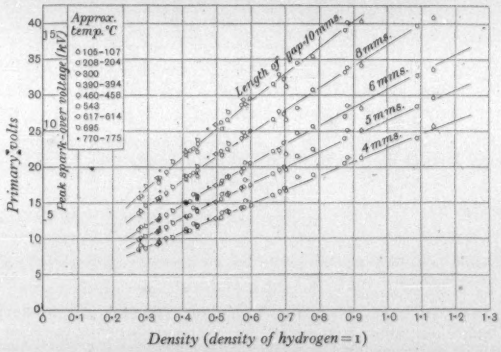

Fig. 4 (2nd series). Copper spheres in hydrogen.

rected for and any discrepancies due to other factors were exposed. After this, the readings were all plotted against density, in figures 3–6, and fell on different curves according to the length of gap. The figures for density give the density of the gas compared with that of the same gas at 0° C. and 760 mm. pressure, the latter density being taken as unity.

It appeared, when the preliminary curves for series (i) and (iv) were plotted, that the readings taken with a gap of 10 mm. were too inconsistent for any conclusions to be based on them. They have, therefore, not been shown among the results plotted in figures 3 and 6, which are confined to shorter gaps.

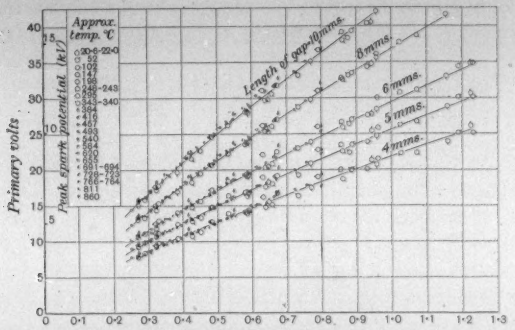

Fig. 5 (3rd series). Nickel spheres in hydrogen.

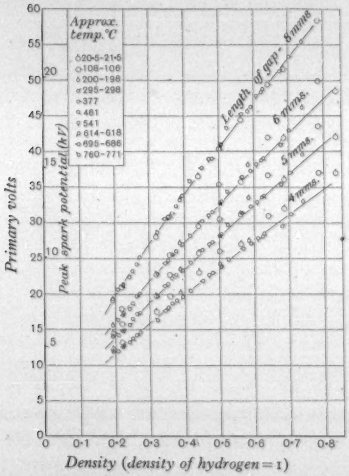

Fig. 6 (4th series). Nickel spheres in nitrogen.

The temperature-readings are shown to the nearest degree, in order to indicate the extent of the temperature-change during each test. In estimating the actual temperature, it is advisable to allow for a possible error of $\pm$ 2 per cent.

Separate curves were also drawn for the 5 mm. gap only, at each temperature, from the data of series (iii). Primary volts, being a measure of spark-potential, were plotted against density as before. The volts at densities 0·4 and 0·5 were read off from these curves, and then plotted against the temperature in figure 7. The points lie approximately along a horizontal line, thus demonstrating that spark-potential is independent of temperature over the range of the experiments.

Effect of temperature on the relation between spark-potential and density. When the results are plotted against density as in figures 3–6, there are, for each gap-length, a number of arrays or sets of points, each set being composed of points obtained at the same temperature. A different sign for each temperature is used to mark the points, so that the sets can be distinguished. Each set, since it represents tests at a given temperature, shows the effect of varying the pressure only at its particular

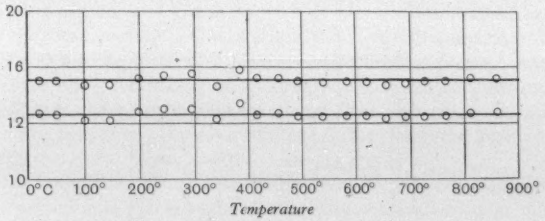

Fig. 7. Relation between temperature and spark-over at constant density (0·4 for lower, 0·5 for upper curve).

temperature. In the method employed the density of the gas was calculated from both temperature and pressure and the points were plotted against density. It was then found that the curves through each set of points, for a given gap, are practically coincident. All the points together produce a smooth curve giving the relation between spark-over and density for all the temperatures, with these particular electrodes.

This agreement leads to two observations about the curve connecting spark-potential with density: (1) that the shape of the curve, and (2) that the position of the curve is unaltered by changes in the temperature, over the range of the experiments. Of course, the points themselves are found on different parts of the curve, in dependence on the density, which does alter with temperature.

These observations lead to the conclusion that, over the range studied, the spark-potential depends on the density of the gas, and is independent of whether the changes in density are caused by changes of pressure or temperature or both. In other words, if the density is kept constant the spark-potential is independent of the temperature. The latter form of the statement follows immediately from an inspection of figure 7, where values of spark-potential for twenty-one different temperatures are plotted for the same density. All the points lie close to a horizontal straight line.

Different gases. The tests of series (iv), in nitrogen, were made in order to find out if there was any effect due specially to the use of hydrogen. The results are similar, the slightly greater irregularity being due probably to the fact that nitrogen is not so good a conductor of heat as hydrogen. It was often some time before the fluctuation of pressure and temperature, caused by changing the pressure, had ceased, and this made the determination of the density difficult. Hydrogen settled down more quickly than the mercury in the manometer.

It might be thought that Paschen's law, which is applicable only to a given gas, might be applied to spark-over in different gases if the term expressing the density were taken to be proportional to either the number of molecules or the mass of gas between the electrodes. Either form applies in any one gas only. By referring to

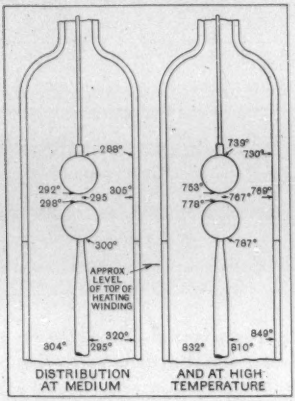

Fig. 8. Approximate temperature distribution in neighbourhood of spark gap.

figures 5 and 6, it is seen, for example, that, with a 4 mm. gap, at density 0·4, the spark-potential in hydrogen is 4·6 kV approximately, and in nitrogen 8·5 kV. These figures, showing the spark-potential in nitrogen to be nearly double that in hydrogen, do not confirm either suggestion. The ratio of the mass of gas in each case is 7 : 1, and if the number of molecules were the basis, the spark-potentials should be identical. It may be mentioned that Paschen and other investigators, who have studied spark-over in nitrogen, hydrogen and other gases, have also obtained results which show that spark-potentials in different gases cannot be related in such a simple manner as that mentioned earlier, i.e. by the taking of the term expressing the density as proportional to either the number of molecules or the mass of gas between the electrodes.

Different metals. Two different metals, nickel and copper, were used as electrodes in these temperature tests. No difference in their behaviour was noticed while the tests were being made. The gas used was hydrogen. When the results for copper in

figure 4 are compared with those in figures 3 or 5, which are both for nickel spheres in hydrogen, practically no difference is observable. Hence it may be concluded that for copper and nickel electrodes, over this range of density, the metal of which the electrodes are formed makes no apparent difference. It is probable that the same results would be obtained with most metals.

At low pressures of the order of 10 mm. or less a number of investigators have obtained different results when using different metals. Dubois[17] found that the presence of impurities on the electrodes caused large differences in the breakdown-voltage, but came to the conclusion that, with perfectly clean electrodes, the sparking-potential is independent of the nature of the metal of which the electrodes are made. Variations, however, are still found with different metals, carefully purified, particularly with sodium[18], and some photo-electric effects have been observed. It seems that these effects are only noticeable when the spark-potential is of the order of a few hundred volts, and no appreciable variation occurs at ordinary densities, when the spark-potential is several kilovolts at least. No alteration in the spark-potential appears to be caused by photo-electric effects at higher pressures and ordinary frequencies, for, according to Reukema[16], at atmospheric pressure and 60 ~ flooding of the spheres with ultra-violet light only serves to increase the accuracy of the sphere gap as a voltmeter, and neither raises nor lowers the spark-over voltage.

Effect of temperature on the initiation of the spark. On comparing voltmeter-readings obtained at high temperatures with those obtained at low or atmospheric temperatures, we at once notice that there is a remarkable regularity among the high temperature-readings, while the low temperature-readings vary slightly. This appears to suggest that at the higher temperatures (say 600° C. upwards) there is a continual supply of ions, which is sufficient to start the discharge directly the applied voltage reaches the spark-potential, but which is not present at low or moderate temperatures. There is, as the curves show, not sufficient ionization to alter the type of discharge, or to reduce the potential required.

When the tests were being made it was found that at high temperatures several readings, practically identical, could be obtained with no difficulty when the voltage was raised gradually. If the voltage was raised to a value slightly below the spark-over voltage and left there no discharge took place, but directly it was raised above this value a spark was produced. At low temperatures, and in the open, it was nearly always found that the voltage could be raised above the value which afterwards appeared to be the spark-potential, and when at last the spark took place a comparatively heavy current passed. The next attempt almost invariably resulted in spark-over at a lower voltage, after which normal values were obtained. If the voltage was raised very slowly it was usually found possible to get well above the normal spark-potential before breakdown occurred. This irregularity at the lower temperatures may be explained by a lack of casual ions or by the presence of some surface effect. The heating of the spheres caused by the spark also accounts for some irregularity, particularly for the low readings.

Reukema[16] mentions a similar irregularity, but overcame it by providing

additional ionization by flooding the spheres with ultra-violet light. He states that, at ordinary frequencies with ultra-violet light, spark-over was obtained directly the voltage reached the required potential, no point differing more than ¼ per cent. from the mean curve, whereas, without ultra-violet light, a difference of about 2 per cent. might occur with any point. The same average value was obtained in both cases.

These results show that at ordinary densities and frequencies some additional ionization does not lower the spark-potential.

§ 6. CONCLUSIONS

Allowance being made for irregularities due to uneven temperature distribution, difficulties in measuring high temperatures, and difficulties in obtaining the correct spark-potential, the results plotted for widely differing temperatures show remarkable agreement. *This indicates that the spark-potential for any particular gap is dependent only on the density, and is independent of any effect due solely to the temperature, over this range*, i.e. *up to* 860° C. The results confirm the part of Paschen's law which relates to density, so far as the range extends; this, in its general form as applied to an individual gas, is: "The spark-potential is a function of the mass of gas between electrodes."

Paschen's law, as applied to different pressures and gap-lengths, of course only holds within a finite but wide range of limits, with uniform fields, and in any one gas. Probably the general conclusion, that changes of temperature and pressure make no difference so long as the density remains constant, holds over a wide range of densities and for non-uniform fields. This conclusion, which applies to both hydrogen and nitrogen, will probably hold for most gases, but the curves relating spark-potential with density, for any two gases, will not be identical in form.

The exceedingly close agreement between series (ii) and series (i) (see § 4 and figures 3 and 4) warrants the conclusion that, so far as copper and nickel* are concerned, *the metal of which the electrodes are made does not directly affect the results*, and it is probable that, if the surfaces are clean and smooth, the results would be practically the same with most metals.

As a result of the deductions in § 5 on the effect of temperature on the initiation of the spark, it appears that additional ionization of a limited extent does not lower the spark-potential at ordinary densities.

§ 7. ACKNOWLEDGMENTS

The author desires to thank the British Electrical and Allied Industries Research Association for their permission to use and publish this information. He would also thank Prof. E. Wilson, of King's College, where the work was carried out, for his continued interest and for his advice, particularly in the difficult early stages. His thanks are also due to Mr H. N. Ridyard for help in connexion with the apparatus.

* It may be mentioned that the melting points of copper and nickel are 1083° C. and 1452° C. respectively. See *Smithsonian Tables*, p. 198 (1927).

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DISCUSSION

Mr S. WHITEHEAD. I should like to ask the author whether he has considered his results in connexion with recent theories put forward to replace or extend the classical Townsend theory. In the theory of Slepian the main causes of the spark-discharge appear to be thermal ionization together with space-charge effects. Although very little thermal ionization takes place at a temperature of 800° or 900° C., yet if the spark normally has a high effective local temperature an increase of 500° or 600° C. ambient temperature, if reflected in the local temperature, would cause a considerable increase in the amount of thermal ionization, if the ionization-potentials were not too high. One would rather expect to find some effect at the temperatures employed by the author if the ionization potentials were of the order of 4 to 10 or 15 volts, which would be more or less true for the gases he employed.

Prof. W. WILSON said that the thermionic effect would not necessarily be large at the temperatures in question: its magnitude would depend on the metal used.

Dr L. F. BATES pointed out that the best-known experiment devised by the founder of the Physical Society was one in which a red-hot ball may lose positive instead of negative charge.

AUTHOR'S reply. In reply to Mr Whitehead: I agree that an effect of the type he described might occur at still higher temperatures, but it is not apparent at 500° or 600° C. If thermal ionization is a cause of the spark-discharge, breakdown will take place when the appropriate temperature is attained, by which time the layers of gas surrounding the inner core, which ultimately becomes the discharge

path, will also be heated, chiefly by conduction from the inner core. The time required to reach this state will be shortened as the ambient temperature is increased, but the final temperature of the inner core will depend on the potential-drop and the properties of the gas.

Thermionic emission, undoubtedly present though small, appears merely to facilitate spark-over.

DEMONSTRATION

Demonstration of an Instrument for Combining Two Curves into One (devised by J. L. HAUGHTON, D.Sc.), *given on November 21, 1930*, by Mr R. PAYNE.

In certain apparatus for recording on a thread recorder the changes of a physical property of a metal with change of temperature, two curves are obtained, one representing change of temperature plotted against time, while the other represents change of the property being measured, also plotted against time. While such pairs of curves can be used to determine the value of the physical property at any temperature, it is often advisable to replot the data obtained, so as to give one curve representing the temperature/property function of the material. The instrument exhibited does this replotting semi-automatically. The thread-recorder drum, containing the two records, is driven slowly below two cross wires which can be moved, by means of lead-screws, in a direction parallel to the axis of the drum. By rotation of the lead-screws while the drum is rotating it is possible to cause the cross-wires to follow the curves. Small lenses are carried above the cross-wires to facilitate reading. One of the lead-screws drives a table whose motion is, therefore, proportional to the ordinates of one curve, say to the temperature. The other lead-screw drives a pencil through the intermediary of a pair of 45° bevel wheels, so that the pencil moves at right angles to, and in proportion to, the ordinates of the other curve. The motion of the pencil relative to the plate will therefore trace out a curve connecting temperature and the physical property being studied.

REVIEWS OF BOOKS

A Text Book of Sound, by A. B. Wood, D.Sc. Pp. xiv + 519. (London: G. Bell and Sons, Ltd.) 25*s*.

This work, as one would expect from the author's reputation, possesses a decidedly practical bias, and, although the mathematical treatment is adequate, mathematics is, throughout the book, a tool used to subserve the needs of the experimenter.

The subject-matter reported is conveniently arranged in five sections, the first of which gives an excellent account of the theory of vibrations and the second of vibrating systems and sources of sound—in particular the maintained tuning-fork and the quartz oscillator. Besides much that is common to most text-books on the subject there are paragraphs on relaxation oscillations, motional impedance, and alternating-current sources, while the analogy between electrical and mechanical oscillating systems is emphasized both here and throughout the book.

The velocity of sound-waves of small and finite amplitude and of normal and ultrasonic frequencies, the effect of change of medium, and the attenuation of waves are among the subjects treated in the next section; here it is satisfactory to find included a comparatively full treatment of diffraction, which is so often considered as peculiar to the domain of physical optics. We note that the author seems to imply that Dixon's method for determining the effect of temperature on the velocity of sound in a gas is a resonance-tube experiment, while recent work by Partington and Shilling using this method is unmentioned.

The section on the reception, transformation and measurement of sound-energy is an important one dealing *inter alia* with the ear, the conversion of sound-energy into electrical energy, directional reception and the analysis of complex sounds, and concluding with the theory of wave-filters in which the line-impedances are "lumped."

In the final section the measurement of distance by sound, the acoustics of buildings, and the reproduction of sound are described, but the author, in view of the magnitude of the subject, has refrained from considering musical instruments. Tables of velocities are given in appendices.

The claim on the jacket of this 25*s*. book that it *amply covers* the requirements of students preparing for university degrees is very modest, inasmuch as the essential requirements in sound for most degrees could be written in an elegant hand on the back of a halfpenny postage stamp and sold for a handsome profit at a penny. The additional claim that it serves as a valuable text-book of reference is justified by the amount of matter reported and a supply of references to original papers. A few tests of the index showed that it was dependable, but disclosed that, although the references to two papers are each given no fewer than three times in the text, the entries for Webster's classical paper introducing the concept of acoustical impedance are omitted.

To sum up: the author has, subject to a few of the clerical errors unavoidable in a first edition, dealt, and dealt well, with "the ever-increasing bulk of new data and methods of investigation which is now available," and has placed upon the market a full, well illustrated and reliable work.

E. J. I.

Lecture Experiments in Optics, by B. K. Johnson. Pp. 112. (London: E. Arnold and Co.) 8*s*. 6*d*.

Those of us who, nurtured on the pages of Lewis Wright, have felt the need for some more modern exposition of lecture methods in optics, heard with pleasure that Mr B. K. Johnson was drawing on his wide experience in these matters in order to prepare such a volume.

In all experimental lecture work, there is a host of detail concerning those practical matters of lighting, dimensions, interchangeable fitments and the like which each lecturer must painfully acquire for himself, or obtain more agreeably from the painful experience of others. Mr Johnson's book is designed to expedite this process, and he has provided his readers with a mass of valuable practical details concerning lecture experiments on reflection and refraction, lenses and mirrors, photometry, the eye, optical instruments, the spectrum, polarization, interference and diffraction.

Some of the experiments seem more suited to the laboratory than the lecture table, and it is with a certain measure of regret that the reviewer finds that Mr Johnson makes practically no use of that very effective method of exhibiting the properties of lenses, mirrors and prisms, which consists in studying the passage through the appropriate system of a series of narrow parallel beams grazing a white vertical board mounted on the lecture table.

Mr Johnson's book can be commended, and we trust that in the near future he will be able to offer to the public a considerably enlarged edition.

A. F.

Applications of Interferometry, by W. Ewart Williams. Pp. viii + 104. (London: Methuen and Co.) 2*s*. 6*d*.

Mr Williams has produced a scholarly but far from elementary book. He groups apparatus for obtaining interference-systems into two broad classes, those based on a division of wave-front, of which the biprism may be taken as typical, and those based on a division of amplitude, as shown in the Michelson interferometer. He discusses carefully and thoroughly the fundamental principles of the methods, devoting special attention to just those difficult points which the average text-book is wont to scamp, and concerning which an inquiring student is apt to put awkward posers.

In so small a volume it is impossible to do more than make a selection of one or two important applications, and Mr Williams has chosen some very apt and interesting instances. In particular, his description of Michelson's method of measuring stellar diameters may be quoted as the clearest account in brief compass with which the present reviewer is acquainted.

Unassuming as it is, the book is a very useful and welcome addition to the literature of physical optics.

A. F.

The Physics of Solids and Fluids, by P. P. Ewald, Th. Pöschl and L. Prandtl. Pp. xii + 372. (London: Blackie and Son, Ltd.) 17*s*. 6*d*.

Among the volumes which have recently appeared dealing, from the point of view of the advanced student, with general properties of matter, this work, which is a translation of certain articles selected from the latest edition of Muller-Pouillet, holds a high place. The book opens with two chapters, from the pen of Professor Pöschl, which deal with elasticity, strength of materials and solid friction. The mathematical treatment, though adequate, is simple, and practical considerations are kept well to the foreground.

Professor Ewald writes on the mechanical structure of solids from the atomic standpoint, and gives, in the compass of some seventy pages, a remarkably interesting account of the lattice theory of polar crystals and the mechanical properties of metallic crystals, concluding with a critical comparison of mechanical properties in metals and non-metals.

The larger portion of the book is devoted to a series of chapters dealing with certain hydrostatic and hydrodynamic problems and is written by Professor Prandtl. Is it necessary to say more than that the reader who opens the book anticipating a thorough

and stimulating treatment of the fundamentals of hydromechanics will not be disappointed? Here again, practical considerations are continually emphasized, and not the least valuable portion of this section is that devoted to problems of aviation.

Those of us who have been nurtured on the arid treatment beloved of the Cambridge school can only sigh and congratulate the more fortunate students of a newer generation.

The translation reads pleasantly, the book is well produced, and the price is modest. It can be recommended unreservedly.

A. F.

Étoiles et Atomes, by A. S. EDDINGTON, F.R.S., M.A., D.Sc., LL.D. Pp. ix + 188 with 13 illustrations. (Paris: Herman et Cie.) Paper cover. 35 fr.

Last summer we learnt from the daily press of a long-haired, frowsy fraternity, anxious to expose their bodies to the sun's radiation in Hyde Park, at the Welsh Harp at Hendon, and elsewhere—sun-bathers, forsooth. Why even Shadrach, Meshech and Abednego, the three youths who, according to holy writ, walked in a fiery furnace, had the "bulge on them" so far as their studies of the effects of radiation on human tissue were concerned. The true cult of sun-bathing is of very recent growth, and its prophets are a select and choice group of our most brilliant physicists, Eddington, Fowler, Jeans, Milne and one or two others. And Sir Arthur Eddington is the champion sun-bather of them all. Listen, "Personnellement je suis mieux chez moi sous la surface du soleil, et j'ai hâte d'y pénétrer" (p. 4). And to think that the temperature beneath the surface may be as high as 40,000,000° (p. 8). Our author is a veritable salamander!

This volume, largely an account in popular terms of Sir Arthur's brilliant contributions to our knowledge of stellar cosmology, is based upon the author's evening discourse before the British Association at Oxford in 1926, and three lectures given at King's College, London, in the same year. The English edition, comprising three chapters, appeared first in 1927. A French translation appeared in the *Bulletin de la Société astronomique de France* in 1928–1929. Since then, our author, following the technique characterizing a Turkish bath, has taken a "cooler" in intersidereal space, where the temperature is of the order of 3° absolute. His experiences there were broadcast as a B.B.C. National Lecture in April 1929. Here they are recounted for the first time in a French edition. It were surely a work of supererogation for me to refer to the contents of the volume. Sir Arthur's contributions to astronomical physics are, or should be, known to all physicists; his style makes one regret the approaching end of the volume as the reading proceeds. Personally, I have been fascinated by his account of the interior of a star (chapter I), recent researches relating to Algol, Sirius, Betelgeuse (chapter II), the age of the stars (chapter III) and the matter of intersidereal space (chapter IV). The translation has been well done—M. Rossignol is a sweet-voiced, capable and well-known translator, and no *rossignol d'Arcadie*—who long since took his *premier essor*. The book is well printed on good paper, is well illustrated and the price is extremely reasonable. I recommend it to physicists generally and, more especially, to examinee students of physics of whom a knowledge of French is required in their examinations. "French, physics and astrophysics without tears" sums up my recommendation of the work.

J. S. G. T.

Band Spectra and Molecular Structure, by R. DE L. KRONIG, Ph.D. Pp. x + 163. (London: Cambridge University Press.) 10*s.* 6*d.*

This book is an elaboration of lectures given at Cambridge in 1929. It is divided into five chapters which deal respectively with the classification of the energy levels of diatomic molecules, the wave-mechanical properties and fine structure of those levels, selection rules and intensity relations, the macroscopic properties (dielectric constants, magnetic susceptibilities, specific heats, etc.) of molecular gases, and the theory of molecular forma-

tion and chemical binding. Generally, the subject matter may be classified into (i) a concise but fairly complete statement of the theoretical results, (ii) an adequate outline of the derivation of these results, and (iii) a very brief indication of the comparison of the theoretical with the experimental results. It is so arranged that certain sections (indicated by asterisks at the beginning and end) in which a knowledge of the wave-mechanics is indispensable, may, with the minimum of interruption of the train of thought, be omitted by the reader who has not the necessary mathematical equipment. After the omission of such sections, very little is left of some of the articles into which the chapters are divided. Of two articles (on the theoretical classification of rotational energy levels), indeed, only the opening paragraph and the closing paragraph remain in each case: so remarkable is Dr Kronig's power of concise expression, and so much of the present theory has the non-mathematical reader to take for granted.

The symbols used conform to the notation recently recommended by Mulliken, for the quantum numbers but not for the energies, frequencies, etc. The following examples of the differences may assist the reader of both authors:

Kronig	Mulliken	Kronig	Mulliken
$W, W_j, W_{j'}$	E, E', E''	ν_0, B_0, ρ_0	ω_e, B_e, r_e
$W^o_{q\Lambda\Omega}, W^o_{q\Lambda}$	E_v	$v, J; v', J'$	$v', J'; v'', J''$
$W^o_{q\Lambda\Omega v}, W^o_{q\Lambda v}$	$E_e + E_v$	even terms (×)	positive terms (+)
$W^o_{q\Lambda\Omega v J}$ (in case *a*) $W^o_{q\Lambda\Omega v K}$ (in case *b*)	$E_e + E_v + E_r$	odd terms (*o*)	negative terms (−)

Particularly, the reader must note that Prof. Kronig still uses the words "even" and "odd" in the sense in which he himself first used them, where Hund and Mulliken now use the words "positive" and "negative" and in quite a different sense from the latter writers' use of *gerade* and *ungerade*.

The published records accessible to the reviewer do not justify the statement on p. 97 that "the fact that the molecules $O^{16}O^{17}$, $O^{16}O^{18}$, and $C^{12}C^{13}$ do not have half of the lines missing like the molecules $O^{16}O^{16}$ and $C^{12}C^{12}$ led Birge and King to the discovery of the isotopes O^{17}, O^{18} and C^{13}." Actually, the discovery of O^{17} and O^{18} was announced by Giauque and Johnston, the fact that $O^{16}O^{18}$ has two lines to every one of $O^{16}O^{16}$ was first pointed out by Babcock, and C^{13} was discovered by King and Birge, who, however, made no mention of there being two lines of $C^{12}C^{13}$ to one of $C^{12}C^{12}$ (though they would be expected, of course). $^2\Pi_1$ on the second line of p. 52 is obviously a misprint for $^3\Pi_1$.

The work closes with a subject-index and a well classified list of 352 references, of which only about half a dozen are to publications earlier than 1923.

With Prof. Kronig's authoritative account of the purely theoretical aspect, Prof. Mulliken's comprehensive "Interpretation of Band Spectra" now appearing in *Reviews of Modern Physics*, the same writer's forthcoming book, and a book which is being written (in English) by a well-known Scandinavian investigator, the English student would appear to be well provided with descriptive works on this subject for several years hence. In addition, there are two recent German monographs (one of which is shortly to re-appear in English), and a forthcoming book by a prominent German band-spectroscopist. At present, however, there is a definite lack of a volume of well-chosen and well-arranged numerical data for the many known band-systems and reliable values of the molecular constants derived from their analyses, with a profusion of energy-level and other diagrams and a bare minimum of descriptive matter. Such a volume, no doubt, would be a useful supplement to the works just mentioned, and would also facilitate the identification of band-spectra in the laboratory. The reviewer is here venturing to express a long-held and considered opinion, for which he has recently found support in conversations with several spectroscopists including Prof. Mulliken himself. But perhaps all this is irrelevant in a review of Prof. Kronig's very welcome book.

W. J.

The Principles of Quantum Mechanics, by P. A. M. Dirac. Pp. x + 258. (London: Clarendon Press.) 17*s*. 6*d*.

In the preface to the *Mécanique Analytique* Lagrange remarks, "On ne trouvera point de figures dans cet ouvrage." Dr Dirac's book is conceived in much the same spirit and on p. 18 we find the significant observation, "One does not anywhere specify the exact nature of the symbols employed, nor is such specification at all necessary." The prejudice of the present reviewer in favour of the wave-mechanical language and symbolism, though not entirely removed, has been greatly shaken by studying this work, and he feels compelled to describe it as a notable achievement and one which is not unworthy of comparison with the great French classic.

The difficulties which have beset the path of the growing quantum theory are very largely due to our inveterate insistence on building up theories in terms of concepts and images borrowed from our immediate experience and observation. It is becoming more and more evident that a logical account of sub-microscopic phenomena cannot be given in terms of familiar concepts in the old-fashioned causal space-time manner. This accounts for the unspecified nature of the symbols in Dr Dirac's symbolic method. They are as it were the expression of new concepts, and the author has pursued the ideal of presenting the new mechanics in terms of them in a rigorously logical form, shorn of all irrelevancies. He claims, perhaps rightly, that his method goes more deeply into the nature of things than does that of wave mechanics; but it would seem to have profited greatly by suggestions from the latter mode of exposition, as the constant use of such terms as "superposition," "interference," "phase" and the like indicates.

While the practical physicist will probably prefer the wave-mechanical form of theory to Dirac's symbolic form (and will probably profit more from it), he cannot afford to ignore the other. Notwithstanding the abstract character of parts of the book it is, as the author maintains, in close contact with physics and contains many important applications. It begins with a chapter on the principle of superposition followed by chapters on the symbolic algebra of states and observables, *eigen* values and *eigen* functions (it is to be regretted that Dr Dirac has not seen fit to render *eigen* by an English equivalent), transformation theory, etc.; while the latter part of the book is devoted very largely to applications.

The book can be recommended to physicists (perhaps also to pure mathematicians, though there is a slight doubt whether they will understand it) not only because the author is one of the chief contributors to the new theory, but because there is much in it of interest and value even for purely experimental physicists. Moreover, once the reader has mastered the first four or five chapters he will find the book extraordinarily fascinating.

W. W.

Geophysical Memoir No. 50. *Practical Examples of Polar-Front Analysis over the British Isles in* 1925–6, by Dr J. Bjerknis, Director of the Geophysical Institute, Bergen. 4to, pp. 21, and 27 plates. (London: H.M. Stationery Office, 1930.) 3*s*. net.

The advances which have been made in dynamical meteorology in recent years by Norwegian meteorologists make it very desirable that their methods should be tested by application to the conditions which are found to exist in other countries, and in the present memoir Dr Bjerknis applies them to the Meteorological Office records of the weather for a period of high pressure, March 31–April 1, 1925; one of moderate pressure, February 10–11, 1925; and one of somewhat lower pressure, January 22–23, 1926. He shows that the theory of the polar front allows the physical processes involved in the weather-changes

experienced to be followed readily, and adds to it in this memoir an explanation of the changes of the fronts from sharp to diffuse or *vice versa* which agrees with that suggested by the late Mr A. Giblett in 1923.

C. H. L.

Geophysical Memoir No. 51. *A Study of Visibility and Fog at Malta.* By J. WADSWORTH, M.A. 4to, pp. 23. (London: H.M. Stationery Office, 1930.) 1*s.* 6*d.* net.

The observations of Mr Wadsworth were made at Valetta at 8 a.m., 2 and 7 p.m. from 1919 to 1925. He finds that mist or fog on the coast is more frequent in winter than in summer and that it tends to disappear by midday. In the absence of fog, visibility extends to 30 miles at least; it is best in winter with N.W. winds and least in summer with E. winds.

C. H. L.

Meteorological Office Reseau Mondial 1923. 4to, pp. xv + 115. (London: H.M. Stationery Office, 1930.) 25*s.* net.

This annual account of the variation of pressure, temperature and precipitation over the surface of the earth from 80° N. to 70° S. has now appeared for 13 years; and for nearly all the observing stations, of which there are two to each 10° square, normal mean values of the three quantities are available. Many years must elapse before sufficient observations have accumulated to justify statements as whether and how they are changing with time.

C. H. L.

Magnetic, Meteorological and Seismographic Observations made at the Government Observatories, Bombay and Alibag in 1926. Reduced and tabulated by Dr S. K. BANERJI. Fsc. pp. iii + 135, and 5 plates. (Calcutta, 1930.) 17*s.* 6*d.*

This volume is on the same lines as the one for 1925 and extends the record of continuous meteorological and magnetic observations to 81 years and of seismological observations to 28 years. Pressure, rainfall, wind-velocity and direction, temperature, and humidity are tabulated for each hour, cloud at seven periods each day, magnetic declination, horizontal and vertical forces for each hour, and the time, amplitude and period of each seismic disturbance.

C. H. L.

THE PROCEEDINGS OF THE PHYSICAL SOCIETY

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PHYSICAL AND OPTICAL SOCIETIES

TWENTY-FIRST ANNUAL EXHIBITION OF APPARATUS

Opening address delivered by the President

SIR A. S. EDDINGTON, F.R.S.

THE present occasion marks the coming of age of this Exhibition, which has become an outstanding annual event in the scientific world, for it is the 21st of the series. It is therefore appropriate to recall the history of this informal congress of the makers and the users of scientific instruments.

It started on December 15, 1905, when the Physical Society held an evening meeting at the Royal College of Science at which certain instrument makers by invitation exhibited their latest productions. Tickets were issued to the members of the Optical Society and of the Institution of Electrical Engineers also. Seventeen firms, most of which are represented on the present occasion, then exhibited. The attendance was about 240—greater than had been anticipated.

The usefulness of and interest in the Exhibition increased year by year. In 1909 the attraction of discourses by leading physicists was added. The first two of these discourses were given by Prof. C. V. Boys and the late Prof. Sylvanus Thompson. This year we are to hear Mr E. Lancaster-Jones and Sir Gilbert Walker.

There was a suspension during the war. On the resumption in 1920 the number of exhibiting firms had risen to 72. Perhaps I should say that this was the number of voluntary exhibitors, because on that occasion there were also a large number of captured German instruments on view.

At this stage the Optical Society joined in. The Exhibition had hitherto been managed by the Honorary Secretary of the Physical Society personally at great sacrifice. It was now entrusted to a Joint Committee of the Physical and Optical Societies, the expense being shared between the two bodies; and I must not leave out a third party whose action makes the holding of this Exhibition possible, namely the Governors of the Imperial College of Science, who have throughout this long

period recognized the value of the Exhibition to workers in Science and have freely placed accommodation at our disposal. It is not only from a financial standpoint that we appreciate this. This is not a trade exhibition in the ordinary sense; and the holding of it within the walls of a great institution of learning and research creates an atmosphere which we should find it hard to imitate in any public exhibition hall. By distribution of the exhibits among the college laboratories use can be made of the facilities for demonstrating the apparatus under working conditions.

In 1926 three new sections were added representing (*a*) typical results of recent research, (*b*) effective but little-known lecture experiments, and (*c*) apparatus of historical interest and repetitions of famous experiments.

The research section is of especial interest and value. Reading the catalogue I cannot but be struck with the conspicuous part taken in it by the research laboratories of the great commercial firms. That is one of the interesting developments of the present day. The field of pure research is no longer a matter solely for university laboratories.

In 1926 it was decided, in view of the general public interest in scientific developments, that the duration of the Exhibition should be increased by one day so that the general public might be freely admitted without detriment to the character of the exhibition as a meeting-place for the makers and users of scientific instruments. This year the public day is Thursday, January 8.

Last year another innovation was introduced, namely a competition for craftsmanship and draughtsmanship, the work of learners in the exhibiting firms. Judges appointed by the Councils of the two Societies viewed the fifty-three exhibits certified as the unaided work of individual apprentices, and awarded prizes subscribed for by the employers. No visitor could fail to be impressed by the high standard attained, and it is believed that the new section will have beneficial results in encouraging and raising the standard of British workmanship and skill. The exhibits in this section on the present occasion will be on view in the Science Museum for a few weeks after the close of the exhibition, by the courtesy of the Director.

An Exhibition like this brings home to us two things—the debt of the scientific worker to the instrument-maker and the debt of the instrument-maker to the scientific worker. I do not know which to place first. I do not think one can be placed before the other; for the instrument-maker provides the resources of the scientific worker, and the scientific worker provides the resources of the instrument-maker. It is of great interest to those of us who are interested in the purely scientific side to see how discovery and progress has been turned to account in these applications, and to see the practical working of scientific method. My instrument-maker friends, I know, like to try to shock me by professing to be unscientific. "We don't really calculate out these curves and dimensions," they say "we go on by trial and error until it comes right." That does not shock me in the least, for "trial and error" can be as scientific as any other method. It is indeed one of the most efficient devices for the harder problems not only in pure physics but in mathematics; and I know well the skill and systematic planning that are needed to make it an instrument of progress. Nor do I think less of the instrument-designer because he sees by trained

instinct what we can only worry out by laborious formulae. I look on him as the master chess-player who grasps the strong and the weak positions of the game intuitively, whilst we inexperienced players have to analyse the situation by moving the pieces one by one.

But there is another less direct debt which the instrument-maker owes to the scientific worker—the stimulus to satisfy new requirements. How often does it not happen that an instrument is designed to meet some new development of scientific research, perhaps with no thought that more than one instrument of the type will ever be needed; a few years later the same instrument is being turned out in hundreds as an indispensable adjunct to commercial operations.

I suppose that the earliest scientific instrument of any high degree of elaboration and delicacy of workmanship belonged to my own science, namely the Astrolabe. We can read in Chaucer of the astrolabe which he obtained for "Litel Lowis my sone, having perceived wel by certeyne evidences thyn abilite to lerne sciencez touchinge noumbres and proporciouns." I cannot forbear quoting a few lines, since it is in a style somewhat different from most of our handbooks on astronomical instruments.

> Thyn Astrolabie hath a ring to putten on the thoumbe of thy right hand in taking the heighte of thinges. And tak keep, for from hennes-forthward I wol clepe the heighte of anything that is taken by thy rewle, the altitude, withoute mo wordes....The est side of thyn Astrolabie is cleped the right side, and the west side is cleped the left side. Forget nat this, litel Lowis.

Ever since the days of the astrolabe, astronomy has been one of the most pressing of the sciences in its demands on the instrument-maker. It is well-known how the telescope was invented by a Dutch spectacle-maker. Testing his lenses by looking through them at a church spire he happened to place two, one behind the other, and saw a greatly enlarged image of the spire. The telescope has had a varied history. At one time before the invention of achromatic lenses, in order to get rid of the colour difficulty, telescopes ran to enormous and unwieldy lengths. Telescopes 300 and 600 feet long were made. The longest telescope of which we have any record of actual use in observation was 212 feet long; it was used in 1722 by Bradley for measurements of the planet Venus. Compared with this, even the 100-inch reflector at Mount Wilson is a handy little instrument. The old Statutes of my own Professorial Chair, which was created about this time, lay it down that the Professor should observe with a telescope not less than 16 feet long—which I suppose was a very modest demand for that period. I am glad to say that, notwithstanding the greater compactness of modern instruments, I fulfil this requirement—but without much margin to spare.

The discovery of achromatic object-glasses in 1733 and the development of reflecting telescopes have changed all that. Achromatism in particular illustrates what I have said as to the way in which an instrumental development produced for some purely scientific purpose is later employed in all kinds of practical applications. I think that at that time the telescope was almost wholly an instrument for astronomical purposes; indeed its unwieldiness almost precluded any commoner

application. And it was the urgent need for the improvement of astronomical observation which called forth the invention of colour-free lenses. Nowadays there is hardly an instrument of any complexity which does not employ an achromatic lens in some part of its system.

As an astronomer I am filled with the deepest gratitude to the practical instrument-maker. Gratitude has been defined as "an expectation of favours to come," and I will not reject that definition in my own case. For a long time to come we shall be asking for more, making those impossible demands which sooner or later the instrument-maker and his research staff bring within the bounds of possibility. At present great hopes are centred on the projected 200-inch reflector which the Carnegie Institution has decided upon, doubling the aperture of the present largest telescope—the 100-inch at Mount Wilson. That will undoubtedly carry us a long way further into the mysteries of the universe. Yet it is interesting to reflect that just the same results might come about by one of those simple, almost accidental, discoveries in a research laboratory. Suppose that whether by further insight into the theory of photochemical action, or by systematic application of the method of "trial and error" which, as I have said, is also a truly scientific procedure, we can produce a photographic plate four times as sensitive as those at present in astronomical use; that would effectively turn the present 100-inch into a 200-inch, and at one stroke bring within reach all that is expected from the great new telescope. In fact for many purposes, such as the exploration of the nebulae, it would be more effective. And not only would such a discovery make the 100-inch equivalent to a 200-inch, but virtually it would double the aperture of every photographic telescope throughout the world.

Our acknowledgment of ever-increasing debt to, and dependence on, the instrument-maker is not without a touch of regret for the good old days. Astronomers, it is true, have long been dependent on highly complex and expensive instruments, which once acquired have to be used to the utmost. For most observatories the question is not so much what kind of astronomical work offers the most promising prospects at the moment as what kind of astronomical work their particular equipment is best suited for. But until lately the physicist was free from that kind of limitation, and designed his apparatus to suit his work rather than designed his work to suit his apparatus. The days are past when one could put together in the workshop some "tin-tacks and sealing-wax," and then set to work and discover something. Times change and we must move with the times. And here we have on view the latest appliances with which we can equip ourselves for further and further advance, to wrest their secrets from the electron, the atom and the star.

Now Ladies and Gentlemen, we are standing before the entrance of a magic cave containing wonders worthy of the Arabian Nights. You will not find therein sacks of gold and diamonds and rubies, for I think that neither the maker nor the user of scientific instruments has very much to display of that kind of wealth. Nevertheless, it is a cave of treasure—the treasure that is evolved by the cunning of the brain and the treasure that is evolved by the cunning of the hand. In this complex civilization of ours, wherever we turn, we meet with the applications of science; and here you

will find gathered together the nucleus of them all—the linking of pure science to technical industry. The eastern story was not far wrong, when it placed the treasure under the charge of the Slave of the Lamp, to be possessed by whoever should learn how to control him. You will find very much in evidence in our treasure-cave the Slave of the up-to-date Lamp—electricity. It remains for me to pronounce the words that shall throw open the doors of the cave. Open Sesame!

A POINT OF ANALOGY BETWEEN THE EQUATIONS OF THE QUANTUM THEORY AND MAXWELL'S EQUATIONS

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ABSTRACT. It is known that there is a close analogy, exhibited by the use of five-dimensional tensor notation, between the electromagnetic equations in free space and the first- and second-order equations of the quantum theory. In the present paper an analogy is traced between two tensors, one of which is related to the magnetic and electric moments of a doublet while the other represents the electric and magnetic field-strengths.

Darwin*, Whittaker†, Frenkel‡ and others have shown that there is a close analogy between the electromagnetic equations in free space and the first-order equations of the quantum theory. These equations were first developed by Dirac from considerations which have nothing to do with this analogy. The resulting second-order equations contain, in addition to those appearing in Schrödinger's equation, terms which are just those required to account for the duplexity phenomena of the atom.

The general form of the electromagnetic equations in four-dimensional space is:

$$\frac{1}{g^{\frac{1}{2}}}\frac{\partial}{\partial x^n}(g^{\frac{1}{2}}F^{mn}) = J^m \qquad \text{.....(1),}$$

$$\frac{1}{g^{\frac{1}{2}}}\frac{\partial}{\partial x^n}(g^{\frac{1}{2}}G^{mn}) = K^m \qquad \text{.....(2).}$$

Flint and Fisher§ have generalized a system of equations in five-dimensional space to take the place of (1). They are written in the following form:

$$\frac{1}{\gamma^{\frac{1}{2}}}\frac{\partial}{\partial x^\nu}(\gamma^{\frac{1}{2}}A^{\mu\nu}) - \frac{2\pi i}{h}\Pi_\nu A^{\mu\nu} = 0 \qquad \text{.....(3),}$$

where J^m is put equal to zero and additional terms have been introduced which correspond to the metric adopted. $\Pi_a = (p_a + e\phi_a)$, where Π_a is a momentum introduced by W. Wilson‖ in the study of the motion of the electron, p_a denotes the component of four-dimensional momentum and ϕ_a is the electromagnetic potential. $A^{\mu\nu}$ is an antisymmetric tensor whose metrical significance has not yet been found. The authors referred to have also developed second-order equations which are

Π_a
p_a
ϕ_a
$A^{\mu\nu}$

* *Proc. R. S.* A, **118**, 654 (1928).
† *Proc. R. S.* A, **121**, 543 (1928).
‡ *Einführung in die Wellenmechanik.*
§ *Proc. R. S.* A, **126**, 644 (1930).
‖ *Proc. R. S.* A, **102**, 478 (1923).

identical with those of Frenkel and Whittaker. As an example of one of these we may write the following:

$$\square\psi_0 - \frac{2\pi ie}{h}(H_1A^{23} + H_2A^{31} + H_3A^{12} - iE_1A^{14} - iE_2A^{24} - iE_3A^{34}) = 0 \quad \ldots\ldots(4),$$

where
$$\square\psi_0 = \Sigma\frac{\partial^2\psi_0}{\partial x^2} - \frac{4\pi ie}{h}\phi_m\frac{\partial\psi_0}{\partial x^m} - \frac{4\pi^2}{h^2}(\Sigma e^2\phi_m{}^2 + m^2c^2)\psi_0 \quad \ldots\ldots(5). \qquad \square$$

The foregoing considerations were based upon a close analogy with the propagation of light as represented in a four-dimensional continuum. The track of a light quantum in this continuum is a nul geodesic

$$ds^2 = 0.$$

The equations of Maxwell for empty space are those which we obtain by writing $J^m = 0$ and $K^m = 0$ in (1) and (2). The view of Flint and Fisher is that the nul geodesic $d\sigma^2 = 0$ for a five-dimensional continuum will give the track of the electron and that the equations in this space corresponding to (1) and (2) will give the first-order equations of the quantum theory. In the paper referred to above they develop this idea.

The object of the present paper is to bring to light another point in the analogy between this system of equations and those of Maxwell. Equation (4) can be written in the following vectorial form:

$$\square\psi_0 - \frac{2\pi ie}{h}(H.a - iE.b) = 0 \quad \ldots\ldots(6),$$

where
$$a_1 = A^{23}, \quad a_2 = A^{31}, \quad a_3 = A^{12}; \qquad a, b$$
$$b_1 = A^{14}, \quad b_2 = A^{24}, \quad b_3 = A^{34}.$$

Writing
$$a = \frac{4\pi im\psi_0}{he}M \text{ and } b = -\frac{4\pi m\psi_0}{he}N \qquad M, N$$

we can put equation (6) into the following form:

$$\square\psi_0 + \frac{8\pi^2m}{h^2}(H.M + E.N)\psi_0 = 0 \quad \ldots\ldots(7).$$

But Schrödinger's wave equation is

$$\nabla^2\psi_0 + \frac{8\pi^2m}{h^2}(W - U)\psi_0 = 0 \quad \ldots\ldots(8),$$

which is a special case of the equation $\square\psi_0 = 0$.

From (7) and (8) we get

$$\square\psi_0 = \nabla^2\psi_0 + \frac{8\pi^2m}{h^2}(W - U)\psi_0 + \frac{8\pi^2m}{h^2}(H.M + E.N)\psi_0 \quad \ldots\ldots(9).$$

The last term of equation (9) represents an additional energy due to a doublet. From this we infer that M is the magnetic moment and N is the electric moment of the doublet. Our antisymmetric tensor $A^{\mu\nu}$ is therefore evidently related in some simple way to these two quantities.

The complete scheme for $A^{\mu\nu}$ is:

$$A^{\mu\nu} = \begin{vmatrix} 0 & \frac{4\pi im}{he} M_3\psi_0 & -\frac{4\pi im}{he} M_2\psi_0 & -\frac{4\pi m}{he} N_1\psi_0 & \frac{4\pi m}{he} N_1\psi_0 \\ -\frac{4\pi im}{he} M_3\psi_0 & 0 & \frac{4\pi im}{he} M_1\psi_0 & -\frac{4\pi m}{he} N_2\psi_0 & \frac{4\pi m}{he} N_2\psi_0 \\ \frac{4\pi im}{he} M_2\psi_0 & -\frac{4\pi im}{he} M_1\psi_0 & 0 & -\frac{4\pi m}{he} N_3\psi_0 & \frac{4\pi m}{he} N_3\psi_0 \\ \frac{4\pi m}{he} N_1\psi_0 & \frac{4\pi m}{he} N_2\psi_0 & \frac{4\pi m}{he} N_3\psi_0 & 0 & i\psi_0 \\ -\frac{4\pi m}{he} N_1\psi_0 & \frac{4\pi m}{he} N_2\psi_0 & \frac{4\pi m}{he} N_3\psi_0 & -i\psi_0 & 0 \end{vmatrix} \quad \ldots\ldots(10),$$

or

$$A^{\mu\nu} \propto \begin{vmatrix} 0 & M_3 & -M_2 & iN_1 & N_1 \\ -M_3 & 0 & M_1 & iN_2 & N_2 \\ M_2 & -M_1 & 0 & iN_3 & N_3 \\ -iN_1 & -iN_2 & -iN_3 & 0 & \frac{he}{4\pi m} \\ -N_1 & -N_2 & -N_3 & -\frac{he}{4\pi m} & 0 \end{vmatrix} \quad \ldots\ldots(11).$$

The complete scheme for F^{mn} is:

$$F^{mn} = \begin{vmatrix} 0 & -H_z & H_y & iE_x \\ H_z & 0 & -H_x & iE_y \\ -H_y & H_x & 0 & iE_z \\ -iE_x & -iE_y & -iE_z & 0 \end{vmatrix} \quad \ldots\ldots(12).$$

Comparing (11) with (12) we notice that $A^{\mu\nu}$ is a tensor in five-dimensional space whose components are related in a simple way to the magnetic and electric moments of the doublet, while F^{mn} is a tensor in four-dimensional space whose components are the electric and magnetic field-strengths.

SOURCES OF ILLUMINATION FOR ULTRA-VIOLET MICROSCOPY

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ABSTRACT. The paper describes experiences encountered in an attempt to find means for reducing the exposures hitherto necessary in ultra-violet microscopy. Amongst the subjects dealt with are quantitative measurements of the relative intrinsic brightness of spectrum lines given by various sources of radiation; methods of producing a monochromatic source suitable for this work; the steadiness of the illuminant; the effect on the definition of the image when a triple spectrum line is used as a source; the study of electrical conditions for the production of increased brightness of the spark, such as the effects of change in frequency, secondary potential, capacity, and energy input; and the production of a compact and inexpensive electrical unit for use with the ultra-violet microscope.

§ 1. INTRODUCTORY

One of the difficulties in ultra-violet microscopy is the provision of a suitable source of radiation which has sufficient luminosity to enable photomicrographs to be taken with a short exposure. Up to the present, when the highest-power immersion monochromat objective is used, the duration of exposure for the transparent type of objects, such as those used in biological and allied work, has been of the order of 10 to 30 seconds, whilst in metallurgical work exposures of 3 and 4 minutes have been found necessary.

In biology, an exposure of the length of 10 seconds necessitates "fixing" of the object in order that Brownian and other movements may be prevented. Moreover such an exposure to ultra-violet radiation of the wave-length at present employed has in some cases a germicidal action on the specimen. For metallurgical objects 3 or 4 minutes' duration of exposure may under certain conditions—for example, instability of the microscope or a change of refractive index of the immersion fluid—prevent precisely focussed photographs from being obtained, and from an economical point of view a reduction in this exposure would seem advisable. In other branches of work also, in which the ultra-violet microscope could be used, the question of shortness of exposure is likely to be of importance, and therefore an attempt has been made to find means for reducing the length of exposure at present required for the taking of ultra-violet photomicrographs generally. An account of the work carried out forms the subject of this paper.

§ 2. THEORY

As the photographic density of the image on the plate is the ultimate criterion, the following relation is the one concerned in this work:

$$\text{Illumination of unit area of image} \propto B \times \left(\frac{\text{N.A.}}{m}\right)^2 \times t,$$

where B is the intrinsic brightness of source,
N.A. the numerical aperture of microscope objective,
m the magnification employed, and
t the transmission factor of the optical parts.

It is evident from this that when once the objective has been decided on and the numerical aperture therefore fixed, the only two variables in the formula are B and m. The latter quantity, however, must not fall below a limiting value if the grain of the plate is to resolve the image, and it is obviously better to use this minimum value where short exposures are desired, rather than to employ high "empty" magnification. This leaves us, therefore, with the increasing of the brightness of the source as the only hope for a reduction in exposure.

The conditions which govern an emission source suitable for this work are as follows: (1) the radiation given by the source must be strictly monochromatic, i.e. must comprise one spectrum line with no other intense lines near it; (2) the source should give maximum possible energy in this region; (3) for existing monochromats the wave-length of the line must be between $0{\cdot}29\mu$ and $0{\cdot}25\mu$.

It is seen, at once, that these conditions place considerable restrictions on the choice of the most suitable source from those which are available. As regards condition (2), it is well known that the amount of energy in the spectra produced by the flame and the arc is greatest in the infra-red region, whilst in the solar spectrum the maximum lies in the visible green, whereas in the case of the spark spectrum the maximum energy is concentrated in the ultra-violet region. The following figures give an illustration of the relative energy distribution in the spark spectrum:

Infra-red	10
Visible	24
Ultra-violet	66

It will, then, be found advisable to employ the spark method for producing the illuminant, for it appears (as described later) that the intrinsic brightness of the spark is proportional to the amount of energy liberated across the spark gap.

Condition (1) enforces the use of some means whereby the desired line can be isolated. This result could be obtained either by dispersion through quartz prisms, by dispersion from a concave reflexion grating, or by use of the illuminant directly with a suitable filter of narrow transmission limits. Up to the present time, however, the first of these methods is the only one which has proved practicable, and therefore a monochromator, consisting of two 60° quartz prisms together with quartz collimator and telescope lens, has to be arranged suitably in conjunction with the source.

As yet it has been necessary to compute ultra-violet microscope objectives for one wave-length only and to use them strictly with this particular radiation; it is, however, permissible to use them with a wave-length which does not differ very widely from the original computed radiation, without introducing any serious aberration effects which might affect the definition of the image. For instance, a monochromat intended for use with $\lambda = 0{\cdot}275\,\mu$ can be used quite satisfactorily with the cadmium monochromatic line $\lambda = 0{\cdot}257\,\mu$. Consequently the experiments here described were confined to a limited range within which it was considered safe to use the objective, namely between $0{\cdot}29\,\mu$ and $0{\cdot}25\,\mu$.

§ 3. EXPERIMENTAL WORK

Attempts were first made to determine the relative brightness of different sources of radiation which might be used under the conditions laid down above. The apparatus for doing this is shown diagrammatically in figure 1. A cadmium

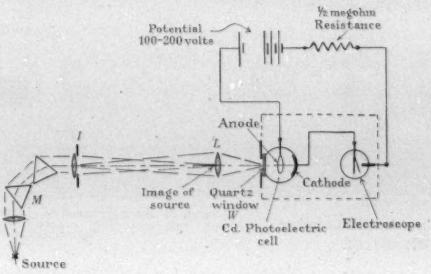


Fig. 1. General arrangement of apparatus employed.

photo-electric cell* sensitive to radiations between $0{\cdot}2\,\mu$ and $0{\cdot}3\,\mu$ was employed with the quartz monochromator M arranged in front of it. The quartz lens L was used to form an image of the iris diaphragm I on the mouth of the cell; by this means fluctuation of the source, a difficulty which was at first experienced when the image of the source was focussed directly on the cell window, was eliminated. All parts of the apparatus were securely positioned and provision was made for ensuring also precise location and width (namely 3 mm.) of the spark gap.

When the cell is used for the comparison of relative energies for a given wave-length, the relation

$$P \propto \frac{1}{T} \times \frac{D^2}{d^2}$$

may be employed, where

P may be the power of the source,
D is the distance between the source (in this case I) and the cell,
d the diameter of cell aperture, and
T the time in seconds for one beat of the electroscope.

* H. D. Griffith and J. S. Taylor, *Lancet*, **209**, 1205 (1925).

In practice five beats of the electroscope were taken, the timing being by stop-watch. *D* and *d* were constant throughout the measurements. The curve showing the sensitivity of the cadmium cell as a function of wave-length is given in figure 2.

A survey of the spark spectra of numerous metals reveals the fact that there are relatively few in which bright, single, and well-isolated lines are to be found. The most hopeful in this direction are those of Cd, Mg, Zn, Tl and possibly Al, spectrograms of which are shown in figure 3; and also that of the mercury arc in a silica tube. It will be seen that the zinc and cadmium spectra in the region between 0·25 μ and 0·29 μ have lines which are single and well separated, but none of them has the intensity of the line at 0·28 μ of the magnesium spectrum; unfortunately this line is a triplet and is not satisfactory in use except for adjustment purposes.

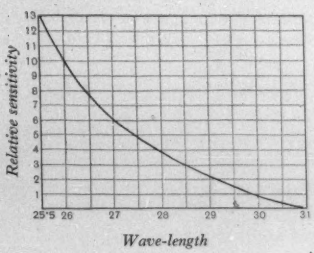


Fig. 2. Sensitivity curve for cadmium photo-electric cell.

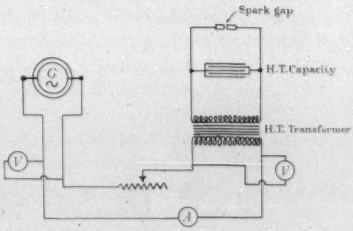


Fig. 4. Arrangement of electrical equipment for spark discharge.

The line of cadmium of wave-length 0·275 μ has been hitherto employed, and the ultra-violet objectives have been computed for it. There is, however, an excellent line at 0·257 μ, although it is not of such intensity as the former line; its isolated character should not be overlooked, for where a longer exposure can conveniently be given its strictly monochromatic nature renders it valuable in critical work. Other lines which might be useful at a future date for obtaining still higher resolution, although they are outside the range we are at present concerned with, are the cadmium line 0·227 μ and the zinc line 0·214 μ. The 0·185 μ line of aluminium is worthy of consideration as representing the shortest usable wave-length in air, and although it is of rather lower intensity than the above-mentioned lines, it offers possibilities for metallurgical work. Also there are lines in the spark spectrum of thallium which might be used, e.g. 0·277 μ, 0·253 μ and 0·230 μ.

Measurements of the relative brightness of the more suitable of these lines were taken, and the results below are given with the correction for the sensitivity of the cell to wave-length already applied. The spark was run under similar electrical conditions, shown diagrammatically in figure 4, throughout this test, namely 78 volts, 2·0 amp., 156 watts, and a capacity in parallel with spark of 0·005 μF. The length of the spark gap was 3 mm.

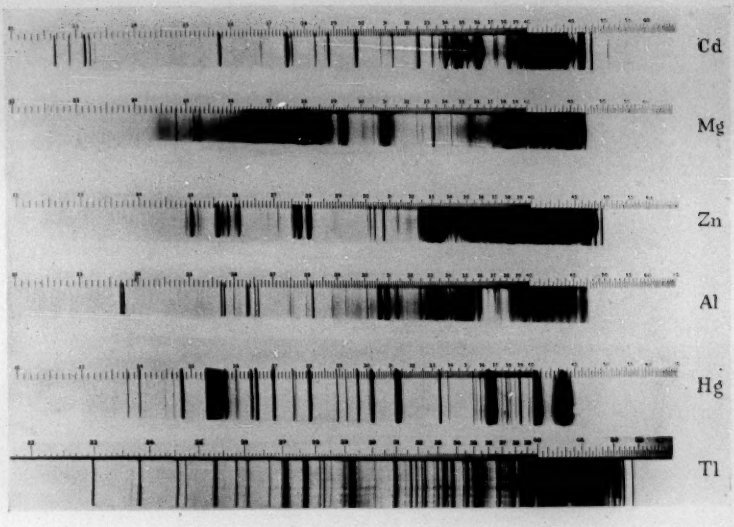


Fig. 3. Spectrograms of metals suitable for this work.

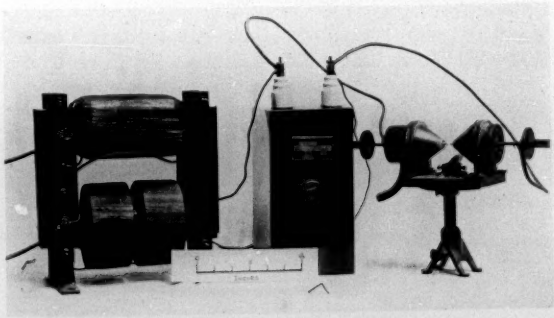


Fig. 10. Specially made transformer and condenser described in the text. Its compactness is shown by the scale.

Table 1. Intensities of spectral lines from 156-watt spark.

Metal	Wave-length (μ)	Relative energy $P = \frac{D^2}{Td^2}$ (arbitrary units)
Zinc	0·280	41·0
	0·256	69·3
Cadmium ...	0·275	112·2
	0·257	79·5
Magnesium ...	0·280 (triplet)	676·0
Aluminium ...	0·282	31·4
	0·263	104·4
Thallium ...	0·277	63·0

From these values it is seen that the magnesium spark at $\lambda = 0{\cdot}28\,\mu$ is by far the brightest source to use in this region but, owing to the present impossibility of achromatizing the ultra-violet objectives even for a limited range of the spectrum, the group-like nature of this line, when used with the microscope, distinctly affects the definition of the image produced on the photographic plate. In figure 5, (*a*) and (*b*) show two photomicrographs taken with the $0{\cdot}275\,\mu$ Cd line and the $0{\cdot}28\,\mu$ Mg line respectively, and illustrate the point in question, showing the undesirability of using the latter line.

The next in order of intensity is the $0{\cdot}275\,\mu$ Cd line, although it has only one-sixth of the luminosity; the aluminium line at $0{\cdot}263\,\mu$ appears fairly strong, but the proximity of several other lines in this region detracts from its usefulness; the $0{\cdot}282\,\mu$ line of the same metal is well isolated but low in intensity. The two zinc lines given in table 1 could be used but they are not particularly bright. The $0{\cdot}277\,\mu$ thallium line would appear from the spectrograms to be distinctly useful, but as these latter are not of equal exposure the density of a line in one spectrogram cannot be compared with the same line in another; and unfortunately the intensity of this line turns out on photo-electric measurement to be relatively low.

It was thought of interest to apply the test to a mercury arc as the source, for although some of its prominent spectrum lines were expected to be less intense than those of a spark emission, the convenience and steadiness in running of such a lamp commends its use very strongly. Table 2 below indicates the relative intensities, expressed in units similar to those used in table 1, of two lines in the named region, the voltage being 60 and the current strength 2·5 amp.

Table 2. Intensities of mercury lines from 150-watt arc.

Source	Wave-length (μ)	Relative energy
Mercury arc (quartz tube)	0·280	23·7
	0·265	39·8

From this it is evident that in spite of the convenience of this type of source, the low intensity of the lines renders it of little use when short exposures are required; nevertheless the author on one occasion took an ultra-violet photomicrograph with the 2 mm. immersion monochromat using the Hg 0·265μ line with a perfectly satisfactory result, though the exposure needed was 4 minutes.

The question of steadiness of the source is of some importance, for it is easy to understand that if the image of the spectrum-line formed on the condenser of the microscope moves about due to fluctuations of the spark, it may happen that at any one instant the condenser and therefore the objective are not wholly filled with light, so that a consequent loss in resolution may occur. An experiment to test this possibility was made. The microscope was set up for visual light with a 2-mm. apochromat and a chemically deposited silver film as an object. Comparison photomicrographs were then taken, first with a pointolite lamp (tungsten

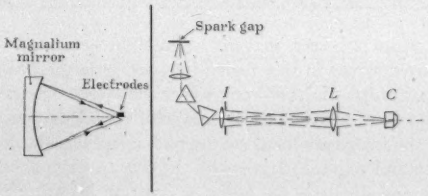


Fig. 7.

arc) as source and then with a cadmium spark, the light in each case being rendered reasonably monochromatic ($\lambda = 0{\cdot}52\mu$) by the use of a filter, but all other conditions remaining the same. In figure 6 (*a*) and (*b*) show the results and indicate that the resolution is distinctly better when the steadier source is used, and therefore any attempt to increase the steadiness of the discharge at the spark gap would appear to be justified. Two plans have been employed in such an attempt. The first of these, due to a suggestion by Mr W. R. Bracey, was to use a concave magnalium mirror behind the spark gap at a distance such that the latter was situated at the centre of curvature of the mirror; by this means any upward or downward movement of the spark would be counteracted by a corresponding opposite movement of the spark image, figure 7 (*a*), and would thus tend to locate the source as a whole. The second method consisted in the use of an auxiliary quartz lens *L*, figure 7 (*b*), so placed in the plane of the spectrum produced by the monochromator as to form an image of the lens and iris *I* on the microscope condenser *C*. Thus the lens *I* acts in effect as the source of light. Both these devices have been found to operate excellently in practice and are to be recommended.

Other points in the attempt to obtain better luminosity of the source, although of minor importance, are worthy of mention. One of these was the use of a reliable form of suction-pump for drawing off the fumes from the spark-housing. Readings taken with the photo-electric cell showed that the cloud forming between the spark

gap and the collimator lens when the pump was not running accounted for a loss of about 35 per cent. of the incident radiation after a period of 4 minutes. Little or no advantage has been found in the oiling of plates at wave-length 0·275 μ. Various makes of photographic plates of different speeds have been tried, but it appears that a so-called "fast" plate for visible radiations differs very little in respect of exposure from a slow one at this wave-length. Hence it is advisable to use a small-grained plate, for instance a process plate, for this work.

Carbon disulphide has a fairly narrow transmission band in the region of $\lambda = 0{\cdot}275\,\mu$ (see figure 8). The possibility of using the Cd spark with such a filter directly instead of using the monochromator at once suggested itself, but unfortunately the transmission of the visible region of the spectrum precludes this—the hope of combining another filter to remove the visible and yet transmit the named ultra-violet region has not been realized*.

§4. ELECTRICAL CONDITIONS

The foregoing results indicate that the spark is the best source to use in order to increase the value B in the formula given on page 128, and also the 0·275 μ Cd line is the best to employ for this purpose. The next step, therefore, was to determine the most suitable arrangement of electrical conditions to give greatest luminosity of the spark with cadmium electrodes. Mr J. J. Holmes, a former research student in this department, has carried out valuable work in this direction with results shortly to be published. The author has had access to his paper which contains measurements and observations on: (*a*) single sparks produced by the charging of a large condenser to a high potential by an electrostatic machine; (*b*) series of sparks produced by means of an induction coil and mercury break; (*c*) series of sparks produced by means of an alternating current.

As it is this last section which has indicated the most hopeful line of approach to the desired aim, the writer has extended the work in this direction. A study of the spark intensity with variation of the three chief factors involved, namely (i) the frequency of the alternating current, (ii) the potential of the secondary current, and (iii) the capacity of the condensers, was made.

Apparatus. The electrical equipment consisted of (1) two alternators, one supplying nominally 500 watts at 100 ~ and the other 500 watts at 250 ~; (2) oil-immersed transformer (by Zenith Electrical Co.) at 100 volts and 0·25 amp, with step-up ratios of 100 : 1 to 200 : 1 variable by five intervals; (3) condensers of various types: (*a*) a variable pattern by Dubilier Co. ranging from 0·0027 to 0·008 μF, (*b*) a battery of tubular Leyden jars by Marconi Co., the individual capacities being 0·00067 μF, (*c*) a fixed type of 0·005 μF; (4) spark gap, with device for ensuring location and fixed width of electrodes; and (5) a special transformer, built to meet requirements suggested by the results of the experiments. The connexions were

* With a film of carbon disulphide 0·25 mm. in thickness it was found that 54 per cent. of the incident radiation was transmitted at wave-length 0·275 μ, so that this liquid might possibly prove a valuable mounting medium for microscope objects.

arranged as in figure 4, and as before the monochromator and photo-electric cell were employed as shown in figure 1.

Experimental work. The first test was made on the two alternators individually, to determine the effect of a change in frequency of the current. The second involved a change in the step-up ratio of the transformer, the capacity remaining the same throughout. Table 3 shows the figures obtained on these two tests.

Table 3. Effect of variation in frequency of current and change in step-up ratio of transformer.

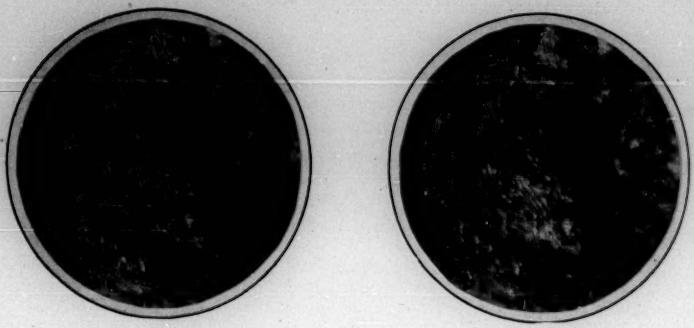
Alternator	Primary			Step-up of transformer	Potential of secondary	Cycles per second	Capacity (μF)	Relative energy by photo-electric cell (arbitrary units)	Relative density by photographic method
	Volts	Amp	Watts						
100 ~	36	4	144	100 : 1	3600	86	·0048	70·8	0·93
	32	4	120	114 : 1	3648	83	·0048	61·4	0·91
	27	3·9	105	135 : 1	3658	79	·0048	54·5	0·72
	24	4	96	150 : 1	3600	74	·0048	46·6	0·66
250 ~	35	4·1	143	100 : 1	3500	228	·0048	68·6	0·92
	30	3·9	117	114 : 1	3420	217	·0048	60·9	0·90
	26	4·0	104	135 : 1	3523	204	·0048	54·1	0·72
				150 : 1	Too much load on alternator				

From these results it will be noticed that the change in frequency of the current, even when it comprises a threefold increase, has little effect on the apparent luminosity of the discharge. Also the potential of the secondary remains almost constant and does not increase with the increased step-up ratio of the transformer. (This is probably due to the overloading of the alternator and the consequent decrease in the power-factor.) There is, however, a direct correspondence between the intensity of the spark and the input wattage. As a check on these figures, photographs were taken with the ultra-violet microscope of a suitable object, and the relative density of the same part of the image was determined when successive exposures were made for variations in the electrical conditions as set out above. In the latter experiment all the exposures* were the same and all plates were given similar conditions of development. The densities were measured with a photo-electric density-meter†. Figure 9 illustrates this test, and the results obtained are appended in the last column of table 3; it will be seen that the relative-density values are in similar proportion to the figures given in the preceding column and thus bear out the results obtained by the first method.

Measurements were then taken with the same step-up ratio of the transformer but with a variation in the amount of capacity put in parallel with the spark. Table 4 shows the values obtained.

* 10 sec. by stop-watch. This exposure was found to give densities within the range of Schwarzschild's law.

† F. C. Toy, *Journ. Sci. Inst.* **4**, 369 (1927).



(*a*) (*b*)

Fig. 5. Contrast in definition when using (*a*) Cd 0·275 μ line, and (*b*) Mg 0·28 μ line.

(*a*) (*b*)

Fig. 6. The effect on resolving power of using (*a*) steady, (*b*) unsteady sources of illumination.

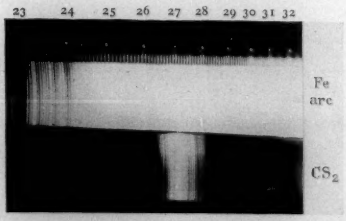


Fig. 8. Transmission of CS_2 in the ultra-violet.

Fig. 9. Change in photographic density with variation of electrical conditions.

From these figures it would appear of considerable importance to increase the amount of capacity to as large a value as possible in order to increase the brightness of the spark. It will again be noted that the frequency of the current has little bearing on the brightness.

Table 4. Effect of change in capacity of condensers.

Alternator	Primary			Step-up of transformer	Potential of secondary	Cycles per second (actual)	Capacity (μF)	Relative energy by photo-electric cell	Relative density by photographic method
	Volts	Amp	Watts						
100 ~	30	3·9	117	100 : 1	3000	85	·0027	47·2	0·68
	30	4·0	120	100 : 1	3000	82	·0037	53·1	0·73
	30	4·0	120	100 : 1	3000	83	·0048	63·0	0·92
	35	4·1	143	100 : 1	3500	81	·0059	79·6	1·02
250 ~	38	4·0	152	100 : 1	3800	230	·0027	48·2	0·70
	38	4·0	152	100 : 1	3800	228	·0037	54·8	0·74
	38	4·0	152	100 : 1	3800	232	·0048	62·5	0·91
	39	4·1	157	100 : 1	3900	229	·0059	78·1	1·01

From the foregoing tests it was apparent that the two most important factors in the improvement of intensity were, firstly, increased input energy, and secondly, high capacity. Steps were therefore taken to increase the wattage hitherto used, namely 150; and a special transformer*, which enabled a larger input energy to be used by reason of an improved power factor, was employed. The particulars of this transformer are as follows: Primary volts 100, amp 5, cycles per second 100. Step-up ratio 50 : 1. Secondary volts 5000, amp. 0·1, kilowatts 0·5.

With this equipment the series of experiments was repeated, but in this case the capacity was gradually increased until the spark refused to jump the gap of 3 mm. Table 5 shows the results obtained.

If we compare line 3 of these results with those in table 3, where the capacity is similar, we see, firstly, that by increase of the input energy the intensity of the spark has been considerably increased. Actually, however, although the wattage is now five times what it was in line 3 of table 3, the relative energy appears only to be trebled; nevertheless this tends to agree with indications given by a graph of the results of table 3, and in any case a distinct increase in brightness has resulted. Secondly, the marked progressive increase in energy with increase of capacity is evident. It will be noted that with the electrical conditions as given in the last line but one in table 5 the brightness of the Cd spark has been increased to nearly six times that obtained at the commencement of these experiments. In practice, however, the discharge is so violent that the cadmium electrodes burn away very rapidly; it is nevertheless possible to use the spark gap under these conditions for about 5 seconds, which, with the increased luminosity thus obtained, is con-

* Designed jointly by Messrs H. Gough, G.I.E.E., and G. K. Johnson, A.M.I.E.E., A.M.I.M.E., and made in experimental form by Messrs Watson and Sons (Electro-medical), Ltd.

siderably longer than the time now required for an exposure with an object of the transparent type. During all adjustments prior to the actual exposure the wattage in the primary is limited by suitably controlled resistances, long usage of the electrodes being thus made possible.

Table 5. New transformer tests.

Primary			Step-up of transformer	Cycles per second	Capacity (μF)	Relative energy by photo-electric cell
Volts	Amp	Watts				
100	5·0	500	50 : 1	100	·0027	84·5
,,	,,	,,	,,	,,	·0037	117·6
,,	,,	,,	,,	,,	·0048	169·0
,,	,,	,,	,,	,,	·0059	208·0
,,	,,	,,	,,	,,	·0069	245·8
,,	,,	,,	,,	,,	·0077	300·4
,,	,,	,,	,,	,,	·0088	314·4
,,	,,	,,	,,	,,	·0098	338·0
,,	,,	,,	,,	,,	·0109	365·4
,,	,,	,,	,,	,,	·0120	409·0
,,	,,	,,	,,	,,	·0131	Would not spark across gap

An ultra-violet photomicrograph taken with the 2-mm. immersion monochromat at a magnification of 840 × with an exposure of only 1 second is now found to give a negative of 0·69 density, a reasonably dense photograph, whereas previously a similar exposure produced a density of 0·11—a value which is too low to reveal detail in the photomicrograph.

§ 5. CONCLUSIONS

The impressions received from this work are: (i) that the spark discharge between cadmium electrodes is the most suitable source of radiation for the quartz monochromat microscope objectives computed for and used with a wave-length in the neighbourhood of 0·275 μ; (ii) that it is still necessary, in spite of considerable loss of light, to employ a monochromator for providing the desired wave-length; (iii) that the intrinsic brightness of the spark appears to be a function solely of the energy-input and the capacity employed. Furthermore the frequency of the alternations of the current seem to be of little importance. In this latter connexion the transformer can be used with alternating mains at 50 ~ so as to save the expense of an alternator; (iv) that a compact and inexpensive electrical unit (see figure 10) can be designed for providing the source of illumination for ultra-violet microscopy.

§ 6. ACKNOWLEDGMENTS

I wish to acknowledge the continued and valuable help of Mr G. K. Johnson, and to express my thanks to Mr H. Gough for his collaboration in the design and manufacture of the new transformer and condenser, and to Professor L. C. Martin for his encouragement and advice throughout the work.

DISCUSSION

Mr T. SMITH suggested that, in view of the difficulties incident to illumination with an extended source, the author should give an idea of the numerical aperture used, for instance, in the production of figure 6. It was rather a pity to include the debatable formulae given on pages 2 and 3: it would be sufficient for the author's purpose to assume merely that the illumination varies as the intrinsic brightness and as the frequency of the beats of the electroscope. The table on page 128, showing the distribution of energy in the spectrum, was not quite to the point: the information required was the energy in particular lines, not regions of the spectrum. The preceding statement that the maximum lies in the visible green of the solar spectrum needed qualification; it was only true on a wave-length basis, whereas if the spectral energy were plotted on a frequency basis the maximum would lie in the infra-red.

Mr J. H. AWBERY. In regard to Mr Smith's remarks, it seems to me that we are not comparing the energy in an arbitrary strip of the spectrum. If the energy in a given spectral line, or in the range called "green," is under discussion, then when the basis of calculation is changed from wave-length to frequency, the range considered has to be altered in such a way that the area (i.e. the energy) is the same whichever basis is adopted.

Prof. L. C. MARTIN. I think the author is to be congratulated on a piece of work which has led to such practically useful results. The result of essential value is the dependence of the effective intrinsic brightness of the spark on the energy or wattage consumed, and its practical independence of the frequency. I had hoped that some additional research work done in the Technical Optics Department by Mr J. H. Holmes would be described at this meeting, but the paper could not be prepared in time. It will throw considerable light on the above result. With regard to the formula used in connexion with the photo-electric cell, I regarded this at first with some suspicion, but a series of practical tests showed that it is quite a close approximation.

AUTHOR'S reply. The photomicrographs in figure 6 were taken with a 2 mm. apochromal objective of numerical aperture 1·2. The formula on page 128, besides showing the way in which the illumination per unit area may vary, also brings out the importance of keeping the magnification as low as its minimum limit will permit, and therefore the inclusion of this relation would appear to be justified. The table referred to is intended to convey an idea as to the distribution of energy in the spectrum given by a spark emission. The enhanced nature of lines in the ultra-violet region of the spark spectrum is likely to make this form of source more useful than one, such as for example the arc or flame, in which the lines of longer wave-length may be, generally speaking, of greater relative intensity.

THE INFLUENCE OF THE CRYSTAL-ORIENTATION OF THE CATHODE ON THAT OF AN ELECTRO-DEPOSITED LAYER

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ABSTRACT. The influence of the crystal-orientation of a cathode on that of an electro-deposited layer is studied by X-ray methods for the cases of copper and nickel, respectively, deposited on rolled copper. The conditions of cathode-surface and current-density which accompany an oriented deposit are determined. The orientation of the copper deposit for small currents is the same as that of the cathode. The nickel, at low current densities, assumes a distinct orientation. As the current is increased there is a region of no orientation, followed, at still higher currents, by an orientation the same as that of the cathode surface below.

§ 1. INTRODUCTION

The very interesting work of a group of investigators* has shown that the minute crystalline grains of an electro-deposited layer tend under certain conditions to arrange themselves in such a way that one particular crystallographic axis sets itself normal to the cathode face. The deposit is said to have assumed an orientation of the fibre type. It becomes, therefore, a matter of further interest to extend the study to the case of deposition upon a surface which, as a result of previous working, itself possesses some degree of orientation. Many materials are in this state in practice, and the relative orientation of layer and base might very reasonably be expected to be a factor of some influence on the properties of the final product. The methods of X-ray analysis provide the most direct approach. They are used in the present work, which deals with the problem of copper and nickel deposited on oriented copper sheet.

§ 2. COPPER ON COPPER

(i) *Experimental.* The material used as cathode was obtained from sheet copper which, after having been rolled to various extents, gave samples possessing different degrees of orientation. This orientation was shown by means of X-ray photographs taken with iron K_α radiation to begin immediately at the surface. This precaution is a necessary one, as the author has shown† that in the case of a wire the action of the die in drawing tends to destroy surface orientation. An anode copper sheet was used. Traces of orientation were removed from the anodes by heat treatment.

The plating bath contained per litre 200 gm. of copper sulphate and 100 gm. of sulphuric acid. In certain cases, as described below, the cathodes were chemically

* G. L. Clark, *Applied X-rays*, p. 241.

† Paper not yet published.

cleaned. This process involved first washing the surface with benzene and alcohol and then placing the specimen as cathode in a solution containing 60 gm. of sodium carbonate, 7 gm. of caustic soda and 7 gm. of sodium cyanide per litre. After being subjected to cathode bombardment for 1 minute the specimen was withdrawn and rinsed in distilled water. Next it was "pickled" by immersion for some 20 seconds in dilute nitric acid. Finally after another rinsing it was hung directly in the plating bath, thereby completing the circuit*. No stage of the process appreciably affected the orientation of the copper.

In each experiment two exactly similar baths were connected in series. In the first the cathode had been cleaned, and in the second not cleaned. The current-density was calculated from the area immersed and the total current passing.

The difficulty of detecting which lines in the X-ray spectrum were due to the original copper and which were due to the deposit disappeared with the observation that, in the region of current-densities employed, the deposited grains produced the characteristic spotted effect which always arises from the presence of crystallites larger than approximately 10^{-3} cm. The interference lines consisted therefore of spotted lines caused by the deposit superimposed upon the smooth continuous lines caused by the original copper. The spectrum of the latter was, of course, weakened by absorption in the layer to an extent depending on the thickness of that layer. In most of the actual experiments the deposit was so thick that the lines due to the original copper were entirely absorbed. It was found very convenient, moreover, to place the specimen at such an angle to the incident X-ray beam that the lines in the spectrum were somewhat out of focus. The sharp diffraction spots due to the layer were thus spread over the increased width of the corresponding lines, and, being therefore distributed over a greater area, they were more easily distinguishable. A "bad" photograph was thus preferable to a good one in which the focussing would cause a concentration of the spots into a line almost as uniform as that from the original copper. Every photograph was taken under the same geometric condition.

(ii) *Observations*. It was found that only in the case of the chemically cleaned surfaces did the electrodeposited layer assume an orientation similar to that of the base. This fact could be utilised as another method of distinguishing the spectrum of the deposit from that of the base metal. Each cleaned specimen was accompanied by one not specially cleaned and both specimens had taken the deposit under exactly the same conditions of time, current-density and solution. Consequently, when the spectrum of one deposit showed orientation and the other none at all, neither spectrum could be put down to the original oriented copper. Apparently the layer of dust and grease present on normal material introduces complications sufficient to prevent the deposit following the orientation below.

The effect depended on the current-density. Using the electroplating bath described above one found that at current-densities greater than 12 mA/cm.^{2} the effect began to disappear and at 15 mA/cm.^{2} the deposited grains were arranged entirely at random. Increasing the current had no effect other than reducing the

* W. E. Hughes, *Modern Electroplating*, chap. 3 and 10.

size of the grains. At 60 mA/cm.^{2} the spotted effect of the grains had gone. Below 12 mA/cm.^{2}, however, the grains were deposited in a way which showed strong orientation. This orientation was the same as that of the copper surface below. These results are illustrated by figures 1, 2, 3. In figure 1 is reproduced the photograph of the original oriented cathode; the lines are not focussed. Figure 2 shows the spectrum of a deposited layer. It can be seen that the diffraction spots due to the deposit concentrate on the intensity maxima shown in the previous figure and absent themselves from the minima. The deposit is oriented like the original. Figure 3 is the photograph of a deposit on an unclean surface and it may be seen that here the spots are scattered equally along the lines. The orientation has gone. For those values of the current-density which permitted the effect to occur, it was found that the time of deposition could be extended to at least 24 hours without causing any appreciable decrease in the degree of orientation. The orientation appeared likely to persist for the thickest layers obtainable under the conditions described.

The introduction of a colloid in the form of gelatin decreased the grain size without, as it seemed, affecting the orientation. The only indication that the observed spectra, now unspotted, were not due to the original specimen, however, was that the comparison photograph from the unclean cathode showed no orientation.

§ 3. NICKEL ON COPPER

(i) *Experimental.* The plating solution contained per litre 300 gm. of nickel sulphate, 6 gm. of boracic acid and 3 gm. of common salt*. As in the experiments on copper, deposition was made under similar conditions on cleaned and unclean copper strip. A piece of sheet nickel used as anode showed no trace of orientation. The identification of the interference lines was simpler than in the case of copper, the pinakoidal spacings of copper being 3·603 A.U., and of nickel 3·499 A.U. Copper K_α radiation was used.

(ii) *Results.* The interference lines of the nickel plate were unbroken; this shows that the deposit possesses the fine-grained structure characteristic of that metal. The effect of time of deposition and of the cleanliness of the surface were the same as for copper.

The orientation of the deposit depended on the current-density in a curious manner. It was observed that for current-densities less than 7·5 mA/cm.^{2} the deposited nickel was very highly oriented. The orientation, however, was different from that of the copper surface. It was of the fibre type and would agree with that found by Clark and Fröhlich† in their work on non-oriented cathodes. The strong maximum in figure 4 illustrates this type of orientation. In that particular photograph the deposit was obtained with a current-density of 6 mA/cm.^{2} passed for 8 hours. At a current-density of 7·5 mA/cm.^{2} the deposit showed no orientation whatsoever. Figure 5 reproduces a photograph of the deposit at this stage. The absence of orientation persisted, moreover, for current-densities up to 10 mA/cm.^{2}.

* W. E. Hughes, *loc. cit.*

† G. L. Clark and P. Fröhlich, *Zeit. Electrochemie*, **31**, 655 (1925).

Fig. 1.

Fig. 2.

Fig. 3.

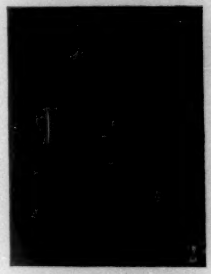

Fig. 4.

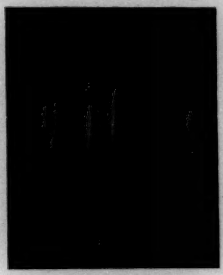

Fig. 5.

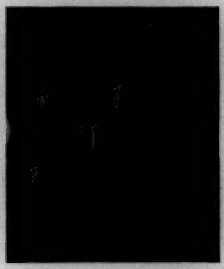

Fig. 6.

For current-densities up to about 40 mA/cm.^{2} the deposit again showed a high degree of orientation. In this case the orientation was the same as that of the original copper surface underneath and, but for the slightly different spacings of nickel, would hardly have been distinguishable from it. Figure 6 shows a typical photograph of a deposit made in this region of current-density. Finally, for still higher current-densities the orientation again tended to disappear and was entirely absent at 60 mA/cm.^{2}. In all the nickel photographs shown the differences of current-densities were so combined with the time of deposition as to give as nearly as possible the same thickness of deposit. Similar results were found for deposition on wires.

§ 4. DISCUSSION OF RESULTS

From the above observations it would appear to be a simple matter to produce a marked orientation in an electrolytic deposit. The current-density must not exceed a certain value such that the ions, as they deposit themselves, are given enough time to take up the orientation. The surface must be clean so as to exclude the possibility of an interposed layer of foreign matter. The orientation actually assumed by an ion depends on the lines of force due to the potential difference between the electrodes and on the localized fields of force due to the particular arrangement of the original atoms on the surface. In the experiments with a copper deposit the latter forces always predominate and the deposit repeats the underlying orientation. In the case of nickel the former forces predominate at very low current-densities and the latter at high current-densities, while for intermediate current-densities the two fields apparently compensate one another and produce a random result. The local forces due to the original orientation are presumably the same for any current-density, so that one must conclude that at the higher currents the influence of the force due to the potential across the electrolyte on the depositing ions is decreased.

The author hopes to extend the work to other metals and to correlate the results with the physical properties of the various deposits.

§ 5. ACKNOWLEDGMENTS

In conclusion, the author wishes to express his thanks to Mr J. R. Clarkson, B.Sc., for his efficient help in the experimental work, and to Dr G. Shearer for his interest in the research.

DISCUSSION

Mr J. GUILD asked whether the optical properties of the metallic films are affected by the orientation of the crystals. For instance, did the reflecting-property of the silver or platinum films vary with this factor?

The AUTHOR said that he had not had facilities for testing this interesting point. Prof. Carpenter had found, however, that the reflecting-properties of single crystals are different for different faces.

THE INFLUENCE OF LOW TEMPERATURES ON THE THERMAL DIFFUSION EFFECT

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ABSTRACT. Measurements of thermal diffusion on the following gas mixtures at temperatures between 15° C. and − 190° C. are described: He—Ne; H_2—Ne; He—A; Ne—A; He—N_2. The experiments confirm and extend the authors' previous work showing that there is a general tendency for k_t to decrease at low temperatures. It is found, however, that for the pairs of gases He—Ne and H_2—Ne, with low liquefying points, the change in k_t between 15° C. and − 190° C. is small. Chapman's theory is used to deduce approximate values of the laws of repulsive force operating between unlike molecules during collisions. At low temperatures molecules tend to become "softer" and their behaviour is less like that of rigid elastic spheres. The "hardest" molecules appear to show the smallest variation in k_t. The influence of helium in this respect, on the value of k_t for a mixture, is clearly shown.

§ 1. INTRODUCTION

An account of a series of experiments on thermal diffusion* at low temperatures was given in a recent paper†. Mixtures of gases were examined with one side of the apparatus at about 15° C. and with the other at temperatures down to − 190° C. It was found that the thermal separation is not always proportional to log (T_1/T_2), T_1 being the absolute temperature of the hot side and T_2 that of the cold side. For the lower values of T_2 the thermal separation falls below the proportional value, which means that k_t, the ratio of the coefficient of thermal diffusion to the coefficient of ordinary diffusion, decreases with temperature. Lugg‡ has recently shown that k_t for a mixture of hydrogen and carbon dioxide increases at high temperatures. These changes in the value of k_t indicate the possibility of changes in the nature of inter-molecular collisions—i.e. the molecules tend to become "softer" at low temperatures; and it was suggested that this might be due to the influence of the attractive forces between molecules. In particular, it was shown that the falling off in k_t for nitrogen-argon mixtures was greater than for such mixtures as hydrogen-nitrogen, or hydrogen-argon. It is therefore of

T_1, T_2

k_t

* D. Enskog, *Phys. Zeit.* **12**, 538 (1911); *Ann. d. Phys.* **38**, 742 (1912). S. Chapman, *Proc. R. S.* A, **93**, 1 (1916); *Phil. Trans.* A, **217**, 157 (1917); *Phil. Mag.* **34**, 146 (1917); **38**, 182 (1919); **48**, 602 (1924); *Proc. R. S.* A, **119**, 34 (1928); A, **119**, 53 (1928); *Phil. Mag.* **7**, 1 (1929). S. Chapman and F. W. Dootson, *Phil. Mag.* **33**, 268 (1917). T. L. Ibbs with a note by S. Chapman, *Proc. R. S.* A, **99**, 385 (1921); A, **107**, 470 (1925). T. L. Ibbs and L. Underwood, *Proc. Phys. Soc.* **39**, 227 (1927). T. L. Ibbs, K. E. Grew and A. A. Hirst, *Proc. Phys. Soc.* **41**, 456 (1929). G. A. Elliott and I. Masson, *Proc. R. S.* A, **108**, 378 (1925). J. W. H. Lugg, *Phil. Mag.* **8**, 1019 (1929).

† *Proc. Phys. Soc.* (1929), *loc. cit.*

‡ *Phil. Mag.* (1929), *loc. cit.*

interest to examine at low temperatures mixtures of a pair of gases for which the attractive forces are regarded as weak. Mixtures of helium-neon and of hydrogen-neon satisfy this condition; the general physical behaviour of these gases indicates a weak attractive field. This conclusion is supported by the theoretical investigation of Lennard Jones*, who has shown that the molecules of these gases can be nearly represented by point centres of repulsive force. For these pairs of gases and for other pairs now examined it has been necessary to work with small quantities of the gases. This has necessitated considerable modification of our experimental methods.

Prof. S. Chapman had previously pointed out to us the need for measurements of k_t for pairs of monatomic gases, and had very kindly obtained the valuable co-operation of Dr F. W. Aston, who devoted considerable time to demonstrating to one of us (T. L. I.) the methods of manipulating and purifying small quantities of gases. Dr Aston allowed us to have full details of the apparatus he has employed for the purpose, and also lent quantities of the rarer inert gases. We have thus been enabled to make measurements on a number of pairs of the monatomic gases which are of special interest owing to the relative simplicity of their molecular structure.

A preliminary investigation was made which showed that our methods could be adapted for use with small quantities of gases. In this early work experiments were made on a number of pairs of inert gases by a comparative method, with the hot side at 100° C. and the cold side at room temperature. In the meantime our work at low temperatures had reached the stage described, where it appeared desirable to develop the method so that measurements could be made on helium and neon. We decided therefore to combine the two series of experiments so as to obtain not only the ordinary value of k_t but also its variation with temperature.

§ 2. METHOD OF EXPERIMENT

Some of the more important parts of the apparatus are shown in figure 1. The "flow" method previously employed, which required considerable quantities of gas, was discarded. The gas mixtures were made in the gas burette† B, of capacity about 80 cm.^{3}, which communicates by a glass tube with the diffusion apparatus shown on the right-hand side of the diagram. The glass bulb A of volume 25·7 cm.^{3} is joined by a connecting tube C of length about 1·0 cm. and internal diameter 0·3 cm. with the katharometer‡ K. Taps T_3 and T_4 communicate with glass bulbs, one of which contained charcoal, used in the purification of the gases. Mr G. O. Harrison gave valuable assistance by his skill in the construction of the apparatus.

The katharometer is employed as before to analyse the gas mixtures and to measure the changes in composition produced by thermal diffusion. The bridge arrangements were the same as those previously described§ and are not shown in

* J. E. Lennard Jones, *Proc. R. S.* A, **107**, 157 (1925).
† Cf. Travers' *Study of Gases*, p. 67.
‡ This instrument was kindly lent by Dr G. A. Shakespear.
§ *Proc. R. S.* A, **107**, 475 (1925).

the diagram. To obtain a calibration curve for any pair of gases a quantity of one of them is drawn into the burette through the syphon tube S and the mercury level in the burette is adjusted to coincide with the point of the glass index I. The pressure of the gas in the burette can be obtained by observation of the difference of levels of the mercury at I and in the reservoir R, and the height of the barometer. A quantity of the other gas is then drawn in and the pressure of the mixture obtained when the volume is the same as before. In this way a mixture of known composition by volume is made. The mixture is then passed through taps T_2 and T_5 into the katharometer and bulb A. The level of R is adjusted to bring the gas to atmospheric pressure and the galvanometer reading for the mixture is observed. By drawing further quantities of gas into the burette the calibration curve for mixtures of a pair of gases is thus obtained, and by reference to this curve any other mixtures of the pair can be analysed.

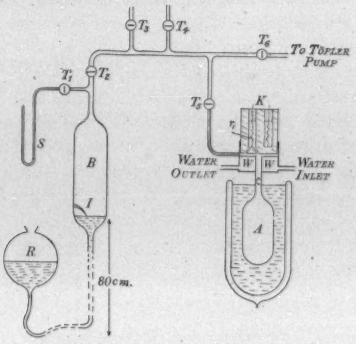

Fig. 1. Diagram of apparatus.

The thermal diffusion measurements. A mixture is passed into the diffusion apparatus at atmospheric pressure as described and tap T_5 is closed, the bulb A being at room temperature. This bulb is now surrounded by a Dewar vessel containing liquid air or cooled pentane, and thermal diffusion is allowed to take place. This causes in the concentration of the gas on the hot side a change which is measured by means of the katharometer. The total volume of the hot side, i.e. the katharometer cell and its connections, is 0·98 cm.^{3} and this side is kept at a steady temperature by the water jacket W supplied from the mains. The small change in composition which occurs on the cold side can be calculated and the total separation obtained. When the gas in A is cooled the pressure is reduced, but previous experiments* have shown that thermal diffusion is not affected by such changes of pressure. By using this fact we have been able to reduce considerably the manipulation required in the experiments, as the same gas mixture can be allowed to remain inside the diffusion apparatus for a series of measurements with the cold side at

* Cf. *Proc. Phys. Soc.* **41**, 468 (1929).

different temperatures. (In practice the bulb A was allowed to warm up to room temperature by removal of the Dewar vessel before each measurement, so that the katharometer zero could be checked. This procedure avoids small errors which may occur owing to fluctuations of the current in the bridge circuit or to changes of temperature in the water jacket.) The changes in pressure produce small changes in the katharometer readings, but the necessary correction for these can be made for each mixture. The correction is obtained by observation of the effect of changes of pressure on the galvanometer zero when the diffusion apparatus is at room temperature. It is worth noting also that if all the diffusion measurements had been made at atmospheric pressure considerably greater quantities of gases would have been required for the experiments at low temperatures. At the end of a series of measurements on one mixture the gas is removed through tap T_6 by means of a Töpler pump and collected over mercury. The gas can then be used again in another mixture. Each pair of gases was generally examined in mixtures of three different proportions.

Temperature control and measurement. Measurements at the lowest temperatures were made as before by use of liquid air in the Dewar vessel. Temperatures down to about $-145°$ C. were obtained by means of pentane cooled with liquid air. To obtain uniformity of temperature the pentane was again stirred by bubbling compressed air through it. The shorter glass bulb A now used was unsuitable for our previous method of obtaining temperatures between $-145°$ C. and $-190°$ C. The absence of observations over this range does not in this case produce any serious difficulty as our previous investigation has shown the general nature of the effect.

Greater precision in the temperature-measurements was obtained by use of a thermocouple instead of a pentane thermometer to measure the cold side temperature. The couple was of copper and copper-nickel alloy ("Ferry" wire). The junctions were silver-soldered. One junction was immersed in ice and water in a Dewar vessel and the other suspended in the pentane near the middle of the diffusion bulb A. The thermo-electric e.m.f. was balanced by the potentiometer method, and the potentiometer readings were calibrated by direct comparison of the thermocouple with a constant-volume hydrogen thermometer. The temperature of the hot side was again measured with a mercury thermometer placed in the outlet of the water bath.

Precision of the measurements. The error in the absolute values of the separation now recorded is probably of the order of about 2 or 3 per cent., while relatively to one another the values for any one mixture are correct to about 1 per cent. The fact that there was now no flow of gas through the apparatus improved the steadiness of the katharometer readings.

An alteration in the mean composition of the gas in the connecting tube to which the temperature gradient is applied occurs as the thermal diffusion effect progresses. With the present arrangement the proportion in the tube of the lighter gas increases with the effect, which will cause a change in the value of k_t. The correction required to make k_t apply throughout to the original mixture has not yet been made in these results, but it may have to be considered as the theory is

further developed. Many of the mixtures examined contained less than 50 per cent. of the lighter gas, and the change of composition in the connecting tube would therefore tend to make the observed separation at low temperatures greater than the true separation for the original mixture, i.e. the falling off in k_t is actually more than is shown by the curves.

Purification of the gases. The gases used were purified in the apparatus by fractionation with charcoal and liquid air. The purity of a gas for our purpose can be examined by testing of the gas, without admixture, for any evidence of the thermal diffusion effect. This provides a very convenient method of testing the purity of the inert gases, as a small quantity of an impurity of considerably different molecular weight—e.g. small quantities of helium as an impurity in neon—can easily be detected.

In our previous work at low temperatures we came to the conclusion that the simple gas conditions postulated in Chapman's theory could be regarded as holding in the conditions of the experiments. Certain experimental tests of this conclusion were then made, and we consider that it is also valid for this work.

§ 3. THE RESULTS OF THE EXPERIMENTS

Five pairs of gases were examined: (1) helium and neon, (2) hydrogen and neon, (3) helium and argon, (4) neon and argon, (5) helium and nitrogen. It is convenient to demonstrate the results in the form of graphs showing the relation between separation and $\log_{10}(T_1/T_2)$, as the characteristic behaviour of different gases can be seen from these curves. All the mixtures were examined down to $-190°$ C.; the points on the first part of the curves are obtained by means of cold pentane, and the point at the end by means of liquid air. As the temperature T_1 for any mixture is practically constant, the value of k_t in a region at a temperature T_2 is proportional to the slope at the corresponding point on these curves and can be calculated from it*.

By means of the formulae of Chapman and Hainsworth† an expression can be obtained, for a pair of gases, for k_t as a function of the proportions of the gases in the mixture, the molecules being regarded as rigid elastic spheres. As predicted by Chapman, the experimental values of k_t are smaller than the calculated values for rigid elastic spheres, and we can obtain a "thermal separation ratio" R_t, where

R_t

$$R_t = (k_t \text{ experimental})/(k_t \text{ for rigid elastic spheres}).$$

The value of R_t appears to be nearly independent of the proportions of the gases in the mixture and can therefore be regarded as a constant for a pair of gases. (A more careful investigation of the constancy of R_t will be made in due course.) Thus when R_t is known it can be used in conjunction with Chapman's formulae to calculate the thermal separation in any practical case. Outside a certain range of temperature R_t will of course vary with temperature, but for all pairs of gases there will be an ordinary value of R_t which can be applied over a considerable tempera-

* Cf. *Proc. Phys. Soc.* **41**, 463 (1929).

† *Phil. Mag.* **48**, 602 (1924).

ture range. A knowledge of this may be of value, as thermal separation must frequently occur in experimental and practical circumstances where gas mixtures are used.

The different mixtures of any pair of gases which have been examined are lettered *A*, *B* and *C* for reference purposes both in the text and in the diagrams. The proportions of the gases in each mixture are given in the tables at the end of the paper.

(1) *Helium and neon.* The results for the three mixtures examined are shown in figure 2. The points obtained by means of pentane, i.e. down to − 145° C., lie on a straight line, and the liquid-air point in two cases lies practically on this straight

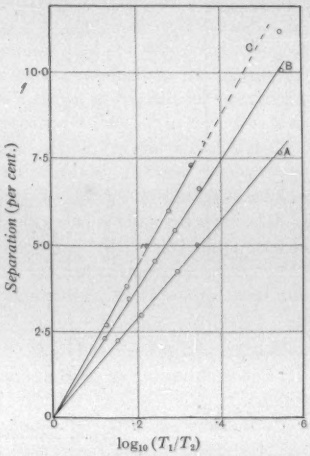

Fig. 2. Helium-neon. Mixtures *A*, *B* and *C*.

line extended, and in the third case a little below it. This means that the experimental value of k_t for a helium-neon mixture can be regarded as nearly constant from 15° C. to − 190° C. This result is of interest as it is the only one of the kind obtained throughout this range of temperature. The correction for the change in composition of the gas in the connecting tube would produce a slight curvature and tend to reduce k_t a little at the lower temperatures. Both helium and neon have low liquefying-points, and it may be worth while later to study them more fully, particularly in the range from − 150° C. to − 190° C.

Using Chapman's formulae we obtain for this pair

$$k_t = \frac{5}{2}\left\{\frac{0{\cdot}255\lambda_1 + 0{\cdot}313\lambda_2}{3{\cdot}062 + 1{\cdot}713\lambda_1/\lambda_2 + 1{\cdot}046\lambda_2/\lambda_1}\right\},$$

where λ_1 and λ_2 are the proportions by volume of the heavier and lighter gases respectively in the mixture, so that $\lambda_1 + \lambda_2 = 1$. λ_1 λ_2

This gives to k_t, for the three mixtures A, B and C examined, the respective values 0·080, 0·110 and 0·124. The experimental values of k_t obtained from the curves are 0·061, 0·080, 0·093, so that R_t has the values 0·77, 0·73, 0·75. This gives a mean value of R_t for the pair of about 0·75, which can be regarded at present as nearly independent of temperature over the range which we have studied.

(2) *Hydrogen and neon.* In the three mixtures examined the proportionality of separation and $\log(T_1/T_2)$ holds over the range of temperature from 15° C. to − 145° C. The separation obtained by means of liquid air is in all cases definitely below the proportional value: this is shown for two of the mixtures in figure 3,

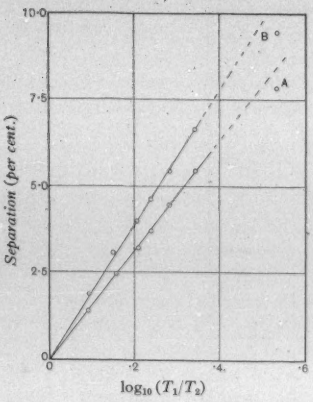

Fig. 3. Hydrogen-neon. Mixtures A and B.

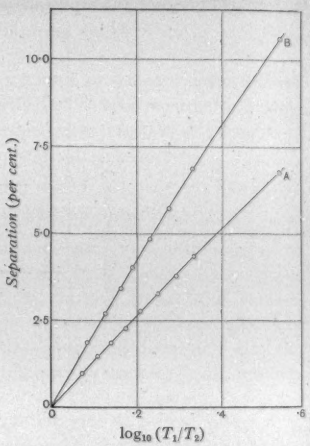

Fig. 4. Helium-argon. Mixtures A and B.

the liquid-air point lying below the straight line produced. The falling off is however small, as the separation with liquid air is only about 8 per cent. below the proportional value. This compares with a falling off in separation of about 20 per cent. previously recorded* for mixtures of hydrogen and nitrogen.

Although the falling off in this case is not great, the definite change produced by substitution of hydrogen for helium in mixtures with neon appears worthy of notice.

Using Chapman's formulae we obtain for hydrogen and neon

$$k_t = \frac{5}{2}\left\{\frac{0{\cdot}177\lambda_1 + 0{\cdot}238\lambda_2}{2{\cdot}213 + 1{\cdot}131\lambda_1/\lambda_2 + 0{\cdot}845\lambda_2/\lambda_1}\right\},$$

where λ_1 and λ_2 have the same meaning as before.

This gives to k_t, for the three mixtures A, B and C examined, the respective values 0·089, 0·113 and 0·123. The experimental values of k_t obtained from the

* *Proc. Phys. Soc.* **41**, 465 (1929).

curves are 0·068, 0·083, 0·090, so that R_t has the values 0·76, 0·73, 0·73. This gives a mean ordinary value of R_t of 0·74 for hydrogen-neon mixtures which is constant from 15° C. to − 145° C.

(3) *Helium and argon.* Argon at atmospheric pressure liquefies at − 186° C. but the reduction of pressure to about 20 cm. of mercury, owing to cooling which occurs in the conditions of our experiments, lowers the liquefying point sufficiently for measurements to be made with liquid air. This proved to be of considerable assistance as it enabled the complete curves for argon mixtures to be obtained without additional experimental difficulty. The results for two of the three mixtures of helium and argon are shown in figure 4. It will be seen that the proportionality of separation and $\log_{10}(T_1/T_2)$ soon begins to fail, the straight line portion of the curve being short. This is in agreement with what has been previously observed* for mixtures of hydrogen and argon for which the falling off in the value of k_t began at about − 80° C. The points obtained for helium and argon mixtures by means of cold pentane lie on a curve which can be smoothly extended to include the liquid-air point. Although the falling off in the effect is soon apparent for helium-argon mixtures, the actual bending of the curves is small. At − 190° C. the falling off in the total separation is about 8 per cent. below the proportional value, which compares with about 16 per cent. previously obtained for hydrogen and argon.

Measurements on mixtures of helium and argon with a higher range of temperature have previously been recorded†. The helium used in the earlier experiments contained a considerable amount of impurity.

Using Chapman's formulae for helium-argon mixtures we obtain

$$k_t = \frac{5}{2}\left\{\frac{0{\cdot}286\lambda_1 + 0{\cdot}244\lambda_2 \ddagger}{2{\cdot}140 + 1{\cdot}879\lambda_1/\lambda_2 + 0{\cdot}448\lambda_2/\lambda_1}\right\}.$$

This gives to k_t, for the three mixtures A, B and C examined, the respective values 0·074, 0·123, 0·151. The corresponding experimental values of k_t obtained from the straight portions of the curves are 0·060, 0·094, 0·105, so that R_t has the values 0·81, 0·76, 0·69. This gives a mean ordinary value for R_t of 0·75. The present values of k_t for this pair are considerably higher than those which would be obtained from our previous measurements.

The curves for helium-argon mixtures, which are continuous throughout the temperature range, yield not only the ordinary values of k_t and R_t but also the values as they diminish at the lower temperatures. From the slope of the curve B, shown in figure 4, we obtain the approximate experimental values of k_t for different temperatures of the cold side. These values are shown in table 1. Similar tables could be prepared from the other curves.

Table 1.

T_2	0° C.	− 50° C.	− 100° C.	− 150° C.	− 190° C.
k_t experimental	0·094	0·094	0·087	0·081	0·074
R_t	0·76	0·76	0·71	0·66	0·60

* *Proc. Phys. Soc.* **41**, 466 (1929).

† *Proc. R. S.* A, **107**, 484 (1925).

‡ Chapman and Hainsworth give $0{\cdot}210\lambda_2$.

Chapman's theory has shown that the value of R_t depends upon the nature of the forces operating during molecular collisions. The changing values of R_t thus provide material for the investigation of the variation of these forces with temperature.

(4) *Neon and argon.* The three curves for this pair of gases given in figure 5 show a marked falling off in the value of k_t at low temperatures. Continuous curves passing through the point given by liquid air can again be obtained. The behaviour is somewhat similar to that shown by the hydrogen-argon mixtures previously

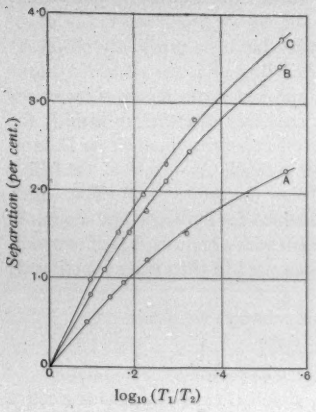

Fig. 5. Neon-argon. Mixtures *A*, *B* and *C*.

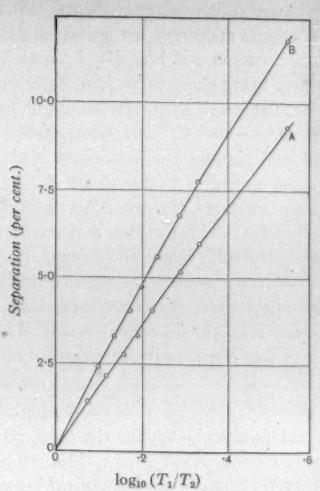

Fig. 6. Helium-nitrogen. Mixtures *A* and *B*.

examined* at low temperatures. The separation at − 190° C. is about 25 per cent. less than the proportional value. The results for this pair show the usual characteristic of argon mixtures, i.e. that the falling off in the value of k_t is soon apparent. It will also be seen that the curvature in the graphs for neon-argon is much more pronounced than in the case of helium-argon.

Using Chapman's formulae with the numerical values inserted for neon-argon mixtures we obtain

$$k_t = \frac{5}{2}\left\{\frac{0{\cdot}241\lambda_1 + 0{\cdot}237\lambda_2}{3{\cdot}616 + 2{\cdot}368\lambda_1/\lambda_2 + 1{\cdot}214\lambda_2/\lambda_1}\right\}$$

This gives to k_t, for the three mixtures *A*, *B* and *C* examined, the respective values 0·043, 0·070, 0·084. The corresponding experimental values for k_t obtained from the straight portions of the curves are 0·024, 0·037, 0·043, so that R_t has the values 0·56, 0·53, 0·51. This gives a mean ordinary value for R_t of 0·53.

* *Loc. cit.*

From the slope of the curve B shown in figure 5 we obtain the approximate value of k_t and R_t for different temperatures. These values are given in table 2.

Table 2.

T_2	0° C.	− 50° C.	− 100° C.	− 150° C.	− 190° C.
k_t experimental	0·037	0·037	0·029	0·022	0·017
R_t	0·53	0·53	0·41	0·31	0·24

(5) *Helium and nitrogen.* Two mixtures of this pair of gases were examined so that the results could be compared with those previously obtained for mixtures of hydrogen and nitrogen. The two curves in figure 6 show that the falling off in the value of k_t is much less than in the hydrogen-nitrogen mixtures. (A similar comparison has already been made of helium-argon mixtures and hydrogen-argon mixtures.) The total separation at − 190° C. is 7 per cent. less than the proportional value, compared with 20 per cent. less for the hydrogen-nitrogen mixtures. The effect of helium in the mixture is thus again well shown. The bending begins to appear in the parts of the curves obtained by means of cold pentane, i.e. before − 150° C., a feature which is characteristic of mixtures containing nitrogen. Continuous curves passing through the liquid-air point can be drawn as before.

Using Chapman's formulae with numerical values inserted for helium-nitrogen mixtures we obtain

$$k_t = \frac{5}{2}\left\{\frac{0{\cdot}322\lambda_1 + 0{\cdot}287\lambda_2}{2{\cdot}584 + 2{\cdot}137\lambda_1/\lambda_2 + 0{\cdot}554\lambda_2/\lambda_1}\right\}.$$

This gives to k_t, for the two mixtures A and B, the respective values 0·112 and 0·149. The corresponding experimental values for k_t obtained from the straight portions of the curves are 0·079 and 0·104, so that R_t has the values 0·71 and 0·70. This gives the mean ordinary value of R_t as 0·70.

From the slope of the curve A shown in figure 6 we obtain the approximate value of k_t and R_t for different temperatures. These values are shown in table 3.

Table 3.

T_2	0° C.	− 50° C.	− 100° C.	− 150° C.	− 190° C.
k_t experimental	0·079	0·079	0·077	0·071	0·064
R_t	0·71	0·71	0·69	0·63	0·57

The value of R_t at − 190° C. is thus only 20 per cent. below the value at 0° C. For hydrogen-nitrogen mixtures the corresponding variation is 50 per cent.

The results of the experiments are given in tables at the end of the paper so that they may be available if required for purposes of calculation.

§4. SUMMARY AND CONCLUSION

The experiments confirm and extend our previous work showing that there is a tendency for k_t to decrease at low temperatures. In no case have we found k_t to increase at these temperatures. A simple examination of the curves has shown that gases have characteristic effects in their behaviour in mixtures, e.g. helium in a mixture tends to make the variation of k_t small, and with argon in a mixture the change in k_t is soon apparent. A feature of the results is that for all mixtures k_t can be regarded as practically constant over a considerable range of temperature. The extent of this range depends of course upon the constituents of the mixture. This means that, within certain limits of temperature, gas-molecules during collision can be regarded as obeying an inverse-power law of repulsion, the index being constant. At lower temperatures the molecules tend to become "softer," their behaviour being less like that of rigid elastic spheres.

For the pairs of gases with weak attractive fields and low liquefying-points the value of k_t changes very little between 15° C. and − 190° C. In helium-neon mixtures the change is very small; and in hydrogen-neon mixtures there is a change below − 145° C. This is in agreement with what would be expected from the general conclusions of our previous work. The behaviour in thermal diffusion is thus related to that in viscosity* and the equations of state. It is perhaps significant that it is the hardest molecules which show the smallest variation in k_t.

Thermal diffusion may possibly provide the most direct method of studying the variations which occur in the nature of inter-molecular collisions as the temperature is reduced. A molecular model which allows for attractive as well as repulsive forces has been successful in explaining the variation of viscosity with temperature, and it may apply with equal success to thermal diffusion†. If such a model is unable to represent the observed effects at low temperatures it may be necessary to consider the possibility of changes in the actual nature of the molecular fields with temperature. Experiments on thermal diffusion at high temperatures now in progress‡ indicate the need for keeping in mind the latter interesting possibility.

Although we shall not attempt to consider in detail the theoretical aspects of the changes in k_t, it is of interest to obtain some idea of the general order of the repulsive forces arising during collision, which may be deduced from thermal diffusion data. Regarding the forces arising during collision as proportional to r^{-q}, where r is the distance between the molecules, Chapman has shown how R_t will depend upon the order q of the repulsive force. Exact values of R_t for different values of q are given in the special case§ where the ratios m_1/m_2 and σ_1/σ_2 are very large; m_1, m_2 being the respective masses and σ_1, σ_2 the respective diameters of the two molecules in the mixture. Although this is obtained for a special case it will

r

q

$m_1, m_2, \sigma_1, \sigma_2$

* Cf. J. Jeans, *Dynamical Theory*, 3rd edit. 286 (1920). Table for helium.

† A theoretical investigation taking account of attractive forces is in progress. Cf. footnote by S. Chapman, *Phil. Mag.* **8**, 1020 (1929).

‡ These measurements are being made by Mr A. C. R. Wakeman.

§ *Phil. Mag.* **48**, 606 (1924).

give approximately the general information we require, shown in table 4. If we assume that repulsive forces only are operating during collision, we can consider the falling off in k_t at low temperatures as directly due to a reduction in the value of q. The effective value of q at different temperatures can thus be obtained from the different values of R_t which we have obtained for any pair of gases.

Table 4. Approximate effective values of q.

Mixture	0° C.	− 50° C.	− 100° C.	− 150° C.	− 190° C.
He—Ne	18	Nearly constant			
H_2—Ne	17	Falling off below − 145° C.			
He—A	18	18	15	13	11
Ne—A	10	10	8	7	6
He—N_2	15	15	14	12	11
H_2—N_2*	10	10	8	7	6
H_2—A*	9	9	7	6·5	6†
N_2—A*	8	8	6·5	5·5	5·2†

The ordinary values of q shown in the first two columns correspond to the ordinary value of R_t for any pair. It will be seen that helium in a mixture appears to result in a high value of q. These ordinary values of q are greater than those which are at present considered to be most suitable in representing the behaviour of gases in viscosity and the equations of state. When we remember the difference in the methods of investigation, and the use of the above-mentioned approximation, the general nature of the results does not appear to be unsatisfactory. In particular, the relative values of q obtained for the different mixtures seem generally to be in agreement with such relative values as might be obtained by other means.

Further measurements are required on these and other pairs of gases. For example, the values of R_t obtained for the three helium-argon mixtures show greater differences than is usual, also the values of R_t for this pair are perhaps higher than we might expect from our knowledge of these gases. Our primary object in this work has been to continue the general study of the effect of low temperatures on thermal diffusion. With our increasing knowledge of the changes which occur in k_t the need for numerous measurements in making a general survey‡ of the phenomenon becomes more apparent. The examination of mixtures containing krypton and xenon will present additional difficulties: in this case a further reduction in the size of the diffusion apparatus will be necessary.

§ 5. ACKNOWLEDGMENT

In conclusion we desire to express our thanks to Prof. S. W. J. Smith for the help he has given by granting full facilities for this work.

* Values obtained from results of previous paper. † Value at − 180° C.

‡ In examining the phenomenon from different aspects some repetition of measurements is necessary. A general comparison and summary of results will be made later.

Table 5. Helium and neon.

A. 25·5 % He $T_1 = 14°$ C.			B. 39·9 % He $T_1 = 14{\cdot}8°$ C.			C. 53·6 % He $T_1 = 15{\cdot}3°$ C.		
T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %
− 191·5	0·546	7·71	− 191·5	0·548	10·03	− 192·0	0·551	11·20
− 144·0	0·347	5·02	− 145·0	0·352	6·64	− 139·6	0·334	7·30
− 128·8	0·299	4·22	− 126·5	0·293	5·43	− 122·0	0·281	5·98
− 115·0	0·260	3·61	− 109·5	0·245	4·54	− 102·1	0·227	4·95
− 96·6	0·212	2·98	− 84·0	0·183	3·41	− 82·0	0·179	3·79
− 73·2	0·158	2·21	− 56·8	0·124	2·29	− 59·2	0·130	2·67
− 47·6	0·105	1·45	—	—	—	—	—	—

Table 6. Hydrogen and neon.

A. 27·8 % H_2 $T_1 = 10{\cdot}3°$ C.			B. 40·6 % H_2 $T_1 = 12°$ C.			C. 49·6 % H_2 $T_1 = 9°$ C.		
T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %
− 191·0	0·538	7·84	− 191·0	0·539	9·44	− 191·5	0·538	10·01
− 145·1	0·344	5·43	− 145·2	0·347	6·66	− 143·7	0·338	7·06
− 125·9	0·284	4·46	− 125·5	0·286	5·40	− 129·1	0·292	5·87
− 111·0	0·242	3·70	− 109·9	0·242	4·63	− 112·0	0·243	5·26
− 100·0	0·214	3·23	− 96·8	0·209	3·97	− 101·5	0·216	4·40
− 76·0	0·159	2·47	− 73·2	0·154	3·10	− 76·3	0·156	3·33
− 44·6	0·093	1·41	− 44·6	0·098	1·90	− 67·5	0·137	2·85
—	—	—	—	—	—	− 51·9	0·106	2·04

Table 7. Helium and argon.

A. 20·85 % He $T_1 = 10°$ C.			B. 38·1 % He $T_1 = 9°$ C.			C. 51·1 % He $T_1 = 10°$ C.		
T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	T_2° C.	$\log_{10}\frac{T_1}{T_2}$	Separation %
− 192·0	0·542	6·79	− 192·5	0·544	10·62	− 192·0	0·542	11·84
− 141·6	0·333	4·34	− 141·2	0·330	6·86	− 143·0	0·338	7·99
− 127·7	0·289	3·76	− 122·5	0·274	5·71	− 125·9	0·284	6·72
− 112·0	0·245	3·22	− 106·5	0·229	4·84	− 110·7	0·241	5·62
− 96·6	0·205	2·74	− 89·0	0·186	4·00	− 96·5	0·205	4·82
− 81·3	0·169	2·25	− 77·7	0·160	3·41	− 85·5	0·179	4·19
− 66·8	0·138	1·85	− 59·6	0·122	2·67	− 66·2	0·137	3·34
− 50·4	0·104	1·43	− 38·7	0·081	1·84	− 43·5	0·092	2·27
− 31·5	0·069	0·95	—	—	—	—	—	—

Table 8. Neon and argon.

A. 19·1 % Ne $T_1 = 10°$ C.			B. 36·1 % Ne $T_1 = 9·5°$ C.			C. 51·7 % Ne $T_1 = 10°$ C.		
$T_2°$ C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	$T_2°$ C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	$T_2°$ C.	$\log_{10}\frac{T_1}{T_2}$	Separation %
− 193·6	0·552	2·22	− 191·5	0·539	3·40	− 192·0	0·543	3·71
− 137·5	0·320	1·52	− 140·3	0·328	2·44	− 143·5	0·339	2·80
− 105·0	0·226	1·21	− 121·5	0·271	2·10	− 121·3	0·271	2·30
− 83·2	0·174	0·96	− 105·0	0·226	1·78	− 101·8	0·218	1·93
− 67·5	0·139	0·79	− 88·7	0·186	1·53	− 78·7	0·163	1·52
− 39·5	0·084	0·51	− 63·7	0·130	1·10	− 46·3	0·097	0·98
—	—	—	− 46·6	0·096	0·82	—	—	—

Table 9. Helium and nitrogen.

A. 34·5 % He $T_1 = 11°$ C.			B. 53·1 % He $T_1 = 11·5°$ C.		
$T_2°$ C.	$\log_{10}\frac{T_1}{T_2}$	Separation %	$T_2°$ C.	$\log_{10}\frac{T_1}{T_2}$	Separation %
− 192·4	0·547	9·35	− 192·4	0·545	11·86
− 141·5	0·334	5·99	− 140·5	0·331	7·77
− 127·3	0·290	5·15	− 126·5	0·288	6·78
− 102·7	0·223	4·05	− 108·2	0·237	5·59
− 87·9	0·187	3·30	− 93·4	0·200	4·72
− 75·4	0·158	2·78	− 81·0	0·171	4·05
− 54·7	0·114	2·16	− 64·5	0·135	3·32
− 33·0	0·073	1·40	− 46·1	0·098	2·44

DISCUSSION

Prof. S. CHAPMAN. I should like to congratulate the authors on this very valuable and comprehensive addition to our knowledge of the facts about thermal diffusion. Dr Ibbs, with the assistance of his collaborators, has made himself the leading experimental expert on thermal diffusion, and in this and another recent paper on the phenomenon as it occurs at low temperatures has gone far to remove the apparent discrepancies that had arisen between the work of various experimenters on the subject. I think the discussion in this paper is as complete and instructive as can be given without entering into complicated mathematical aspects that are not yet fully worked out. I hope and anticipate that before long the theory may be further developed to the extent necessary for the fuller discussion of the many results now accumulated by Dr Ibbs and his colleagues.

Prof. A. O. RANKINE. I would like to ask whether the effect of the partial liquefaction of one of the components of the mixture is to be regarded as an extreme case of thermal diffusion. Clearly, if the cold bath is cold enough the less volatile of the

constituents will liquefy in part first, thereby reducing largely its concentration in the warmer chamber. Is this separation true thermal diffusion? On this point it should perhaps be noticed that the separation by differential liquefaction may be either of the same sign as that to be expected from the relative masses of the molecules, or of the opposite sign. Take water vapour and oxygen, for example. The water-vapour molecule is of smaller mass than the oxygen molecule, yet its liquefying point is much higher. Presumably in this case the two effects would be oppositely directed.

Prof. Sir A. S. EDDINGTON said that according to table 1 of the paper the molecules get softer and softer as the temperature falls. There must come a point, therefore, where the sign of the thermal-diffusion effect is reversed.

Prof. S. CHAPMAN. The reversal of the thermal-diffusion effect does not occur till the force-index falls below 5, a state of things of which there is no sign in these experiments. With reference to Prof. Rankine's remarks about liquefaction, one would expect that the gas composed of the larger and heavier molecules, which go preferentially to the cold end, would be the first to liquefy; hence it seems unlikely, though perhaps not impossible, that the two effects should work in the same direction.

AUTHORS' reply. Although we have not yet observed a case of the reversal of the thermal-diffusion effect as mentioned by Sir Arthur Eddington, we have kept in mind this interesting possibility during the progress of the work.

The question raised by Prof. Rankine regarding the possibility of partial liquefaction of one of the gases in the cold vessel is an important one, although any separation as the result of such condensation should not be considered as due to thermal diffusion. We have already considered this possibility with some care in our previous paper*, and concluded that the simple gas conditions could be regarded as holding throughout the experiments. At low temperatures we observe a decrease in the value of k_t, which means that the separation obtained is less than normal, whereas any partial condensation of the heavier gas would tend to increase the separation and make the apparent value of k_t greater than its true value. The example of a mixture of water vapour and oxygen, mentioned by Prof. Rankine, appears to provide a rather special case in which the lighter molecules would be the first to condense.

* *Proc. Phys. Soc.* 41, 468 (1929).

FURTHER EXPERIMENTS ON MAGNETOSTRICTIVE OSCILLATORS AT RADIO-FREQUENCIES

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ABSTRACT. Work on magnetostrictive oscillators at radio-frequency is continued from a previous paper. It is shown that the coil surrounding the bar can be in either branch of a simple tuned anode circuit. When the bar-coil is in the inductive branch the circuit may be operated as a series or parallel arrangement; in the latter case the direct plate current does not pass through the inductive branch of the fly-wheel circuit. The variation in either the anode or the grid current can be used to indicate resonance. Comparative experiments with corronil, nickel and glowray suggest that glowray is the most suitable of these materials for high-frequency oscillators. The experiments are carried on to the case of a glowray oscillator 1·9 mm. long which has a frequency of 1280 kc./sec. and weighs less than a fiftieth of a gramme.

§ 1. INTRODUCTION

This paper is a continuation of one of similar title* and records some further experiments on small magnetostrictive oscillators.

§ 2. THE OSCILLATOR AND COIL IN EITHER BRANCH OF THE TUNED CIRCUIT

In the discussion on the former paper, Dr D. Owen suggested that it would seem preferable to include the coil surrounding the oscillator in the condenser branch of the tuned circuit rather than in the inductance side, since in the latter case the variation of the plate current constitutes an unnecessary variation in the polarization of the oscillating bar. In my printed reply I described some work which had been done on this point but too recently to be included in the body of the paper. This showed that the coil surrounding the oscillator could be included in either branch and afforded an example in which the stability had a greater range with this coil in the condenser branch. Figure 1 gives the arrangement used in these experiments. The pivot P of the two-way switch is joined to the negative terminal of the high-tension battery and the ends of the oscillator coil L_3 are connected to the other two electrodes of the switch. By changing the switch arm from PR to PQ the experimenter could transfer the coil L_3 from the inductive branch to the capacitative branch of the oscillatory circuit. In the position PR the coil carries the steady plate current; in the position PQ the coil L_3 is traversed by alternating current only. It was found that the mutual inductance between L_1 and L_2 had to be increased in order to maintain the circuit in oscillation when the position was PQ.

* *Proc. Phys. Soc.* **41**, 476–86 (1929).

§ 3. BAR COIL IN THE INDUCTIVE BRANCH

The assemblage shown in figure 2 was set up in order to investigate the effect of the steady anode current when the oscillator coil is still in the inductive side of the fly-wheel circuit. It allows the plate current to traverse L_3 when the switch arm is in the position AB. This will be called the series arrangement. When the switch is in the position AC the anode current returns to A through R_2, L_3 being traversed by alternating current only. This will be called the parallel arrangement. The direct-current resistances of these alternative paths were made identical by the resistances R_1 and R_2. In either case the direct component of the current supplied by the high-tension battery was measured by A_m.

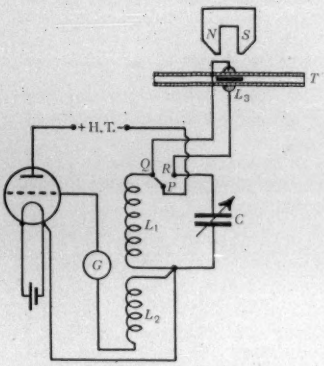

Fig. 1.

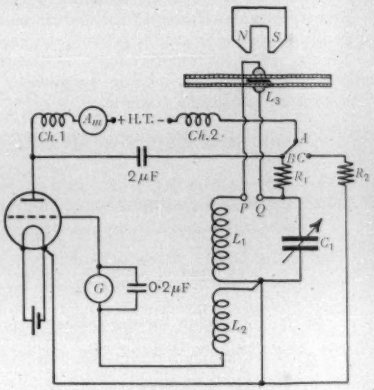

Fig. 2.

In figure 2, P and Q are small mercury cups into which the wires leading to L_3 dip. By exchanging the position of these wires the direction of the steady anode current could be changed when the switch was in the position AB. The coil L_3 has 400 turns of No. 42 enamelled copper wire (diameter 0·1 mm.) wound on a thin-walled glass tube in which was placed the glowray oscillator, 6 mm. in length, used in the previous paper. L_1 is a close-wound single-layer coil of 60 turns of No. 24 d.w.s. copper (diameter 0·56 mm.) on a cylindrical former 9 cm. in diameter; the grid coil L_2 is similarly constructed of 75 turns of No. 26 d.w.s. copper (diameter 0·46 mm.) on a former 8·2 cm. in diameter. L_1 and L_2 were mounted coaxially with the distance between their centres kept at 6·5 cm. during the experiments described below. The valve is an R.C. 210 B.T.H. and was operated with 90 volts on the plate and 1·9 volts on the filament, with no grid bias.

§ 4. ANODE CURRENT PASSING THROUGH L_3

Figure 3 gives a set of readings of the grid galvanometer G, the anode galvanometer A_m and the condenser of the absorption wavemeter, plotted against the values of C_1 as abscissae, for the series arrangement in which the steady current through L_3 assists the polarization due to the permanent magnet NS.

It is seen from figure 3 that with this disposition of apparatus the indications of either the grid or the anode galvanometer can be used to indicate resonance. Oscillation-hysteresis is indicated by arrows as in the previous paper.

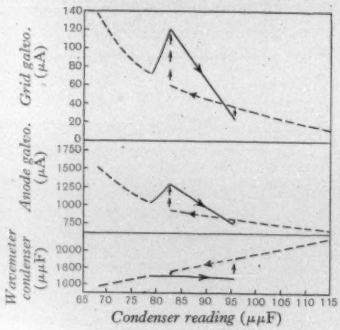

Fig. 3. Series arrangement.

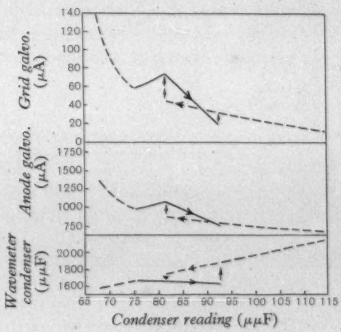

Fig. 4. Series arrangement with bar coil leads reversed.

If now the leads to P and Q from L_3 be interchanged, all else being kept the same, the results (shown in figure 4) are of the same general character; in particular the range of variation of the condenser C_1 over which the set is stabilized by the magnetostrictive oscillator is sensibly unaltered. The peaks of the curves of the galvanometer readings are, however, much less pronounced when the anode current is sent through L_3 in the direction to oppose the polarization due to the magnet.

§ 5. ANODE CURRENT NOT PASSING THROUGH L_3

When the leads to L_3 are in their original position so as to assist the polarization in the series arrangement, change of the switch from the AB to the AC position, which alters the mode of operation of the set from series to parallel working, produces no appreciable change in the curves. This is shown in figure 5, taken with the switch in the AC position and the leads to L_3 in the same position as for figure 3.

It might be expected that on reversal of the leads to L_3 no effect on the curves would be now found. But figure 6, taken after this reversal, shows that the current peaks are depressed very similarly to the case in which the steady anode current

flows through L_3. This shows that with the particular conditions of these experiments the reversal of phase of the oscillatory current in L_3 with respect to that in L_1 is not immaterial. This is probably due to a lack of symmetry in the wave-form of the oscillating current.

The wave-length corresponding to 1700 $\mu\mu$F on figures 3, 4, 5 and 6 is 719 m. and the frequency 417 kc./sec.

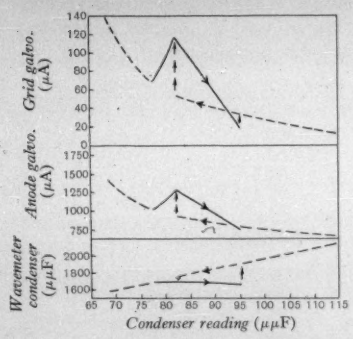

Fig. 5. Parallel arrangement.

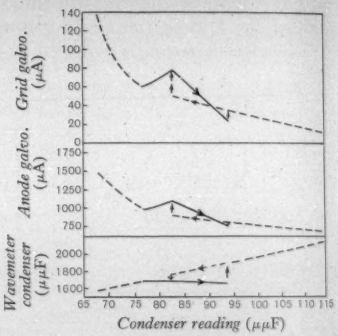

Fig. 6. Parallel arrangement, bar coil leads reversed.

From these experiments we see that the indicating galvanometer may be in the anode or grid circuit and the bar coil L_3 may be in the inductive or capacitative branches of the fly-wheel circuit.

§ 6. EXPERIMENTS WITH CORRONIL AND NICKEL

Replicas of the 6 mm. glowray bar were made in corronil and nickel. Corronil is an alloy, similar to monel, of the approximate percentage-composition 71 Ni, 25 Cu, 4 Mn. Pierce* found monel to be a very powerful oscillator. The bars of nickel and glowray behaved very similarly but the effects produced by corronil were very much more pronounced. Corronil, however, was not found to be a suitable material for high-frequency oscillators as its behaviour shows a marked instability due to change of temperature. In order to obtain reproducible curves it is necessary to use very small oscillatory fields. This effect shows itself as follows.

When the current in the grid galvanometer (and thus when the oscillatory current) is high, the deflection increases with time. If now the capacity is changed so as to reduce the deflection to a small value the deflection slowly decreases to a steady value. This instability affects all parts of the grid-galvanometer/capacity curve; the top of the resonance peak, however, is less affected than the remainder

* *Proc. Amer. Acad. of Arts and Science*, **63**, 1–47 (1928).

of the graph and soon attains a steady value. These effects are probably due to the magnetic permeability of corronil changing rapidly with temperature. The frequency corresponding to the peak value does not change noticeably so that these drifts are due to magnetic rather than mechanical changes. Cooling the oscillator by allowing a few drops of ether to evaporate from the bar coil and glass tube lowers the deflection of the galvanometer. If the cooling be carried lower than the dew point, the deposit of water acts as a clamp on the oscillator and the peak disappears from the curves but is restored when the tube and oscillator are dried. Glowray is much freer from these thermal drifts than corronil, as also is nickel, but nickel does not possess the same stabilizing properties as glowray. Pierce* remarks on the lack of stabilizing power of nickel; by comparison of the 6 mm. bars of glowray and nickel in the same apparatus, the range of variation of capacity over which the frequency was stabilized was found to be about 1·7 times as great for glowray as for nickel.

§ 7. CIRCUITS WHICH OSCILLATE ONLY AT THE CONTROLLED FREQUENCY

Since corronil is such a vigorous oscillator it is possible to repeat at these frequencies (378 kc./sec.) an experiment already described with longer bars. This is the reduction of the grid coupling to such an extent that the passage through

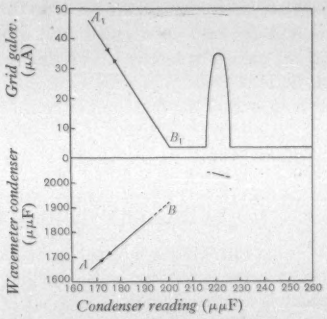

Fig. 7.

resonance is shown by a peak which is quite isolated, being flanked in either capacity-direction by a region in which the set does not oscillate†. This experiment is illustrated by figure 7 which shows a set of readings of the grid galvanometer (a unipivot of resistance 43 ohms) plotted against the capacity C, the apparatus being arranged as in figure 1. A large condenser not shown in this figure was connected across the plate battery. The valve was R.C. 210 B.T.H. operated at 100 volts with no grid bias. The 400-turn coil surrounding the corronil bar (6 mm. in length and 2·135 mm. in diameter) was in the inductive branch of the fly-wheel circuit. L_1

* *Loc. cit.*

† *Electrician*, **102**, 11 (1929).

was a single-layer coil of 40 turns of No. 24 d.w.s. (diameter 0·56 mm.) closely wound on a former 9·2 cm. in diameter. The grid coil L_2 was a similar coil of 75 turns of No. 26 d.w.s. (diameter 0·46 mm.) on a former 8·1 cm. in diameter. The mutual induction between these coils was reduced by trial until the resonance peak was isolated.

The upper part of figure 7 shows the way in which the steady grid-current changes as the capacity is decreased from a high value. The set is not oscillating at first but begins to oscillate when the capacity is reduced to 226 $\mu\mu$F; the deflection reaches a maximum at 222 $\mu\mu$F and returns to its former low value at 216 $\mu\mu$F, when the set is again quiescent. Throughout this range the set is approximately stabilized, the frequency falling slightly as the capacity is decreased. This is shown on the lower portion of the figure in which the ordinates are the readings of the condenser in the wavemeter circuit, the condenser value 2040 corresponding to a frequency 378 kc./sec. On further decrease of the capacity the set begins to oscillate again at about 200 $\mu\mu$F and now the stabilization is entirely absent, as is indicated by the line BA in the figure. In the part B_1A_1 of the grid-current curve the effects of change of temperature on the magnetic properties of the oscillator are pronounced, the recorded deflections depending on the time occupied at each setting of the condenser. This thermal drift does not sensibly affect the frequency, so that, although the curve A_1B_1 does not repeat itself, AB is quite definite. Apart from this thermal effect, oscillation-hysteresis is absent and the isolated peak is found in the same position when the capacity changes are made in the reverse direction.

Experiments of this type are easily performed with a corronil oscillator, and with different valves and bar coils. For instance, with this particular oscillator it is quite easy to obtain the isolated peaks even with a bar coil of only 50 turns, the bar being located in either branch of the fly-wheel circuit.

§8. EFFECT OF DECREASING THE NUMBER OF TURNS IN THE COIL SURROUNDING THE OSCILLATOR

When the number of turns surrounding a nickel oscillator is reduced, the peak on the grid-current/capacity curve is reduced and the falling portion of the curve becomes steeper as capacity is increased, while indications of oscillation hysteresis decrease until they become negligible. At the same time the range of stabilization becomes less until one finally gets a mere discontinuity, coinciding with the crevasse, in the frequency curve. This is illustrated by two pairs of curves, figures 8 *a* and 8 *b*. In figure 8 *a* a 6-mm. nickel oscillator is surrounded by a 400-turn coil, while in figure 8 *b* the coil has only 50 turns. The wavemeter capacity of 1650 $\mu\mu$F corresponds to a frequency of 420 kc. The two pairs of curves were drawn under nearly the same conditions, the mutual inductance between L_1 and L_2 being less in the case of figure 8 *b*. Figure 8 *a* shows that the stabilizing power of the nickel oscillator is limited to the falling part of the peak on the grid-current curve, and does not include the rising part of the peak on its low-capacity side as in the case of glowray oscillators. (See figures 2 and 3 of the former paper.) Curves similar

to those of figure 8 *b* could be obtained with different valves by proper adjustment of the coupling between the coils L_1 and L_2.

If a pair of telephones are inserted in the plate circuit under the conditions for producing a curve like that of figure 8 *b*, then if the capacity be decreased regularly a slight, ill-defined click is heard on passage through point *A* immediately to the left of the bottom of the crevasse. No sound was produced in the telephones when the curve was traversed in a reverse direction.

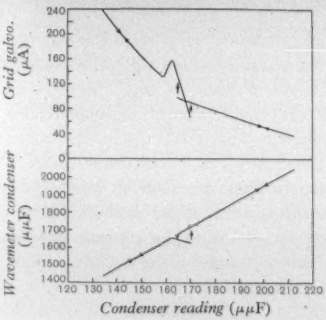

Fig. 8*a*.

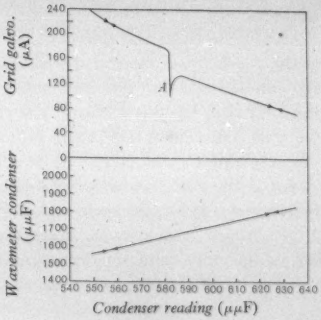

Fig. 8*b*.

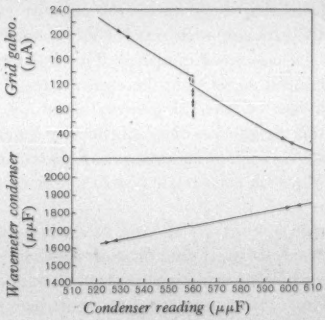

Fig. 9.

When a similar glowray bar is substituted for the nickel bar in figure 8 *b* marked oscillation-hysteresis is evident. This is illustrated in figure 9. When the curve is traversed from left to right a distinct peak, followed by a deep crevasse, is shown, while on the return path from right to left the deflection suddenly increases when the capacity has fallen to a certain value, and the crevasse and peak are missed entirely.

The possibility of obtaining a sharp crevasse in the grid-current/capacity curve with oscillators about 6 mm. in length suggested a further trial of shorter oscillators. The 4·5-mm. glowray oscillator (used in the former paper) was tried in a bar coil of 50 turns with an R.C. 210 B.T.H. valve. This gave a sharp crevasse when the curve was traced in the direction of increasing capacity, the measured frequency at the left-hand side of the top of the crevasse being 540 kc./sec. This value is identical, within the limits of accuracy of measurement, with the value obtained previously under very different circumstances, the valve, bar coil and polarizing field all being different.

When the curve was traced in the opposite direction a very small dip occurred in the curve at the value of the condenser corresponding to the right-hand side of the crevasse and was followed closely, on a slight decrease of the capacity, by a sudden rise in the grid current. The crevasse was thus apparently eliminated from the trace taken with decreasing values of capacity. This small dip in the curve, which precedes a sudden rise on decrease of the capacity, is a very usual feature of such curves, but is more noticeable when experiments are made at much lower frequencies. At low frequencies, however, the dip is not changed in character even if the changes in capacity are made exceedingly slowly. With the 4·5-mm. glowray bar, on the other hand, the dip is transformed into a crevasse if the trace from right to left is made with very small decrements of capacity.

§ 9. EXTENSION OF THE EXPERIMENTS TO HIGHER FREQUENCIES

These results suggested that if one varied the capacity very gradually it would be possible to detect the crevasse with much shorter oscillators, even when the number of turns in the bar coil was diminished. This was found to be the case and no difficulty was experienced in carrying the experiments down by easy stages to the case of an approximate cylinder of glowray about 1·9 mm. long and about 1·2 mm. in diameter: this weighs less than a fiftieth of a gram. The frequency of the bar was 1280 kc./sec., corresponding to a wave-length of 234 m. There does not seem to be any reason why such experiments could not be extended to much higher frequencies.

As the bars were made successively shorter, oscillation-hysteresis and stabilization of frequency decreased. It was found necessary also to increase the polarizing magnetic field and to work with oscillating currents of smaller amplitude.

It was possible to detect resonance with these short bars when the bar coil was in either branch of the fly-wheel circuit and with circuits differing very widely from each other. The frequency corresponding to resonance was, to a close approximation, constant throughout such changes.

In the case of the bar just mentioned, when it was surrounded by a coil of 25 turns of No. 42 enamelled copper wire (diameter 0·1 mm.) in the inductive branch of the circuit, the whole irregularity in the grid-current/condenser graph (including the crevasse and the slight departures from uniformity outside it on both sides) occurred within a range of $2\,\mu\mu$F in a total capacity of $575\,\mu\mu$F.

§ 10. ACKNOWLEDGMENTS

I have great pleasure in thanking my students, Messrs M. S. Hoban and G. E. Lloyd, for assistance in taking readings and Mr Lloyd for drawing the diagrams.

DISCUSSION

Mr S. BUTTERWORTH. I wish to congratulate Dr Vincent on having succeeded in maintaining a magnetic rod in oscillation at a frequency of more than $10^6 \sim$. The practical value of these mechanical oscillators is that they serve to supply accurate standards of frequency in a compact and convenient form. The phenomena shown can be imitated with purely electrical circuits, but in order to obtain sharp and precise indications these circuits must have very small damping which can be achieved only with bulky coils. The suitable circuit consists of a resistance and conductance shunted by a resonant circuit consisting of resistance, capacity and inductance.

THE EQUIVALENT CIRCUIT OF THE MAGNETOSTRICTION OSCILLATOR

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ABSTRACT. The Joule effect in magnetostriction is defined by the relation $S = \lambda B$, where S is the stress produced by a small increment of induction B of an existing induction B_0; and the converse effect by the relation $H = \kappa\xi$, where H is the magnetic field produced by a small strain ξ. Conservation of energy requires that $\kappa = 4\pi\lambda$. These equations are used in investigating the motion of a magnetostriction oscillator consisting of a closed circular ring inside a toroid, eddy currents and hysteresis being taken into account. It is shown that the oscillator can be represented by an equivalent electrical circuit comprising a pair of parallel impedances Z_c, Z_m, in series with an impedance Z_l, where Z_l represents leakage impedance and Z_c impedance due to core flux in the absence of motion, while Z_m is a resonant shunt to Z_c and represents the effect of the motion. Expressions for the elements of the circuit in terms of the fundamental constants of the material are given. The circle-diagram of impedances is deduced, and the modifying effects of eddy currents and hysteresis are investigated. Some simple geometrical relations between the vectors in the diagram are derived. It is shown that the size of the circle diagram for solid material having a large value of λ depends mainly upon permeability and resistivity and only slightly upon λ, and that a high degree of resonance may be associated with poor magnetostriction quality; the circle-diagram may, in fact, lead to erroneous conclusions unless interpreted in the light of an adequate theory. An experimental investigation of the resonant radial vibrations of solid and laminated nickel rings verifies the theoretical deductions and leads to numerical values of λ and κ in agreement with values deduced from the work of Masumoto and Nara and of McKeehan and Cioffi. For nickel in the annealed state, $\lambda = 1{\cdot}76 \times 10^4$ and $\kappa = 22{\cdot}1 \times 10^4$ at a point on the curve corresponding to $H_0 = 14{\cdot}5$ gauss.

§ 1. INTRODUCTION

The work hitherto published on the magnetostriction oscillators, proposed by Pierce[1] as standards of frequency, has been mainly of an experimental nature. The only theory available is based on initial assumptions which avoid the complications arising from eddy currents and hysteresis. With solid magnetostrictive materials, eddy currents are important since their effect is to confine the alternating magnetic flux to the surface of the vibrator, and it is desirable to develop a theory which takes this into account.

In common with other mechanical vibrators maintained in vibration by electrical means, the magnetostriction oscillator can be represented by an equivalent electrical circuit. When this equivalent circuit is known, the magnetostriction oscillator with the associated valve circuits can be considered as a single electrical circuit from

which the conditions for the maintenance of mechanical vibrations and the optimum values for the components of the valve circuit can be found by standard methods. Formulae for calculating the elements of the equivalent circuit from the fundamental properties of the magnetostriction oscillator are derived in the course of the theoretical analysis; these formulae show the effect of any changes in the material, dimensions or coil of a magnetostriction oscillator and are of some assistance in arriving at the best oscillator for any specific purpose. It is to be noted, however, that the magnetostriction oscillator should always be considered in association with the external circuit maintaining it in vibration. The problem of finding the best oscillator to use under a given set of circumstances cannot be solved by mere substitution of rods or rings of various metals and sizes in a single circuit, as the circuit may be nearer optimum conditions for one rod than for another. Considerations of this kind may account for a statement by Pierce[1] that nickel is not the best material to use when employing magnetostrictive systems to control the frequency of a valve oscillator. Nickel shows a large magnetostriction effect and it is possible that a smaller effect was required to satisfy optimum conditions in his circuits.

A sound fundamental theory is therefore essential if the best results are to be obtained without an excessive number of experiments. This paper attempts to set out this theory and to show how it may be used to derive the fundamental properties of magnetostrictive materials from motional-impedance measurements.

§ 2. FUNDAMENTAL PRINCIPLES

If a magnetized material undergoes a small cyclic change B of induction, an increment in stress S is produced which is proportional to B, so that we may write

B
S

$$S = \lambda B \qquad \text{.....(1).}$$

The factor λ may be positive or negative and its value depends upon the steady induction about which the small change B takes place.

λ

Conversely, if the magnetic material undergoes a small cyclic fractional extension ξ, a small magnetizing force H is induced which is proportional to ξ so that

ξ, H

$$H = \kappa\xi \qquad \text{.....(2).}$$

κ

If we ignore the complication of hysteresis it may be shown from the principles of the conservation of energy that the factors λ and κ are not independent, and if we use the electromagnetic system of units we have the relation

$$\kappa = 4\pi\lambda \qquad \text{.....(3).}$$

In this system it is interesting to notice also that the square of λ (or κ) has the dimensions of a stress.

§ 3. EQUATION OF MOTION OF MAGNETOSTRICTIVE RING

Consider a ring of magnetostrictive material such as that shown in figure 1.

r, b, d The ring has mean radius r and rectangular cross-section b, d. If the material has

E, σ Young's modulus E and density σ, then for a peripheral strain ξ the increase in

x mean radius is x, or $\frac{x}{r}\xi$, so that the potential energy due to the strain is V, where

γ
$$V = \tfrac{1}{2}E\,(x^2/r^2).\,2\pi rbd = \tfrac{1}{2}\gamma x^2, \text{ say} \qquad \text{.....(4)},$$

in which
$$\gamma = 2\pi bdE/r \qquad \text{.....(5)}.$$

T Also for varying strains the ring has kinetic energy T given by

$$T = \tfrac{1}{2}\sigma \times 2\pi rbd\dot{x}^2 = \tfrac{1}{2}\alpha\dot{x}^2 \qquad \text{.....(6)},$$

α in which
$$\alpha = 2\pi rbd\sigma \qquad \text{.....(7)}.$$

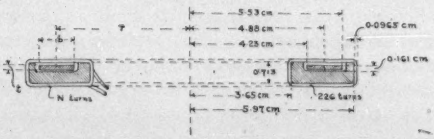


Fig. 1.

F If, further, we assume the dissipation function F to be given by

$$F = \beta\dot{x}^2 \qquad \text{.....(8)},$$

the equation of motion follows from the usual dynamical theory and is

$$\alpha\ddot{x} + \beta\dot{x} + \gamma x = \psi \qquad \text{.....(9)},$$

ψ in which ψ is the force producing the strain.

E, β The modulus E and the damping coefficient β must be carefully defined. Any straining of magnetostrictive material produces magnetic force and, if this strain varies, eddy currents which absorb energy from the motion and produce damping are set up in the material. The factor β in equation (9) is not supposed to include this damping, which will be taken into account as part of the force ψ in the later equations. Also the stress produced by the induction caused by the above magnetic force is proportional to the strain and so may legitimately be regarded as producing a modification of Young's modulus. It would be so included if the modulus in question were measured with the material under magnetization, and in the case of oscillators its value would depend not only upon the state of magnetization but also on the rate of alternation of the strains. The Young's modulus to be used in finding the γ factor of equation (9), however, is supposed to exclude this variable factor. Its effect will be included as part of the force ψ in the later equations.

For the present applications the force ψ is due to the total flux across the section of the ring, which is produced primarily by a variable current flowing in the surrounding toroidal coil. Owing to eddy-current effects the induction will vary in magnitude and phase across the ring-section and, if ϕ be its integral value, the

integral stress will be $\lambda\phi$ by (1). This stress produces a radial component of force $\lambda\phi . \delta\theta$ on an element subtending an angle $\delta\theta$ at the centre of the ring. Since each element is subject to a similar radial force, the value of ψ is given by $\delta\theta$

$$\psi = 2\pi\lambda\phi \qquad \text{.....(10).}$$

Using this value of ψ in equation (9) and assuming alternating flux, we now have for the equation of motion

$$(\gamma - \alpha\omega^2 + j\omega\beta)\, x = 2\pi\lambda\phi \qquad \text{.....(11),}$$

in which x and ϕ now represent rotating vectors having pulsatance ω, and j is the operator denoting anti-clockwise rotation through a right angle on the vector diagram. x, ϕ, ω, j

For the purposes of abbreviation we may write (11) in the form

$$\nu x = 2\pi\lambda\phi \qquad \text{.....(12),}$$

in which

$$\nu = \gamma - \alpha\omega^2 + j\omega\beta \qquad \text{.....(13).} \qquad \nu$$

§ 4. RELATION BETWEEN INTEGRAL FLUX, DISPLACEMENT AND CURRENT

If the ring is surrounded by a toroidal coil having N turns and carrying a current $I\epsilon^{j\omega t}$, the magnetizing force due to this current has amplitude $2NI/r$. In addition, there is the magnetizing force due to the strain which, by equation (2), is $\kappa x/r$. Hence the total amplitude of the magnetizing force is given by N I, t

$$H = (2NI + \kappa x)/r \qquad \text{.....(14).}$$

For very low frequencies the integral flux produced by H may be written

$$\phi = \mu H b d \epsilon^{-j\eta} \qquad \text{.....(15),}$$

in which μ is the reversible permeability and η the angle of lag, due to hysteresis, at the working point on the magnetization curve of the ring. μ, η

The formula requires modification when eddy currents are appreciable and then we must replace (15) by

$$\phi = \mu H b d \epsilon^{-j\eta} \chi \qquad \text{.....(16),}$$

in which χ is the eddy-current vector defined by its amplitude χ_0 and lagging phase ζ so that χ, χ_0 ζ

$$\phi = \mu H b d \chi_0 \epsilon^{-j(\eta+\zeta)} \qquad \text{.....(17).}$$

The in-phase and quadrature components of χ will be denoted by χ_R and χ_I which are related to χ_0 and ζ by χ_R, χ_I

$$\chi_R = \chi_0 \cos\zeta, \quad \chi_I = \chi_0 \sin\zeta \qquad \text{.....(18).}$$

Although χ is obviously a "dimensionless" quantity, yet it depends upon the linear scale of the ring section since one of the lengths involved in the determination of χ is the wave-length of propagation of electromagnetic disturbances in the material of the ring, and the linear size enters as a multiple of this wave-length.

If the ring section is such that d/b is small, then we can obtain an approximate value of χ from the ordinary theory of penetration into an infinite plane sheet of alternating magnetic forces. This theory gives

$$\chi = \theta^{-1} \tanh \theta \qquad \text{.....(19),}$$

θ, f, ρ where $\theta = \pi d\,(2j\mu f/\rho)^{\frac{1}{2}}$, f being the frequency and ρ the resistivity of the material.

L Since $(\rho/\mu f)^{\frac{1}{2}}$ is the wave-length, L, appropriate to electromagnetic waves in the material, we may denote the dimensionless nature of θ (and therefore of χ) by writing

$$\theta = \pi . (2j)^{\frac{1}{2}}\, d/L \qquad \text{.....(20).}$$

χ_0, ζ, χ_R, and χ_I are thus real functions of the real variable z, where

z $$z = \sqrt{2} . \pi d/L.$$

Table 1, which gives these functions, has been obtained from Kennelly's *Tables of Complex Hyperbolic and Circular Functions*[9].

Table 1. Table for computation of eddy-current factors.

z	χ_0	ζ degrees	χ_R	χ_I
0	1·000	0·0	1·000	0·000
0·2	1·000	0·8	1·000	0·014
0·4	0·998	3·0	0·997	0·052
0·6	0·990	6·8	0·983	0·117
0·8	0·970	11·9	0·949	0·200
1·0	0·931	18·0	0·885	0·288
1·2	0·871	24·4	0·793	0·360
1·4	0·796	30·5	0·686	0·404
1·6	0·713	35·8	0·578	0·417
1·8	0·632	39·9	0·485	0·405
2·0	0·560	42·9	0·410	0·381
2·2	0·497	44·8	0·353	0·350
2·4	0·445	46·0	0·309	0·320
2·6	0·402	46·5	0·277	0·291
2·8	0·367	46·6	0·252	0·267
3·0	0·338	46·5	0·233	0·245

For large values of z, χ_0 settles down to the value $1/z$, ζ becomes $45°$ and $\chi_R = \chi_I = 0{\cdot}707/z$.

The above formulae and table 1 apply not only to a single ring of solid material but also to a composite ring built up of a number of flat rings. d then refers to the thickness of each ring and bd in (17) must be replaced by the total metallic section of the composite ring.

Using (14) in (17) we now have for the relation between integral flux, current and displacement

$$\phi = \mu bd\,(2NI + \kappa x)\,\chi_0 \epsilon^{-j(\eta+\Omega)}/r \qquad \text{.....(21).}$$

§ 5. CONTRIBUTION OF RING TO IMPEDANCE OF TOROIDAL COIL

The alternations of ϕ induce in the toroidal coil a back electromotive force v of amount $j\omega N\phi$, and if x is eliminated between the two equations (12) and (21), we find, on putting $j\omega N\phi = v$, that the relation between I and v may be written

v

$$I = v\,(1/Z_e + 1/Z_m) \qquad \text{.....(22),}$$

in which

$$Z_e = 2j\omega N^2 \mu bd\chi_0\, \epsilon^{-j(\alpha+\zeta)}/r, \qquad Z_e$$

$$Z_m = -j\omega N^2 \nu/\pi\lambda\kappa \qquad \text{.....(23).} \qquad Z_m$$

Thus the ring behaves as if it consisted of two impedances Z_e and Z_m in parallel.

The impedance Z_e is independent of the magnetostrictive and motional properties of the material. It is, in fact, the core impedance that would be obtained if the ring could be rigidly clamped. The factor $2N^2\mu bd/r$ is the core inductance at very low frequencies when hysteresis is neglected; we may denote it by L_0. Z_e may, then, be analysed into a resistance component R_e and a reactance component ωL_e. These components are given by the formulae

L_0

R_e, L_e

$$R_e = \omega L_0 \chi_0 \sin(\eta + \zeta), \quad L_e = L_0 \chi_0 \cos(\eta + \zeta) \qquad \text{.....(24).}$$

The impedance Z_m is independent of eddy currents and hysteresis and represents the reaction of the ring motion on the toroid. Using the value of ν in equation (13) we find that

$$Z_m = \frac{\omega^2}{\omega_0^2}\frac{N^2\gamma}{\pi\lambda\kappa}\left\{\frac{\beta}{\alpha} + j\omega\left(1 - \frac{\omega_0^2}{\omega^2}\right)\right\} \qquad \text{.....(25),}$$

in which $\gamma/\alpha = \omega_0^2$, and ω_0 is to be interpreted as the free pulsatance of the ring. We may write (25) thus:

ω_0

$$Z_m = R_m + j\,(\omega L_m - 1/\omega C_m) \qquad \text{.....(26),}$$

provided that

$$L_m = N^2\gamma\omega^2/\pi\lambda\kappa\,\omega_0^2, \quad R_m/L_m = \beta/\alpha, \quad L_m C_m = 1/\omega_0^2 \qquad \text{.....(27).}$$

L_m, R_m, C_m

It will be noticed that L_m is proportional to the square of the pulsatance ω, but as in all practical cases the value of Z_m is very high except when the pulsatance is in the neighbourhood of ω_0, we may without much error regard ω as equal to ω_0 in (27), and then with the help of (5) we get

$$L_m = N^2\gamma/\pi\lambda\kappa = 2N^2 bd\,E/\lambda\kappa r = L_0 E/\lambda\kappa\mu \qquad \text{.....(28).}$$

The ratio L_m/L_0 is thus an intrinsic property of the material of the ring whatever its linear dimensions.

Equation (26) shows that motional-impedance measurements do not distinguish between an actual moving system and an electrical circuit, consisting of a resistance R_m in series with an inductance L_m and capacity C_m, the whole of which is shunted by the resistance R_e in series with the inductance L_e. As any measurements on the toroidal coil must also include the leakage inductance L_e and the copper resistance R_e the complete electrical circuit revealed by alternating current measurements is that shown in figure 2.

§6. IMPEDANCE DIAGRAM OF MAGNETOSTRICTION OSCILLATOR

A usual means of representing the properties of electromagnetic oscillators is the impedance diagram. The resistance and reactance (or impedance and phase angle) are measured for a series of frequencies and these are plotted in rectangular (or polar) co-ordinates, resistance being measured horizontally and reactance vertically. The points so obtained, when connected by a curve, form the impedance diagram for the oscillator. For a magnetostriction oscillator the curve obtained in the neighbourhood of resonance is practically a circle as shown in figure 6, and if measurements are extended so as to include frequencies well below and then above resonance, the out-of-resonance points lie upon a smooth curve to which the circle is attached. The diameter of the circle which passes through the point of attachment usually slopes downwards at a considerable angle, although this slope may be made small by the use of highly laminated material.

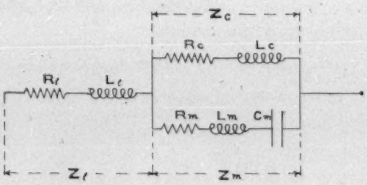


Fig. 2.

We will now show that the electrical system of figure 2 will give this type of impedance diagram provided that Z_m is very large, except in the neighbourhood of mechanical resonance. Since resistances are measured horizontally and reactances vertically on the impedance diagram, we will denote resistances by X symbols and reactances by Y symbols, while polar distances will be denoted by r and angles with respect to the horizontal by ϕ. The subscripts l, c and m will as before refer to leakage, core and motion, while the subscript T will refer to the overall components.

X
Y, r
φ, l, c, m
T

The total impedance in vector notation is given by

$$\begin{aligned} Z_T &= Z_l + Z_m Z_c/(Z_m + Z_c) \\ &= Z_l + Z_c - Z_c^2/(Z_m + Z_c) \\ &= X_l + jY_l + r_c\epsilon^{j\phi_c} - r_c^2\epsilon^{2j\phi_c}/(X_c' + jY_c') \qquad \text{.....(29),} \end{aligned}$$

in which

$$X_c' = X_m + X_c, \quad Y_c' = Y_m + Y_c \qquad \text{.....(30).}$$

In the immediate neighbourhood of resonance the only factor in (29) which varies considerably with frequency is Y_c', occurring in the last term of (29). Also if the non-resonant value of Z_m is large compared with Z_c, the whole of the effect of this last term occurs in the immediate neighbourhood of resonance. We will assume that the non-resonant value of Z_m is so large that the remaining terms in (29) may be

regarded as practically constant during the range of frequency-change in which the last term is appreciable. From (24) and (25) we have

$$Y_e' = \omega (L_m + L_e) - 1/\omega C_m \qquad \text{.....(31).}$$

Writing

$$(L_m + L_e) C_m = 1/\omega_1^2 \qquad \text{.....(32),}$$

and supposing $(\omega - \omega_1)/\omega$ small, we find that

$$Y_e' = 2p (L_m + L_e) \qquad \text{.....(33),}$$

in which $p = \omega - \omega_1$.

The pulsatance ω_1 is slightly lower than ω_0, the relation between ω_0 and ω_1 being given by

$$\omega_0/\omega_1 = (1 + L_e/L_m)^{\frac{1}{2}} \doteq 1 + \tfrac{1}{2} L_e/L_m \qquad \text{.....(34).}$$

In the electrical system shown in figure 2, ω_0 is the resonance pulsatance of the arm Z_m and ω_1 the resonance pulsatance of the mesh $(Z_m + Z_e)$. Mechanically ω_0 is the resonance pulsatance calculated from a value of Young's modulus which is free from the effects of magnetostriction and eddy currents, while ω_1 would be obtained if we included these effects in estimating the modulus. For the purpose of interpretation of the impedance diagram it is preferable to measure variations in pulsatance from ω_1 rather than from ω_0.

If we use the relation

$$\tan \phi_e' = Y_e'/X_e' = 2p (L_m + L_e)/X_e' \qquad \text{.....(35)}$$

to eliminate Y_e' from (29), we obtain

$$Z_T = X_1 + jY_1 + r_e \epsilon^{j\phi_e} - \epsilon^{j(2\phi_e - \phi_e')} \cos \phi_e' \, r_e^2/X_e' \qquad \text{.....(36),}$$

and ϕ_e' is the only factor which varies with frequency.

On the impedance diagram, figure 3, we first obtain the point M, giving the extremity of the impedance vector at the pulsatance ω_1. Following (36) for the case $\phi_e' = 0$, we draw OL to represent the leakage impedance $X_1 + jY_1$ and LC to represent the core impedance $r_e \epsilon^{j\phi_e}$. The last term of (36) is then represented by a line CM of length r_e^2/X_e', $L\hat{C}M$ being equal to ϕ_e. This direction is clearly obtained by rotation of a negative vector through $2\phi_e$ in the positive (or counter-clockwise) direction, as is demanded by the form of the phase factor in (36), when $\phi_e' = 0$.

If axes LX', LY' are drawn through L parallel to OX, OY, and CM cuts LX' in P, then $LP = CP$.

When ϕ_e' is not zero the extremity of the impedance vector is shifted to M', which is obtained from M by rotation of CM through ϕ_e' in the negative (or clockwise) direction about C, and reducing its amplitude by the factor $\cos \phi_e'$. Thus the angle $CM'M$ is a right angle and M' traces out a clockwise circle on the diameter CM as ϕ_e' increases.

Through Q, the centre of this circle, draw QS parallel to PL to meet LC in S. Then since $LP = CP$, $QS = QC$ so that the circle cuts LC at S in the vertical direction. The point S is thus the point of minimum resistance of the system.

Again, $CM = r_e^2/X_e' = r_e^2/(X_m + X_e)$, so that CM is always less than r_e^2/X_e or $r_e \sec \phi_e$. But since $CL = r_e$, $CP = \frac{1}{2} r_e \sec \phi_e$, and therefore $CQ < CP$. It follows that the point of minimum resistance always lies above L.

Physically, of course, this result is inevitable, since theoretically X_l may be made as small as we please and the total resistance of the system can never be negative.

The existence of dip of the resonance diameter of the motional-impedance circle is well known, but its inevitable consequence, the limitation of the circle diameter, does not seem to have been sufficiently emphasised. This limitation has an important bearing upon the interpretation of circle diagrams. If the mechanical damping, here represented by X_m, is negligible, then the centre is at P whatever the values of the magnetostriction constants, and this condition is approximated to most closely in the case of solid materials having good mechanical qualities. Mere information as to the sizes of the impedance circles then throws no light upon the magnetostrictive properties of the material. In fact, in the extreme case, the diameter of the impedance circle merely gives a measure of the damped-core impedance. This argument does not apply to measurements upon highly laminated specimens, as in these cases the eddy-current damping, the main factor governing the size of X_e, has been deliberately made small, and the motional factor X_m then has a large measure of control of the circle diameter.

In order to get the value of X_m from an experimental circle diagram it is necessary to know the ratio CP/CQ, which is $(X_m + X_e)/X_e$. This involves a knowledge of the damped-core impedance and therefore of X_e, and the value of X_m follows from CP/CQ.

If the damped-core impedance is not known, we can measure (or calculate) the copper resistance X_l and, knowing CM, we can draw CL bisecting the angle between CM and the horizontal, and make CL cut the vertical through X_l at L. The actual determination of the leakage inductance need be made only in order to verify the adequacy of the theory.

The remaining elements in the arm Z_m may be found if we know the pulsatance change ρ which occurs on passage from M to M', which yields L_m by (35), while the actual pulsatance ω_1 at the point M gives C_m by (32).

§ 7. DETERMINATION OF MECHANICAL RESONANCE CURVE FROM IMPEDANCE DIAGRAM

I_m

If I_m be the current flowing through the arm Z_m of figure 2, then since this is produced by the core voltage v, which $= j\omega N\phi$,

$$I_m = j\omega N\phi / Z_m \qquad \text{.....(37).}$$

Using the value of Z_m given by (23), and the value of ϕ given by (12), we find that I_m is related to the displacement x by the formula

$$I_m = -\kappa x / 2N \qquad \text{.....(38).}$$

Thus with the exception of the constant multiplier $2N/\kappa$, I_m gives a measure of x

for all phases of the motion. Now if I be the total current flowing through the system of figure 2, we have

$$I_m/I = Z_c/(Z_c + Z_m) = r_c \epsilon^{j(\phi_c - \phi_c')} \cos \phi_c'/X_c' \qquad \text{.....(39).}$$

If, on a current diagram, we draw the vector $r_c \epsilon^{j\phi_c}/X_c'$, this will represent in magnitude and phase the maximum value of I_m/I, and the vector values of I_m/I on either side of this maximum have the locus of their extremities lying on a circle of which the diameter is the maximum vector. We may adapt the impedance diagram of figure 3 to represent the I_m circle by taking the point C as origin and letting the scale be such that CL is numerically equal to I. For in the impedance diagram

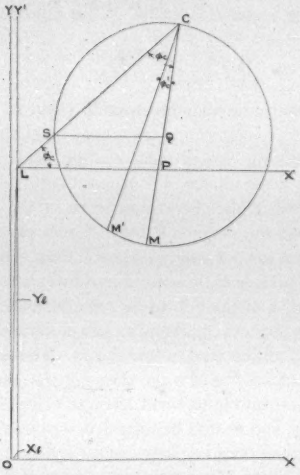


Fig. 3.

$CL = r_c$ and $CM = r_c^2/X_c'$, so that CM/CL is equal to the maximum value of I_m/I. Moreover, the vector CM leads the vector CL by the correct phase-angle ϕ_c, so that if the direction of CL indicates the phase of I, then $L\hat{C}M'$ will give the phase-difference between I_m and I. By (38) the circle represents also the displacement diagram for constant total current.

§ 8. DETERMINATION OF THE MAGNETOSTRICTION CONSTANTS

By (28) we know that $L_m = L_0 E/\lambda\kappa\mu$ and $L_0/\mu = 2N^2bd/r$, while E is related to the pulsatance ω_0 by the formula

$$\omega_0 = (E/\sigma)^{\frac{1}{2}}/r \qquad \text{.....(40).}$$

We have shown how L_m may be found from the impedance diagram, and since the remaining quantities may be determined by simple measurement, we can deduce

the value of $\lambda\kappa$. If we accept the relation $\kappa = 4\pi\lambda$ we have thus a means of determining λ. Otherwise we require further information. A determination of the value of x for a known total current would enable κ to be estimated separately by the methods of the last section. The determination of κ, however, in the case of a ring would not be easy, and the experiments of the second part of this paper have been directed to the determination of λ on the assumption that $\kappa = 4\pi\lambda$. The determination of μ and ρ under the working conditions is not too easy for solid rings, but a proper analysis of the change of damped-core impedance with frequency enables these quantities to be estimated. This analysis is described in the second part of the paper and has been carried out in order to see whether the whole theory can be fitted to the facts, while reasonable values of the fundamental constants are still retained.

§ 9. EXPERIMENTAL

The magnitude of the Joule magnetostriction effect in ferromagnetic materials is usually defined by a curve connecting the steady magnetizing field applied to the specimen with the resulting proportional change of length [5, 6]. These curves provide some guidance in the choice of a material suitable for a metrical study of the phenomena described in the theoretical work of the preceding section. The changes of induction and the magnetostrictive forces produced by an alternating magnetic field superposed upon a steady magnetic field, which are to be considered here, are not, however, the same as those produced by a small increment of an existing steady magnetizing field, a distinction to be remembered in considering the published curves; the difference between the two cases is somewhat akin to the difference between reversible and differential permeability. The largest magnetostriction effects are observed in nickel [5] and some alloys of the permalloy group [6]. These alloys are particularly susceptible to small changes of heat treatment, composition and mechanical working, and cannot be regarded as suitable for a metrical investigation. Nickel has therefore been employed in the following work, since it has been widely studied and many of the properties relevant to this investigation have been measured and can be reproduced by suitable heat treatment.

§ 10. HYSTERESIS AND EDDY CURRENTS IN NICKEL

Part of the theory is concerned with the influence of eddy currents and hysteresis on the magnetostriction effects, and the first step in the experimental work must be to establish the magnitude of these effects for nickel. A circular ring, cut from a sheet of nickel to the dimensions given in figure 1, was annealed at 1000° C. for two hours and cooled slowly during twelve hours. It was covered with a single layer of silk tape and tightly wound toroidally with 231 turns of copper wire of No. 22 s.w.g. insulated to an overall diameter of 0·038 in. The tight winding served a double purpose in that it reduced leakage flux between the wire and the nickel to a minimum and tended to prevent resonance of the ring under the action of the forces produced by magnetostriction. A steady magnetizing field of 14·5 gauss was applied to the

ring by direct current flowing in another toroidal winding, after the usual precaution of demagnetizing the ring had been taken. Measurements of the impedance of the tightly wound toroid were made on an alternating-current bridge, described elsewhere[7], over the frequency range 640–41,200 ∼. The leakage inductance and the copper resistance were estimated from published formulae[8] and subtracted from the total inductance and resistance in order to obtain the inductance and resistance due to core flux only. This core inductance and resistance can be expressed in the form

$$L_c = L_0\chi_0 \cos(\eta + \zeta),$$

$$R_c = \omega L_0 \chi_0 \sin(\eta + \zeta) \qquad \text{.....(41)}$$

from § 4 (24), and thence

$$R_c/\omega L_c = \tan(\eta + \zeta) \qquad \text{.....(42).}$$

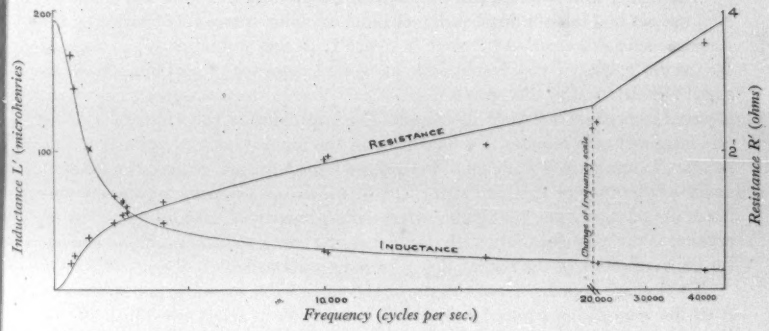


Fig. 4.

Here η and ζ are the angles of lag due to hysteresis and eddy currents respectively. The dimensions and properties of the nickel ring are such that flux is confined to a thin surface-layer at frequencies over 5000 ∼, and at such frequencies the angle ζ must be 45°. The measured values of $R_c/\omega L_c$ above 5000 ∼ averaged 1·15, which is tan 49°, whence $\eta = 4°$. η, ζ

Now by expanding equation (41), and writing $\sin\eta = \eta$ and $\cos\eta = 1$, we can show that, if small terms in η^2 are discarded,

$$L' = L_0\chi_0 \cos\zeta = L_c + R_c\eta/\omega, \qquad L'$$

and

$$R' = \omega L_0 \chi_0 \sin\zeta = R_c - \omega L_c \eta \qquad \text{.....(43),} \qquad R'$$

where the resistance R' and inductance L' calculated from R_c, L_c and η should be of the same form as the functions $\chi_0 \sin\zeta$ and $\chi_0 \cos\zeta$ respectively. Values of R' and L' are represented by the marked points in figure 4, and values calculated from the functions $\chi_0 \sin\zeta$ and $\chi_0 \cos\zeta$ are represented by the smooth curves. The calcu-

lated curves have been brought into the best general agreement by the adoption of the following numerical values for μ and ρ*:

Permeability (reversible)	μ	42·3
Resistivity	ρ	$1{\cdot}00 \times 10^4$ e.m.u.

from which the inductance L_0 at zero frequency is 194 μH and

$$z = 0{\cdot}047\ \sqrt{f}.$$

It is clear that the theoretical expressions (41) represent the damped impedance of the ring adequately when the symbols have the numerical values given.

§ 11. MOTIONAL IMPEDANCE

The tightly wound toroid and the insulating tape were unwound and the nickel ring was set in a loose-fitting bakelite channel in which it was free to vibrate; the dimensions of this channel are given in figure 1. It was wound with 226 turns of No. 22 s.w.g. copper wire insulated to an overall diameter of 0·038 in. An additional toroidal winding was provided as before to carry a steady magnetizing current adjusted to produce a field of 14·5 gauss. The impedance of the 226-turn winding was measured at a number of frequencies in the range 12,000 to 20,000 ~. The measured impedance is made up of the leakage impedance and the core impedance, and the latter may be regarded as the sum of the damped impedance and the motional impedance†. The leakage impedance is a property of the toroid and has no relation to the magnetostrictive effects; in fact, the core impedance only is required for the verification of the results of the previous section.

The leakage impedance can be estimated from suitable formulae and subtracted from the total damped impedance, but as it amounts to about two-thirds of this impedance the method is not accurate. It is preferable to make use of the results obtained in § 10 and to calculate the part of the damped inductance due to core flux from (41), the appropriate numerical values being used. The two methods give results in satisfactory agreement, thereby providing some check on the accuracy of the measurements. The total impedance due to core flux may now be represented by the smooth curves and the damped impedance due to core flux by the dotted

* In order to estimate the values of μ and ρ from the observed values of L' and R' the following formulae, deduced from the theory given in § (3), are useful. At high frequencies

$$R' = 2\pi f L' = 2N^2 b\ \sqrt{(\rho\mu f)}/r \qquad \text{.....(A).}$$

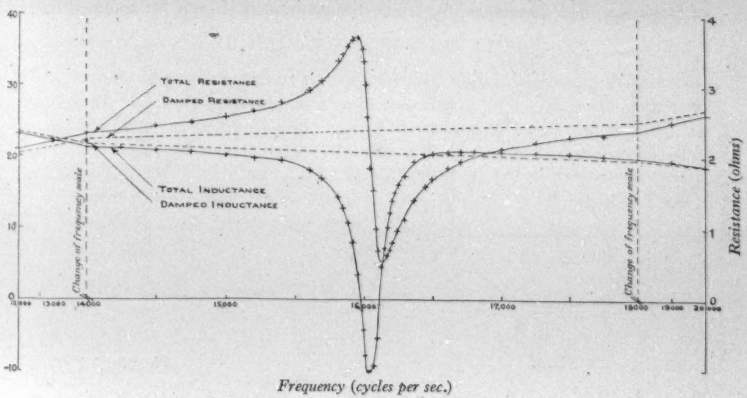
At low frequencies

$$\left.\begin{aligned} L' &= L_0\,(1 - \pi^4 t^4 \mu^2 f^2/5\rho^2) \\ R' &= 4\pi^3 t^2 \mu L_0 f^2/3\rho \end{aligned}\right\} \qquad \text{.....(B).}$$

From (A) the value of $\rho\mu$ may be found by plotting the high frequency values of R' (or fL') against $\sqrt{f}$. From (B) the value of μ may be found by plotting the low frequency values of L' against f^2. This gives L_0, from which μ may be found. The initial frequency coefficients in (B) can be used as checks.

† The terms "motional impedance," "motional resistance," etc., used in this section must not be confused with the elements of the parallel arm Z_m developed in the theory. They refer to the equivalent series system and are represented in the theory by the last term of equation (36).

curves in figure 5, the difference between corresponding pairs of curves being the motional impedance. The motional inductance or, strictly speaking, the motional reactance increases as the frequency is raised from 12,000 ~, reaching a maximum negative value at about 16,040 ~. It vanishes at 16,490 ~, increases to a small positive value and then falls asymptotically to zero. At 15,970 and 16,110 ~ the motional reactance annuls the damped reactance and the total impedance reduces to a pure resistance; between these frequencies the total reactance is negative, that is, it can be represented by a condenser. The changes of motional resistance can be studied in a similar manner from the curves and do not call for special comment.



Frequency (cycles per sec.)

Fig. 5.

§ 12. GEOMETRY OF THE CIRCLE DIAGRAM

The motional impedances corresponding to pairs of marked points in figure 5 can be represented by vectors drawn from the point C in figure 6. The ends of these vectors, which are represented by marked points in the diagram, lie on a circular locus with centre at a point Q and radius QC. The departures of the measured points from the circle are an index of the errors to which the measurements are liable. The resonance diameter CM has been provisionally fixed by the method of plotting, but, as an error in the estimate of damped inductance leads to a movement of C round the circle, it is desirable to devise a more sensitive test for the determination of the exact inclination of CQM to the axes of co-ordinates. If M' is a point on the circle corresponding to a frequency f', equation (35) shows that a straight line should result when $\tan MCM'$, that is $\tan \phi_c'$, is plotted against f' for M' and similar points. A diagram of this kind is shown in figure 7 for the correct position of the resonance diameter CM. An error of 0·5° in the inclination of CM destroys the straight-line relation and yields a curve, concave or convex to the axis of frequency according to the sign of the error. The correction to the provisional position

of *CM* which results from this test is only 0·5°, which serves as an index of the errors to which the measurement of damped impedance are liable. The final estimate of the angle of depression of *CM* below the horizontal is 98·8°.

A vector *LC* can now be drawn representing the damped impedance at the resonance frequency and, if *L* is taken as origin, vectors drawn from *L* to points on the circle in the neighbourhood of resonance represent the total impedance due to core flux. The restriction is necessary because the length of the vector *LC* is a function of frequency and, strictly speaking, *L* has a different position for every point on the circle. However, a large part of the circle corresponds to a narrow band of frequency, near resonance, for which the position of *L* may be assumed fixed.

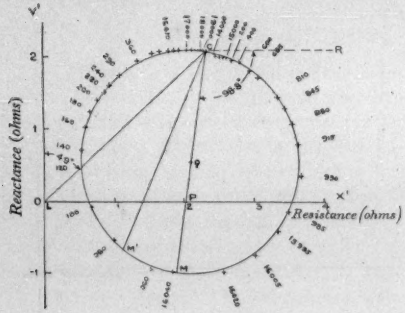


Fig. 6.

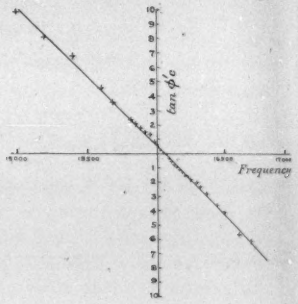


Fig. 7.

The angle *CLY'* should, according to the theory, be half the angle *RCM* and the triangle *LPC* should be isosceles. The values of the measured angles are as follows: $\angle CLY' = 49°$, $\angle RCM = 98{\cdot}8°$, $\angle CLP = 41°$, $\angle LCP = 40{\cdot}2°$, so that the geometrical relations required by the theory are very nearly realized.

§ 13. THE EQUIVALENT ELECTRICAL CIRCUIT

The elements of the equivalent electrical circuit can be derived from figures 6 and 7 with the help of the last two paragraphs of § 6. From figure 6, $X_c = 2{\cdot}37$ ohms, $L_c = 20{\cdot}6\,\mu$H and

$$CP/CQ = (X_c + X_m)/X_c = 2{\cdot}12/1{\cdot}57 = 1{\cdot}35,$$

whence $X_m = 0{\cdot}829$ ohms. Now

$$\tan \phi_c'/2p = (L_m + L_c)/(X_m + X_c) = 7{\cdot}75 \times 10^{-4}$$

from figure 7, and thence $L_m = 2{\cdot}459$ mH. Again,

$$\omega_1^2 = 1/(L_m + L_c)\, C_m = (6{\cdot}28 \times 16{,}050)^2 = 1{\cdot}017 \times 10^{10}$$

and

$$C_m = 39{\cdot}6 \times 10^{-6} \text{ F}.$$

Finally $L_l = 41\,\mu$H and $R_l = 0{\cdot}8$ ohms from bridge measurements.

The magnetostriction constants can be derived from the circle diagram or the equivalent electrical circuit. In the latter method the calculation proceeds as follows. From equations (5), (7), and (34)

$$\omega_0^2 = \omega_1^2/(1 + L_c/L_m) = \gamma/\alpha = E/\sigma r^2,$$

which, with a measured value 8·73 of σ, leads to a value for E, the modulus of elasticity, of $2{\cdot}15 \times 10^{12}$. Equations (28) and (3) give the relation

$$L_m/L_\sigma = E/4\pi\lambda^2\mu,$$

whence

$$\lambda = 1{\cdot}76 \times 10^{4}.$$

§ 14. THE MAGNETOSTRICTION CONSTANT

The circle diagram characteristic of a laminated resonator provides an instructive contrast to that already given for a solid resonator. Nickel rings cut from sheet 0·0025 in. thick, to external and internal diameters of 11·05 and 8·55 cm. respectively, were annealed at 1000° C. for two hours and cooled to room temperature in twelve hours.

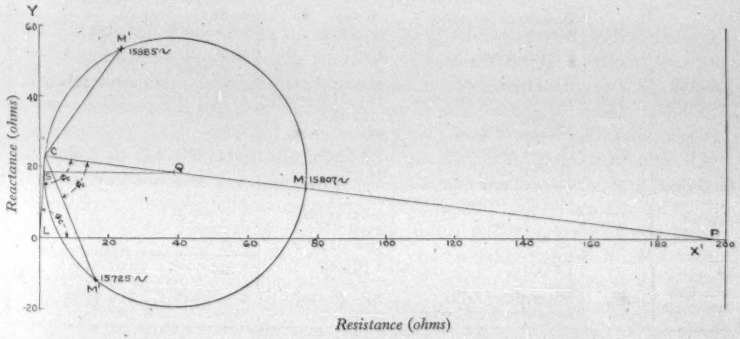


Fig. 8.

The annealed rings were covered on both sides with a thin layer of paraffin wax, piled alternately with paper rings cut from condenser-paper 0·0005 in. thick, and clamped between two thick metal plates. These plates were heated sufficiently to melt the wax and, after slow cooling, the consolidated laminated ring was extracted and freed from superfluous wax. The finished ring was 0·33 cm. thick and weighed 91 gm., corresponding to a density of 7·16 gm./cm³.; it was inferred from the known density of nickel that the composite ring contained 78 per cent. by volume of nickel, the remainder being wax and paper.

The laminated ring was set on three small pieces of spongy rubber at the bottom of a circular-channelled wooden ring. The wooden ring was wound toroidally with 220 turns of No. 24 s.w.g. d.c.c. copper wire insulated to an overall diameter

of 0·031 in., the cross-section of this winding, taken to the centre of the wire, being 3·2 cm. in the radial and 1·9 cm. in the axial directions. A second toroidal winding was provided to carry a direct current adjusted to produce a magnetizing field of 14·5 gauss at the mean diameter of the nickel ring.

The impedance of the 220-turn winding was measured at frequencies near resonance on an alternating-current bridge; the technique of this measurement has been described elsewhere by one of the writers[7]. The leakage impedance, calculated from the dimensions of the coil, was subtracted from the measured impedances to

Table 2.

	1 Symbol	2 Definition	3 Solid ring	4 Laminated ring	5 Published values
Mechanical properties		Inside diameter	8·46 cm.	8·55 cm.	
		Outside diameter	11·06 cm.	11·05 cm.	
	r	Mean radius	4·88 cm.	4·90 cm.	
	d	Axial thickness	0·161 cm.	0·33 cm.	
	b	Radial width	1·30 cm.	1·25 cm.	
	σ	Density	8·73 gm. per c.c.	7·16 gm. per c.c.	8·70–8·91 gm. per c.c.
	f_1	Resonance frequency	16,050	15,807	
	E	Young's modulus	$2{\cdot}15 \times 10^{12}$	$1{\cdot}69 \times 10^{12}$	$2{\cdot}02$–$2{\cdot}35 \times 10^{12}$
Electrical properties	μ	Permeability	42·3	29·7 38 (nickel)	
	ρ	Resistivity	$1{\cdot}00 \times 10^4$ e.m.u.		$1{\cdot}00 \times 10^4$ e.m.u.
	L_0	Inductance at zero frequency	185 μH (for 226 turns)	234 μH (for 220 turns)	
	N	Number of turns	226	220	
Equivalent electrical circuit	R_1	Copper resistance	0·8 ohms	1·43 ohms	
	L_1	Leakage inductance	41 μH	119 μH	
	R_c	Damped core resistance	2·37 ohms	1·36 ohms	
	L_c	Damped core inductance	20·6 μH	234·6 μH	
	R_m		0·829 ohms	5·65 ohms	
	L_m		2·459 mH	12·2 mH	
	C_m		$39{\cdot}6 \times 10^{-9}$ F	$8{\cdot}33 \times 10^{-9}$ F	
Magneto-striction constants	λ	§ 1 (1)	$1{\cdot}76 \times 10^4$	$0{\cdot}934 \times 10^4$	$1{\cdot}77 \times 10^4$* $1{\cdot}53 \times 10^4$†
	H_0	Magnetising field	14·5 gauss	14·5 gauss	14·5 gauss
	B_0	Induction for $H_0 = 14{\cdot}5$	3700	2100	3900*

* See references (5) and (10). † See reference (11).

obtain the core impedance. The latter is represented by the circular locus in the impedance diagram of figure 8, in which the letters correspond to those in figures 3 and 6. The frequencies corresponding to the points M' on the circle such that $CM' = \frac{1}{2}CM$, were found by measurement of a beat note of about 1000 ~ between the valve-oscillator supplying the alternating-current bridge and another valve-oscillator constant in frequency. The effect of eddy currents in the thin sheets can be estimated from the permeability and resistivity of nickel. If $\mu = 38$, $\rho = 10^4$, $f = 15807$ and $d = 0{\cdot}00635$ cm., then $z = 0{\cdot}219$, and from Kennelly's *Tables*[9], $\chi_0 = 0{\cdot}9998$ and $\zeta = 0{\cdot}92°$. The effect of eddy currents is to reduce the integral flux in the ratio 1·0 : 0·9998 and to retard its phase by 0·92°. The angle of depression of the resonance-diameter CM below the horizontal is 6·75° and, since this angle

should be $2(\eta + \zeta)$, it appears that the retardation of phase due to hysteresis is $2{\cdot}45°$.

The elements of the equivalent electrical circuit can be derived from the circle diagram as before, with slight modifications in the calculations to allow for the composite nature of the material. The numerical values for solid and laminated material are summarized in table 2, in which the first column gives the symbol, the second its meaning, or a reference to its first appearance in the text, the third and fourth columns numerical values for solid and laminated material respectively, and the last column numerical values obtained from published tables of physical constants and papers. The numbers above the bar in the fourth column refer to the laminated material and those below the bar have been calculated for nickel from the properties of the laminated material.

Two values of the constant λ are given in the last column, one derived from the work of Masumoto and Nara[5, 10] and the other from the work of McKeehan and Cioffi[11].

§ 15. CONCLUSION

We are now able to review the whole of the experimental work with the help of the diagrams and the preceding table. The values of density, elasticity, and resistivity obtained by measurements on the solid ring show satisfactory agreement with published values. The change of density and elasticity brought about by lamination is clearly indicated in the table, from which it appears that the velocity of sound in the laminated ring is slightly less than that in the solid ring. The magnetostriction constant λ for the solid ring agrees almost exactly with that deduced from the Japanese measurements. The laminated ring gives a smaller value. Some clue to the discrepancy is given by the ordinary B/H curves for the samples, which indicate that the laminated ring is somewhat hardened by mechanical working. The values, given in the table, of induction B_0 when $H_0 = 14{\cdot}5$ gauss show this clearly and justify the view that the small value of B_0 for the laminated specimens may be attributed to this hardness. The authors believe, however, that the close agreement between the value of λ for solid material and that inferred from the Japanese measurements is to some extent fortuitous. The assumption underlying the comparison, that a small change of magnetic induction produces the same magnetostrictive effect independently of the manner in which the change is brought about, can be supported by arguments which are reasonable and plausible rather than strict. It is known that a small alternating field superposed upon a steady field produces an average magnetic induction very slightly different from that appropriate to the steady field acting alone, and similar phenomena are to be expected in the magnetostriction effects, since these are so closely related to magnetic induction. The amount and nature of the impurities present in the samples, the heat treatment and the mechanical working are known to affect the magnetostrictive properties. In fact, precision in magnetostriction measurements can only be attained by use of the purest materials, carefully annealed and free from the effects of cold working. These

conditions have been fairly closely fulfilled in the experimental work described here, with the exception of the hardened condition of the laminated specimen, produced by the process of manufacture.

The elements of the equivalent electrical circuit given in the table bring out very clearly the effect of lamination. The corresponding change in the circle diagrams, figures 6 and 8, is also striking. The damping of the solid ring is mainly attributable to eddy currents, that in the laminated ring to mechanical friction, and it is interesting to observe that the solid specimen is not so resonant as the laminated specimen. The circle diagram for the solid specimen, figure 6, approaches the maximum size possible. The limitations imposed by considerations of energy-conservation prevent the excursion of Q along CM beyond the point P, and the maximum radius possible is therefore CP. If we postulate an imaginary material having the same permeability and resistivity but with, say, the magnetostriction constant increased ten times, it would yield a circle but little larger; the size of the circle would not, in fact, reveal this large change in properties. The frequency-spacing round the circle would however change, and the specimen would appear to be much less resonant. Under some conditions, then, sharpness of resonance may be an index of poor magnetostrictive quality. A similar change in the properties of the laminated material would be revealed by a large increase in the size of the circle and only a slight change in the sharpness of resonance, in marked contrast to the solid material. These considerations show that some care is necessary to interpret the circle diagram correctly and that it is essential to discriminate between the parts of the damped impedance due to leakage flux and core flux; that is, the circle diagram should be a diagram of core impedance rather than the usual diagram of total impedance.

The agreement between theory and experiment is uniformly good, such small discrepancies as appear being within the limit of experimental error. The constants for nickel which can be checked against published values are also in good agreement, and it may be concluded that the theory gives an adequate representation of the behaviour of a magnetostriction oscillator in the form of a ring.

§ 16. ACKNOWLEDGMENT

The authors desire to express their indebtedness to the Admiralty for permission to publish this paper.

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DISCUSSION

Dr. D. W. DYE. I wish to congratulate the author upon the elegant manner in which he has developed the equivalent electric circuit of the magnetostriction oscillator. These equivalent circuits are of the greatest value to those who have to work in terms of associated mechanical vibrating systems and electrical circuits. Experiments which I communicated to the Society a few years ago on piezo-electric resonators proved that the equivalent circuit for this case can be applied with very great accuracy.

The ring form of oscillator, such as that described in the paper, is of a particularly satisfactory kind since, in an isotropic material, the stresses and strains are uniform throughout the ring from section to section. For this reason I have been experimenting with rings cut from natural quartz. These cut with plane perpendicular to optic axis have uniform properties in any azimuth in the ring, and when supplied with suitably disposed electrodes can be made to vibrate radially in the fundamental mode. It was found however that the damping was surprisingly large. On examination of the numerical values given in the paper in table 2 it will be found that for the magnetostriction resonators the logarithmic decrement given by the ratio $Rm/2L_m n$ has the value 0·01 for the solid ring, and about the same value for the laminated ring.

This is about the same as for a quite moderately small electrical circuit. I have found that in the case of the quartz ring the damping was nearly all due to the air-viscosity between the cylindrical surfaces and the closely embracing electrodes. When the ring was placed in a vacuum this damping all disappeared and the ring behaved as a first-class resonator. It occurs to me that a large part of the heavy damping of the magnetostriction ring might be due to the same cause since the ring is pocketed in an enclosure closely embracing it.

THE THEORY OF THE MICROSCOPE

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ABSTRACT. The paper examines the diffraction-effects produced by (*a*) two adjacent apertures, and (*b*) a series of apertures in an opaque screen situated in the focal plane of a lens system, when the illuminating system is projecting the elementary image of a point-source of light into this object plane. The diffraction-effects and geometrical resolving power of the grating are shown to be independent of the concentration of the light in the object plane; they depend rather on the number of apertures free to transmit light. The theory is then extended to the case where the illumination of the object is produced by a source of finite area. Both the equivalence and Abbe principles appear in the analysis of the mode of formation of the image in such critical illumination, but subject to the difficulties encountered in integrating for the effects of finite sources of light. Confirmatory experimental work is in hand.

§ 1. INTRODUCTION

The recent renewal of interest in photomicrography by ultra-violet radiation has re-directed attention to the problem of the interpretation of the microscope image, and hence to the question of the mode of formation of the image. The work of E. Abbe, and the theory associated with his name, sprang from the realization that the physical process involved in the formation of the image of a non-self-luminous object must (in the microscopic sense at least) differ radically from the action in the case of a self-luminous body. Practical experience leads, however, to the conclusion that in the final result there may be a great measure of equivalence between the two cases. Thus the late Lord Rayleigh investigated* the image of a row of points and of a grating, in cases both where they are assumed to emit light of equal intensity in the same phase, and where they are self-luminous. The case where the phases of the successive elements have a constant difference is also discussed. In two or three paragraphs he discusses the function of the condenser when it projects an image of the source of light into the object plane: "...To a point of the flame corresponds in the image, not a point, but a disk of finite magnitude....The radius of the disk has the value $\frac{1}{2}\lambda/\sin\alpha$, where α is the semi-angular aperture of the condenser....It seems fair to conclude that the function of the condenser in microscopic practice is to cause the object to behave, at any rate in some degree, as if it were self-luminous, and thus to obviate the sharply-marked interference-bands which arise when permanent and definite phase-relations are permitted to exist between the radiation which issues from various

* *Phil. Mag.* **42**, 167 (1896).

points of the object." This opinion, which was not the result of any detailed investigation but was clearly expressed as an opinion, is the accepted basis of the use by practical microscopists of the so-called "critical illumination" in which the image of the radiant is formed in the plane of the object; it is further considered by many that the object behaves under such conditions as if there were then practically no phase-relations amongst the light emitted from the various parts and that *all* interference effects are thereby avoided.

A further conclusion is sometimes drawn that the Abbe principle is valueless in the practical circumstances of observation in the microscope. For these reasons it seems that perhaps a closer discussion of this case of "critical illumination" is desirable.

§ 2. THE CASE OF TWO APERTURES

The case of two indefinitely small apertures may be recalled for the purpose of illustrating the present discussion. Let the dispositions be indicated in figure 1, where S is the object plane and C is the condenser supposed free from aberration.

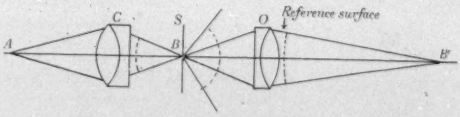

Fig. 1. Critical illumination (diagrammatic).

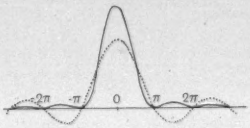

Fig. 2. Relative amplitude ($\sin u/u$) (dotted) and intensity ($\sin^2 u/u^2$) (full line.)

Consider first an elementary axial source of light at A. The image projected into the object plane is not merely a disc; it consists of the well-known distribution of light, first worked out by Airy, in which the amplitude variation along a radial direction is represented by

$$J_1(d)/d,$$

where d is proportional to the radius. The relative intensity and amplitude are very similar to those plotted in figure 2; the amplitude in the first bright ring surrounding the central disc is as much as one-eighth of the central maximum; although the phases of the odd number rings differ by π from that of the central disc and even rings, yet the phase is otherwise very nearly constant* over the various regions. This is equivalent to the assumption of negative amplitudes for the odd rings†.

* Rayleigh, *loc. cit.* p. 1; L. C. Martin, *An Introduction to Applied Optics*, p. 95 (Pitman and Sons, London, 1930).

† This attribute of the Airy disc is so generally misunderstood that a simple discussion of the point has been added to the paper as an appendix.

Given two elementary apertures M and N situated symmetrically near B, figure 1 (see also figure 5), and assuming that the illuminating disc falls on them symmetrically, we see that they will radiate in the same phase and with equal intensity, while the radiation into various angular directions will be fairly uniform if the holes are small enough. If the objective O is free from aberration, each aperture by itself produces an image of the Airy disc type in the plane through B'. If two co-phasal images overlap, the displacements (and thus in our case the amplitudes) have to be added in order that we may realize the resultant effect; if the light in the two apertures M and N had been non-coherent the intensities would have had to be added directly.

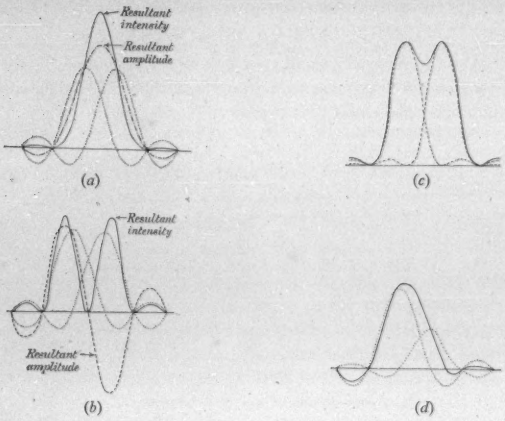

Fig. 3. Superposition of two elementary images. (*a*) Light in same phase; amplitudes are added. (*b*) Light in opposite phase; amplitudes subtracted. (*c*) Light not coherent; intensities are added. (*d*) Light in same phase; amplitudes unequal.

Let the case be chosen when the separation of the centres of the images is equal to the radius of the first dark ring of the Airy disc. The result (co-phasal case) is illustrated in figure 3*a* for the intensity-distribution along the line through the two centres. If the original point-source is moved to a place such that M and N are in opposite phase, the amplitudes are to be subtracted, giving the amplitudes of figure 3*b**.

It will easily be seen that there is no separation of maxima in the resultant if the apertures emit light in the same phase. A little graphical work allows the contour diagram of intensity in the plane of B' and for the area immediately surrounding B' (due to the light from the two apertures M and N when they are symmetrically

* The amplitude-distribution curve in the figures actually relates to the distribution characteristic of a rectangular aperture; this fact produces no important alterations in the general results. Figure 4 is computed for the true circular aperture distribution.

illuminated) to be drawn, figure 4. The patch of light is indeed elongated in one direction but shows no sign of its double nature as it does in the case when the light in the images is incoherent, figure 3*c*. On the other hand there is a marked division between two maxima in the resultant if the apertures emit light in opposite phases. Figure 3*d* suggests the case of light in the same phase but unequal intensity.

With these results in mind we may illustrate the application of something like the Abbe principle for this case. From the above condition of separation of the image points it follows that the separation h of the corresponding object points is given by

h

$$h = 0{\cdot}61\lambda/\mathscr{N}\mathscr{A}_0,$$

λ,

where $\mathscr{N}\mathscr{A}_0$ is the numerical aperture of the objective. The distribution of light in the posterior principal focal surface of the objective (constructed for very close parallel rays derived from the neighbourhood of B) now resembles that of a system

$\mathscr{N}\mathscr{A}_0$

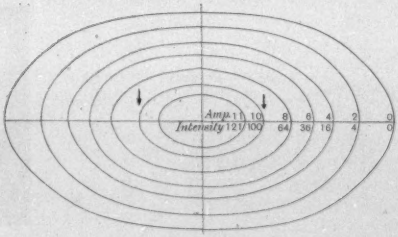

Fig. 4. Contour map of intensity distribution in the final image of adjacent small apertures illuminated by coherent light. Separation of apertures $=0{\cdot}61\lambda/\mathscr{N}\mathscr{A}$. Positions of individual maxima indicated by arrows. (Circular apertures.)

of simple interferences. Let α be the angle made by such rays with the axis in the axial plane containing the apertures; then the resultant amplitude A in the corresponding point F of the focal surface, figure 5, is given by

α

$$A = 2a_1 \cos\{(\pi nh \sin\alpha)/\lambda\} \qquad \ldots\ldots(1),$$

where a_1 is the amplitude due to one hole and n is the refractive index of the object space. Naturally the amplitudes of adjacent bright fringes alternate in sign.

a_1, n

We may now go a step further. The mutual angular divergence of the rays passing through F is so small that the relative path-differences of the disturbances meeting in the corresponding points of a spherical reference surface centred at B' will not be appreciably different from those at F. Since the lens would form at B' the image of an elementary source at B, the phases of the disturbances from such a source at B would be identical all over the reference surface. Also, owing to the symmetry, the phase of the resultant disturbance due to the two apertures M and N will be identical with that due to an imaginary source at B, the mid-point; hence the phase of the resultant will be uniform over the reference surface.

A proportion of this interference system, depending on the aperture as suggested
u in figure 6*a*, 6*b* is focussed behind the objective. When u is the extreme divergence for a ray entering the objective $n \sin u = \mathcal{N}\mathcal{A}_0$, so that, if we take the value of h given above, we obtain

$$\text{(Marginal) } A = 2a_1 \cos(0{\cdot}61\pi),$$

in which case a small part of the second bright fringe on each side of the central one begins to be included. The integration below is carried out for a wider aperture.

The calculation of the resultant energy distribution in the image plane on the basis of Huygens's principle can be carried out without difficulty for a rectangular aperture with edges parallel and perpendicular to the fringes. In order to obviate some difficulties the aperture parallel to the directions of the fringes is considered small. In a familiar manner, the effect of the whole aperture is reduced to that of a strip in the plane of the aperture and parallel to the diametral direction along

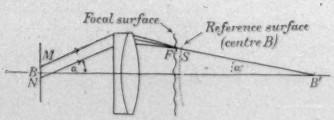

Fig. 5.

Fig. 6.

Fig. 5. Focal surface and reference surface.

Fig. 6. To illustrate diffraction or interference maxima found in reference surface of objective when two apertures on stage are illuminated by coherent light.

which the integration is to be effected—i.e. perpendicular to the fringes. Such a strip is shown by PAQ, figure 7, in which A is the centre of the strip and B' is the centre of curvature. The points R and S show equal and equidistant elements
dy dy, and the point C in the focal surface is conveniently defined by the difference of
ϕ phase 2ϕ with which disturbances from P and Q meet in it. Again the resultant phase for all pairs of elements giving equal contributions, such as those at R and S, will be sensibly constant, since there is equal lag and lead for the phases of the disturbances from elements symmetrically situated*.

y, Y Let the distances AR, AP be y, Y; the effective amplitude of the displacement
A_y at R and S be A_y. The phase-difference of the disturbances arriving at C from these
δ points is δ, where

$$\delta = 2\phi y/Y.$$

Hence the resultant amplitude at C due to the elements at R and S is

$$\text{constant} \times 2A_y dy \cos(\phi y/Y).$$

* See Appendix.

But A_y must be calculable from the above expression in equation (1). Since the optical sine relation gives

$$nh \sin \alpha = n'h' \sin \alpha', \qquad n', h', \alpha'$$

then

$$A_y = 2a_1 \cos \{(\pi n'h' \sin \alpha')/\lambda\}^*,$$

thus for the purposes of our problem, since $\sin \alpha'$ is proportional to y, we can replace $(\pi n'h' \sin \alpha')/\lambda$ by $\Delta . y/Y$ and write

$$A_y = 2a_1 \cos (\Delta . y/Y),$$

where Δ is half the phase difference, at the ordinate Y, of the disturbances from Δ
M and N, figures 1 and 5, and the final expression for the amplitude at C is

$$\text{const.} \times \int_{y=0}^{y=Y} \cos (\Delta . y/Y) \cos (\phi y/Y)\, dy.$$

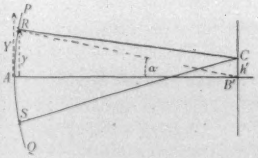

Fig. 7.

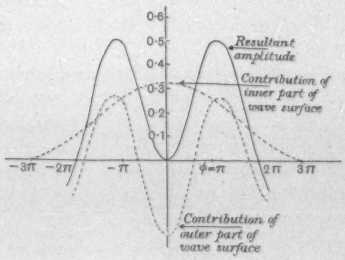

Fig. 8.

Fig. 8. Showing the amplitude-contributions of the inner and outer parts of the wave surface for points near the axis when the object consists of two small apertures symmetrically illuminated by co-phasal light.

The case now chosen for integration is that in which the aperture extends to the first lateral maximum on each side in the reference surface, i.e. $\Delta = \pi$, so that h becomes equal to $\lambda/\mathcal{N}\mathcal{A}_0$. The integral can then be written

$$\text{Amplitude} = \frac{1}{2}\int_0^Y [\cos \{y(\pi - \phi)/Y\} + \cos \{y(\pi + \phi)/Y\}]\, dy$$

$$= \frac{Y}{2}\left[\frac{\sin \{y(\pi - \phi)/Y\}}{\pi - \phi} + \frac{\sin \{y(\pi + \phi)/Y\}}{\pi + \phi}\right]_0^Y$$

$$= \frac{Y}{2}\left\{\frac{\sin (\pi - \phi)}{\pi - \phi} + \frac{\sin (\pi + \phi)}{\pi + \phi}\right\} \qquad \ldots\ldots(2),$$

for which the values are easily computed for a series of values of ϕ. They are plotted in figure 8 in full lines.

* See Appendix.

The effect of the central and marginal parts of the aperture can be illustrated by separation of the integral into the two parts

$$\int_0^Y = \int_0^{Y/2} + \int_{Y/2}^Y,$$

with the following result

$$\frac{Y}{2}\left\{\frac{\sin\frac{1}{2}(\pi-\phi)}{\pi-\phi} + \frac{\sin\frac{1}{2}(\pi+\phi)}{\pi+\phi}\right\}$$
$$+ Y\left\{\frac{\cos\frac{3}{4}(\pi-\phi).\sin\frac{1}{4}(\pi-\phi)}{\pi-\phi} + \frac{\cos\frac{3}{4}(\pi+\phi).\sin\frac{1}{4}(\pi+\phi)}{\pi+\phi}\right\}.$$

The two parts are represented for various values of ϕ by the dotted lines in figure 8, from which the inner part of the wave-surface is seen to produce no resolution of the images. The co-phasal regions at the margins produce an interference-system of which the maxima are spaced by a separation rather greater than the

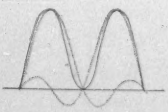

Fig. 9.

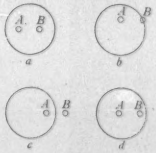

Fig. 10.

Fig. 9. Superposition of two elementary images separated by $\phi = 2\pi$.

Fig. 10. Showing how the illuminating Airy disc (first dark ring indicated by the large circle) may fall on two small adjacent apertures A and B.

separation of the maximum and minimum in an elementary image resulting from the action of the whole aperture. Also the side fringes in the focal plane of O will have a phase difference of π from the central fringe*. Hence at B' the contribution of the outer part of the wave-surface is in opposite phase to that due to the central region. Thus it is that the interference system passes for the resolved "images" of the two elementary apertures.

We reach precisely the same result if the amplitude of two co-phasal elementary images, having their maxima separated by an interval ϕ of 2π, are simply added together. The amplitude in such a single image (due to a rectangular aperture) is represented by the general function $\phi^{-1}\sin\phi$. The addition for the separation 2π is shown in figure 9. It is also clear from equation (2).

The above work illustrates one of the main conceptions of the Abbe theory, namely the production of an interference phenomenon in the posterior focal plane of the objective from which the nature of the final image may be deduced. In the above simple case, however, the actual areas characterized by uniform phase are finite, and not merely sets of corresponding lines or points as are the "homologous points" usually discussed in the Abbe theory.

* See Appendix.

§ 3. ILLUMINANT OF FINITE AREA

It is seldom or never the case in practice that the source of illumination approximates to a point. In "critical illumination" the source consists of an assembly of the point-sources, as considered above, of which the elementary images will be assumed to be in focus in the plane of the "object": let the latter again consist of two elementary apertures. Let the separation of these be so close that they are on the limit of resolution, and let the aperture of the condenser be increased to equal that of the objective. The radius of the Airy disc corresponding to each elementary point of the source will then be approximately equal to the separation of the apertures, which will be illuminated by discs falling over them in all possible ways. Some cases are shown in figure 10, which suggests various ways in which the Airy disc may fall upon the two apertures A, B; the line represents the dark minimum (circular aperture). These cases are as follows:

(*a*) Both apertures illuminated by light of equal intensity and in the same phase, figure 10*a*.

(*b*) One aperture only effective, the other being situated on the dark circle, figure 10*b*.

(*c*) Equal intensity; phases opposite*, figure 10*c*.

(*d*) Unequal intensity; phases agree, figure 10*d*.

We should expect that, when the images are on the point of resolution, the central maximum of one image would fall on the first dark ring of the other if the object points were self-luminous. A number of cases corresponding to the conditions above are shown in figure 3. Figure 3*a* corresponds to condition (*a*). The phases agree, the intensities are equal, and there is no division of the maxima. Figure 3*b* corresponds to case (*c*), in which the phases are opposite. The amplitudes of the elementary images are now subtracted, and it is evident that there is a sharply marked separation of the maxima. Figure 3*d* shows a case of unequal intensity with agreement of phase.

The final result will be the statistical average of all possible cases, and without closer discussion it will be seen that the ultimate effect is likely to be similar to that characteristic of two independent elementary sources, the resolution being made possible by the fact that some at least of the light from A and B emerges in opposite phases. The sum of the intensities for two elementary images is shown in figure 3*c*.

It will be understood from the above that, while the action can be discussed much more simply from the point of view of the equivalence principle, the emergence of this principle has not involved any new physical conceptions. The more detailed analysis, first contemplating the effects in the principal focal surface of the objective, then proceeding to the effects of the final image, still remains valid.

* With regard to (*c*), note that the phase of the first bright ring in the elementary image is opposite to that of the first dark minimum and the point B is supposed to lie on the first bright ring while A is still on the central disc.

§4. CASE OF THE GRATING; POINT-SOURCE OF ILLUMINATION

The next problem concerns the case of the grating, represented in an ideal scheme by a row of elementary apertures. This object lies on the stage of the microscope and we will assume that the illumination is furnished by a condenser of rectangular aperture with its principal direction parallel to the line of elements. The source of illumination first considered is an axial elementary point-source of which the image falls into the object plane, giving an amplitude-distribution of the type plotted in figure 2, and represented by an equation of the general form

$$A = \frac{\sin \phi}{\phi}.$$

We shall consider later how the results are affected by the use of an extended source of illumination. Note that the first lateral minimum occurs where the difference of phase 2ϕ between the disturbances arriving in the illumination image from the extremities of the condenser aperture is equal to 2π, and the linear distance of this minimum from the central maximum will be given by

$$h = 0{\cdot}5\lambda / \mathcal{NA}_c,$$

$\mathcal{NA}_c$ where $\mathcal{NA}_c$ is the numerical aperture of the condenser.

As a first simple case, assume the object B to consist of a row of apertures situated in the object plane symmetrically about the central maximum. The relative amplitudes of the light issuing from successive apertures in both directions must therefore be given by such terms as

$$\frac{\sin u}{u}, \quad \frac{\sin 3u}{3u}, \quad \frac{\sin 5u}{5u}, \text{ etc.},$$

where u depends on the spacing. The final resultant of the light diffracted in the plane containing the apertures and the axis into any given direction will be found from

$$A^2 = [\Sigma \{a \sin \delta + a \sin (-\delta)\}]^2 + [\Sigma \{a \cos \delta + a \cos (-\delta)\}]^2,$$

since there will be equal lag and lead in the phases of elements on each side of the centre. Hence as usual,

$$A = \Sigma 2a \cos \delta.$$

x Let x be the grating interval, then the first pair contribute an amplitude (into the direction making an angle θ with the axis) given by

$$2a_1 \cos \delta = 2\left(\frac{\sin u}{u}\right) \cos\left(\frac{\pi}{\lambda} x \sin \theta\right),$$

and the second pair give

$$2\frac{\sin 3u}{3u} \cos\left(\frac{\pi}{\lambda} 3x \sin \theta\right).$$

The whole effect of the grating is that

$$\text{Amplitude} = 2\left\{\frac{\sin u}{u} \cos\left(\frac{\pi}{\lambda} x \sin \theta\right) + \frac{\sin 3u}{3u} \cos\left(3\frac{\pi}{\lambda} x \sin \theta\right) + \frac{\sin 5u}{5u} \cos\left(5\frac{\pi}{\lambda} x \sin \theta\right) + \text{etc.}\right\},$$

the number of terms included depending on the number of open apertures, or if v be written for $(\pi/\lambda)\, x \sin\theta$,

$$\text{Amplitude} = 2u^{-1}(\sin u \cos v + \tfrac{1}{3}\sin 3u \cos 3v + \tfrac{1}{5}\sin 5u \cos 5v + \text{etc.})$$

$$= u^{-1}[\{\sin(u+v) + \tfrac{1}{3}\sin(3u+v) + \text{etc.}\} + \{\sin(u-v) + \tfrac{1}{3}\sin(3u-v) + \text{etc.}\}] \quad \ldots\ldots(3).$$

These series represent well-known Fourier expansions. The value of the first bracket, if an infinite number of terms are included, is $\pi/4$ from $(u+v) = 0$ to $(u+v) = \pi$, and then $(-\pi/4)$ from $(u+v) = \pi$ to $(u+v) = 2\pi$ and so on. The second bracket, depending on $(u-v)$, is similarly evaluated. In order to understand the result we may select special cases. Take for example $u = \pi/2$, which means

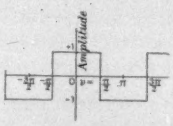

Fig. 11.

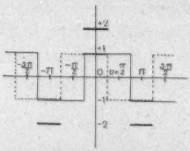

Fig. 12.

Fig. 11. Distribution of amplitude in the back focal surface of the objective when $u = \pi/2$.

Fig. 12. Distribution of amplitude when $u = \pi/4$.

that the value of u corresponding to the first lateral aperture is equal to $\pi/2$, so that this aperture falls well within the illuminating disc. The effect is most quickly seen by substitution in the single form of the series above, whence we obtain

$$\text{Amplitude} = 4\pi^{-1}\left(\cos v - \frac{\cos 3v}{3} + \frac{\cos 5v}{5} - \text{etc.}\right),$$

i.e. the cosine development of unity, of which the value is $(+1)$ from $v = -\pi/2$ to $+\pi/2$, and (-1) on each side, figure 11. We have, in fact, the case where the distribution of light in the focal surface of the objective shows the broad central maximum representing the direct image of the entrance pupil of the condenser system, sharply bordered by a succession of lateral diffraction maxima having amplitudes of alternating sign.

If the aperture of the condenser is reduced, the images will no longer touch each other. The way in which this arises is illustrated in figure 12, exhibiting the case when $u = \pi/4$, corresponding to the case of half the previous condenser aperture. The value of the first series in equation (3) is $\pi/4$ from $v = (-\pi/4)$ to $v = 3\pi/4$, and is $(-\pi/4)$ from $v = 3\pi/4$ to $v = 7\pi/4$ and so on; the value of the second series is $\pi/4$ from $v = (-3\pi/4)$ to $v = \pi/4$ and so on. Adding the amplitudes, we arrive at regions of amplitude 2, alternatively positive and negative, separated by regions of no disturbance. These regions correspond to the separated images of the condenser stop found in this case.

On the other hand it is interesting to study the effects of overlapping of the diffraction images. When $u = 3\pi/4$, the first series represents $\pi/4$ from $v = (-3\pi/4)$ to $v = \pi/4$, and is periodic in the usual way; the second series is $\pi/4$ from $v = 3\pi/4$ to $(-\pi/4)$ and so on. The addition of these results seems at first sight to contradict the expected results; it shows separated maxima as in figure 12 and does not seem to indicate any overlapping, such as is frequently observed when diffraction images are actually made to overlap in the microscope by opening of the stop of the condenser. We have only, however, to consider what will happen if the overlapping images (width $3\pi/2$) are of equal intensity and opposite phase to realize that the result of the analysis is correct. The usual appearance in the microscope arises because there is not the complete correspondence of phase characteristic of this ideal case, nor the exactly equal intensities now imagined between the direct and diffracted beams.

The above cases contemplate a position of the grating which has the centre of a dark space on the axis. If a bright aperture is situated on the axis the series is no longer the same. We then obtain a series of the form

u_1, v_2

$$A = 1 + u_1^{-1}(\sin u_1 \cos v_2 + \tfrac{1}{2}\sin 2u_1 \cos 2v_2 + \tfrac{1}{3}\sin 3u_1 \cos 3v_2 + \text{etc.}),$$

where v_2 is now $(2\pi/\lambda)\, x \sin\theta$.

Expanding as before we can separate two series from the odd coefficient terms and two from the even:

$$A = 1 + 2u^{-1}\{(\sin \overline{u_1 + v_2} + \tfrac{1}{3}\sin 3\overline{u_1 + v_2} + \text{etc.}) + (\sin \overline{u_1 - v_2} + \tfrac{1}{3}\sin 3\overline{u_1 - v_2} + \text{etc.})$$
$$+ (\tfrac{1}{2}\sin 2\overline{u_1 + v_2} + \tfrac{1}{4}\sin 4\overline{u_1 + v_2} + \text{etc.}) + (\sin 2\overline{u_1 - v_2} + \tfrac{1}{4}\sin 4\overline{u_1 - v_2} + \text{etc.})\}.$$

The first two series are the familiar Fourier expansions involving finite discontinuities of the type dealt with above. The second two series are not so easily discussed mathematically, but involve sudden discontinuities of sign at values 0, π, 2π, etc., of the parameter. In view of the general theory to follow they need not be discussed, but when $u = \pi/4, \pi/2, 3\pi/4$ etc., they reduce to specially simple forms, which will illustrate the case. Clearly when $u_1 = \pi/2$, or any integral multiple of $\pi/2$, the series of even terms will cancel. Again when $u_1 = \pi/4$ the even terms of the series in its first form are

$$= \tfrac{1}{2}\cos 2v_2 - \tfrac{1}{6}\cos 6v_2 + \tfrac{1}{10}\cos 10v_2 + \text{etc.}$$
$$= \tfrac{1}{2}\{\cos 2v_2 - \tfrac{1}{3}\cos 3\,(2v_2) + \tfrac{1}{5}\cos 5\,(2v_2) + \text{etc.}\},$$

a Fourier expansion again involving sudden discontinuities. The same series with reversed sign appears when $u_1 = 3\pi/4$. The various particular cases are very interesting to work out in detail. When $u_1 = \pi/4$, the presence of the constant positive term of value unity suppresses the maxima of negative amplitude, and the positive ones only are left. Absent spectra may thus occur, even in the presence of critical illumination, and may furnish the explanation of an experiment shown to the writer by Mr Conrad Beck, in which the resolution of a grating entirely disappeared when

the illumination was exactly critical, reappearing for other adjustments of the condenser. The particular cases when u_1 is any integral multiple of π are the only cases in which the central aperture alone is illuminated and the amplitude is uniform in the back focal surface. This is the complete realization of the equivalence principle, but in a very particular case.

§ 5. ASYMMETRICAL ILLUMINATION OF THE GRATING

In general, the apertures of the grating will be asymmetrically illuminated by the condenser, even though a single elementary illuminating element is considered. If the maximum illumination lies asymmetrically with regard to any two elements we may suppose that the apertures lie at the points given by

$$u, \quad u \pm v, \quad u \pm 2v, \quad \text{etc.},$$

where v is now a measure of the grating interval. The series representing the resultant amplitude must now take account of the phase differences of the disturbances from the successive apertures, which is directly proportioned to the sine of the angle of obliquity and to the spacing. Hence the series can be written

$$\text{Amplitude} = \frac{\sin u}{u} + \frac{\sin (u+v)}{u+v} e^{-imv} + \frac{\sin (u-v)}{u-v} e^{+imv} + \frac{\sin (u+2v)}{u+2v} e^{-2imv} + \text{etc.},$$

where m depends on the angular inclination; see below. It is of interest to note that exactly the same series was employed by Lord Rayleigh* to represent the resultant amplitudes in the microscope image when the object is a grating illuminated by plane waves incident at various angles, although the terms then bear another significance. In that paper Rayleigh points out that if the series be multiplied by the factor e^{-imu} it becomes periodic with respect to u in period v, and hence can be developed by Fourier's theorem in periodic terms, in the form

$$A_0 + iB_0 + (A_1 + iB_1) \cos (2\pi u/v) + (C_1 + iD_1) \sin (2\pi u/v) + \ldots$$
$$+ (A_r + iB_r) \cos (2\pi ru/v) + (C_r + iD_r) \sin (2\pi ru/v) + \ldots.$$

The multiplication of the terms of the series by e^{-imu} (interpreted in our case) simply refers the phase of the resultant to that of an imaginary disturbance propagated from the centre of the illuminating disc, which may not, however, be exposed. The series is then

$$\frac{\sin u}{u} e^{-imu} + \frac{\sin (u+v)}{u+v} e^{-im(u+v)} + \frac{\sin (u-v)}{u-v} e^{-im(u-v)}$$
$$+ \frac{\sin (u+2v)}{u+2v} e^{-im(u+2v)} + \text{etc.}$$

The full discussion cannot be reproduced here, but the resulting expansion appears as a series of terms which exist only between certain values of the variables. Writing

$$s_r = 2\pi r/v,$$

we find that B_r and C_r both become zero, so that the series, being left as a succession of terms involving (cos $\pm$ i sin), can be expressed exponentially.

* *Loc. cit.* p. 1.

The rth term of the above Fourier expansion appears as

$$\frac{\pi}{2v}\left[e^{is_r u}\{(1+m+s_r)^*+(1-m-s_r)^*\}+e^{-is_r u}\{(1+m-s_r)^*+(1-m+s_r)^*\}\right],$$

* in which the meaning of the asterisk against each bracket is that this bracket is subject to a certain limitation; it only appears as (± 1), the sign being that of the bracket. Hence the term in $e^{is_r u}$ in the series vanishes unless $(m+s_r)$ lies between $(+1)$ and (-1), and the term in $e^{-is_r u}$ vanishes unless $(m-s_r)$ lies within the same limits. The first term has

$$B_0 = 0,$$
$$A_0 = (\pi/2v)\{(1+m)^*+(1-m)^*\},$$

A_0 vanishing unless m lies between $(+1)$ and (-1), as above. Then when A_0 exists, its value is π/v, so that the whole series is represented by

$$(\pi/v)\,(1^*+e^{is_1 u^*}+e^{-is_1 u^*}+e^{is_2 u^*}+\text{etc.}),$$

where $s_1 = 2\pi/v$, $s_2 = 4\pi/v$, etc. The meaning of the asterisk is now that every term is subject to the limitations above. It only appears when $m \pm s_r$ lies between $(+1)$ and (-1). We may now proceed to interpret the series in our case, which is entirely distinct from Lord Rayleigh's application.

Each term in the expansion represents a sharply bounded diffraction maximum at the back of the objective, corresponding to the effect of the infinite series of indefinitely small apertures. The first term represents the direct image. It exists only from $m = +1$ to $m = -1$. To interpret this, note that m is the measure of the angle of obliquity of the diffracted light. The phase angle is given by

$$mu_1 = (2\pi/\lambda)\, nh_1 \sin\theta,$$

n, h_1, θ where n is the refractive index, h_1 is the linear measure corresponding to u_1 in the object plane, and θ is the angle of obliquity. To connect h and u, note that the first dark minimum $(u = \pi)$ is distant from the central maximum of the illuminating disc by the interval

$$h = 0{\cdot}5\,\lambda/\mathcal{NA}_c,$$

$\mathcal{NA}_c$ where $\mathcal{NA}_c$ is the numerical aperture of the condenser. We thus obtain

$$m = \frac{n\sin\theta}{\mathcal{NA}_c} = \frac{\text{numerical aperture of diffracted beam}}{\text{numerical aperture of condenser}}.$$

Thus the existence of the term 1^* above between $m = +1$ and $m = -1$ simply means that it extends over that numerical aperture of the objective which is filled by the condenser. The other diffraction maxima have the same breadth in the above sense, and the terms denote the appearance of maxima in symmetrical positions on each side of the central maximum.

For example, if

$$s_1 = 2\pi/v = 2, \text{ and } s_2 = 4, \text{ and so on,}$$

the first term appears from $m = -1$ to $m = +1$,

second ,, $m+2 = +1$ to $m+2 = -1$, i.e. $m = -1$ to $m = -3$,

third ,, $m-2 = +1$ to $m-2 = -1$, i.e. $m = +1$ to $m = +3$,

and so on, the maxima touching in the present case, which is the one that was mentioned above for a particular arrangement of the illumination; but the discontinuity is shown (in general) by the difference in the phase angles. Thus when $s_1 = 2$ and $u = \pi/2$, the phase angles $(s_1 u)$ of the first symmetrical lateral maxima are seen to be π and $(-\pi)$ respectively. This result therefore confirms that reached in the previous particular case. The amplitudes of the maxima are independent of the positions of the apertures relative to the point of maximum illumination. We must notice that the above results are strictly true only when a very great number of grating apertures is open or available. If the number is limited, the sharpness of the maxima disappears, and the secondary grating maxima have to be considered.

The above investigation relates, of course, to indefinitely narrow apertures. The expression must be integrated for finite apertures to make it of more practical significance†. If we have a series of vibrations expressed by

$$ae^{ip\theta}d\theta, \quad ae^{ip(\theta+\delta\theta)}d\theta, \text{ etc.,}$$

p

the sum is

$$ad\theta\{\cos p\theta + i\sin p\theta + \cos(p\theta + p\delta\theta) + i\sin(p\theta + p\delta\theta) + \text{etc.}\},$$

or

$$d\theta\{\Sigma(a\cos\Delta) + i\Sigma(a\sin\Delta)\},$$

where Δ stands for the phase angle. The resultant amplitude may thus be obtained by formal integration of the exponential expression. Thus

Δ

$$\int_{\theta=u_1}^{\theta=u_1+U} ae^{ip\theta}d\theta = \left[\frac{a}{ip}e^{ip\theta}\right]_{u_1}^{u_1+U} = \frac{a(e^U - 1)}{ip}e^{ipu_1}.$$

Hence the effect of a finite width U of the apertures may be obtained by integration between $u = u_1$ and $u = (u_1 + U)$. Thus

U

$$\begin{aligned}
\text{Amplitude} &= \frac{\pi}{v}\int_{u_1}^{u_1+U} du\,(1^* + e^{is_1u^*} + e^{-is_1u^*} + e^{is_2u^*} + \text{etc.}) \\
&= \frac{\pi}{v}\left[u^* + \frac{e^{is_1u}}{is_1}^* - \frac{e^{-is_1u}}{is_1}^* + \frac{e^{is_2u}}{is_2}^* - \text{etc.}\right]_{u_1}^{u_1+U} \\
&= \frac{\pi}{v}\left[U^* + \frac{e^{is_1(U+u_1)} - e^{is_1u_1}}{is_1}^* - \frac{e^{-is_1(U+u_1)} - e^{-is_1u_1}}{is_1}^* + \text{etc.}\right] \\
&= \frac{\pi U^*}{v} + \Sigma\frac{\pi(e^{is_rU} - 1)e^{is_ru}}{is_rv}^*,
\end{aligned}$$

where the terms are taken for positive and negative integral values of r subject to the same limitation that $(m \pm s_r)$ lies between $(+1)$ and (-1). If s_r be replaced by $2\pi r/v$, the expression becomes

$$= \frac{\pi U}{v} + \Sigma\frac{(e^{i2\pi rU/v} - 1)e^{i2\pi ru_1/v}}{2ir}^*.$$

The second member becomes

$$\Sigma\left\{\sin\frac{2\pi rU}{v} + i\left(1 - \cos\frac{2\pi rU}{v}\right)\right\}e^{i2\pi ru_1/v}\Big/2r^*.$$

† This type of two-dimensional discussion must be independent of considerations relating to the polarization of the light, etc., etc., which would be relevant in a thorough three-dimensional treatment.

This is of the well-known form $(a + ib)\, e^{i\psi}$, from which the square of the resultant amplitude due to any term is given* by $(a^2 + b^2)$. Thus for one term

$$(\text{Amplitude})^2 = \frac{\sin^2(2\pi r U/v) + 1 - 2\cos(2\pi r U/v) + \cos^2(2\pi r U/v)}{4r^2}$$

$$= \frac{1 - \cos(2\pi r U/v)}{2r^2} = \frac{\sin^2(\pi r U/v)}{r^2}.$$

The amplitude of the rth maximum is written $r^{-1}\sin(\pi r U/v)$ while that of the central one is $\pi U/v$. This is the same proportion as that found by elementary theory† for the case of plane waves diffracted by a grating; although it seems very surprising at first sight that even when an elementary source of light is focussed in the grating the proportional amplitude of the central maximum‡ should depend simply on the ratio of the width of the grating aperture to the width of the aperture plus bar, especially when the distribution of light in the region of the focus is observed; the difficulty only arises from our ingrained habit of thinking in terms of point foci, etc., and forgetting that disturbances actually exist far outside the region of easily visible light.

The appearance of the central diffraction maximum within the limits of numerical aperture set by the condenser, and the sharply-bounded lateral maxima at angles of obliquity depending solely on the aperture of the condenser, the grating space, and the wave-length of the light, has now been established, within the theoretical limits of the discussion, as a consequence of the wave-theory. The geometrical resolving power of the grating has been shown to be independent of the apparent concentration of light, whether one or many elements are visibly illuminated.

The theoretical discussion assumed (i) uniform radiation in all directions from the elementary apertures; (ii) an unlimited number of apertures in the object plane; (iii) an objective which is free from aberration for the back focal surface. In practice, none of these assumptions will be strictly valid, but the effect of the falling off of the radiation with increasing obliquity will only modify the results of the theory for large angles, and in the majority of cases the number of elements in regular structures will be fairly large, so that the sharpness of the boundaries of the maxima will not be seriously affected. The question of the location of the diffraction maxima is discussed below. Actually, these maxima are formed by an objective which, being of finite aperture, can focus only a certain proportion of the whole pattern. The number of maxima thus taking part in the formation of the final image will thus be definitely limited.

* Schuster, *Theory of Optics*, 2nd ed. p. 17.

† Schuster, *loc. cit.* p. 121.

‡ It must be remembered that the *actual* light in the central region of the diffraction pattern may be due to overlapping maxima. In cases where a dark "bar" obscures the illuminating disc, it may be found that the lateral maxima are in opposite phase to the central one and tend to destroy the central light, that is of course from the analytical standpoint.

§6. CASE OF GRATING WITH ILLUMINANT OF FINITE AREA

So far we have assumed a point-source of illumination but in practice the grating will be illuminated in critical illumination, not by one elementary image, but by many such images. All these, however, will produce series of coincident maxima, since the distribution and relative intensity of such maxima have been shown to be independent of the situation of the grating with regard to the illumination. Hence the appearance of the sharply-bounded diffraction maxima is quite general under the conditions of our problem, in which the row of apertures is parallel to one side of the rectangular aperture of the condenser.

It is also to be noted that the aperture of the objective has been considered to be restricted to a small amount except in the direction of the row of apertures.

The removal of these restrictions and the discussion of circular apertures for condenser and objective leads to a Fourier expansion involving Bessel functions, and will not be dealt with in this paper. Moreover, the general case of illumination by a disc with its centre not in the line of apertures is more complex than the comparatively simple condition dealt with, i.e. the case of the rectangular aperture when the $u^{-1} \sin u$ amplitude-distribution holds in any line parallel to a diameter of the aperture.

There seems no room for doubt, however, that Huygens's principle can be applied to such cases as this without fear. If, in figure 1, S be regarded as the grating, the dotted wave-surface to the left could have been taken as the origin of elementary disturbances each of which would be diffracted by the whole width of the grating available. Looked at in this way, the sharp definition of the maxima is comprehensible in a manner that is not realizable when the distribution of energy in the plane of the grating is considered, because the condition of the sharp concentration of the energy, as it appears to the eye, seems to imply quite vitally different conditions of diffraction; in fact, many seem to find difficulty in realizing that diffraction can occur at all.

This discussion, however, shows the perfect consistency of Huygens's principle even in this somewhat difficult case. It may even appear that the argument is one of a rather circular nature, inasmuch as the distribution of energy in the elementary image has to be calculated from Huygens's principle. The correspondence of theory and fact in this case is, however, well known, and it appeared better to start from this point.

The Abbe theory (or "principle" as it might better be termed) has long been known as an elementary method of discussing the formation of the image of a regular structure in the microscope. If the incident illumination is imagined to be resolved into groups of plane wave-trains inclined at all angles to the object plane within the aperture limits of the condenser, then the diffraction maxima appear as points or lines in the principal focal surface of the objective. Such maxima are regarded as coherent sources giving rise to interferences of the Young type. If each group of wave-fronts produces an independent set of interferences, these agree in reproducing the image pattern in that plane only which is conjugate to the

object. Lord Rayleigh's paper was largely concerned with this condition, and Conrady and others have shown how the theory accounts for the increasing geometrical correspondence between the details of object and image consequent on the increase of the aperture of the objective and the inclusion of higher-order diffraction maxima.

The difficulties encountered in the application of this theory to explain certain experimental results lead to the formulation of the "equivalence principle" which would claim that, in the case of critical illumination of a grating, the object would behave as if self-luminous, as far as the resulting image is concerned. In the light of the above work we can gain some insight into the physical mechanism.

Refer again to figure 5; regard MBN as the grating plane, and consider the amplitude at a point S not in the principal focal surface of the lens, but on a spherical reference-surface struck with centre B', the conjugate to B, where B represents the centre of the illumination disc. If it were necessary to compute the amplitude at S, it would be essential to compute the optical paths from S to all the apertures in the grating. A simple discussion based on Fermat's principle would establish path differences very nearly proportional to the distances of the elements from B for a considerable range on each side. Then it would be found that, provided that the range of the grating was small in comparison with the width of the objective, the expressions used to calculate the amplitudes at S would be almost exactly the same as those given above. Only the higher-order terms would be affected. We may also consider that the phase terms appearing in the expressions above give the phase relative to that of a disturbance derived from the centre of symmetry B of the illuminating disc, which phase is uniform in the reference surface if the objective is free from aberration. Therefore the calculated maxima, hitherto referred vaguely to the principal focal surface, are now seen to have an exact meaning with respect to the reference surface.

The discussion of the resultant effects in the final image plane is straightforward for any particular elementary image-disc, but the total effect of illumination by a finite source would have to be computed on a statistical-average basis, regard being had to the possible positions of the individual discs with reference to the grating. In the present paper we shall indicate the lines of procedure for only the two-dimensional aspect of the problem.

§7. APPROXIMATE CALCULATION OF EFFECTS IN THE FINAL IMAGE PLANE

The position of points in the final image plane relative to the point of symmetry of the illuminating disc may be given in terms of u, and since the extent of the regions of uniform phase in the reference surface is determined by the aperture of the condenser, any such region considered alone will give a distribution of light represented by $W^{-1}\sin W$ where $W = \pi h' Y/\lambda f'$, h' is the linear displacement corresponding to W, Y the diameter of a region of uniform phase in the reference surface, and f' the radius of the reference surface.

W, h', Y, f'

It is simplest to assume that the lateral maxima give the $W^{-1} \sin W$ type of distribution in inclined planes, normal to the radii from the centres of the regions of the lateral maxima and intersecting at the central reference point, see figure 13. Then since there will be a fair depth of focus effect in practical cases, we may add the amplitudes, due to the successive maxima, for points in the normal plane, if we take account of their phase differences.

The disturbance from a lateral maximum will show a phase difference in the normal plane proportional to the distance from the axis. To select a simple case, assume that $s_1 = 2\pi/v = 2$ as above, so that the maxima in the reference surface touch at their edges. The phase difference at a distance h' from the centre for the disturbance due to a first-order lateral maximum will then be

$$\delta = 2\pi h' Y/\lambda f' = 2W.$$

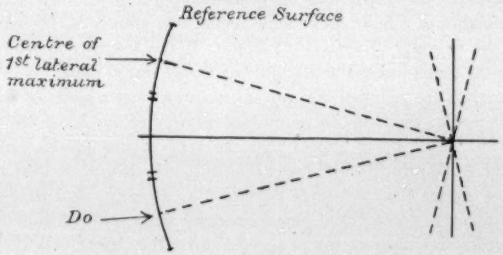

Fig. 13. To illustrate the approximate method to integrate the effects of the diffraction maxima for regions near the axis.

Hence if we go back to the elementary case where the successive maxima are represented by

$$1^* + e^{is_1 u^*} + e^{-is_1 u^*} + \text{etc.},$$

we may consider the central aperture of the grating to lie at the point u_1 in the illuminating disc.

The distribution of light due to these maxima in the final focal plane is therefore given by

$$W^{-1} \sin W \{1 + e^{2i(W+u_1)} + e^{-2i(W+u_1)} + \text{etc.}\},$$

and the effect of the central and diffraction maxima becomes

$$W^{-1} \sin W \{1 + 2 \cos 2(W + u_1) + 2 \cos 4(W + u_1) + \text{etc.}\}.$$

The symbol W represents the value of $(u - u_1)$. Hence the expression becomes

$$(u - u_1)^{-1} \sin (u - u_1)(1 + 2 \cos 2u + 2 \cos 4u + \text{etc.}),$$

which represents the contribution of a single illuminating point. It is a discontinuous distribution appearing only at $u = 0$, $u = \pi$, $u = 2\pi$, etc., and the first term outside the bracket shows that these isolated maxima will have considerable values near the region $u = u_1$ only. This is what we should expect if the objective could have an indefinitely great aperture, but the method of the discussion is not really

applicable here. If, however, the aperture only includes the first two or three diffraction maxima, the distribution will not be discontinuous.

Let us, however, consider the effect of a multiplicity of illuminating points. We must integrate the intensities for all values of u_1 from $-\infty$ to $+\infty$. The intensity corresponding to the case above is

$$(u-u_1)^{-2}\sin^2(u-u_1)\,(1+2\cos 2u+2\cos 4u+\text{etc.})^2,$$

and the intensity in the image plane is therefore represented by

$$I=(1+2\cos 2u+2\cos 4u+\text{etc.})^2\int_{-\infty}^{+\infty}\frac{\sin^2(u-u_1)}{(u-u_1)^2}\,du_1$$

$$=\pi\,(1+2\cos 2u+2\cos 4u+\text{etc.})^2,$$

so that the isolated maxima now all have the same intensity. An exactly similar expression appears, no matter how many terms appear in the bracket. The effect of the illumination of a finite area of the object will simply be to remove the restriction of the $(u-u_1)^{-2}/\sin^2(u-u_1)$ factor. If the aperture is wide enough to include only the direct image the illumination in the image plane is uniform; if the first order maxima are included the appearance consists of a series of main maxima at $u=0$, $u=\pi$, etc., separated by secondary maxima of one-ninth the intensity of the main ones. Thus it will appear that the type of discussion, explaining the working of the Abbe principle employed by Conrady*, which was developed on the assumption of groups of plane waves diffracted by a grating and which traced the increasing similarity of image and object resulting from the inclusion of more and more of the diffraction maxima, can be applied also to the case of critical illumination.

The full discussion of the correspondence of image and object in more general conditions will be relegated to a separate paper. One or two points, however, may be emphasized.

In this way of analysing the mechanism of the image-formation a unit of the whole system would be seen by use of (i) critical illumination, (ii) an optically perfect condenser system, and (iii) a source of light so small that its image in the object plane approximates to the elementary type. Unless this illuminating disc is very small in relation to the spacing of the object details, the image cannot be regarded as other than an assembly of interferences. In practice the numerical aperture and the resolvable object detail in the microscope are found to be connected by the approximate equation

$$h=0{\cdot}5\lambda/\mathcal{NA},\ \ldots\ldots\ \text{(rectangular aperture form)}$$

and if the aperture of the condenser is equal to that of the objective the diameter of the illuminating disc is $\lambda/\mathcal{NA}$. The majority of cases of fine resolution (only attained by a condenser of somewhat smaller aperture) are therefore attributable to interference effects, but the application of the equivalence principle to the broad features of the imaging is fairly clear; for this is safeguarded by the appearance of the first term in the series expansion. The image is formed in part by simple convergent spherical waves of which the intensity depends simply on the transmission

* *Journ. R.M.S.* **163**, 610–633 (1904); **168**, 541–553 (1905).

of the object plane around the corresponding region of illumination. The aperture of these waves is limited simply by the aperture of the illumination. Within this range the equivalence is perfect, provided that no diffraction maxima disturb the uniformity of the convergent light. We can easily see, in the light of these ideas, why the aperture of the illumination must be increased to resolve certain fine structures.

It appears that the main advantage of critical illumination is that we can by its means most easily realize the uniform distribution of light into the objective. The interference phenomena in the field (although we still largely depend on them to secure the finest resolution) are then extremely localized. If the object is a grating the maxima are likely to have uniform intensities over their areas of the reference surface and therefore will be in the best condition to manifest interference.

The relations of the equivalence principle and the Abbe principle are now more clear. The first gives an account of the broad features of the image-formation, while the second is still a perfectly general and legitimate means of analysis of the mechanism of the image-formation, as is shown, for example, by the final form of the expression given above for the distribution of light in the image plane, when the restrictions of the illumination have been removed.

Looking at this in another way, we see that we could produce just the same effect by putting a self-luminous surface in the first principal focal plane of the condenser; then with a reasonable aperture, the light passing through the object plane can be regarded as composed of independent groups of parallel wave-trains, and we then contemplate sets of coherent homologous points in the diffraction maxima. In order to get the results in the final image plane we have to integrate the effects of all the groups of homologous points. This is a different mode of analysis, but one which must ultimately lead to a precisely similar result.

It seems necessary to point out, however, that neither the Abbe nor the equivalence principle are more than aids to the discussion of complex cases. Both have their legitimate uses. It cannot be agreed that the Abbe principle is only of interest, then, in the extreme case of illumination by indefinitely narrow pencils; it represents a mode of analysis which remains valid in all conditions, notwithstanding the difficulties occurring in its application. In both cases it is the final integration of the effects which is the real difficulty.

Credit is due to Vasco Ronchi* who pointed out the persistence of the diffraction under the conditions of critical illumination for his own observations, and drew attention to this point in connection with the theory of the microscope. In view of the practical importance of the question, it was judged advisable to examine the consistency of Huygens's principle in this connection, and also to arrange for further confirmatory experimental work. The importance of the experimental work is very considerable because all deductions based on the elementary forms of Huygens's principle are bound to be approximations. The exact calculations present problems of the greatest difficulty. It may be said, however, that the results found above are well confirmed by careful experiments which have been carried out by one of my pupils and which will be published in due course.

* *Zeit. f. Phys.* **46**, 594 (1928).

APPENDIX

The constancy of phase for the Airy disc

With reference to figure 7, let PAQ be a horizontal diameter of a symmetrical convergent wave-front, free from aberration, and centred in the point B'. Let $B'C = h'$ be the radius of the first dark ring of the diffraction pattern, then

$$h' = \cdot 61\lambda f/Y,$$

where $AB' = f$ and $AP = Y$. If we join AC, and drop a perpendicular from B' on AC, we see that we may put, within very close approximations,

$$AC - AB' = h' \sin C\hat{A}B' = h'^2/f = \cdot 4\lambda^2 f/Y^2.$$

This will be a very small quantity in comparison with λ, even for values of h' corresponding to many interference rings. Hence the wave-disturbances arriving from the point A at points in the focal plane surrounding B will be sensibly in the same phase for a distance very large in comparison with the visual limits of the Airy disc.

Considering a disturbance arriving at C from R, figure 7, we can, by dropping a perpendicular from C on to the line RB', find that (very nearly)

$$RB' - RC \doteq h' \sin \alpha'.$$

Hence disturbances from R will reach the point C with a lead in phase as compared with those arriving at B' from any point of the wave-front. This lead in phase will be

$$2\pi n h' \sin \alpha'/\lambda,$$

n where n is the refractive index.

On the other hand, disturbances arriving at C from the point S will have a numerically equal lag in phase as compared with those arriving at B' from any point of the wave-front, say from the point A. In practical cases the slightly different distances of R and S from the point C will not cause any noteworthy difference of amplitude in the disturbances from equal elements at R and S.

Now two disturbances having equal amplitudes but differing phases must produce a resultant having a phase which is the mean of the phases of the components. Therefore the phase of the resultant at C is the phase of the disturbance from A, which we have shown to be sensibly constant. We may note, however, that if the two phase-angles lie between 90° and 270°, then the resultant amplitude will have a negative sign as compared with the positive amplitude-contribution from A.

This discussion applies so far to the disturbances actually derived from the line through R and S. If we visualize the spherical segment of the wave-surface and $B'C$ as the polar axis of this sphere, then we can divide up the segment into strips of latitude (perpendicular to the diagram in figure 7) and realize that points such as C will be equidistant from all points on such a strip. The action will be as though the whole strip were concentrated at its mid-point, and the effect produced at C by the whole segment is reduced to that of a line source with constant phase but with

amplitude diminishing from the centre outwards symmetrically on each side. The amplitude at the symmetrical points R and S being equal, we may now apply the above argument. Since any pair of symmetrical points can occasion no difference of phase in the resultant, the same consideration must apply to the effect of the whole wave-surface.

It is worth noting that a similar argument applies to many other cases in optics, such as a system of interference fringes. The phase is constant over the system, but the amplitudes of successive fringes alternate in sign.

DISCUSSION

Mr T. Smith. Explanations of distinctly varied characters have been offered to account for the effects seen with microscopes, but a mathematician will not wish for anything but a wave theory, i.e. a theory which allows the light-intensity seen at any point to be regarded as the effect due to disturbances of suitable amplitudes and phases imposed at an arbitrarily selected surface surrounding that point. The choice of surface is merely one of convenience, and it is possible to obtain very approximate results under certain conditions by neglecting the disturbances over all but a part of the surface. One of the novel features of Abbe's principle lay in selecting a different surface from that which might have been thought convenient. With certain types of object the distribution of light in this surface is markedly irregular, and its appearance affords the microscopist a guide as to the suitability of his optical system for such objects. Abbe's view, however, does not appear to me to be in general an advantageous way of regarding the action of a microscope. It is easy to understand how it came about that incorrect conclusions on the relative advantages of direct and dark-ground illumination were drawn from it, though Abbe's adherents do not seem to have realized that the inferences were wrong. The disagreement of experiment with this view is responsible, if I understand aright, for the appearance of the equivalence theory. If this theory goes so far as to claim that no difference exists between the images of self-luminous and illuminated objects, it must be rejected: if it goes less far it is useless unless it points out the nature and magnitude of the differences.

The real difficulties in constructing a theory of the microscope lie in our ignorance of what occurs when the light reaches the object. We have in consequence to consider a variety of ideal objects with assumed properties, which may be very far from those of the natural objects the microscopist wishes to examine. The characteristic peculiarity of the microscope is that large angular apertures are involved, and this makes it necessary for an adequate general theory to take account of vibrations in three dimensions at the object. With certain objects the final result at the retina of the observer may be the same or nearly the same as that which follows from a two-dimensional treatment such as that of the present paper, but in others the results may call for appreciable modification. Prof. Martin does well to emphasize the importance of amplitudes where the energy may be small. It may be recalled that on these grounds Abbe's theory seemed to the late Lord Rayleigh to

require theoretical justification before the correctness of its precise doctrine was evident.

In considering the mathematical results of the paper it may be helpful to remember that two factors are involved in optical imagery, viz. a point-to-point correspondence between object- and image-space, and adequate contrast in the vicinity of the image. With some instruments the two go together almost automatically, but this is not invariably the case. The distinction is allied to that between a simple and a complex system of waves. Any investigations which help us to interpret more accurately the appearances presented by microscopic images should be most gratefully received, and most critically examined.

Mr J. RHEINBERG. I should like in the first place sincerely to congratulate the author on a fine and very useful piece of work.

There have probably been few subjects on which during a period of half a century there has been more controversy than that of image-formation in the microscope. The well-known Abbe or diffraction theory was assailed at the outset; other theories based on the work of Airy, Rayleigh and others were considered to show that it could not hold good under the ordinary working conditions of the microscope, and even as recently as 1929, when a paper by Dr H. Moore, of the British Scientific Instrument Research Association*, led to many papers and a general discussion on the subject by the Royal Microscopical Society, it was again shown how much confusion still exists, and how the various theories, or (as the author more correctly terms them) principles, are supposed to contradict one another. By mathematical treatment, the author has now shown us that there is no want of harmony, no necessary discord, and no contradiction between these principles.

As long ago as 1902 I published a paper, the title of which ("The Common Basis of the Theories of Microscopic Vision Treated Without the Aid of Mathematical Formulae")† shows that already at that time I was arriving at similar conclusions, and the abstract of my contribution to the 1929 discussion at the Royal Microscopical Society‡ starts as follows: "It is shown that, as regards the formation of the image by the microscope, the Abbe theory and the Airy theory and its variants do not in any way conflict. The Abbe theory, by reason of the way it originated, deals in the first place with objects so illuminated as to lack self-luminosity to the maximum extent. The Airy theory deals with completely self-luminous objects. These are the two limiting cases. A gradual but strictly limited approach towards self-luminosity is presented by the structural elements of objects according to the way they may be illuminated." I have never used other means than careful experiments, physical concepts and analytical methods to arrive at my conclusions; I was not aware that the author was going into the subject mathematically, and am therefore the more pleased that there should be so great a correspondence in the conclusions arrived at.

* *J. R. Mic. Soc.* **48**, 133–143 (1928).
† *Zeit. für wissenschaftliche Mikroskopie*, **19** (1902).
‡ *J. R. Mic. Soc.* **49**, 132–143 (1929).

One or two matters arise out of the author's work to which I should like to call attention. The chief one concerns the relative applicability of the so-called "equivalence" and the Abbe principles. The "equivalence theory" is a term that has sprung up only of recent years. I believe that Dr Moore imported it from Germany from the papers of Prof. Berek of Marburg, but what it denotes has not, so far as I am aware, before now been properly defined. The author has made clear in his paper, and in the few words which he spoke in introducing his paper to the meeting, that he understands the term as meaning the equivalence between the behaviour of an illuminated object and a self-luminous one, and that is the sense in which I regard it in the following remarks. Now the concept of a self-luminous point is one which emits coherent rays in all directions, giving rise to a wave-front of regular amplitude and light-intensity; a self-luminous area consists of an aggregation of such points each preserving its own characteristics, whilst they do not bear any established phase-relations amongst themselves. The focussed image of a point in the object plane of the microscope is, however, a diffraction or Airy disc, i.e. an area consisting of points all of which are in phase-relationship with one another, besides being of variable amplitude and light intensity; and the focussed image of an illuminant of finite area consists of the overlapping of such diffraction discs. The area covered by a single diffraction-disc may cover many elements of the object itself, so that in an object of regular structure, such as a grating, many elements of the object may be emitting or passing coherent rays, instead of the non-coherent rays which would be emitted by self-luminous points similarly spaced. The author's figure 10, for example, clearly shows how the Airy disc (taken only as far as the first dark ring) may cover many object elements. To make our object as nearly self-luminous as may be possible, we must therefore contract the size of the Airy discs. This can be done only by increase of the aperture of the condenser-lens which is forming the Airy disc in the object plane, but the point to be noted is that even so we can get nowhere near the ideal of self-luminosity. Therefore, in the vast majority of cases we are going far more safely if we apply the Abbe principle than if we apply the so-called equivalence principle.

Any natural object, even if of supposedly regular structure, is so complicated and intricate in its constitution—the refraction, absorption and polarizing properties and the colour of its elements all play their part—that it simply defies adequate mathematical treatment. It has been one of the most frequent complaints against Abbe that his theory or principles were based upon experiments on artificial preparations under special conditions of illumination which are not found to be the best in practice. That was inevitable, and I would only note here that others who have tackled the problem of image-formation in the microscope, whether by experimental, analytical, or mathematical methods, including Rayleigh, Johnstone Stoney, and now the author have also found it absolutely unavoidable to adopt the same course. The working microscopist, however, cannot enter into involved mathematics, even if he has the ability; for him the necessity is to have a quick means of estimating to what degree the image of the object which he sees may be likely to correspond to the structure of the object itself, and to find out rapidly

whether that image can be improved. Herein, to my mind, lies the supreme value of the Abbe principles, for the laws established from these have furnished us with a reference-plane, namely the back focal plane of the objective, easily examined, in which light phenomena occur (or can readily be made to occur) which afford us the best clue to the interpretation of the image formed in the final image-plane of the object, and also tell us in many cases how to illuminate our object to the best advantage. Microscopists are familiar with the light distribution of maxima and minima occurring in this plane when the natural or the artificial object viewed is one having approximately regularly repeating elements; with care, analogous phenomena may be observed with objects consisting of only one or two elements. These phenomena are due simply to the fact that the light in the object-plane does not consist of an assemblage of points emitting non-coherent rays, as it would if this object were self-luminous; we are dealing with *images* of self-luminous points in the object plane, consisting of an assemblage of points which pass on coherent rays that can mutually interfere. Even if we have arranged for so-called "critical illumination" of our object, so that the phenomena are obscured, we only require to cut down the angle of the illuminating cone temporarily, by means of the iris of the condenser, to obtain the required information.

The equivalence principle is helpless to afford the microscopist any such experimental insight or information as to what he may be doing. So long as this principle which, as the author says, has not involved any new physical conceptions, is held to be in opposition to the Abbe principles, it just prolongs and accentuates the confusion already too pronounced as to the mode of formation of the microscopic image. If in the hands of competent physicists and mathematicians the equivalence principle proves a tool to throw light upon some isolated problems which are difficult to solve by the Abbe principles, so much the better, although personally I doubt whether it will do so. It seems to me too limited in its application in relation to the problems of the microscope. The great merit of the author's work is that by mathematical treatment he shows that there is no opposition between the so-called equivalence principles and the Abbe principles, and for my part I think he has rendered a notable service to microscopy in doing this, because it helps to dispel the doubts—of those microscopists who have had such doubts—about the methods hitherto employed as their surest guide in the interpretation of the microscope image.

Dr H. Moore. The results obtained by the author are of interest on theoretical as well as on practical grounds. In the first place they throw considerable light on the vexed question of the validity or otherwise of the Abbe theory. Briefly, Abbe pointed out that any object having a regular structure would so diffract the light that, if the aperture of the object-glass were sufficiently large, diffraction-images of the source would be formed in or near the back focal plane of the object-glass and, as a result, interference-fringes would be produced in the image-plane. Abbe's theory, as expounded by Dippel and others in statements made with Abbe's express approval, stresses the importance of these interference-effects to the extent

of claiming that they are the only physical processes involved in the formation of the images of non-self-luminous objects in the microscope, irrespective of the conditions of illumination and irrespective, also, of the structure of the object. In other words, according to the Abbe theory, the ordinary processes involved in the formation of the image of a self-luminous object play no part in the formation of the image of a non-self-luminous object.

As is to be expected, the author's mathematical analysis shows clearly that the interference effects to which Abbe drew attention do arise, but it shows also that, with critical illumination, "The image is formed in part by simple convergent spherical waves of which the intensity depends simply on the transmission of the object plane at the corresponding point of illumination," i.e. by the processes involved in the production of images of self-luminous objects. This result is a direct contradiction of the Abbe theory.

From the practical point of view the results will be of special interest to those who, for any reason, may have to work under conditions in which the illumination of the object must be restricted in some way or other. Mathematical analysis of the distribution of light in the image plane, such as that given in the paper, should afford material assistance in deciding, in such cases, which of the available methods of illumination will reduce to a minimum the visibility of interference-effects arising from any regular structure in the object and which, therefore, will give rise to an image having the closest resemblance to the object. From the practical point of view, also, particular interest attaches to one of the conclusions in the present paper, viz.: "The majority of cases of fine resolution are therefore attributable to the interference effects...." This is in conflict with the views of many microscopists, provided the illumination of the object be properly arranged. It will be interesting to see the further papers which the author has promised us, in which he proposes to deal with more general conditions of illumination, and to see how far this conclusion needs modification as a result of more general treatment.

AUTHOR's reply. I am fully aware of the divergence between the complexity of practical cases and the artificial simplicity of the theory. Also it is not easy to realize what the implications of the formulae really are. An example of this is shown in my use of a misleading word in a sentence which is quoted by Dr Moore, and which I intend to change in the paper. Instead of the words "at the corresponding point of illumination," I really meant "around the corresponding region of illumination," because, as Dr Moore will agree, the transmission at a single point cannot be significant. Thus, in the case of the grating, the simple direct spherical wave of the theory has the relative intensity $\pi U/v$ which is the average transmission of the object plane. I think we ought to be very careful in applying these results to cases involving quite other conditions, and I hope that those interested in the subject will think over the question again in the light of the present work. I do not think my results contradict the older principles, but they do reveal their limitations.

SOME STUDIES IN PYROMETRY AND ON THE RADIATION PROPERTIES OF HEATED METALS

Summary of lecture delivered on October 17, 1930

By Dr RUDOLF HASE,
Dozent für Wärmemesstechnik an der Technischen Hochschule, Hannover

THE investigation of the spectral energy radiated by solid and liquid metals at high temperatures is of double interest. Firstly, from a technical point of view, radiation in both the visible and the infra-red portion of the spectrum seems to be the simplest (and in many cases the only) means possible for high-temperature measurement, provided that we know the laws connecting temperature with distribution of spectral energy. Secondly the modern development of physics has proved that the study of energy emitted by matter is of increasing importance for research on atomic problems.

Ideal black-body conditions, for which the Wien-Planck relation holds true, will be met with rarely in practical pyrometry. This is a well known fact as far as measurements of iron ingots, molten metals in crucibles and the like are concerned, but it is necessary to draw attention to the fact that even the radiation of closed ovens, such as Martin furnaces, is not a totally black one. Even if corresponding measurements in different wave-lengths of the visible spectrum with two pyrometers, one having a red and the other a blue filter, give the same temperature reading, thus apparently indicating the blackness of the enclosure, difficulties arise as soon as we alter the method by measuring the total radiation. Deviations from black radiation are known to become more noticeable the farther we move towards the longer wave-lengths. In the case mentioned the deviations may obviously be ascribed to the absorption bands of vapour and carbon dioxide. The problem of an industrial furnace, uniformly heated by gas or oil, has not been solved yet. In fact there are always differences of temperature between iron bath, cover, side-walls and flames, and it is easily understood that the selective emissivity of the hot gases must have great influence, to say nothing of the absorbing cooler gases in the channel-shaped sight-holes, which attenuate the infra-red radiation but have no influence in the visible spectrum. Similar considerations hold true for the majority of industrial temperature-measurements.

This interpretation explains in a satisfactory manner the divergent results obtained by different methods, if we correct the hitherto unchallenged opinion regarding the black-body conditions in industrial furnaces. Evidently modern pyrometry is not a problem of instrument-making, the essential point being a critical understanding of the heat-exchange between the body to be measured and its surroundings.

Difficulties increase if we consider the radiation conditions of glowing metals outside the furnace in the open air. In this case the surrounding temperature is

so low relative to the heated body, that we can neglect the influence of absorbed or reflected radiation and speak of the proper radiation of the metal in question, or better still of its surface radiation. Excepting in the case of transparent materials, such as molten glass, the emission is now to be attributed to the outer atomic layers of a heated solid or liquid body.

Since most of our industrial metals oxidise at higher temperatures one has to ascertain whether the radiation observed belongs to the pure metal or to its oxide. But even if we know the spectral energy curves for both substances, a further uncertainty is involved by the existence of different oxides. In the case of iron the amount can vary between a scarcely visible film of oxide and a dark layer of scale. The latter, although having a greater emissivity, forms a solid crust of bad thermal conductivity, being often separated from the metal by insulating layers of air,

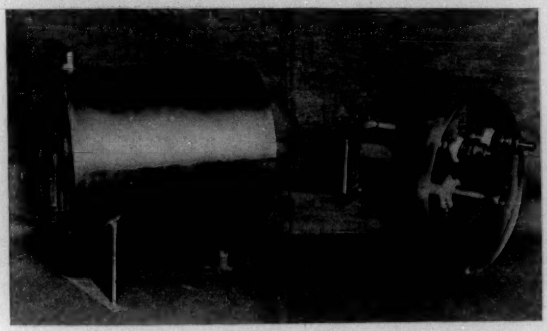

Fig. 1.

and so gives the effect of too low a temperature. The same may be said of liquid metals. Still, in the case of iron my own investigations* have shown that beginning at about 1350° C. the oxide layer gets more and more dissolved in the liquid iron, the latter thus with increasing temperature assuming more and more the radiation-properties of the pure metal. Below this limit, between 1200 and 1400° C., the oxidising power is so effective that the molten iron, leaving the tap-hole of the cupola in a fairly pure state with an emissivity of only 0·35, after one metre's run in contact with the open air has acquired an emissive power of nearly 1·0, i.e. that of a black body. Further details may be found in the publication mentioned.

To obtain proper and well-defined experimental conditions I investigated various metals in the form of U-shaped sheets mounted on a frame, figure 1, inside a water-cooled box, which could be evacuated or filled with argon. The true temperature of the electrically-heated metal sheet was determined optically, by observation of the narrow space between the two planes†. The radiometric observation was carried out

* *Stahl und Eisen* (1930).

† Mendenhall, *Phys. Rev.* **33**, 74 (1911).

with a rocksalt spectrometer with vacuum thermocouple and string-galvanometer, through a rocksalt window sealed into an opening in the front wall of the metal box.

By Maxwell's theory, Drude and Planck found* the relation

$$R = \frac{2\sigma\tau - 2\sqrt{(\sigma\tau)} - 1}{2\sigma\tau + 2\sqrt{(\sigma\tau)} + 1},$$

where R is the reflectivity,

σ the electric conductivity

and τ the frequency of the radiation,

which leads by approximation to the equation

$$R = 1 - 2/\sqrt{(\sigma\tau)} = 1 - A.$$

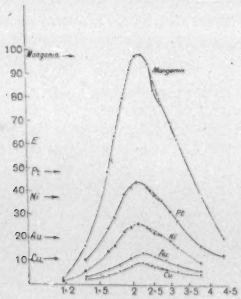

Fig. 2. $E_{max.}$ (T=2640).

As, according to Kirchhoff, emissivity is proportional to absorptive power, we obtain with specific resistance w and wave-length λ

$$E = 2/\sqrt{(\sigma\tau)} = \text{constant} \times \sqrt{(w/\lambda)}$$

on the assumption that we confine ourselves to the longer wave-lengths. That means that the emissivity of a pure metal is to be expected to be equal to the root of the specific electrical resistance. Corresponding observations of the metals gold, copper, nickel, platinum and manganin gave the isochromatic curves of figure 2. It will be seen that the ordinates of the energy maxima increase in the same order as the specific resistance. The true positions of the values of $\sqrt{w}$ are indicated by little arrows, in most satisfactory agreement with the data obtained, as the deviations must be ascribed to inevitable impurities of the metal surfaces. The most striking fact brought out by these observations is the validity of the Planck-Drude relation for wave-lengths down to 2μ.

Combining this equation with the Wien-Planck law for the spectral energy-distribution, we find for the intensity-maximum

$$E_{max.} = \text{constant} \times \sqrt{w_0} \cdot T^8.$$

* P. Drude, *Physik des Aethers*, **2**, 575 (1894).

That is to say, the maximum of the energy emitted by a radiating metal is proportional to the root of the specific resistance at zero and to the sixth power of the absolute temperature, instead of to the fifth power as in case of a black body.* This sixth-power law has been stated hitherto only for platinum by Lummer and Pringsheim, but it appears to be a universal one. Figures 3 and 4 give the energy curves for iron and nickel obtained with the rocksalt spectrometer previously mentioned.

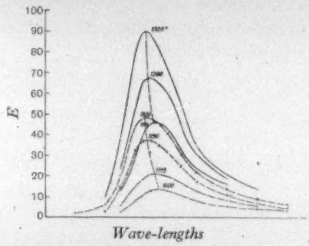

Fig. 3. Isochromes of iron:
Black body •—•—•—•
Clean iron in argon +—+—+—+

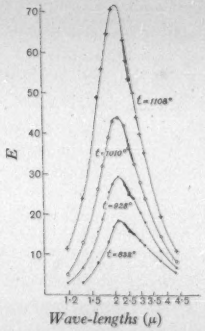

Fig. 4. Isochromes of nickel.

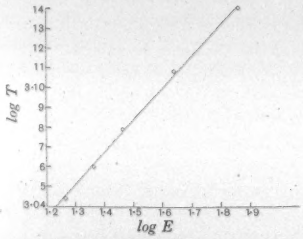

Fig. 5. Relation between log T and log E for nickel.

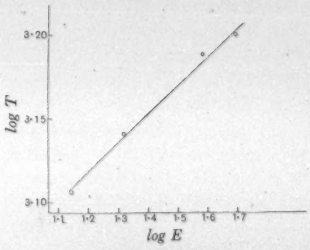

Fig. 6. Relation between log T and log E for iron.

In the case of iron, I succeeded at last in sustaining a clean surface in a current of argon for at least the duration of each curve-measurement. For comparison the black-body curves for the same temperatures are plotted. It must be added that all observations were carried out by immediate comparison with a black body, to eliminate errors due to atmospherical and optical absorption. While the iron isochromes, figure 3, show fairly well the displacement of the maximum similar to that given by Wien's law, this appearance seems not so marked in the nickel curves, figure 4. But if we make use of the data given by those curves and plot the logarithm

* Aschkinass, *Ann. der Phys.* **17**, 960 (1905).

of the absolute temperature against the logarithm of the maximal emitted energy, we shall find remarkably straight lines, figures 5 and 6, following the equations

$$\log E_{\max.} = 6 \log T - 17{\cdot}5,$$

$$\log E_{\max.} = 6 \log T - 17{\cdot}0$$

respectively. This means that the sixth-power law of Aschkinass is obeyed not only by platinum but also by pure iron and nickel in the temperature-range observed. Corresponding observations are being carried out on other metals.

In any case the results of the present investigation are in good agreement with the theoretical predictions, based on Maxwell's theory, which assert an interesting relation between electric resistivity and emissivity of metals.

DEMONSTRATIONS

Demonstration of plug and ring gauges given on January 16, 1931, by F. H. Rolt, National Physical Laboratory.

The object of the demonstration was to show how the degree of fit between a highly finished cylindrical plug and a ring gauge, both of hardened steel, varies with the difference between their sizes. A ring and a series of plugs of graded sizes were used for the experiments. The actual sizes of the various gauges were as follows:

Ring	... ...	1 + 0·00005 in.
Plugs	*D*	1 − 0·00010 in.
	H	1 + 0·00000 in.
	J	1 + 0·00005 in.
	F	1 + 0·00015 in.

With the surfaces of the gauges quite clean, the plugs were inserted in turn into the ring. Plug *D* was an easy fit, and with the difference of 0·00015 in. between them it was possible to spin the ring freely upon the plug when the latter was held horizontally. The next plug *H* could still be inserted freely by hand, but plug *J* (which had the same measured size as the ring) required gentle hand-pressure to fit it into the ring. The last plug *F*, which was 0·0001 in. larger than the ring, could not be inserted by hand-pressure, but the operation became possible after the surfaces of the gauges had been lubricated with vaseline. This apparent fitting of a plug into a ring smaller than itself is explained by the fact that the ring expands, assisted largely by the powerful wedging-effect brought into play by even moderate axial pressure, as the result of the usual slight taper at the advancing end of the plug. The increase in size of the ring has been verified by measurements taken on its outside diameter before and after insertion of the oversize plug.

It was further demonstrated that when the surface of the smallest plug *D* also was coated with vaseline, this plug became almost as tight a fit in the ring as the oversize plug *F*. A film of grease thus has the effect of masking entirely the presence of both positive and negative differences of a small order.

Demonstrations were given also of the adherence which can be obtained between gauges of the Johansson type, which consist of rectangular blocks of hardened steel, one opposite pair of faces of each block being finished optically flat and parallel.

A small set of these gauges ranging from 0·1000 in. to 0·1001 in. by steps of 0·0001 in. was shown. It was demonstrated that the difference between two neighbouring gauges of this set could be seen against a strong light by the wringing of the two gauges side by side on a truly flat surface and the laying of a knife-edge straight-edge across their upper surfaces. In such a test, the slit of light between the straight-edge and the smaller of two gauges can no longer be seen if the difference is less than about 0·00003 in. It was shown, however, that a difference as small as 0·00001 in. can be detected by preliminary light smudging of the bright surfaces of the gauges with a finger and subsequent scraping of the straight-edge

gently across them. From the extent of the bright mark thus produced on the surfaces it was possible to say which was the larger of a pair of gauges differing by only 0·00001 in.

Demonstration of some stroboscopic effects, *given on December* 5, 1930, *by* Prof. G. B. Bryan, Royal Naval College, Greenwich.

The demonstration was intended to show the properties of a form of stroboscope developed by Prof. Bryan during the last three years. It consists simply of a gas vacuum-tube attached radially to a disc driven by the shaft whose speed is to be determined. The discharge through the tube is produced by a small induction coil with a tuning-fork interrupter. The outer electrode of the tube is earthed to the shaft and the inner one is connected to an insulated axial pin on which rests an insulated metal brush. The coil produces f almost instantaneous flashes per unit time, where f is the frequency of the fork, and when the disc is revolved a spoked pattern is formed owing to the persistence of retinal vision (about $\frac{1}{15}$ sec.), and at certain speeds this pattern is quite steady. The number of spokes in the pattern and the frequency of the fork determine the speed of the shaft. The great advantage of this form of stroboscope is that the whole brightness of the tube is made use of to form the image on the retina, and as this brightness is considerable, especially if the gas be neon, the patterns formed are easily seen in a sunlit room.

Two forms of vacuum tube were shown. One consists of an ordinary spectrum tube with a capillary about 3 in. long and having a tap at one end so that it can be pumped out *in situ*. It can be filled with any gas, but air gives very good results. This form gives a fine-line pattern, but owing to its size is not satisfactory for speeds above 1500 r.p.m.

In the second form the small neon tube from a sparking-plug tester is used. This is a thin-walled tube, about 1·5 in. long and $\frac{3}{16}$ in. in diameter, having external electrodes. It is mounted radially in a small metal box 3 or 4 in. in diameter and $\frac{3}{4}$ in. deep, with a glass front. The axial end of the tube is carried in a spring socket of ebonite and the outer end in a small socket in the rim of the box. This mounting is very like that of a pump on a bicycle and the tube is easily detachable. The ordinary ignition coil is too strong for this tube but gives excellent results if arranged with a parallel spark gap about 1 mm. long and a series gap about 0·5 mm. long. The series gap cuts out the faint discharge that occurs on make. This mounting can be run with safety at 4000 r.p.m.

The following experiments were made to illustrate the use of the stroboscope: (1) measurement of speed of a d.c. motor, (2) control of speed of motor, (3) measurement of slip of induction motor, (4) demonstration of falling into step of an auto-synchronous motor; displacement of motor as load is put on, and falling out of step when maximum load was reached, (5) deceleration of motor.

REVIEWS OF BOOKS

Radiations from Radioactive Substances. By LORD RUTHERFORD, J. CHADWICK and C. D. ELLIS. Pp. xi + 588. (London: Cambridge University Press.) 25*s*.

The advent of this important work inevitably suggests comparisons with its distinguished forerunners—Rutherford's *Radioactivity* in its editions of 1904 and 1905, and the *Radioactive Substances and their Radiations* of 1912. No strict comparison is, however, possible. The two—or rather three—earlier treatises were concerned almost entirely with the progress of radioactive science under the inspiration and guidance of the transformation theory, and with its phenomenally rapid expansion into the whole range of classical atomics. By contrast, the new treatise derives its inspiration mainly from the nuclear theory, which was in its infancy when the *Radioactive Substances* went to press.

It is therefore in all essentials an entirely new work, and it supplements rather than supersedes the treatise of 1912. The properties of the α, β and γ radiations are described and discussed in the greatest detail, and with particular reference to their bearing upon problems of atomic and nuclear structure. Brief accounts are added of recent work on the penetrating radiation in the atmosphere and on the non-radioactive isotopes. As is indicated by the title, there is only the briefest description of the properties of the radio-elements; the book is, however, made largely self-contained by the insertion of concise but adequate summaries of the transformation theory and of the most recent values of the fundamental radioactive constants.

Some of the more important and more beautiful of the earlier experiments—such as the first α-particle counting—are included, but in the main the book deals with the advances of the past few years. The distribution curve of the numerous references to original papers gives no indication of having traversed a maximum—a large proportion of the references is to papers published since 1925—and even if the work had been no more than a compilation, it would clearly have been difficult of fulfilment by a single hand.

The senior author, on every occasion on which he has undertaken a comprehensive account of the current position in radioactive science, has been committed to the writing of a new treatise rather than to the revision of an existing work—a penalty mitigated only by the circumstance that it has so largely been a result of the activities of his own research schools in Montreal, Manchester and Cambridge. On this occasion he has taken into collaboration two of his colleagues, both actively engaged in research on the problems under review. Their close cooperation has resulted in a book which is singularly free from the discontinuities which are commonly associated with a composite work.

The new book will be welcomed as an authoritative statement of the present positions of the many sides of modern radioactive research. The success of its appeal to specialists in radioactivity was assured from the outset, but the book can also be recommended confidently to a wider circle of readers—namely, to all who are interested in the trend of atomic theory. Its appearance is doubly opportune, both for the specialist and for the general reader, in view of the continuous rapid expansion of the subject and of the developments which may be expected to result from the application of the new mechanics. And, in conclusion, the book will bring much encouragement to those who still persist in regarding physics as a predominantly experimental science.

The Physical Principles of the Quantum Theory. By WERNER HEISENBERG. Translated into English by CARL ECKART and FRANK C. HOYT. Pp. xii + 186. (London: Cambridge University Press.) 8*s*. 6*d*.

This volume is based on lectures delivered by Professor Heisenberg at the University of Chicago in the spring of 1929. The leading part which the author has taken in the development of the new quantum mechanics is recognised by all those who have attempted to follow its growth and, as Professor A. H. Compton remarks in the foreword: "the *uncertainty principle* has become a household phrase throughout our universities." Classical theories assume the possibility of observation without perturbation of the object under investigation. Modern quantum theory denies this possibility, and on this denial may be based the main argument of Heisenberg. "Every experiment destroys some of the knowledge of the system which was obtained by previous experiments."

The book contains five chapters, and an appendix in which the formal mathematical apparatus has been collected. The first chapter is of an introductory nature and contains an account of the fundamental concepts of the quantum theory. It is shown that both matter and radiation possess a remarkable duality of character, as they exhibit the properties sometimes of waves, at other times of particles. The solution of this difficulty here proposed is that the two mental pictures are both incomplete and have the validity of analogies which are accurate only in limiting cases. Mathematics is not subject to the limitations imposed by the formation of mental pictures, and it has been possible to invent a mathematical scheme which seems adequate for the treatment of atomic processes. The limitations of the concept of a particle are obtained by considering the concept of a wave, and conversely one may derive the limitations of the concept of a wave by comparison with the concept of a particle. Chapter II deals with the critique of the corpuscular theory, and chapter III with the critique of the wave theory, the uncertainty relation being made the basis of each discussion. The next chapter deals with the statistical interpretation of quantum theory, and the last chapter contains a discussion of certain important experiments. The author concludes that the progress of physics is far more likely to result in further limitations on the applicability of classical concepts than in the removal of those already discovered. In spite of its difficulties this is a book which cannot be neglected by the serious student of modern physics.

H.S.A.

(1) *Recueil d'Exposés sur les Ondes et Corpuscules;* (2) *Introduction à l'Etude de la Mécanique Ondulatoire*. By LOUIS DE BROGLIE. Pp. 80. (Paris: Herman et Cie.) 29 fr.

In these two volumes Louis de Broglie gives an account of the new wave-mechanics which he did so much to initiate some six years ago. The first contains five articles which appeared between 1927 and 1929, giving a more or less popular account of the origin and development of the new theories. Whether a particular entity is to be regarded as corpuscular or undulatory in character depends far more than we had imagined on the phenomenon investigated. The same physical entity can present itself to us, according to the experiment to which it is submitted, under either the one aspect or the other. In the language of Bohr these two aspects of the entity are to be regarded as complementary rather than as contradictory. Of special interest is the last article of the series which deals with determinism and causality as affected by the physics of to-day. The author has been very successful in his aim of presenting to a wider public a general idea of the questions which are at present occupying the minds of physicists.

The second volume is more weighty and contains a detailed account of the methods of the new mechanics. Its importance is sufficiently shown by the fact that it has been trans-

lated into English by T. H. Flint and published by Methuen (1930). The French edition contains a portrait of the author and some striking reproductions, due to M. Ponte, of diffraction rings obtained by the passage of streams of electrons through thin films of zinc oxide.

H.S.A.

Conférences d'Actualités Scientifiques et Industrielles, Année 1929. Pp. viii + 270. (Paris: Herman et Cie.) 35 fr.

This most interesting volume contains nine lectures delivered in 1929 under the auspices of the Conservatoire Nationale des Arts et Métiers. These lectures were arranged to provide an outline of some of the important problems now attracting the attention of physicists, and the lecturers chosen have been very successful in presenting the philosophical principles of their subjects without entering into minute details or experiments, or into elaborate mathematical calculations. L. de Broglie explains how he was led to associate waves and quanta and he describes the rise of wave-mechanics. Foex gives an account of "les substances mésomorphes," a term including the liquid crystals discovered by Lehmann about 1890 and denoting a state distinct from and intermediate between the crystalline and amorphous states (Friedel). The magnetic properties of such substances are described and are explained with the aid of Langevin's theory. E. Bloch gives an exceptionally clear account of the theory of quanta, leading up to the most recent developments in connection with the wave properties of the electron and the principle of indeterminacy of Heisenberg. Dunoyer describes the construction and properties of photoelectric cells and gives an account of recent applications of such cells which illustrates in the most striking way how research in pure science can and does lead to most unexpected practical results. G. Ribaud supplies a lecture on the radiation from incandescent bodies and shows not only how the radiation from an ideal "black body" can be realised experimentally, but also how the measurement of radiation can be utilised in optical pyrometry. An article of exceptional interest on the electric production of sound is contributed by Colonel Jullien, who assembles a mass of information not available elsewhere with regard to the production of "electric music." Those who are not familiar with the subject will be surprised at the number of different arrangements and possibilities in this field. L. Bloch discusses the relation between the structure of spectra and the structure of atoms, emphasising the point that according to the new mechanics an atom can be revealed to us only by its radiations, so that apart from the frequency and intensity of the rays which it emits an atom has no existence for us. There is not merely correspondence, there may be said to be identity between the structure of an atom and the structure of its spectrum (Heisenberg). A more technical subject, that of high vapour-pressures and their applications in industry, is dealt with in an authoritative way by V. Kammerer. Finally M. R. Mesny discusses directed electromagnetic waves and their applications, as in the beam system of Marconi. He mentions the interesting fact that communication has been established over a distance of 8 km. with a wave-length of only 17 cm. The volume as a whole is to be recommended as providing a stimulating series of lectures on subjects of interest alike to the physicist and to the scientific layman.

H.S.A.

Modern Physics, A General Survey of its Principles. By THEODOR WULF. Translated from the German by C. J. SMITH, Ph.D., A.R.C.S., M.Sc., with 202 diagrams. Pp. xi + 469. (London: Methuen and Co.) 35*s*.

The author informs his readers that this book has been written for "non-technical experts" who, without wishing to become physicists, desire an insight into the realm of physics. The aim is to present not mere facts but their inter-relationships, and to unfold a

clear view of the structure of matter. The philosophical aspect is to be emphasised, the book being indeed based upon lectures given by the author to students of philosophy over a period of twenty years.

It is divided into four parts: Part 1, the Material World; Part 2, the Atomic Structure of Matter; Part 3, the Structure of the Atom; and Part 4, the Physics of the Ether. The range traversed is very wide, and some account will be found of supra-conductivity, the Compton, Zeeman and Stark effects, electron waves and wave-mechanics, and many other subjects of comparatively recent investigation; as well as of the traditional phenomena usually treated under physics.

The aim of the author is certainly an ambitious one, for this enormous range is offered for the comprehension of the student without special equipment or training. Therein must lie our chief criticism of the book. It is not surprising to find that at many places space is taken up by a quite elementary detailed treatment of ordinary subject-matter, as, for instance, in the proof that the focal length of a spherical mirror is half the radius, or in the laborious treatment of the laws of electromagnetic induction; and that, on the other hand, in the treatment of the more abstruse parts, the stages of exposition of the theme are often glided over, as, for instance, in the theory of metallic conduction, where the failure of the simple electron theory is pointed out, and a merely nominal reference is given to the work of Pauli and Sommerfeld.

The choice of subject-matter is, for the same general reason, arbitrary in its inclusions and omissions. Many interesting descriptions occur in the course of the volume: e.g. of supra-conductivity, of the variation of specific heat with temperature, and of the distribution of elements through the earth's crust. As instances of omission, on the other hand, may be cited the facts that on the subject of the value of the velocity of sound in water no work later than that of Colladon and Sturm is mentioned; that in the section on X-rays there is no description of the spectrometer, nor indeed is the work of the Braggs mentioned; and that in regard to the dispersion of light no account is given of the electron theory—indeed the inference might be drawn that no such theory of dispersion had been developed.

Serious errors of statement are not infrequent: for example, that characteristic X-rays are excited by incident rays of the same frequency, that the Peltier effect is proportional to the absolute temperature, and that nitrogen is a paramagnetic body.

British physicists on the whole receive scant notice; no reference will be found to the work of Rayleigh, O. W. Richardson, Bragg, Lodge, Townsend, or Fleming, while the name of Sir J. J. Thomson occurs once, though Boyle is mentioned three times and Maxwell nine times.

Despite the criticism here offered it seems, nevertheless, quite likely that the book will prove of decided interest and stimulus to the discriminating reader with a fair basic knowledge of physics at his command. The translation is ably carried out.

D.O.

Manual of Meteorology. Vol. III. *The Physical Processes of Weather.* By Sir NAPIER SHAW, LL.D., Sc.D., F.R.S., with the assistance of ELAINE AUSTIN, M.A. Pp. xxviii + 446. (London: Cambridge University Press.) 36*s*.

It is now eleven years since the first part of this manual appeared as a volume of 160 pages in paper covers. It dealt with the relation of the wind to the distribution of atmospheric pressure in a way which indicated that the manual when complete would probably be for some time the standard work on meteorology. The subsequent appearance of vol. I, containing a historical introduction and general account of meteorological problems, and vol. II devoted to a survey of the globe and its atmosphere, strengthened this belief, and vol. III establishes it as a fact. The new volume treats of gravitational, sound and light waves and their effects in determining meteorological phenomena in considerable detail. Greater

use is made of entropy than has been customary in meteorological works, and the authors show how valuable the idea is. The frequent appearance in the thermodynamic formulae of tt in place of the usual symbol T for absolute temperature is likely, however, to lead to misunderstandings. Quotations from original sources and in their original languages are introduced very freely, and the style is more ambulatory than in the earlier volumes.

Vol. IV, which is promised for this winter, is to deal with the applications of dynamics to meteorology.

C.H.L.

Four-Place Tables of Logarithms and Trigonometric Functions. Unabridged edition. By E. V. HUNTINGTON. Pp. 33. (London: Allen and Unwin, Ltd.) 3*s.* 6*d.* cloth, 2*s.* paper.

This is a book of four-place tables as convenient and comprehensive as any that the reviewer has handled. Besides the ordinary tables of logarithms and of logarithmic and natural circular functions, the work contains conversion tables, decimal equivalents of common fractions and of sexagesimal parts of a degree, tables of squares and cubes, square roots and cube roots, reciprocals, areas of circles, logarithms of radian measure, exponentials, Napierian logarithms, tables of radian measure, and a selected table of useful constants.

Although the present reviewer has a prejudice for eighteenth-century numerals in books of tables, he has to confess that the generous spacing of the type of uniform height employed in printing this book makes its use very easy and pleasant. Printing and production leave nothing to be desired, and the book is remarkably good value for the money.

A.F.

Les Statistiques Quantiques et leurs Applications. By Professor LÉON BRILLOUIN. 2 volumes, pp. 404. (Les Presses Universitaires de France, 1930.) 125 fr.

The advent of the Bose-Einstein statistics and the Fermi-Dirac statistics and their close relation to wave-mechanics had recently given to the subject of statistical mechanics a much more important place in physical theories than it previously held. One of the most important applications which have been made is to the theory of the electrical and magnetic properties of metals by Sommerfeld, Bloch and others, and those who are anxious to keep abreast of these modern theories cannot ignore the developments of statistical theory.

The treatise by Professor Brillouin gives an adequate account of the foundations of statistical theory and of recent applications without a wide knowledge of mathematics. Assuming an elementary knowledge of thermodynamics, he outlines the attempts to deduce by classical methods the distribution of energy in a radiative field and shows the inadequacy of classical ideas. The Planck formula is deduced not by the original method of Planck but by the recent statistical method of Bose, in which radiation is definitely regarded as made up of photons with quantised energies. He considers in some detail the two theories of light and their relation to one another, but lays considerable emphasis on the corpuscular or photon conception of light. He shows how the quantum conditions give to photons properties similar to those of waves, and how the law of diffraction of light by a plane grating can be deduced by this method (after Duane). It is interesting to note at what point the concept of photons and the quantum conditions permit an interpretation of the physical laws of optics and approach the wave point of view. The essential assumptions involved in the Bose-Einstein statistics are clearly stated and compared with those of the Fermi-Dirac statistics, the analogue of the Pauli exclusion principle in statistical theory. Heisenberg's indeterminary principle is also described and its significance in statistical applications of the new mechanics is pointed out.

The second volume deals with some of the recent applications of the new statistics. The electron theory of metals as developed by Sommerfeld and Bloch is described at some length, but the author does more than merely reproduce the works of these authors. By simple illustrations he clothes the bare bones of mathematical theory in a presentable garment of physical theory. The distribution of electrons in atoms as indicated by statistical theory is given by the method of Thomas and Fermi, and the book ends with a long discussion of the problems of ionisation of atoms at high temperatures, such as occur in astrophysical theory.

The book is well written and aptly illustrated and can be confidently recommended to those who want a simple but sound introduction to statistical mechanics.

Radioactivity and Radioactive Substances. By J. CHADWICK, M.Sc., Ph.D. Pp. xii + 116. (London: Sir Isaac Pitman and Sons.) 2*s.* 6*d.*

There is no need to dilate on the good qualities of Dr Chadwick's little volume. It has stood the test of ten years' use and now passes into a third edition. In this edition "the text has been in some places corrected, in others amplified, so as to bring it into agreement with the later developments. The numerical data and tables have been revised." The book, which possessed original features in that it cut quite away from the historical order of development, and assumed at the outset modern notions of the structure of the atom, still retains these features. It is proverbially dangerous to generalise, but it can safely be said that the book is the best introductory treatise of its size extant.

A Text-Book on Spherical Astronomy. By W. M. SMART, M.A., D.Sc. Pp. xi + 414. (Cambridge University Press.) 21*s.*

The recent growth of stellar astronomy during the last half-century has had the effect of largely attracting the attention of students away from the classical methods of the older "Spherical" Astronomy, which is concerned fundamentally with the geometry of celestial bodies rather than with their physical state. The result has been that, while a large number of treatises which bring modern astrophysics within the range of the non-technical or semi-technical reader is now available, the classical works of Chauvenet, Godfray and Ball, designed for the student of mathematical astronomy, have had few or no successors.

Nevertheless this branch of astronomy remains as fundamental as ever it was; for example, all the recent indirect methods of determining the distances of stars, clusters and nebulae, such as spectroscopic parallaxes, Cepheid periodicity, mass-luminosity law, etc., derive ultimately their justification from the trigonometric or dynamical parallaxes obtained by the methods of spherical astronomy.

On the other hand, far-reaching developments and improvements have taken place in this branch of astronomy since the above treatises were written. To name only three, we have (i) the displacement of visual by photographic observations, involving a whole new technique with elaborate mathematical machinery, in which connection, incidentally, the methods of projective geometry might often be applied with advantage; (ii) the determination of the orbits of visual, spectroscopic and occulting binaries, a subject little noticed in the older books; (iii) the calculation, which again was usually omitted, of heliographic coordinates for physical observations of the sun.

Methods of numerical computation, also, have altered and are altering. With the growing use of calculating-machines of the Brunsviga type, the importance which used to be attached to formulae "adapted to logarithms" is rapidly disappearing.

Certain classical observations, such as the determination of the solar parallax by the transit of Venus, which bulked largely in the older treatises, are now of only historical interest.

For all the above reasons, Dr Smart's new work, which brings, so to speak, the student's spherical astronomy up to date, is to be warmly welcomed. It will prove invaluable to all those who desire to make any serious study of astronomy; and it is to be noted that there is a growing tendency in universities to recognise the importance and educational value of this subject. In this, as in many other matters, London has given a lead; for many years now, it has granted a B.Sc. degree to astronomical specialists, and that this is being taken advantage of is evidenced by the many questions from London examination papers to be found among the examples in the present book.

In addition to what may be termed the classical chapters on spherical astronomy, in which the changes are mostly of arrangement and order of exposition, Chapter VII deals partly with heliographic coordinates, Chapter XI contains a very full discussion of the proper motions of the stars, including the determination of the solar motion and statistical parallaxes, Chapter XII gives a very clear account of the theory of astronomical photography and Chapter XIV of binary star orbits. Chapter XIII, on the determination of position at sea, is a very interesting reminder of the fact that astronomy still has important applications to practical life and has special value in view of the author's practical experience of navigation with the Grand Fleet during the War.

The absence of certain matters may be noticed. Thus, no attempt is made to deal with the problem of orbits of occulting variables, which receives a mere mention. Again, the book does not venture outside the old Euclidean lines and neither the three famous Einstein effects nor the influence of the curvature of the universe on observations of very distant objects are discussed. Probably the relativity theory is beyond the real grasp of a student at this stage and the omission in this case is a wise one. The problem of the occulting variable, on the other hand, is of great interest and might, one would have thought, have been presented in a fairly simple manner. Perhaps Dr Smart may consider including it in a later edition, which it is to be hoped will come soon.

The notation of the International Astronomical Union has been followed, fortunately not slavishly, and we congratulate Dr Smart on having had the courage to part company with the Nautical Almanac and introduce the term "Greenwich mean astronomical time" for the hour angle of the mean sun at Greenwich, a nomenclature that to some extent obviates the confusion introduced by a convention which no longer measures every kind of time by the hour angle of the indicating body or point.

One almost wishes, indeed, that Dr Smart had found it in his heart to disregard another recent innovation also, to wit, the reversal of sign of the equation of time, a reversal which has broken the hitherto universal rule (a rule which was a most useful memory-aid for the student) that corrections or "equations" in astronomy are always to be added to a datum of observation in order to obtain a theoretical quantity.

The examples at the end of each chapter have been carefully selected and provide an indispensable adjunct for the proper appreciation of the principles. Of the printing and diagrams it will be sufficient to say that they are fully up to the usual high standard of the Cambridge University Press. L.N.G.F.

Photoelectric Cells. By Dr N. R. Campbell and Dorothy Ritchie. Pp. vii + 209. (London: Sir Isaac Pitman and Sons, Ltd.) 15*s*.

As the authors remark in their preface, the two devices which make use of the effect of light in producing or varying electric currents have had singularly different histories. The selenium "cell" has been overworked by the inventor, neglected by the theorist; while the cell based on the photoelectric effect, an effect of fundamental importance in modern physical theory, has but lately attracted the attention of the inventor, and many of its practical possibilities remain yet unexplored.

The book before us is written in a spirit which realises these possibilities and realises

too that a growing audience exists which is clamorous for detailed information concerning the properties and uses of photoelectric cells. That Dr Campbell is associated with the book is sufficient guarantee that the work will be clear, thorough, and, should occasion demand it, emphatic. The book is certainly remarkable in the manner in which it conveys detailed and advanced information with the assistance of a minimum of mathematics. The authors indeed 'swither,' as the Scots say, between an exposition suitable to the professional physicist and one likely to appeal to the enquirer, earnest enough in all conscience, whose mathematical equipment is limited to the proverbial four books of Euclid and algebra up to (and including) quadratic equations. This is, perhaps, in the nature of the case, inasmuch as photoelectric cells and circuits are likely to be used by all sorts and conditions of men.

The book is divided into three sections which deal respectively with the theory, use and applications of photoelectric cells. The first section discusses the photoelectric effect, photoelectric emission, the electric discharge, the voltage current characteristic and intermittent currents. The second section opens with an abstract but none the less valuable discussion of some general principles used in the measurement of physical quantities, and then proceeds to the consideration of electrostatic methods of measurement and of valve amplification. The third section is concerned with the applications of photoelectric cells, and it is not easy to find a consistent set of headings under which the many uses can be conveniently grouped. The authors, on consideration, find that the "thing studied or used, and the ultimate cause measured, compared, or detected" may be classified under one of four headings, namely, luminous flux, illumination, colour and absorption, and to each of these headings a chapter is devoted.

The volume is a storehouse of ordered information on an important and growing subject; it will, we hope, attain the distinction, rare among scientific books, of inclusion in the class of best-sellers.

THE PROCEEDINGS OF THE PHYSICAL SOCIETY

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THE REFLECTING POWERS OF ROUGH SURFACES AT SOLAR WAVE-LENGTHS

BY H. E. BECKETT, B.SC., Building Research Station.

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ABSTRACT. A study of the reflecting properties of building materials for solar radiation has been made, a hemispherical mirror being used for integrating diffusely-reflected energy upon a thermopile receiver. The paper follows the work of Coblentz and Royds and deals at length with the errors inherent in the method and with the adjustment of the apparatus. The theory of the method is simplified by the introduction of an auxiliary specimen which, in particular, renders the observations independent of the degree of blackness of the thermopile receiver. Reflecting powers for a large number of building materials are given for four radiation bands within the solar range, and also for the composite radiation obtained by the screening of a pointolite lamp with a thin gold film. The latter radiation somewhat resembles that of the sun in range and distribution, and enables the reflecting power of a surface for total sunlight to be determined approximately in a single reading. The apparatus has been tested with magnesium carbonate surfaces and yields reflecting powers in the visible spectrum in good agreement with those found photometrically.

§ 1. INTRODUCTION

IN the course of an investigation of the heating effect of the sun's radiation upon buildings it was found that the reflecting characteristics of building materials had hitherto received little attention. As no apparatus capable of measuring the reflecting powers of rough surfaces over the solar wave-length range was available, it was decided that apparatus upon which such measurements could be made should be set up at the Building Research Station.

The methods previously used in examining rough surfaces are well discussed by Coblentz* and need not be detailed in this paper. That now adopted was developed concurrently by Royds† in Germany and Coblentz* in America. The incident radiation passes normally on to the specimen through a slot in a hemispherical mirror. The latter collects all radiation reflected from the specimen and

* W. W. Coblentz, *Bull. Bur. Stand.* **9**, 283 (1913).
† T. Royds, *Phil. Mag.* **21**, 167 (1911).

focusses it on the receiving surface of a thermopile. The necessary integration of reflected rays is thus performed by the apparatus, and the determination of reflecting powers becomes a routine operation when once certain errors inherent in the method have been evaluated. By the use of filters, radiation from various spectral regions may be allowed to fall on the specimen and the variation of its reflecting power with wave-length determined.

When an attempt was made to set up and use apparatus of this type, following very closely the specifications of Coblentz, numerous experimental difficulties were encountered. Although great accuracy was not essential in view of the variable nature of the materials to be examined, the opportunity has been taken to investigate certain errors which seem to have escaped previous notice. It is therefore with the adjustment of the apparatus and with the correction of the results that the present paper mainly deals.

§ 2. DETAILS OF THE APPARATUS

The general lay-out of the apparatus is shown in figure 1. The image of a slit S suitably illuminated from behind is projected by means of the concave mirror M_1 and the plane mirror M_2 on to the specimen, which is placed alongside the thermopile in the diametral plane of the hemispherical mirror M_3.

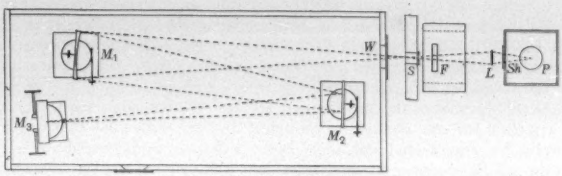

Fig. 1. Plan of apparatus.

The source and slit. The tungsten bead of a 100 c.p. pointolite lamp P provides a convenient source of high intrinsic brilliancy. The lamp operates normally with a current of 1·32 amp. and by substitution of an adjustable rheostat for part of the series resistance it is possible to maintain this current constant in spite of considerable fluctuations in the supply voltage. The necessary current check is provided by a high-grade ammeter whose scale is illuminated and viewed through a convex lens. In this way the current can be maintained constant to 0·002 amp. or about 0·15 per cent. The control-rheostat and ammeter are placed near the galvanometer scale and it is found that, with practice, observations can be made by a single observer. The lamp is mounted in an asbestos-cement screen which carries a brass shutter *Sh* operated through a string-and-pulley system.

As readings beyond the wave-length 3·0μ have not been required, a glass lens L of 10 cm. focal length is used to throw an image of the pointolite bead upon the slit. It may be shown by a simple analysis that the energy available in the apparatus is greatest for a given lens if the latter is placed at such a distance behind the slit

that the solid angles subtended at the slit by the lens and the concave mirror are equal. The source is placed so that its magnified image falls upon the slit. The lens used measures 5 cm. in diameter and is 50 cm. distant from the slit, thus allowing ample room for the insertion of a stand for filters.

The slit measures 5 mm. by 1 mm. and is mounted on a brass plate blackened on the side facing the concave mirror to prevent the reflection of stray light into the optical path. The image cast upon the thermopile measures 8 mm. by 1·6 mm.

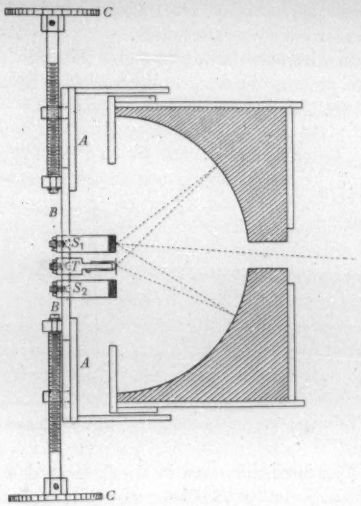

Fig. 2. The hemispherical mirror.

The mirror system. To prevent tarnishing, the three mirrors, which are surface-silvered, are mounted inside a wooden box whose atmosphere is dried with calcium chloride and kept free from sulphides with lead acetate. The dimensions of the curved mirrors are as specified by Coblentz: the concave mirror is 17 cm. in diameter and of 100 cm. focal length, and the hemispherical mirror 10 cm. in diameter with a slot 2 cm. by 1 cm. to admit the incident radiation. The hemispherical mirror is fixed in a cylindrical brass housing, figure 2, which is screwed to the base of the box. The other mirrors also are screwed to the box, but each has two rotational movements which enable accurate adjustment of the position of the reflected image to be made.

During the preliminary trials it was discovered that changes of temperature caused slight movements in the wood of the box which produced corresponding errors in the adjustment of the mirrors. The trouble was removed by an arrangement whereby any necessary adjustments could be made from outside the box

immediately before readings were taken. A white card placed over the slot of the hemispherical mirror has a hole which is exactly filled by the pencil of radiation when all is in adjustment. The card may be viewed through a window in the side of the box, and any shift of the mirrors is immediately apparent by the streak of light which shows at the edge of the hole. Readjustment is effected with two keys which are pushed through small holes in the side of the box and engage with square section pins on the two controls of the plane mirror. If the box were opened for this operation the uniformity of temperature within it would be destroyed and the thermopile disturbed for some time afterwards.

Radiation from the slit enters the box through a thin glass window *W*, figure 1, which also serves to prevent low-temperature radiation from the slit-mounting from reaching the thermopile.

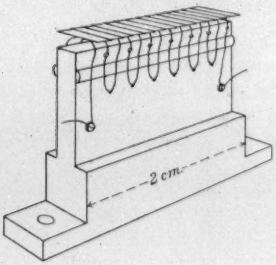

Fig. 3 *a*. The thermopile.

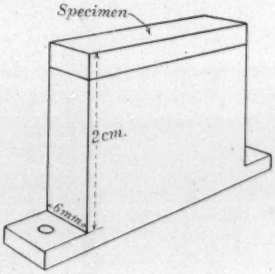

Fig. 3 *b*. A mounted specimen.

The thermopile. The thermopile used in this investigation differs from that of Coblentz in certain details. Its form will be most readily appreciated from figure 3 *a*. It is of compact construction, the wires being bent round and anchored so that no part projects beyond the blackened receiving-surface. It can therefore be placed close to the brass block upon which the specimen is mounted.

The receiving-surface is continuous in order that no error may result from a difference of intensity-distribution in the incident image and in that produced by reflection from the hemispherical mirror. To this end Coblentz's method was adopted, small strips of tinfoil being attached to the hot junctions and overlapped slightly like the slats of a venetian blind. Each strip was insulated from its neighbours by shellac and the receiver was blackened when complete. Coblentz's instrument was constructed of 22 silver/bismuth thermocouple elements mounted on an ivory block. Owing to the great difficulty of working with fine bismuth wire, the present instrument has been constructed of copper (46 s.w.g., diameter 0·061 mm.) and constantan (40 s.w.g., diameter 0·122 mm.). For soldering together the fine wires an electrically-heated bent wire was used, and the process was carried out in a pool of Canada balsam. The tinfoil receivers measure 5 mm. by 1·3 mm. by 0·02 mm. and are soldered to the hot junctions. The complete receiver, which measures

5 mm. by 15 mm., embodies 12 elements. The method of assembly is discussed elsewhere*.

In blackening the receiver a thin coat of camphor black suspended in turpentine was applied first and the instrument was finally smoked over the flame of a sperm candle. A copper cooling funnel was used in the last operation to avoid damaging the delicate tinfoil surface. Even when blackened in this manner the thermopile still possessed a reflectivity sufficient to make visible an intense yellow image thrown upon the blackened surface. The error due to incomplete blackness and its correction are discussed later.

The block upon which the thermopile is mounted is of erinoid and is cut away at each side to reduce the total width of the instrument and to minimize the risk of damage to the fine wires. The thermopile is very robust and, in spite of the lower electromotive force of the copper/constantan couple, has proved more sensitive than a similar instrument of bismuth and silver constructed earlier, chiefly owing to its smaller resistance—3·7 ohms as against 15·5 ohms.

The thermopile receiver has an unobstructed view of the whole of the hemispherical mirror. Coblentz's instrument seems to have been shaded from peripheral portions of the mirror by projecting brasswork.

Much disturbance was at first caused in the thermopile circuit by waves of air pressure from closing doors and gusts of wind. This effect has been considerably reduced by inclusion in the circuit within the box of a second exactly similar thermopile, connected in the opposite sense and not exposed to radiation. Impulsive effects in the two thermopiles should then be equal and opposite. Some unsteadiness still remains, however, and has so far prevented the galvanometer from being used with a period greater than 4 seconds.

The galvanometer. The thermopile is connected to a Paschen galvanometer. The instrument is of recent design, and is screened from electromagnetic disturbances by two small shields, one of mumetal and the other of Swedish iron. It is mounted on a brick pier which stands on a separate clay foundation and has proved very free from vibration. The galvanometer lamp has a filament wound in a fine spiral, a direct image of which is cast upon the scale. The lamp and scale are about 1½ metres distant from the galvanometer and the position of the image can easily be read to the nearest millimetre.

The specimens and their mounting. The specimens are mounted upon standard brass blocks, figure 3*b*, similar to the erinoid block that carries the thermopile, but not cut away at the sides. After being cut roughly to size and ground at the back to a suitable thickness, the specimens are fixed to the blocks with a cellulose cement and their projecting portions are ground away.

In every case the test surface is arranged to be 2 cm. above the projecting lugs by which the blocks are fastened in the apparatus. All surfaces studied then fall into the same vertical plane as the receiver of the thermopile, and one adjustment of the apparatus suffices for the whole series of tests.

The casing of the hemispherical mirror, figure 2, carries an adjustable plate *A*

* H. E. Beckett, *Journ. Sci. Inst.* **6**, 169 (1929).

furnished with two screw-operated slides *BB*. The thermopile *T* and specimen S_1 are inserted through notches in these and are bolted in position. The slides are linked by the sprockets *CC* and chains to a rod which passes through the box and ends in an external control-knob. By this means the incident radiation can be received upon the specimen or the thermopile as desired, when once the relative position of the two slides has been determined. The slide carrying the thermopile is also slotted to accommodate a second specimen S_2 similar to S_1 and symmetrical with it about the thermopile.

Each specimen block is provided with a tapped hole in the back into which a handle may be screwed to facilitate insertion and removal. These operations are effected through a small door in the end of the box. A glass panel in the door allows the positions of the slides to be read without opening the box. The plate *A* upon which the slides are mounted can be screwed down upon springs so as to bring the surfaces of the specimen and thermopile into the diametral plane of the hemispherical mirror.

§ 3. THE ADJUSTMENT OF THE APPARATUS

For this purpose the thermopile is replaced by a light coloured specimen which enables the adjustment to be carried out visually. Reference to the thermopile will, however, be made in order to avoid confusion. The relative positions of the slides are first altered until a space of 2 mm. is left between the specimen and the mounting-block of the thermopile. The auxiliary specimen is at the same distance from the thermopile. A bright image of the slit is then allowed to fall on the main specimen and adjustments are made, without alteration of the relative positions of the slides, until a sharply focussed image falling on the centre of the specimen produces a sharp secondary image upon the centre of the thermopile receiver, by reflection in the hemispherical mirror.

It then only remains to determine accurately the working positions of the slides. The thermopile is replaced for this operation and readings are obtained at various slide-settings as the slides are traversed through the expected positions. Owing to the slightly greater sensitivity of the thermopile receiver in its centre line (immediately over the hot junctions) the curves of deflection against slide-setting show maxima from whose positions the correct settings are obtained. The rapid adjustment of the apparatus at any subsequent time is made possible by the white card already described.

§ 4. THE REGIONS OF THE SPECTRUM STUDIED

The solar energy curve extends approximately from $0{\cdot}3\mu$ to $3{\cdot}0\mu$ and has been investigated in greatest detail by Abbott and his co-workers. His mean smoothed curve for Washington with the sun at an elevation of 60° has been presented in convenient form by Johansen* and for the present purpose has been adopted as a standard of summer sunshine. The curve becomes more symmetrical when plotted on a frequency scale, and is shown in this form in figure 4.

* E. S. Johansen, *Strahlentherapie*, **6**, 45 (1915).

The four filters which have been used were chosen so as to give fairly narrow transmission-bands equally spaced on a frequency scale throughout the solar spectrum. The radiation from the pointolite lamp has been computed from known figures for tungsten*, on the assumption that the temperature of the bead is 2920° K (the manufacturer's figure), and the radiation bands transmitted by the various filters are plotted in figure 4 on various ordinate scales. Filter I consists of Chances' blue-green contrast filter No. 6, 3·3 mm. thick, with their orange contrast filter No. 4, 2·7 mm. thick. The blue-green band transmitted by the first glass is stopped by the second, a single band in the infra-red with energy centred about the wave-length 1·78μ being left. Filter II comprises a 2-cm. water cell, Chances'

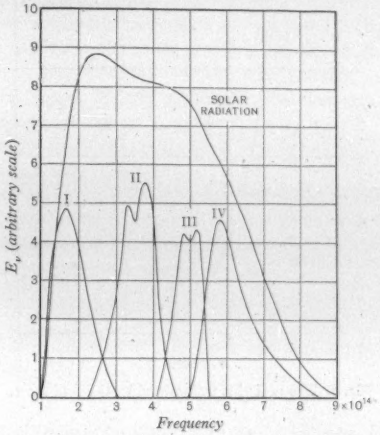

Fig. 4. The radiation transmitted by various filters.

orange contrast filter No. 4 and a cobalt blue glass, 1·8 mm. thick. The blue glass has three transmission-bands. The long-wave band is removed by the water cell and the short-wave band by the orange glass, a band centred at the wave-length 0·84μ being left. Filter III comprises a 1-cm. cell of potassium dichromate solution (72·0 gr. per litre) and a 1-cm. cell of copper sulphate solution (57·0 gr. of the hydrated salt per litre). The transmissions of these solutions overlap slightly and give a band centred at 0·61μ. Filter IV is a 2-cm. cell of copper sulphate solution, saturated at 14·2° C., which gives a band centred at 0·50μ.

The transmissive properties of the above materials in the visible spectrum have been found spectrophotometrically. The infra-red transmissions of the coloured glasses were supplied by the manufacturers, while that of water has been measured by Coblentz†.

* W. E. Forsythe and A. G. Worthing, *Astrophys. Journ.* **61**, 146 (1925).

† W. W. Coblentz, *Bull. Bur. Stand.* **7**, 619 (1911).

More filters would have been available if the thermopile could have been enclosed in an airtight case, so as to enable the galvanometer to be used at a higher sensitivity. This modification would, however, have entailed the redesigning of the hemispherical mirror-mounting and would have added considerably to the experimental difficulties.

A search has been made for a filter which, in combination with the pointolite lamp, would transmit radiation similar to that of the sun. This would enable the reflecting power of a surface for sunlight to be determined approximately in a single reading.

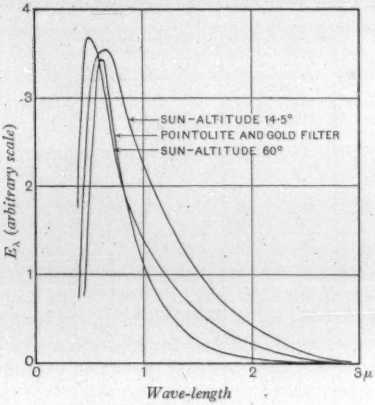

Fig. 5. The radiation transmitted by the gold filter.

The filter finally adopted consists of a thin gold film* sputtered on glass and transmitting 23 per cent. at the wave-length 0·5μ. The curve of transmitted energy, based on the maker's calibration and shown in figure 5, has maximum energy at the wave-length 0·6μ, between the maxima for the high and low sun shown in the same figure. The filtered radiation is somewhat deficient in infra-red energy but yields reflecting powers for solar radiation in fair agreement with those obtained by computation from the results with the other filters. The computed values given in § 6 are based on the energy curve of the sun at 60° elevation.

§ 5. THE CORRECTION OF THE READINGS

In finding the reflecting power of a surface at a particular wave-length two observations are necessary, with the radiation incident upon the specimen and thermopile respectively. The ratio of these readings will be called the apparent reflecting power of the specimen.

* W. W. Coblentz, W. B. Emerson and M. B. Long, *Bull. Bur. Stand.* **14**, 653 (1918).

Certain correcting factors obviously must be applied. Part of the energy diffusely reflected from the specimen is lost through the slot which admits the incident beam, while a further loss occurs by absorption in the hemispherical mirror. Another source of error, which has apparently been neglected by Coblentz and Royds, lies in the incomplete blackness of the thermopile receiver. If the thermopile reflects appreciably and no auxiliary specimen is used, there is an important difference between the optical systems for the two readings. When the radiation falls upon the specimen the thermopile and the specimen lie at conjugate foci of the hemispherical mirror, and any radiation reflected from the thermopile is in part returned to it by subsequent multiple reflections. The thermopile is therefore effectively blacker in this position than in the second, where the radiation is incident directly upon the thermopile and there is no surface at the conjugate focus.

The use of the auxiliary specimen provides similar conditions for the two observations, as will be seen from the following brief analysis.

Let the reflecting powers of the thermopile, the specimen and the silvered mirror for the particular wave-length considered be r_t, r_x and r_s respectively. It will be assumed that the fraction of radiation $(1 - p)$ is lost through the mirror slot at each reflection of diffused radiation. r_t, r_x, r_s p

When the radiant energy E falls upon the specimen the reflected radiation is handed backwards and forwards between the specimen and the thermopile in rapidly decreasing amount, and the total energy absorbed by the thermopile is the sum of a geometrical progression and equal to E

$$Epr_x r_s (1 - r_t)/(1 - p^2 r_x r_s^2 r_t) \qquad \text{.....(1).}$$

Similarly, when the radiation E falls upon the thermopile, the total energy absorbed at its surface is

$$E (1 - r_t)/(1 - p^2 r_x r_s^2 r_t) \qquad \text{.....(2).}$$

The ratio of the expressions (1) and (2) gives the apparent reflecting power of the specimen in the simple form

$$pr_x r_s \qquad \text{.....(3),}$$

which would obtain if the thermopile were perfectly black and no auxiliary specimen were used.

In the absence of the auxiliary specimen the apparent reflecting power is in the form

$$pr_x r_s/(1 - p^2 r_x r_s^2 r_t) \qquad \text{.....(4),}$$

and the quantity r_x cannot be evaluated without a knowledge of r_t.

The assumption of the simple form (3) in this case introduces an error the magnitude of which depends on the various reflecting powers involved. Representative examples are shown in table 1. The error varies linearly with change in the value of r_x.

Tests with blackened specimens show that the reflecting power of the thermopile receiver when freshly smoked is about 0·03, but this value doubtless increases with age.

The use of the auxiliary specimen not only renders the observations independent of the blackness of the thermopile but simplifies the correction of other errors.

Table 1. The error caused by reflection from the thermopile.

p	r_x	r_s	r_t	% error in r_x
0·97	0·50	0·98	0·03	1·4
0·97	0·50	0·98	0·05	2·3
0·97	0·50	0·98	0·10	4·5
0·97	0·50	0·85	0·03	1·0
0·97	0·50	0·85	0·05	1·7
0·97	0·50	0·85	0·10	3·4

The value of r_s is readily found by insertion in the apparatus of two glass specimens, surface-silvered at the same time as the hemispherical mirror and subsequently stored beside it in the protecting box. When the incident radiation falls upon the silvered specimen it is reflected specularly and for this reason the factor p is missing from the numerator of expression (1). The apparent reflecting power then gives the value of r_s^2 without further correction. Periodical checking of r_s must be performed in case the mirror tarnishes. Actually the change over a period of 2 years has been found extremely small, not more than 1 per cent. at any wave-length studied. The recent values of r_s for various types of radiation are shown in table 2.

Table 2. The reflecting power of the silver mirror.

Filter	r_s
I	0·977
II	0·940
III	0·915
IV	0·851
Gold film	0·919

For the broad spectral band transmitted by the gold filter it is possible that when the radiation has first been reflected from a highly selective specimen the calibrated value of r_s, namely 0·919, may be somewhat incorrect. Calculation shows, however, that far greater selectivity than that shown by building-materials is necessary to produce appreciable error in this way.

The value of the slot factor p is found by putting in the hemispherical mirror a small blackened screen alongside and of the same area as the slot. With this arrangement apparent reflecting powers are of the form

$$qr_xr_s \qquad \text{.....(5),}$$

q where

$$q = 2p - 1 \qquad \text{.....(6),}$$

and by comparison with the normal values p is readily evaluated.

The factor p must be expected to vary for different specimen surfaces. As, however, the figures obtained for a number of representative surfaces at four wave-lengths all fell within the range 0·96 to 0·98, the standard figure 0·97 has been used

in all determinations of reflecting power. For the greatest accuracy p should, of course, be determined separately for each surface studied. The reflecting powers for magnesium carbonate quoted in the final section are corrected in this manner. The mirror slot is actually bigger than it need be and might well be reduced to one-third of its present area. A standard correcting factor of, say, 0·99 could then be applied with still less possibility of error.

The apparatus is so disposed that radiation specularly reflected from the specimen does not strike the slot.

In the analysis it is assumed that p is constant in the various multiple reflections which occur in the system. This strictly is incorrect but the error introduced is negligible.

There remains the possibility of error due to the dependence of the reflecting powers of the specimen and of the thermopile upon the geometrical distribution of the radiation incident upon them. The greater error is likely to be caused by the thermopile. When total radiation is being measured the incident beam falls normally upon the instrument. In the measurement of reflected radiation, however, the radiation is first diffused by reflection at the specimen, and falls at all angles upon the thermopile receiver. The blackness of the thermopile is likely to be less in this case owing to the increasing reflectivity of the black as grazing incidence is approached. From experiments with a rotatable thermopile it has been calculated that the difference between the reflecting powers of the blackened surface for normally incident radiation and for radiation such as that reflected from a perfect diffuser is about 2 per cent. This is the maximum error to be expected, as rough surfaces which do not obey the cosine law of reflection reflect a smaller proportion of radiation at high angles of incidence.

In the absence of detailed knowledge of the reflecting characteristics of the various specimens, all values of r_s have been multiplied by the full correcting factor 1·02. The correction would be unnecessary for a specimen possessing a large component of specular reflection. In every measurement the mean of six observations has been obtained. The separate observations are read to the nearest millimetre, and zeros are taken as the mean of successive readings to eliminate drift errors. The observational accuracy depends on the size of the deflection measured, and is lowest for surfaces of low reflecting power. The deflection produced when the radiation fell directly upon the thermopile was usually about 40 cm. The reflecting powers in the final section are quoted to the nearest figure in the second decimal place and are likely to be accurate to this figure.

No disturbance due to re-radiation from the specimens has been experienced in this work. An intense beam of radiation thrown for five minutes upon a dark brick specimen, reflecting about 10 per cent., produced no change in a deflection of 21 cm. due to the reflected radiation.

§6. RESULTS

The apparatus has been tested with specimens of magnesium carbonate, for which the reflecting power in the visible spectrum seems well established. The specimens were cut from the commercial block form of the material and were about 5 mm. thick. The figures obtained for surfaces prepared with fine glass-paper are shown in table 3, together with all necessary corrections. The high reflectivity found in the visible spectrum agrees closely with the figure 0·981 for white light at normal incidence obtained by McNicholas* at the U.S. Bureau of Standards (probably the most accurate determination yet made).

Table 3. The reflecting power of magnesium carbonate.

	I (1·78 μ)	II (0·84 μ)	III (0·61 μ)	IV (0·50 μ)	Gold film
Apparent reflecting power ($p r_x r_s$)	0·582	0·885	0·854	0·771	0·839
Slot constant (p)	0·970	0·969	0·972	0·965	0·970
Silver reflecting power (r_s) ...	0·977	0·940	0·915	0·851	0·919
Obliquity correction	1·02	1·02	1·02	1·02	1·02
True reflecting power (r_x) ...	0·63	0·99	0·98	0·96	0·96

Apparently the reflecting power of magnesium carbonate is not constant through the visible spectrum, but rises slightly towards the red.

The tests with blackened surfaces, mentioned previously, showed a reflecting power of 0·03 with all filters except filter I. In this region (1·78μ) the reflecting power was 0·02. The surfaces were blackened to resemble the receiver of the thermopile, tinfoil being painted with camphor black in turpentine and finally smoked over a sperm candle.

In table 4 are given the reflecting powers of a representative collection of building materials. The figures speak for themselves and need little discussion. One point worthy of note is the low reflectivity for total sunlight shown by almost all roofing materials. The good effect of whitening a roof surface where undue absorption of heat is undesirable is well seen from the results for galvanized iron. An even greater improvement is obtained with darker materials such as asphalt and slate.

§7. ACKNOWLEDGMENT

The author wishes to take this opportunity to express his thanks to Mr A. F. Dufton, at whose suggestion this investigation was undertaken and whose assistance and supervision have been most helpful.

* H. J. McNicholas, *Bur. Stand. Journ. Res.* **1**, 29 (1928).

Table 4. The reflecting powers of building materials.

No.	Specimen: Description	Reflecting power: I (1·78 μ)	II (0·84 μ)	III (0·61 μ)	IV (0·50 μ)	Gold film	Computed
	Clay tiles						
29	Dutch: light red	0·68	0·66	0·56	0·21	0·57	0·52
31	Machine-made: red	0·72	0·42	0·34	0·11	0·38	0·38
25	,, red	0·55	0·38	0·31	0·11	0·34	0·33
33	,, lighter red ...	0·52	0·40	0·32	0·13	0·34	0·33
34	,, dark purple	0·22	0·22	0·19	0·13	0·19	0·18
28	Hand-made: red	0·60	0·47	0·37	0·12	0·40	0·39
24	,, red-brown ...	0·55	0·33	0·28	0·13	0·31	0·31
	Concrete tiles						
27	Uncoloured	0·37	0·38	0·36	0·27	0·35	0·33
32	Brown	0·13	0·17	0·15	0·09	0·15	0·13
26	Brown: very rough	0·08	0·13	0·13	0·10	0·12	0·11
30	Black	0·06	0·09	0·09	0·09	0·09	0·08
	Slates						
42	Dark grey: smooth	0·09	0·11	0·11	0·11	0·11	0·10
43	,, fairly rough ...	0·10	0·11	0·10	0·09	0·10	0·10
46	,, rough	0·09	0·10	0·11	0·11	0·10	0·10
44	Greenish grey: rough ...	0·16	0·11	0·12	0·13	0·12	0·13
45	Mauve	0·14	0·16	0·13	0·10	0·14	0·13
47	Blue-grey	0·20	0·16	0·13	0·12	0·13	0·15
48	Silver-grey (Norwegian) ...	0·22	0·21	0·21	0·19	0·21	0·20
	Other roofing materials						
1	Asbestos cement: white ...	0·35	0·42	0·41	0·36	0·41	0·39
2	,, red ...	0·33	0·33	0·29	0·14	0·31	0·26
36	Enamelled steel: white ...	0·35	0·53	0·53	0·57	0·52	0·52
37	,, green ...	0·26	0·34	0·17	0·13	0·24	0·25
38	,, red	0·24	0·26	0·18	0·08	0·19	0·19
39	,, blue ...	0·23	0·27	0·17	0·18	0·20	0·23
40	Galvanized iron: new ...	0·58	0·30	0·34	0·34	0·35	0·35
41	,, very dirty ...	0·10	0·09	0·09	0·09	0·09	0·09
62	,, whitewashed	0·63	0·79	0·79	0·76	0·78	0·74
49	Special roofing sheet: brown	0·20	0·15	0·12	0·07	0·13	0·13
50	,, green	0·13	0·20	0·12	0·12	0·14	0·15
8	Bituminous felt	0·10	0·12	0·11	0·11	0·12	0·11
60	Aluminized felt	0·67	0·60	0·61	0·57	0·62	0·60
59	Weathered asphalt	0·12	0·12	0·11	0·09	0·11	0·11
61	Roofing lead: old	0·46	0·20	0·19	0·15	0·21	0·23
	Bricks						
19	Gault: cream	0·74	0·69	0·64	0·43	0·64	0·61
16	Stock: light fawn	0·56	0·47	0·38	0·19	0·44	0·39
10	Fletton: light portion ...	0·67	0·61	0·57	0·35	0·58	0·52
9	,, dark portion ...	0·54	0·46	0·37	0·15	0·41	0·37
15	Wire cut: red	0·56	0·48	0·41	0·15	0·44	0·39
17	Sand-lime: red	0·41	0·37	0·30	0·11	0·32	0·30
18	Mottled purple	0·33	0·26	0·22	0·15	0·23	0·23
14	Stafford: blue	0·21	0·12	0·11	0·08	0·11	0·12
20	Lime-clay (French)	0·57	0·63	0·52	0·29	0·54	0·49

DISCUSSION

Dr Ezer Griffiths. I wish to congratulate the author on the manner in which he has persevered to overcome the numerous difficulties in the technique of this class of work. There is one question I would like to ask him: Would it be possible to modify the apparatus so as to obtain average values for areas of the order of 1 ft.^{2} or so? In studying commercial materials such as building-bricks and roofing-tiles it is desirable to obtain average values for a number of samples, owing to the heterogeneous nature of the materials. To obtain the necessary data from test surfaces of very small area would involve much labour.

Dr J. H. Coste said that it would be a pity if such a good piece of work were to be spoilt by lack of sufficiently varied data. It would be useful if a great many specimens of each material could be examined: an idea of the variability of the material could be obtained in this way.

Mr J. Guild. The author is to be congratulated on the care and ingenuity with which he has eliminated the various systematic errors to which the measurements are liable on account of incomplete reflecting power of the silver mirror, incomplete absorption by the thermopile, etc. In the latter connexion, has he had an opportunity of trying bismuth black, recently recommended by Pfund as having a considerably higher absorption-coefficient than the blacks usually employed on thermopiles? With regard to the check measurements on magnesium carbonate, I do not think it is quite correct at the present time to regard this as the material of which the constants are best known. A recent paper by Preston in the *Transactions of the Optical Society* gives values for magnesium oxide which are probably more reliable than any that exist for magnesium carbonate. If the oxide is smoked on to a silver surface no great thickness of deposit is necessary to obtain constant properties.

The author could, apparently, make use of greater sensitivity than he obtains at present, were it not for the disturbing effects of draughts and adiabatic pressure-waves affecting his thermopile. I suggest that these effects could be greatly reduced in one or both of two ways. If the mounting which carries the thermopile and specimens were constructed to close completely the hemispherical mirror, and a window of thin glass or mica were placed over the slot in this mirror, the pile would be protected from the air currents which inevitably circulate in the larger box. The second improvement is on the lines which the author has already utilized in principle, namely the connexion in opposition to the main thermopile of another similar pile not exposed to the radiation. In order to obtain much advantage from this mode of compensation it is, however, necessary for the two piles to be in close juxtaposition, so that any local eddies may affect them to similar extents.

It would not be possible, without destroying the symmetry which is essential in the author's system, to mount one pile beside the other, as is done in an apparatus described by me in the January number of the *Journal of Scientific Instruments*, but a very satisfactory alternative would be to make the compensating pile in two

portions, one at each end of the main pile, on the same mount. If this were done, in addition to restricting the space by closing up the spherical mirror, it is probable that the air-disturbance effects would be negligible at the highest sensitivity of which the galvanometer is capable. The difficulty of hermetically sealing the space within the spherical mirror without interfering with the lateral adjustment of the thermopile and specimens could readily be overcome by allowing the back plate to slide over a flange on the mirror mount and sealing the joint with vaseline or a light tap grease.

A secondary advantage which would accrue from enclosing the spherical mirror would be a further reduction in its rate of tarnishing.

Dr J. S. G. THOMAS enquired whether it was correct to refer to the *reflecting* power of the specimens as what was measured. Should it not rather be the *scattering* power of the specimen? He also enquired whether the data contained in table 4 could not be set out in such manner as to be of more immediate use to architects and builders, more especially in regard to the specification of the materials and the arrangement of the specimens in the different classes in either ascending or descending order of scattering power.

AUTHOR'S reply. I agree with Dr Griffiths that the use of larger specimen surfaces is desirable where average values are required. With Coblentz's method the use of a small specimen is unavoidable. Had any other method capable of dealing with larger surfaces appeared practicable it would certainly have been adopted. With every material as representative a sample as possible has been selected. Where great variability occurs it may be necessary to repeat measurements with a number of samples.

In reply to Mr Guild: My apparatus was working satisfactorily when Pfund's paper relating to bismuth black appeared, hence I have not tried a thermopile blackened in this way. I considered magnesium oxide for the purpose of check measurements but rejected it in favour of the carbonate. Preliminary tests showed that the oxide had to be used in a considerable thickness in order to yield repeatable results, and that, moreover, its reflecting power was subject to a slight change due to carbonation. The thermopile could doubtless have been steadied still further by boxing-in of the hemispherical mirror. As, however, the use of the compensating thermopile had reduced the disturbances sufficiently for the tests in hand, such a change in design, which could not have failed to have made the apparatus more difficult to adjust and handle, was not embarked upon.

The term "scattering power" suggested in preference to "reflecting power" by Dr Thomas, is not, I think, applicable to the quantity which I have measured. For the ratio of the total reflected radiation to the total incident radiation "reflecting power" is surely correct, even where the reflection is not specular.

The information given in table 4 is of minor importance in a scientific paper dealing essentially with the method of measurement. The results will be published by the Building Research Station in a form more suitable for the attention of architects and builders.

THE VELOCITY OF SOUND-WAVES IN A TUBE

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ABSTRACT. The apparent velocity of sound in a tube of diameter 2 cm. has been measured at temperatures up to 400° C. and with frequencies of from 3000 to 14,000 ~. The reduction in velocity below the free-air value is discussed, and the suggestion is put forward that this reduction, for a single tube and gas, depends on the wave-length rather than on the frequency. The theoretical expression found by Helmholtz and Kirchhoff for the reduction in velocity does not appear to be valid, at any rate for the conditions obtaining in these experiments. The method used by Dixon and by Partington and Shilling for correcting for the influence of the tube receives support from the present results.

The experiments reveal the complication which ensues when the wave-length falls below a certain multiple of the tube diameter.

§ 1. INTRODUCTION

Rather more than sixty years ago, Helmholtz and Kirchhoff investigated theoretically the reduction in velocity suffered by plane waves of sound advancing along a narrow tube. The former author considered only the effect of the viscosity of the air, whilst Kirchhoff took into account also the effects of thermal conduction, but both reached a formula which may be written

$$V = V_0 (1 - A/r\sqrt{n}) \qquad \ldots\ldots(1),$$

V, V_0, r, n, A

where V is the velocity in the tube, V_0 is the velocity in free air, r the radius of the tube, n the frequency and A is a constant with different meanings in the two investigations, but in any case depending on the gas and on the material of the tube. In the intervening years the equation of Helmholtz and Kirchhoff has been studied by numerous observers, some of whom find it correct, although several

B

report that the observations agree better with a formula $V = V_0 (1 - B/rn^{\frac{3}{2}})$.

The purpose of the present paper is to investigate the velocity-reduction afresh. The whole of the observations were carried out with one tube and with pure dry air as the gas, so that no complications arose owing to variations in the roughness, etc. of the tube walls. The velocity was obtained by direct measurement of the internodal distance when standing waves were set up in the tube. The frequency was varied over a large range of values, and measurements were carried out at four temperatures, 14° C., 163° C., 320° C. and 424° C.

The observations brought out incidentally the fact that no simple formula can deal with waves of all frequencies in a given tube. This seems to be connected

with a phenomenon pointed out by Rayleigh,* who showed that waves set up in a cylindrical tube will ultimately become plane, provided that the frequency is less than that of the natural transverse vibration of the tube, i.e. if the wave-length exceeds 3·4 times the radius. Otherwise, the sound pattern is immensely complicated by the transverse waves, and the apparent internodal distance is liable to error.

§ 2. DESCRIPTION OF APPARATUS

The apparatus is shown in figure 1. It consists of a furnace-tube of pyrex glass of internal diameter 2·0 cm. and wall-thickness 0·20 cm., a listening-tube being attached near one of its ends and a movable reflector supported inside it. The source of sound is situated beyond the end of the furnace-tube remote from the reflector,

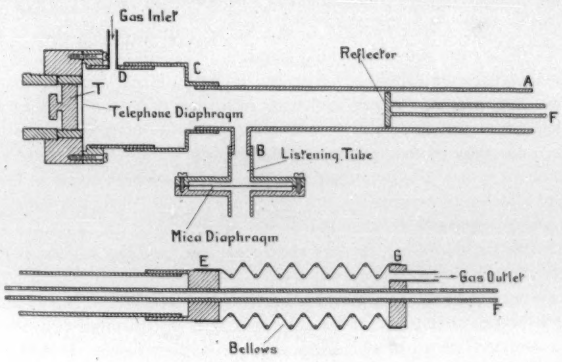


Fig. 1. Sectional view of apparatus.

and before observations are made, the distance between the telephone and the side tube is adjusted until maximum sharpness of the nodes is reached. Successive nodes can then be brought under the listening-tube by movement of the reflector; the internodal distance is directly measured from this movement.

On reference again to figure 1, it will be seen that on the side tube is fixed a brass mounting carrying a 5·0-cm. mica diaphragm of thickness 0·003 cm. This is to prevent the escape of gas and still allow the passage of sound. On the same end of the furnace-tube is fastened a brass collar and flange carrying a brass tube *C* of internal diameter 5·3 cm. and wall-thickness 0·6 mm.

The telephone *T* is mounted in a brass case carried by a brass tube *D* which makes a sliding fit inside tube *C*. Tube *D*, of the same material as *C*, carries the inlet gas-tube, which is branched to communicate with the outside air. To render the tubes gas-tight, a rubber tube of slightly smaller diameter than tubes *C* and *D* is stretched over them. One end of the rubber tube is cemented on to tube *C*

* *Theory of Sound*, 2, § 301, p. 161 (1894).

and the other on to tube *D*. This allows free movement of one tube within the other, while preventing the entrance or escape of gas. The telephone used is a Brown A earpiece with ebonite cap and cone removed. The construction of the telephone-mounting and the method of stretching the steel diaphragm can be seen from the figure. On the other end of the furnace-tube is a brass collar *E* drilled with a central hole of diameter 9 mm. This supports a pyrex tube *F* carrying the *Pt-Pt-Rh* thermocouple with which the temperature is read, and has on its end a solid pyrex disc, making a loose fit in the tube. It is about 3 mm. thick except in the centre, where it is thinner to allow of the close approach of the thermo-junction to the face of the disc.

The other end of the movable tube is supported in a wooden block fixed to a cursor engraved with a fine line, and sliding along a steel metre rule. Close to its end support, tube *F* carries a brass disc *G* which is drilled to take the exit gas-tube. Between the collar *E* and the disc *G* and fitting tightly on to both is a rubber bellows to obviate the use of a packing gland.

A steel tube of length about 2 ft. 6 in. and of slightly larger diameter than tube *A* constitutes the furnace. It is wound with nichrome wire, the turns being more widely spaced in the centre, to give a more uniform temperature-distribution. There is uniformity of temperature to within $\pm$ 1° C. at 500° C. over a central length of about 30 cm. The temperature was measured at three points in this region before and after every experiment.

The whole apparatus is mounted on an optical V-bench, separate supports being used for the telephone, furnace and metre rule, and the various parts being accurately aligned.

The gas used in these experiments is dry air, free from CO_2. It is passed through three wash-bottles containing strong sulphuric acid and through two others containing a saturated solution of caustic potash. It then passes through tubes of calcium chloride and phosphorus pentoxide into the apparatus via tube *D*. A mercury manometer is included in the circuit, one arm being open to the atmosphere. A pressure of about 6 cm. of mercury is required to send a slow stream of air through the apparatus, on account of the air having to pass between the collar *E* and the pyrex tube *F*. This collar is made to have a close fit, to prevent the motion of cold air into the hot region when the bellows are compressed. At the exit end the gas is led from tube *G* into a spherical glass vessel 30 cm. in diameter, containing calcium chloride, and from this it passes to a wash bottle containing sulphuric acid. The effect of this sphere (which is not shown in the figure) is to minimize the changes in pressure caused by movement of the bellows.

To dry the furnace tube, the temperature was raised to about 600° C. and air was passed through it. The bellows were heated from the outside by means of a stream of hot air to prevent the deposition of moisture. The moisture was thus driven out into the large glass sphere where it was absorbed by the calcium chloride. This process was continued till no more moisture was deposited in the sphere. The internal pressure can at any time be reduced to atmospheric by momentarily opening the tap on tube *D* and allowing excess gas to escape.

§ 3. OSCILLATOR

The chief features of the oscillator are the number of different frequencies obtainable and their extreme constancy. It is in two stages:

(*a*) *A crystal-controlled multivibrator*. This consists of an oscillating circuit with a quartz crystal between grid and filament, coupled to an Abraham-Bloch multivibrator. The latter circuit is very unstable and very rich in harmonics and the crystal exercises its control through one of these harmonics. The arrangement is shown in figure 2. It was found that, by a suitable choice of the inductance L_1, the capacity C_1 could be dispensed with and the circuit would still oscillate quite readily. Also the condenser C_2 was replaced by two fixed 0·001-μF condensers which could be coupled in series or in parallel. The different sub-harmonics of the crystal frequency were then obtained by variation of C_3 alone. They are far enough apart to be easily identifiable. The crystal used had a natural frequency

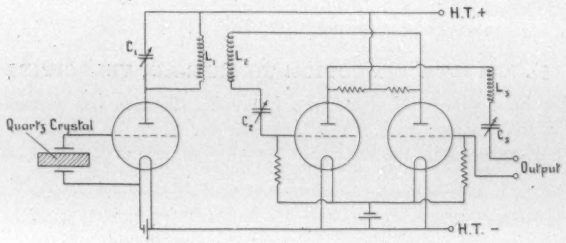


Fig. 2. Crystal-controlled multivibrator.

of 27,422 ~ and dimensions 10 × 1 × 1 cm. The frequency was measured with the crystal mounted exactly as in the following experiments, and it was verified that the switching on or off of the multivibrator or the variation of condenser C_3 made absolutely no difference to its frequency. A previous measurement, some weeks before, when the crystal was in a different mounting, had given the value 27,423 ~ and indicated a negative temperature-coefficient of frequency of about 1 part in 10^6 per ° C.

(*b*) *A two-stage amplifier*. The amplifier is of conventional design, two AF5 Ferranti transformers being used. This was quite satisfactory when the higher notes were being used, but for the lower ones a difficulty was encountered, due to the presence of the overtones of the fundamental. The procedure eventually adopted when one of the lower notes was required was to replace the two Ferranti transformers by two comparatively inefficient ones. Also the telephone diaphragm was relieved of all tension and its response to high notes was thereby weakened. By these means a note free from overtones could be obtained. On the other hand, if such a frequency as 4/13 of the crystal frequency were required, the two Ferranti transformers were retained and the multivibrator was tuned to 1/13 of the crystal

N frequency. If the crystal frequency be denoted by N, the telephone would then be emitting a fundamental note of frequency $N/13$ together with some of the overtones, i.e. notes of frequency $2N/13$, $3N/13$, etc. It was sometimes possible so to magnify one of these overtones, by tuning the telephone diaphragm to the correct frequency, that the other notes were hardly audible by comparison. In this way frequencies such as 2/5, 2/7, 3/7 and 3/10 of the crystal frequency were obtained and used in experiments. A rheostat is included in the filament circuit of the amplifier, so that when the maxima have been obtained sharply and distinctly, the volume of the sound can be diminished, until nothing at all is audible in the side-tube except over a very small range on either side of a maximum. In the case of the high notes, this range could be reduced with care to about 0·5 mm. In the taking of measurements, each maximum was approached from both sides, and the mean position was taken. Several pairs of readings were taken for each maximum. Attention was paid principally to the maxima at each end of the uniform temperature region, the intermediate maxima being measured only once, chiefly as a check on their equidistance.

§ 4. RESULTS. REDUCTION TO FREE-AIR VELOCITIES

As has been previously stated, the internodal distance was measured at a number of frequencies at each of four temperatures. From the internodal distance (i.e. the half-wave-length) the velocity of the sound under the particular conditions

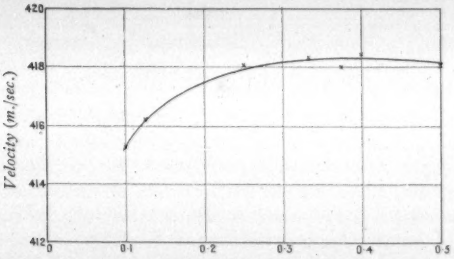


Fig. 3. Sound-velocity as a function of frequency at 163° C.

λ concerned is immediately obtained as the product of wave-length, λ, and frequency n n. Even at one temperature and in one tube these velocities are not absolutely constant, because each is reduced below the free-air value by an amount varying with the frequency. The results at 163° C. are shown in figure 3 as an example. Regarded as a function of frequency the velocity appears to pass through a maximum, although the fall is hardly sufficient to establish the fact with certainty. It is, however, supported by a similar variation in the graphs for each of the other three temperatures.

The object is to find from these observations the true free-air velocity at each of the four temperatures and, if possible, to establish the correct form of the law governing the reduction in velocity.

Method (1). A natural method for calculating V_0, and one which has been used by several of the early observers, is to use the observations to deduce the constant A in the Helmholtz-Kirchhoff formula quoted above. We have adopted this method, using, however, a generalized formula $V = V_0(1 - B/n^x)$, where B will in general depend on r, the radius of the tube. x r

The constants V_0, B and x were obtained from three values selected from below the maximum on a smoothed curve such as that shown in figure 3.

The results found are set out in table 1.

Table 1.

Temperature	x	B	V_0 (m./sec.)
14° C.	1·92	0·00008	339·7
163	1·41	0·00038	419·2
320	1·26	0·00052	487·2
424	0·49	0·0063	532·7

There is no evidence that the power x is in general either 0·5, as in the theory, or 1·5 as certain observers have reported, although it may well occur that one or other of the values would be found under special conditions. In so far as they provide a basis for a general method of correction, the observations indicate that x decreases rather rapidly with increasing temperature, and that B increases even faster. Thus the Helmholtz-Kirchhoff law would appear to have no general validity, even with slight modifications of form.

Method (2). A somewhat different method of correcting the observations to obtain free-air values has been used by Dixon* and by Partington and Shilling† in their extensive work on the specific heat of gases. In this method the Helmholtz-Kirchhoff equation is again taken as a basis, but modified in a different manner from that just suggested in method (1).

Kirchhoff's theoretical value being adopted for the constant A, equation (1) becomes

$$V = V_0\{1 - [\mu^{\frac{1}{2}} + \nu^{\frac{1}{2}}\gamma^{-\frac{1}{2}}(\gamma - 1)]/2r\sqrt{(\pi n)}\} \qquad \text{.....(2),}$$

where μ is the kinematic viscosity, μ

γ is the ratio of the specific heats, γ

and ν is the thermal diffusivity of the gas. ν

Dixon writes this in the form $V = V_0(1 - kc)$, where c is the numerator in the above fraction, and k^{-1} is theoretically $2r\sqrt{(\pi n)}$. Accepting the theoretical value for c, c he obtains an empirical value for k from an accurate experiment with a gas for k which V_0 is accurately known at the temperature of the experiment. This k is then

* *Proc. R. S.* A, **100**, 1 (1921).

† *The Specific Heats of Gases*, p. 53 (London, E. Benn, Ltd., 1924).

regarded as a constant associated always with the particular tube and frequency. The value of c is taken from the formula, and regarded as a function of temperature for the particular gas. For experiments at other temperatures, or with other gases, the observed velocity is corrected by division by the factor $(1 - kc)$, with the above value of k and with the appropriate value of c. No particular justification is given for the assumption that k does not vary with temperature nor with the gas used.

The data obtained in the present work were corrected by this method. From the smoothed curves of velocity against frequency, values were read off at frequencies 13711, 6856 and 4570 ~. The value at 14° C. was taken (from the work of other observers) to be 339·8 m./sec. Thus at any one of the frequencies mentioned, V as observed $= 339{\cdot}8\,(1 - kc)$, where c is known, so that k can be evaluated. The values found for k were 0·00328, 0·00246 and 0·00492 for the three frequencies mentioned. It is to be remarked that at all temperatures the last two frequencies fall to the left of the maximum on the velocity/frequency curve, and the first one falls to the right.

With these values of k, and the appropriate c for each temperature, the corrected velocities found were as shown in table 2.

Table 2.

Temperature (C.)	14°	163°	320°	424°
Frequency 13711 ~	339·8*	419·2	487·6	529·0
6856 ~	339·8*	418·85	487·1	527·3
4570 ~	339·8*	418·7	487·4	527·4

* Assumed value.

It appears that by this method of correction the results taken at the highest frequency (i.e. beyond the maximum of the frequency/velocity curve) are definitely high, but for the other two frequencies the results show no significant variation with frequency.

§ 5. GENERAL DISCUSSION OF THE CORRECTION

The only assistance which theory gives in evaluating the tube correction is that afforded by the Helmholtz-Kirchhoff formula. Both methods of correction which have been described in the paper are based on modifications of that formula. If the correct free-air velocities were known it would be possible to evaluate the actual corrections and to discuss the laws which they follow, but it is obviously impossible to conduct experiments in free air at these temperatures, so that the true values have to be obtained from the observations in a tube. Even if the methods used are regarded as empirical, it is likely that where they agree in giving the same estimate of the free-air velocity this must be substantially accurate. Table 3 shows the agreement between the two methods, the values obtained at the highest frequency, where the waves are probably not plane, being rejected.

True open-air values (independently of any correction) have been determined for sound at air temperature by Hebb and others, and we may take the correct value at 14° C. to be 339·8 m./sec. At temperatures of 163° and 320° C. our corrected values by the two methods are in substantial agreement, with mean values of 418·9 and 487·2 m./sec. respectively.

Table 3. Corrected values of velocity (m./sec.).

Temperature °C.	Method (1)	Method (2)	
		Frequency 6856 ~	Frequency 4570 ~
14	339·7	—	—
163	419·2	418·85	418·7
320	487·2	487·1	487·4
424	532·7	527·3	527·4

Adopting these values, but setting aside for the present the values at 424° C. on the ground that the true value is not defined with sufficient accuracy, we have calculated the difference between each observation and the free-air value at the same temperature. On examination of the law followed by these corrections, it appeared probable that the percentage reduction in velocity was more likely to be related to the wave-length than to the frequency, since then the wave pattern is likely to be unaltered. It is true that at a fixed temperature the substitution of one for the other is immaterial, but this is not so when the temperature is allowed to vary. Moreover, the evidence from the calculations of method (1) shows that there is no simple law, valid for all temperatures, connecting velocity-reduction with frequency.

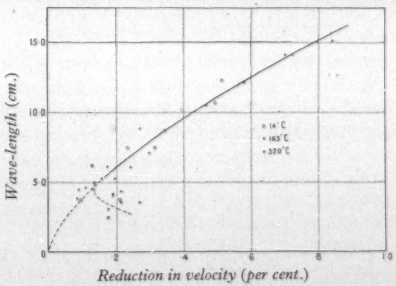


Fig. 4. Percentage reduction in velocity as function of wave-length.

Accordingly, the corrections expressed as percentages of the corrected velocity have been plotted against the wave-length in figure 4. The points for each temperature are shown separately, and it will be observed that the points for all temperatures lie on one curve. This curve shows in the lower left-hand corner a large irregular group of points which correspond to the points at and beyond the maximum of

the velocity/frequency curve, where the waves are probably not plane. The general course of the curve is doubtless somewhat as shown in the right-hand dotted line, figure 4, and must be due to a failure of the ordinary theory of sound-propagation when the wave-length is too small. Rayleigh has in fact pointed out that the wave will ultimately become plane if the wave-length exceeds 3·4 times the radius, and although he did not investigate the motion for wave-lengths shorter than this, we assume, as was remarked in the introduction, that transverse waves would exist and introduce subsidiary nodes. The numerical agreement with Rayleigh's result is not exact, for the minimum reduction of velocity in figure 4 occurs when the ratio of wave-length to radius is about 4·5, i.e. somewhat greater than the theoretical figure. It would hardly be expected that the transition to plane waves would be completed in a negligible distance, until the ratio exceeds 3·4 by some substantial amount, a fact which is implied in Rayleigh's investigation*.

Returning to consideration of the remainder of the curve, which, it will be seen, is well defined, we note that there appears to be no effect of temperature greater than the experimental error. The corrections follow approximately a 1·5-power law of the wave-length, and the curve drawn represents in fact the law

$$\text{percentage error} = 0{\cdot}0136\ \lambda^{1{\cdot}5}.$$

As far as we are aware, this suggestion that wave-length is the true criterion has not previously been put forward. It will of course require further experimental work, both over more extended ranges of temperature and with tubes of other materials and of different diameters. Should it be substantiated, however, it opens up a new method of applying the correction for the reduction of the velocity of sound in a tube. The apparent velocity having been determined at a temperature where the true velocity is known, the constant in the law

$$\text{percentage error} = \text{constant} \times \lambda^{1{\cdot}5}$$

is known, and all observations with this tube can be corrected by this law, whatever the temperature. It may be, however, that the particular form of the law found by us is incidental to the experimental arrangements adopted. In this event it would be necessary to determine the whole of the λ/error curve by experiments at different frequencies, but the method would still have the advantage of allowing these experiments to be carried out at some convenient temperature. It would be necessary, of course, to know the true value at that temperature, however it may be derived.

An interesting test of the suggested law is to apply it to our observations at 424° C. which were not used in computing the λ/error curve. The results found are shown in table 4.

The corrected values show no systematic variation with wave-length, and the mean value is 527·6 m./sec. This agrees well with the value 527·4 obtained from the same data by method (2), and suggests that the use of that method is not liable to cause serious error.

* *Loc. cit.* p. 161: equation (12).

Table 4.

Wave-length (cm.)	Uncorrected velocity (m./sec.)	Corrected velocity (m./sec.)
15·27	523·4	527·7
11·47	524·3	527·1
9·59	525·7	527·9
7·67	525·8	527·3
5·76	526·9	527·9
4·80	526·9	527·6
3·84	527·1	527·6

§ 6. ACKNOWLEDGMENTS

We have to thank Dr G. W. C. Kaye, Superintendent of the Physics Department, for his interest and the facilities for carrying out this work. For the crystal used we have to thank Dr Dye, whose continued interest has been most helpful on many occasions. Mr A. R. Challoner, Observer, constructed practically the whole apparatus and contributed materially to the design, particularly as regards the mounting of the telephone diaphragm.

DISCUSSION

Dr A. B. Wood referred to the high degree of precision attained in the measurement of the velocity of sound to 1 part in 4000. He pointed out, however, that the amplitude of movement of the particles might amount to 1 or 2 mm. and their velocity to 1 or 2 m./sec. It might be well, therefore, if the authors would state the exact figures for these quantities and indicate any allowance made for them. Peirce, in the course of an investigation carried out by means of the quartz oscillator, ascribed to changes of frequency the variations which he observed in the apparent velocity of sound. These variations might be more easily explicable, however, as due to the effects of particle-velocity.

Mr N. Fleming. The authors state in their introduction that "the velocity was obtained by direct measurement of the internodal distance when standing waves were set up in the tube." If I understand their procedure correctly, this statement is not strictly accurate. The internodal distance was inferred from measurements of the displacements of the piston which were necessary to give a succession of nodes at the listening-tube. The point is of some importance, for if they had measured directly the distance between nodes they would have avoided one of the possible sources of error in their method; this is the error which may arise through imperfect reflection of sound at the piston, due both to leakage of sound round the piston (which makes only a loose fit in the tube) and to longitudinal vibration of the free portion of the piston-rod. I see that Mr Henry has treated both these points

in his paper, but it may be of interest to consider qualitatively the variation with frequency of the error due to the latter cause. If reflection from the piston is not perfect, i.e. if the terminating impedance of the tube is not infinite, there may be a shift in the position of the whole nodal system, though the distance between nodes remains unaltered. The magnitude and direction of this shift vary with the terminal impedance and so with the free length of the piston-rod. Figure 1 of Mr Henry's paper illustrates the variation of the shift with l/λ', the ratio of the free length of the piston-rod to the wave-length of longitudinal vibrations in it, negative values corresponding to a shift towards the piston. For a low frequency and a small value of l, l/λ' is small and there is a slight shift of the nodes towards the piston. As the piston is moved towards the listening-tube l/λ' increases and the shift increases. Consequently the piston must be moved through a greater distance than would be required in the absence of such a shift, and the velocity deduced is slightly too great. At a somewhat higher frequency the difference in the shifts at the two positions is greater. It is true that this error is now distributed over a greater number of internodal distances, but this number does not increase with frequency as rapidly as does the error. The velocities inferred therefore increase at first too rapidly with frequency. At some still higher frequency the free length of the piston-rod may pass through the value $\lambda'/4$ between the two positions at which measurements are taken. The shift in the first position is then towards and in the second position away from the piston. The error is reversed in sign and the velocity deduced is too low. Do the authors consider that such an effect could account for the maximum which they obtain in the variation of velocity with frequency? Can they give any information as to the natural frequencies of the free lengths of the piston-rod used by them?

Mr D. A. Oliver. I should like to refer to some of the practical points in the authors' apparatus. Is the diameter of the side-tube B in figure 1 to scale? If so, it would appear that the acoustical impedance of the main tube at this place would be profoundly modified at some frequencies by the presence of this coupled subsidiary acoustical system, which conceivably might affect, by a measurable amount, the standing-wave system in the main tube. I would suggest that the diameter of this tube should not exceed 1 mm., but any effect due to this cause can easily be checked experimentally.

In a precise discussion of the errors to which tube measurements are subject, would the authors agree that the source and air column could, with advantage, be treated as a coupled mechanical-acoustical system, and the conditions for resonance, involving the mechanical impedance of the source in the correction terms, deduced? The type of source used by the authors is likely to have a mechanical impedance very variable with frequency. An air-damped electrostatic source would be much more uniform with respect to variation of impedance with frequency. As has already been pointed out by Mr N. Fleming, the movable piston also must be taken into account, and I would add, from my own experience, that little or nothing can be assumed to be rigid at frequencies ranging between 3000 and 30,000 ~.

Dr D. Owen. It is interesting to note the high accuracy of setting of the piston, in view of the large opening to the listening tube. Is not the accuracy dependent on the frequency, since the intensity of the disturbance is likely to be different at the different frequencies, and the sensitivity of the ear itself varies with the frequency?

Mr P. S. H. Henry. Is there any evidence of the wave-length readings becoming definite again if the frequency is increased beyond the point at which the readings become indefinite? This would probably be the case if radial resonance were the cause of the variations; for radial resonance would be but little damped, and would only have a serious effect in the neighbourhood of certain definite frequencies corresponding to the various modes of radial vibration of the gas.

Authors' reply. In reply to Dr Wood and Dr Owen, we would point out that the intensity was definitely reduced to such a point that no sound was audible except over a small range in the immediate neighbourhood of a maximum. To reduce the range to a minimum, the telephone had to be carefully adjusted with respect to the side-tube. With alternate reduction of the intensity and adjustment of the telephone, the range of audibility could be greatly reduced. It was, of course, smaller with the higher frequencies. Remembering that Rayleigh has found that amplitudes as small as 10^{-8} cm. give audible notes, we do not think that there can be any difficulty in this direction.

The point referred to by Mr Fleming is of great interest. We are unable to give the frequency of the longitudinal vibrations in the reflector, if only for the reason that the latter could not be regarded as rigidly clamped by the collar *E*. In any case it is clear that vibrations in the reflector cannot appreciably affect the average result determined with a number of different frequencies. It is, however, just possible that they are responsible for the maximum in one of the velocity/frequency curves, in which case the maxima in the remaining curves would be unlikely to be due to the same cause, occurring as they do at about the same wave-length at all temperatures. We agree with him and with Mr Oliver that the system can be regarded as a coupled mechanical-acoustical system and, indeed, would obviously have to be regarded as such if the position of the nodes with respect to the telephone were of importance. It is, however, the internodal distance which is measured, and if the region over which the reflector is moved is far enough from the side-tube, then only a negligible error will be introduced by having the side-tube of large diameter. The diameter in our case was approximately 4 mm.

In reply to Mr Henry, we regret that it was impossible to trace the wave-length error curve beyond the point reached in figure 4.

A BALLISTIC RECORDER FOR SMALL ELECTRIC CURRENTS

By E. B. MOSS, B.Sc.

Received November 21, 1930, *and in amended form December* 11, 1930.
Read and discussed February 6, 1931.

ABSTRACT. The standard thread recorder is so modified that it records ballistic throws in place of the usual steady deflection. By this means the current-sensitivity may be increased at least twenty-five times.

§ 1. INTRODUCTION

Robust and comparatively simple instruments are readily available for the purpose of recording graphically currents greater than 1μA, and it is when currents of smaller magnitude are to be studied that recourse must be had to elaborate apparatus. Either a photographic method is employed in conjunction with a mirror galvanometer, or the current is amplified, for example with a thermionic valve, and then recorded on a suitable instrument. Any elaboration such as this cannot but reduce the degree of accuracy which may be obtained, and the following paragraphs describe a modification of the standard thread recorder of the Cambridge Instrument Company, whereby this instrument is made to record currents of the order of 10^{-7} amp.

§ 2. DESCRIPTION

Although the thread recorder must be a familiar instrument to many, a brief résumé of its operation may not be out of place here.

The galvanometer coil is freely suspended and the pointer moves above the clock-driven drum carrying the chart. A little above the pointer and parallel to the drum-axis is a chopper-bar, while just below the pointer, i.e. between the pointer and the drum, an inked thread is stretched under slight tension. At half- or one-minute intervals the chopper-bar is released, falling on to the pointer and pressing it down momentarily, with the inked thread, on to the chart. The dot which results on the chart is therefore a register of the pointer-deflection when the chopper-bar fell.

As the title implies, the modified recorder operates on a ballistic principle and some additions are made to the sequence of events detailed above. The circuit used in conjunction with the instrument and shown in figure 1 is not in any sense novel and is that generally employed for measuring small currents ballistically, although its particular application to photoelectric measurements has been mentioned by P. Selényi*. The circuit includes three contacts, *A*, *B* and *C*. While *B* and *C* are

* *Photoelectric Cells and their Application*, p. 41 (The Physical and Optical Societies, 1930).

closed the galvanometer coil is short-circuited and therefore damped. During this time the photoelectric current charges the condenser so that when B is moved over into contact with A and away from C a ballistic impulse is imparted to the galvanometer, and it is the resultant throw that is recorded in place of the usual steady deflection.

The standard thread recorder has the coil wound on a copper former and, with a coil of 2000 ohms' resistance, the current required to produce a full-scale deflection is about $5{\cdot}0 \times 10^{-6}$ amp. Such an instrument has a ballistic sensitivity of 24 microcoulombs for the same deflection. This charge accumulating during 60 seconds is equivalent to a steady current of $0{\cdot}4 \times 10^{-6}$ amp. Therefore, to use this instrument ballistically increases the current sensitivity 12·5 times. If the time interval between successive contacts is increased the current sensitivity will go up proportionally.

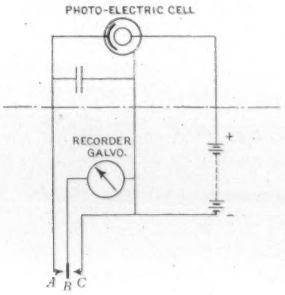


Fig. 1. The electrical circuit.

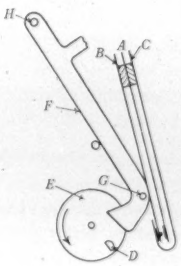


Fig. 2. Contact arrangement.

The movements of the suspended coil are damped almost critically by the copper former, so that an improvement results from its removal; in fact it is found that only 12 microcoulombs are necessary to give a full-scale deflection with a formerless coil. This coil, being undamped, does not return slowly to zero after being released by the chopper-bar, but oscillates about the zero position and would not come to rest during the succeeding minute unless some damping were provided, and it is for this purpose that contact C in figure 1 is employed.

The mechanical details are illustrated in figure 2. In accordance with standard practice the cam E carries the pin D and rotates in the direction indicated by the arrow. In the course of its rotation the pin raises the release arm F (which is pivoted at H and is subsequently permitted to drop back into its rest position) releasing the chopper-bar so that it falls on to the pointer. Having been released, the chopper-bar is raised by auxiliary clockwork during the course of one or two seconds. The additions to the recorder are the spring contacts, A, B and C which correspond to those bearing the same letters in figure 1. While the condenser is charging, which in practice is for about 56 seconds during each period of one minute, contacts B

and C are closed, short-circuiting the galvanometer and damping its movements. As F is raised by the rotation of the cam E the insulated pin G, on the former, presses B out of contact with C, leaving the galvanometer undamped, and then moves B further and into contact with A, thus connecting the galvanometer to the condenser. A ballistic throw results, but E has still to rotate a little before D releases the arm F, and during this time the galvanometer-pointer is travelling to the maximum extent of its throw. The interval is therefore adjusted so that the chopper-bar falls at the instant when the pointer has zero velocity. Immediately on the fall of the chopper-bar B and C come into contact and the galvanometer is damped during the entire period occupied by its return to zero. The operations are then repeated, the usual period being one minute for each cycle.

§ 3. THEORY

There is little need to enter into the theory of the instrument here, as the treatment follows that found in the standard text-books on ballistic galvanometers. One point, however, is of interest, and that concerns the error in the record which is produced by an incorrect setting of the interval between the initiation of the ballistic throw and the fall of the chopper-bar. Once the throw has been started the displacement x of the pointer at any instant t is given by the expression

$$x = b \sin nt,$$

in which b is the maximum throw of the pointer, and $n = 2\pi/T$ where T is the period of one oscillation of the galvanometer.

Substitution in this expression of values of $\sin nt$ corresponding to errors of ± 5 per cent. in the timing, that is when $nt = 90° \pm 4{\cdot}5°$, shows that the resultant error in the reading would be $\pm 0{\cdot}31$ per cent.

§ 4. APPLICATIONS

The principal application of the instrument at present seems to be for the recording of illuminations photoelectrically, and its great advantage over previous instruments is that the gain in sensitivity enables a vacuum cell to replace one with gas filling without having to be of large size. Since comparatively small illuminations are necessary, troublesome heating of the cathode is avoided. The vacuum cell has many well-known advantages over the gas-filled type; perhaps the most important is its constancy of sensitivity. The second advantage is that a steady accelerating-potential is unnecessary, since the cell is worked at saturation, so that mains supply may be used. The third advantage concerns particularly the method of recording which has been described. If a sufficiently high potential is applied to the cell, the current through it will be unaffected by the potential building up in the condenser, a convenient capacity for which is 2μF. Each dot on the recorder, therefore, represents a true integration of the light which has fallen on the cell during the preceding minute.

In practice it is found that the arrangement shown in figure 1 is much less liable to be affected by leakage troubles than a straightforward circuit, provided that the condenser is placed near the cell. In other words, that part of the arrangement shown above the dotted line must be regarded as the light-sensitive unit. Three leads connect this unit to the recorder and battery, both of which may be removed to any distant position convenient for their housing.

The cells which have been used for this work are Osram type KV6, and no troubles have been experienced when the cell was kept reasonably dry. The condenser must of course be of good quality, although mica insulation is not necessary, and for this work a Ferranti type C2 has been used successfully. For some time a cell has been on a roof at Cambridge mounted in a water-tight metal box and exposed to daylight through a glass window incorporating a deep red screen. A typical

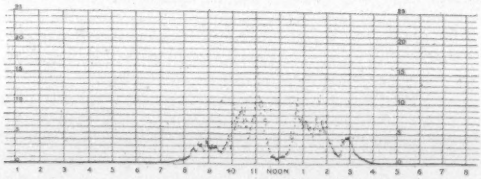


Fig. 3. Record of daylight through a red filter (Wratten No. 26).

record is reproduced in figure 3 and shows two points which are noteworthy: firstly the complete absence of any deflection during the period of darkness, and secondly the smoothness of the record resulting from the fact that the record is a succession of integrations over periods of one minute. There can be little doubt that the combination of a vacuum cell with the recorder described above gives records which are far more accurate than any that can be obtained with the aid of valve-amplifiers and other recording devices. Furthermore, the apparatus is so extremely simple.

Although the application of the recorder to the measurement of photoelectric currents has been described in detail it may be applied also to the recording of any d.c. leakage current where the voltage applied to the circuit is large compared to that reached by the condenser.

§ 5. ACKNOWLEDGMENT

Finally, my thanks are due to the Directors of the Cambridge Instrument Co., Ltd., for permission to publish this paper.

DISCUSSION

Mr R. W. WHIPPLE congratulated the author on the development of a piece of apparatus that might prove useful for the measurement of extremely small electric currents. It might eventually prove of service as a means of recording solar radiation by means of records from two or three photo-electric cells screened with glasses of varying spectral densities. Three or four different records obtained in this manner could be recorded on the same instrument.

Prof. A. O. RANKINE enquired what damping occurred during the throw of the galvanometer.

Mr A. F. DUFTON said that his experience suggested that the pointer would stick to the chopper bar unless these parts were cleaned.

The AUTHOR replied that the damping during the throw was negligible, and that if the bar were thoroughly cleaned a thread-recorder would work for 3 or 4 months without trouble due to sticking.

THE INSTRUMENTAL PHASE-DIFFERENCE OF SEISMOGRAPH RECORDS; AN ILLUSTRATION OF THE PROPERTIES OF DAMPED OSCILLATORY SYSTEMS

By F. J. SCRASE, M.A., B.Sc., Kew Observatory

Received January 14, 1931. *Read and discussed February* 6, 1931.

ABSTRACT. A discussion is given of the method of interpretation of the maxima shown on the records of earthquakes during the surface-wave phase. The usual procedure is to treat the waves (which actually appear as beats) as being truly simple-harmonic and to apply the formulae which are derived on this assumption. It is shown that, in general, this procedure does not necessarily lead to the correct interpretation. In the case of direct registration the true earth-maximum may have occurred one half-period later than the time obtained by the usual correction. With galvanometric registration the maximum may have occurred either one, two, or three half-periods earlier than the time indicated by the usual formula due to Galitzin. Some curves are included to illustrate these points, and an attempt is made to obtain a mathematical explanation.

It is shown that there is no easy method of eliminating an ambiguity of one half-period. For direct registration, therefore, the phase-correction at present in use appears to be as good as the one alternative. In the case of galvanometric registration, although there are altogether four forms of phase-correction, the number of alternatives for any particular period cannot exceed two. The final recommendation in this case is that the correction suggested by Somville and which is one half-period less than Galitzin's, be adopted for general use.

§ 1. INTRODUCTORY

The data concerning important earthquakes published by seismological stations usually include some details of the prominent maxima which occur during the long or surface-wave phase. Most stations give times of maxima to the nearest second, and state whether the displacements are positive or negative. When mechanical or direct optical registration is employed one presumes that the times of maxima refer to actual earth-motion and that they include the usual correction for the phase-difference introduced by the pendulum: few stations explicitly state whether the corrections have been applied. With electromagnetic registration there enters a further complication, for there is great uncertainty as to what is the right phase-correction to apply. Various authors have attempted to settle the question once and for all, but differences in practice still remain; some stations use one form of correction, some use another, while at least two stations give the times of maxima as read directly from the records, and leave the reader to apply whichever correction he favours. These variations in practice lead to confusion, so this paper is written with the object of bringing the question to the light once more, and of reaching some conclusions which may lead to the adoption of a uniform procedure at all stations.

§ 2. MOTION OF PENDULUM FOR SIMPLE-HARMONIC EARTH-MOTION

The motion of the pendulum under the influence of earth-movement may be represented by the usual equation:

$$\ddot{\theta} + 2e\dot{\theta} + n^2\theta + \frac{\ddot{x}}{l} = 0 \qquad \text{.....(1),}$$

θ, x where θ is the angular displacement of the pendulum, x the displacement of the earth,
e, n, l whilst e, n, and l are characteristic constants of the instrument. Actually $n = 2\pi/T$
T where T is the free period of the pendulum. It is convenient to introduce another
μ symbol μ for which

$$\mu^2 = 1 - e^2/n^2.$$

The solution of equation (1) when the earth-motion is simple-harmonic is quite straightforward. Thus, if

x_m $$x = x_m \sin(pt + d) \qquad \text{.....(2),}$$

p, t, d then $$\theta = Q \sin\{p(t - \tau) + d\} \qquad \text{.....(3),}$$

Q where $$Q = x/l(1 + u^2).\sqrt{\{1 - \mu^2 f(u)\}} \qquad \text{.....(4),}$$

τ and $$\tau = \frac{T_p}{2\pi} \cdot \tan^{-1}\left\{\sqrt{(1 - \mu^2)} \cdot \frac{2u}{u^2 - 1}\right\} \qquad \text{.....(5).}$$

T_p, u, f Here $T_p = 2\pi/p$, $u = T_p/T$ and $f(u) = \{2u/(1 + u^2)\}^2$. If the pendulum is critically damped, $e = n$ and therefore $\mu^2 = 0$.

Equation (5) represents the phase-difference, expressed as time, between the motion of the pendulum and the motion of the earth. This time-lag τ is always positive and for critical damping it varies from zero, when u is infinitely large, to one half-period, when u is zero; when u is unity τ amounts to a quarter-period.

In view of what follows in this paper it is very important to draw attention here to the sign of θ. Suppose, for example, we are dealing with the vertical component and that, therefore, an earth-movement which is directed towards the zenith is regarded as being of positive sign. Then we must also regard the displacement θ of the pendulum as being positive if it is directed upwards. Equation (1) shows that for a sudden upward or positive impulse of the ground θ must be negative, i.e. the pendulum shows a displacement downwards. On the other hand, for sinusoidal earth-motion, if there is a positive maximum earth-displacement at a time t, then the corresponding maximum displacement of the pendulum τ seconds later than t is also positive. There is therefore a subtle distinction between the case of a sudden impulse and that of simple-harmonic motion, and it is thought that this distinction is not always realized or is sometimes neglected.

O. Somville* has drawn attention to the fact that, when the earth-motion is represented by equation (2), the differential equation (1) offers an alternative solution, which can be written as follows:

$$\theta = -Q \cos\{p(t - \tau') + d\} \qquad \text{.....(6),}$$

where $$\tau' = \frac{T_p}{2\pi} \cdot \tan^{-1}\left\{\frac{1}{\sqrt{(1 - \mu^2)}} \cdot \frac{1 - u^2}{2u}\right\} \qquad \text{.....(7).}$$

* O. Somville, *Annales de l'Observatoire Royal de Belgique*, Brussels, 1918.

Since

$$\tan^{-1}\{\sqrt{(1-\mu^2)}.2u/(u^2-1)\} = \tan^{-1}\{(1-u^2)/2u\sqrt{(1-\mu^2)}\} + \pi/2.$$

Somville's expression only amounts to an alternative way of writing the usual solution.

He points out, in fact, that both solutions can be combined in a general expression.

$$\theta = Q\sin\{p(t-\tau_m) + d + m\pi/2\} \qquad \text{.....(8)},$$

$$\tau_m = \tau + mT_p/4, \qquad \tau_m$$

m being a whole number, positive, zero, or negative. The ordinary formula corresponds to $m = 0$. When both earth-motion and recorded motion are continuous trains of simple-harmonic waves, it matters little which form of correction is applied, so long as due regard is given to signs. m

§ 3. MOTION OF GALVANOMETER COIL FOR SIMPLE-HARMONIC EARTH-MOTION

For the complete theory of electromagnetic registration we are indebted to Galitzin*. The differential equation for the motion of the galvanometer coil is

$$\ddot{\phi} + 2n_1\dot{\phi} + n_1^2\phi + k\ddot{\theta} = 0 \qquad \text{.....(9)},$$

where ϕ is the angular displacement of the coil, θ the angular displacement of the pendulum, and n_1, k are galvanometer constants. The constant n_1 is given by ϕ, θ n_1, k

$$n_1 = 2\pi/T_1,$$

where T_1 is the free period of the galvanometer, which is assumed to be critically damped. To arrive at a solution when the earth-motion is of the form T_1

$$x = x_m\sin(pt + d),$$

Galitzin first obtains the formula (3), inserts this in equation (9) and finally obtains the following expression:

$$\phi = Q_1\sin\{p(t-\tau-\tau_1) + d\} \qquad \text{.....(10)},$$

where $$Q_1 = kx_m T_p/l.2\pi.(1+u_1^2)(1+u^2)\sqrt{\{1-\mu^2 f(u)\}} \qquad \text{.....(11)},$$

and $$\tau_1 = \frac{T_p}{2\pi}.\left\{\tan^{-1}\left(\frac{2u_1}{u_1^2-1}\right) + \frac{\pi}{2}\right\} \qquad \text{.....(12)}.$$

Here $u_1 = T_p/T_1$ and τ as before represents the phase-difference, expressed as time, between the pendulum-motion and the earth-motion, while τ_1 is the phase-difference between the galvanometer and the pendulum. We shall write $(\tau + \tau_1)$ as τ_2. τ_2

In interpreting the galvanometer record, it is important not to have any misconception of the sign of the displacement. The galvanometer should be connected to the pendulum coils in such a way that the transmission coefficient k is positive. When the ground suffers a sudden impulse in a positive direction (upwards if we are considering the vertical component), the pendulum starts to move in a negative direction but the galvanometer records the impulse in the same direction as ground movement. If the earth-motion is simple-harmonic with a positive maximum at zero time, then the pendulum shows a positive maximum at a time τ, as we have already seen, and the galvanometer shows a positive maximum at a time τ_2.

* Fürst B. Galitzin, *Vorlesungen über Seismometrie*, Teubner, Leipzig, 1914.

The paper by Somville is devoted mainly to putting forward evidence and arguments in favour of the adoption of an alternative formula for general use in determining the time of the earth-maximum from the time as given by a galvanometric record. A discussion of this alternative has been given by H. P. Berlage, Junr.* The formula may be written as follows:

$$\phi = -Q_1 \sin\{p(t - \tau - \tau_1') + d\} \qquad \text{.....(13),}$$

τ_1' where $$\tau_1' = \frac{T_p}{2\pi} \cdot \tan^{-1}\{(1 - u_1^2)/2u_1\} \qquad \text{.....(14).}$$

This again is no more than an alternative way of writing the usual solution given by formulae (10), (11), and (12), for

$$\tan^{-1}\{(1 - u_1^2)/2u_1\} = \tan^{-1}\{2u_1/(u_1^2 - 1)\} - \frac{\pi}{2},$$

and the negative sign of equation (13) implies a further change of one half-period. The Galitzin correction refers to a recorded maximum which occurs with a lag given by τ_1 and is of the same sign as the earth-maximum. Somville's formula refers to a recorded maximum which occurs one half-period earlier than the Galitzin maximum and is of opposite sign to the earth-maximum. The evidence given in Somville's paper is regarded by him as being sufficiently conclusive for the complete rejection of the Galitzin formula. Unfortunately, however, he appears to ignore the reversal of sign which is implied by his formula. If the waves under consideration are truly simple-harmonic, then it is clear that both formulae are valid, provided that due regard is paid to the sign of the displacement. The two formulae, as Somville points out, can be combined into a general expression:

$$Q = -Q_1 \sin\{p(t - \tau - \tau_m') + d + m\pi/2\} \qquad \text{.....(15),}$$

τ_m' where $$\tau_m' = \frac{T_p}{2\pi}\left\{\tan^{-1}\left(\frac{1 - u_1^2}{2u_1}\right) + \frac{m\pi}{2}\right\} \qquad \text{.....(16).}$$

§ 4. GRAPHICAL SOLUTIONS FOR BEAT-WAVES

As a general rule the oscillations which occur during the surface-wave phase of an earthquake are not truly simple-harmonic. They appear to consist of a series of beat-waves which work up to a maximum amplitude soon after the commencement of the phase and then die down very gradually. The usual practice at observing-stations is to measure up the maximum amplitudes of a few of the well-marked beats. Since no convenient theoretical expression is available which will apply rigidly to the whole of the surface-wave phase and satisfactorily interpret both the beats and their waxing and waning, the usual procedure is to treat the oscillations as being simple-harmonic and to apply the theory which has been outlined in the foregoing sections for converting the recorded amplitudes and times into values which refer to the actual earth-motion. So far as the amplitudes are concerned, no appreciable errors are introduced, but it can be shown that this application of the simple theory to the case of beat-waves does not always give the correct answer for the time of the maximum amplitude in a beat.

* H. P. Berlage, Junr., *Seismische Registrierungen in De Bilt*, No. 9. (Koninklijk Nederlandsch Meteorologisch Institut, 1921.)

A slightly more rigid interpretation of the records can be obtained if the earth-motion is regarded as being produced by the interference of two trains of simple-harmonic waves of slightly different periods, thus:

$$x = a\cos(pt + d) + b\cos(p't + d'). \qquad p, d, p', d'$$

Now we can deduce the recorded motion by treating each of the component waves separately, applying the formulae for simple-harmonic waves and summing the motions so obtained. In the case of the pendulum for instance:

$$\theta = A\cos\{p(t-\tau) + d\} + B\cos\{p'(t-\tau') + d'\}, \qquad A, B$$

where A and B involve the appropriate magnification factors while τ and τ' are the corresponding instrumental phase-differences, given by equation (5). The case of the galvanometer is quite similar but the magnifications will be different and the instrumental phase-differences will be $(\tau + \tau_1)$ and $(\tau' + \tau_1')$, given by equations (5) and (12).

The effect of this treatment is most readily seen by consideration of some numerical examples. In each of the figures accompanying this paper there are two curves, one representing the earth-motion, the other showing recorded motion. The curves are obtained in the manner indicated above and are drawn to scale as accurately as possible. The instruments are assumed to be critically damped and to have free periods of 25 seconds. In general it is not difficult to pick out the maximum displacement in each group of waves; if there is any doubt the numerical values will of course settle which peak is the largest. As would be expected, the instrumental phase-differences cause some change of character in the motion; for example, if the original motion is symmetrical about the maximum of a beat, the recorded motion is not necessarily so. On account of this, it cannot strictly be said that any particular point on the recorded-motion curve corresponds with some particular point in the original motion. The best we can do is to admit a correspondence between the maximum displacements for earth-motion and recorded motion in each beat; that, at any rate, interests us from the practical point of view. What we want to discover, therefore, is how far the time-differences between the corresponding maxima fit in with the differences which are obtained by the ordinary formulae for simple-harmonic waves.

§ 5. GRAPHICAL SOLUTIONS; MOTION OF PENDULUM

In figure 1 the earth-motion is assumed to be

$$x = \cos(2\pi t/36) + \cos(2\pi t/44).$$

The greatest displacement in the beat is therefore a positive one at zero time, and the quasi-period T_p of the resultant oscillations is 40 sec. Since the free period T of the pendulum is 25 sec., u, which is T_p/T, is 1·6. The appropriate relative magnification factors and phase-differences give for the pendulum-motion:

$$\theta = 319\cos\{2\pi(t - 6{\cdot}9)/36\} + 239\cos\{2\pi(t - 7{\cdot}1)/44\}.$$

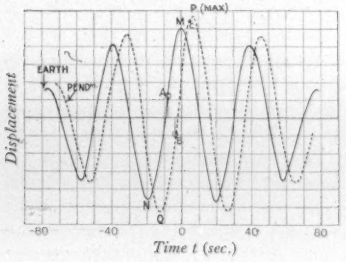
It will be seen from the curve that the greatest pendulum displacement P is of the same sign as the earth-maximum, and occurs 7 sec. later. Now this is the phase-

difference τ (equation 5) for simple-harmonic waves of 40-sec. period. The application of the usual formula therefore gives the correct answer in this particular case.

The earth-motion represented in figure 2 is the sum of the two cosine waves of periods 4·5 and 5·5 sec. respectively and, since no initial phase-difference is assumed, the greatest maximum is at zero time. The quasi-period of the resultant is 5 sec. and u is 0·2. The pendulum-motion is given by:

$$\theta = 96 \cos \{2\pi (t - 2{\cdot}0)/4{\cdot}5\} + 94 \cos \{2\pi (t - 2{\cdot}4)/5{\cdot}5\}.$$

In this case the greatest pendulum displacement Q is a negative one and is 0·3 sec. in advance of the earth-maximum. The usual formula indicates a lag of 2·2 sec. If, therefore, we were interpreting the recorded motion, we should measure the time of the maximum Q and say that the earth-maximum occurred 2·2 sec. earlier. This



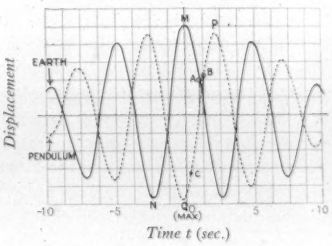


Fig. 1. Earth-motion and pendulum-motion; $T = 25$ sec., $T_p = 36$ sec., $T_{p'} = 44$ sec., $d = 0$ sec.

Fig. 2. Earth-motion and pendulum-motion; $T = 25$ sec., $T_p = 4{\cdot}5$ sec., $T_{p'} = 5{\cdot}5$ sec., $d = 0$ sec.

would indicate the peak N as being the greatest earth-maximum. Obviously this procedure is faulty. The correct procedure in this particular case is to apply a time-correction given by $(\tau \pm \frac{1}{2}T_p)$, which in this case is $-0{\cdot}3$ sec., and to reverse the sign. This corresponds to the case where $m = (-2)$ in the general formula suggested by Somville. Therefore when we turn from simple-harmonic waves to beat-waves, the difference between the alternative formula becomes significant. Some attempt will be made later to discover when it is better to use one formula and when the other.

§ 6. GRAPHICAL SOLUTIONS: MOTION OF GALVANOMETER COIL

In figure 3 the earth-motion is the same as for figure 1. The motion of the galvanometer coil is given by:

$$\phi = 174 \cos \{2\pi (t - 22{\cdot}8)/36\} + 123 \cos \{2\pi (t - 25{\cdot}1)/44\}.$$

The recorded-maximum S is of opposite sign to the earth-maximum M and has a retardation of 4 sec. The Galitzin formula for an oscillation of 40-sec. period

($u = 1{\cdot}6$) gives a retardation of 24 sec. Using this formula we should interpret S as corresponding to the earth-maximum N. This particular case fits in with the Somville formula (14) or with $m = 0$ in his general formula (15); we should therefore apply a time-correction of $(\tau_2 - \frac{1}{2}T_p)$ sec. and reverse the sign.

The earth-motion in figure 4 has a quasi-period of 5 sec. ($u = 0{\cdot}2$) and the recorded curve is given by

$$\phi = 198 \cos \{2\pi (t - 5{\cdot}1)/4{\cdot}5\} + 238 \cos \{2\pi (t - 6{\cdot}1)/5{\cdot}5\}.$$

The greatest recorded maximum T is of the same sign as the earth-maximum M and has a retardation of 0·6 sec. Using the Galitzin formula, which gives a lag of 5·6 sec., we should expect the earth-maximum to be at O, whereas using Somville's formula, which gives a retardation of 3·1 sec., we should expect the maximum to be at N. It will be seen that the appropriate correction to be made is $(\tau_2 - T_p)$ sec. and that there is no reversal of sign. This corresponds to $m = (-2)$ in the general formula (15).

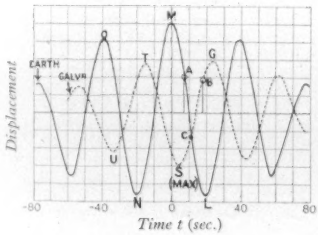


Fig. 3. Earth-motion and galvanometer-motion; $T = T_1 = 25$ sec., $T_p = 36$ sec., $T_p' = 44$ sec., $d = 0$ sec.

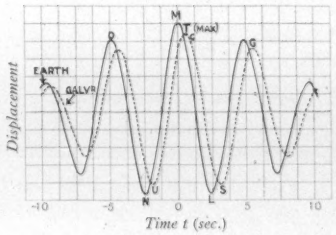


Fig. 4. Earth-motion and galvanometer-motion; $T = T_1 = 25$ sec., $T_p = 4{\cdot}5$ sec., $T_p' = 5{\cdot}5$ sec., $d = 0$ sec.

In figure 5 an initial phase-difference has been introduced into the earth-motion, which is given by:

$$x = \cos \{2\pi (t - 1)/36\} + \cos \{2\pi t/44\}.$$

The curve for the galvanometer coil is given by:

$$\phi = 174 \cos \{2\pi (t - 1 - 22{\cdot}8)/36\} + 123 \cos \{2\pi (t - 25{\cdot}1)/44\}.$$

The earth-maximum M occurs at 0·5 sec. and the recorded maximum G is 24 sec. later. This case corresponds exactly to the Galitzin formula, which gives a retardation of 24 sec. for $u = 1{\cdot}6$. Obviously the Somville formula would lead to an incorrect interpretation by indicating L as the earth-maximum.

The earth-motion in figure 6 is represented by

$$x = \cos \{2\pi (t + 0{\cdot}1)/2{\cdot}25\} + \cos \{2\pi t/2{\cdot}25\},$$

while the motion of the galvanometer coil is:

$$\phi = 216 \cos \{2\pi (t + 0{\cdot}1 - 2{\cdot}65)/2{\cdot}25\} + 257 \cos \{2\pi (t - 3{\cdot}20)/2{\cdot}75\}.$$

The earth-maximum M occurs at $-0{\cdot}05$ sec., while the galvanometer-maximum U,

which is of opposite sign, is 0·85 sec. in advance of M. In this case the Galitzin correction τ_2, which for $u = 0{\cdot}1$ is 2·9 sec., would indicate the peak P on the earth curve as being the maximum. The Somville correction, which is one half-period less than the Galitzin correction, i.e. 1·65 sec., would indicate the peak O. Both, therefore, lead to incorrect interpretations, and the suitable correction for this particular case is three half-periods less than the Galitzin correction, i.e. $(\tau_2 - 1\frac{1}{2}T_p)$. A reversal of sign must be made. The corresponding value of m in Somville's general formula is -4.

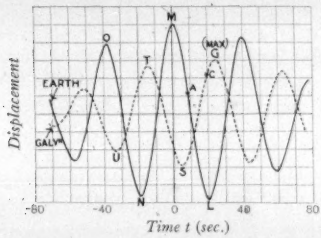


Fig. 5. Earth-motion and galvanometer-motion; $T = T_1 = 25$ sec., $T_p = 36$ sec., $T_{p'} = 44$ sec., $d = 1$ sec.

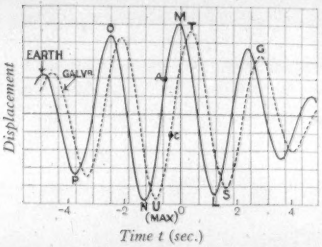


Fig. 6. Earth-motion and galvanometer-motion; $T = T_1 = 25$ sec., $T_p = 2{\cdot}25$ sec., $T_{p'} = 2{\cdot}75$ sec., $d = 0{\cdot}1$ sec.

It is seen then that there are at least four different forms of phase-correction which may apply to the case of beat-waves. Can we formulate some rules as to when each form is applicable?

§ 7. MATHEMATICAL ANALYSIS OF PENDULUM MOTION

The mathematical interpretation of the results which we have obtained graphically can be made more clear if, at first, we restrict ourselves to the simple case in which the two trains of simple-harmonic waves forming the beats have no initial phase-difference. Suppose then that the earth-motion is represented by:

δp
$$x = a \cos\{(p + \delta p)\, t\} + b \cos\{(p - \delta p)\, t\}.$$

This can also be written in the following form:

$$x = \sqrt{(a^2 + b^2 + 2ab \cos 2\delta p\,.\,t)}\,.\cos\left\{pt + \tan^{-1}\left(\frac{a-b}{a+b}\tan \delta p\,.\,t\right)\right\} \ldots\ldots(17).$$

The resultant can be regarded as approximately a simple-harmonic motion whose quasi-frequency p is the arithmetic mean of the component frequencies and whose amplitude alternates between the sum of and the difference between the component amplitudes, the variation having a frequency, $2\delta p$, equal to the difference between the component frequencies. If both the difference between the frequencies and the difference between the amplitudes are small, the variable phase-difference will be very small for oscillations near the middle of a beat and we can neglect it. The beat,

represented by the first member of the product (17), is a maximum at zero time, and therefore coincides with one of the maxima given by the second member, thus making this maximum the largest in a beat. This, of course, we expect, because there is no initial phase-difference.

Now the motion of the pendulum is given by:

$$\theta = A\cos[(p+\delta p)\{t-(\tau+\delta\tau)\}] + B\cos[(p-\delta p)\{t-(\tau-\delta\tau)\}] \ldots(18),$$

where A and B are obtained from formula (4), while $(\tau+\delta\tau)$ and $(\tau-\delta\tau)$ are the phase-differences given by formula (5). We wish to find the time of the largest amplitude in the beat. It is best to express θ as a product:

$$\theta = \sqrt{\{A^2+B^2+2AB\cos 2(\delta p.t-\overline{\delta p.t+p.\delta t})\}}$$
$$\cos\left[pt-\overline{p\tau+\delta p.\delta\tau}+\tan^{-1}\left\{\frac{A-B}{A+B}\tan(\delta p.t-\overline{\delta p.\tau+p.\delta\tau})\right\}\right] \ldots(19).$$

Here again, under the conditions specified, we can neglect the variable phase-difference introduced by the inverse tangent term. It should be noted, however, that this phase-difference differs by a fixed amount $(\delta p.t+p.\delta\tau)$, from that of the original oscillations. The term $\delta p.\delta\tau$ is small enough to be left out of consideration. We are left with:

$$\theta = \sqrt{[A^2+B^2+2AB.\cos 2\{\delta p.t-(\delta p.\tau+p.\delta\tau)\}]}\cos p(t-\tau) \ldots(20).$$

The second member of this product represents the oscillations of mean frequency p (mean period T_p) and they have a phase-difference τ which is given by the usual equation (5) for simple-harmonic waves of period T_p. The first member of the product will tell us which of the oscillations has the largest amplitude. Thus the beats themselves reach a maximum when

$$t = \tau_b = \tau + p\delta\tau/\delta p = \tau - T_p\delta\tau/\delta T_p \qquad \ldots\ldots(21),$$

and this is the phase-difference, expressed as time, of the beats of frequency δp. It can be seen that the question which is the greatest maximum in a beat rests on the value of $\delta\tau/\delta T_p$. If this is zero, then the maximum which has a retardation given by τ is the greatest; in other words, the application of the ordinary formula for simple-harmonic waves gives the correct interpretation. If, however, $\delta\tau/\delta T_p$ is sufficiently large the beats themselves may be retarded (or advanced) by an amount so different from the retardation τ of the oscillations, that an earlier or later maximum (positive or negative) than that given by the usual formula may become the largest in a beat.

If the formula (5) for τ is plotted against T_p, it becomes obvious that there are an infinite number of pairs of periods which give the same value for $\delta\tau/\delta T_p$. So long as we restrict ourselves to pairs which have not large differences, these pairs will, to a good approximation, all have the same mean, and the corresponding value of $\delta\tau/\delta T_p$ will be the value of the differential coefficient $d\tau/dT_p$ when T_p is the mean period of the component waves. If $d\tau/dT_p$ is, say, $\frac{1}{4}$ when T_p is 20 sec., then $\delta\tau/\delta T_p$ will also be $\frac{1}{4}$ for pairs of periods given by (T_p+n) and (T_p-n) so long as n is not

too large. The differential coefficient of τ is easily obtained. For simplicity let us assume that the pendulum is critically damped. Then equation (5) becomes

$$\tau = (T_p/2\pi) \tan^{-1} \{2u/(u^2 - 1)\}.$$

This can be written:

$$\tau = (T_p/2\pi) (\pi - 2 \tan^{-1} u).$$

Therefore we have

$$d\tau/dT_p = \tfrac{1}{2} - \pi^{-1} \tan^{-1} u - u/\pi (1 + u^2),$$
$$= \tau/T_p - u/\pi (1 + u^2).$$

The lag of the beats is then (from 21)

$$\tau_b = T_p . u/\pi (1 + u^2).$$

When there is no initial phase-difference in the original components $T_p d\tau/dT_p$ is the time by which the beat maximum is in advance of the particular maximum that falls at time τ. If it is less than one quarter-period, then the maximum at τ will be the greatest in the beat. If it is greater than a quarter-period, then the negative maximum at $(\tau - \frac{1}{2}T_p)$ will be the maximum maximorum. The two will have equal amplitudes when $T_p d\tau/dT_p$ is exactly one quarter-period. This occurs when:

$$\tfrac{1}{4} = \tfrac{1}{2} - \pi^{-1} \tan^{-1} u - u/\pi (1 + u^2).$$

Writing $\tan^{-1} u = \psi$, we find that

$$\pi/4 = \psi + \sin \psi,$$

whence $\psi = 22°{\cdot}8,$

and $u = 0{\cdot}421.$

Table 1 gives some other values of the phase-differences of the oscillations and of the beats expressed as fractions of the mean period T_p of the oscillations. All of the values are positive: i.e. they are retardations. This table affords an explanation of the differences found by the graphical method. With reference to figure 1 for

Table 1.

u	τ/T_p	$d\tau/dT_p$	τ_b/T_p
0	0·50	0·50	0
0·2	0·44	0·48	0·06
0·3	0·41	0·32	0·09
0·42	0·37	0·25	0·12
1·0	0·25	0·09	0·16
1·6	0·17	0·03	0·14
∞	0	0	0

which u is 1·6, the point C on the pendulum curve is marked as being the time of the maximum of the beat given by τ_b. In this case it is $0{\cdot}03T_p$, i.e. 1·2 sec. in advance of P. P therefore is the greatest oscillation. In figure 2, on the other hand, the point C corresponding to maximum of the beat is $0{\cdot}48T_p$, i.e. 2·4 sec. in advance of P or $0{\cdot}06T_p$, i.e. 0·3 sec. behind the corresponding earth-maximum at zero time. It is clear then that the peak Q will have the greatest amplitude. Remembering that we are at present confined to the case in which there is no initial phase-

difference between the original cosine components, we can say that so long as $d\tau/dT_p < 0{\cdot}25$, i.e. when $u > 0{\cdot}42$, the greatest recorded maximum of the beat will be that indicated by the usual formula for τ. If $d\tau/dT_p > 0{\cdot}25$, i.e. if $u < 0{\cdot}42$, then the greatest maximum is recorded one half-period earlier and is of opposite sign to the earth-maximum. When $u = 0{\cdot}42$ the beat-maximum falls half-way between the peaks P and Q, which then have equal amplitudes.

Unfortunately the rule breaks down when an initial phase-difference between the two earth components is introduced. The effect is that the maximum of the earth beat no longer coincides with a maximum of an oscillation. It seems unnecessary to work through the theoretical expressions again, for they are simply an extension of those already given but with added fixed phase-differences. It suffices to state that, whatever the initial phase-differences, the retardation of the oscillations is still given by the usual formula for τ and the retardation of the beats themselves behind those of the earth-motion is still given by the τ_b, equation (21). We can no longer say, however, that the beat-maximum is in advance of the oscillation by $T_p d\tau/dT_p$. If the initial phase-difference is such that the beat-maximum of the earth occurs at a time $\pm rT_p$ in advance or in retard of the maximum oscillation, then on the record the beat-maximum will occur at a time $T_p(d\tau/dT_p \pm r)$ from the maximum oscillation that is indicated by the usual formula τ. Thus in figure 1 we might have chosen an initial phase-difference such that the maximum of the beat-motion (i.e. the wave of frequency $2\pi/\frac{1}{2}(44 - 36)$) occurred at a time given by the point A. The peak M would still have the maximum amplitude. Since, however, the beats themselves are retarded by $0{\cdot}06\,T_p$, the corresponding point on the pendulum curve is given by B. Although $u > 0{\cdot}44$ the peak Q would have the maximum recorded amplitude. Similar reasoning can be applied to figure 2 to show that the maximum amplitude may occur at a time which conforms to the ordinary formula. It appears therefore that we cannot decide, from a knowledge of the mean period T_p alone, which correction, τ or $(\tau - \frac{1}{2}T_p)$, is applicable. We must have some information concerning the phase-difference between the original components. Unfortunately this is not easily obtained from the records; the work involved is far too troublesome for routine procedure. The best course we can adopt is to use one correction throughout, always bearing in mind the fact that the maximum earth-oscillation of opposite sign which occurs one half-period earlier or later, according to whether we use τ or $(\tau - \frac{1}{2}T_p)$, may be slightly greater than that which occurs at the time indicated by the correction.

It should be pointed out that in the case of direct registration there can be only two alternative corrections. All cases of initial phase-difference are covered by the limits $\pm \frac{1}{4}T_p$ for the time between the beat-maximum and the largest oscillation of the earth-motion. The corresponding time-difference on the pendulum curve therefore must lie between $T_p(d\tau/dT_p + \frac{1}{4})$ and $T_p(d\tau/dT_p - \frac{1}{4})$. From table 1 it can be seen that so long as the oscillations are of finite period this time-difference must be less than $+\frac{3}{4}T_p$, and greater than $-\frac{1}{4}T_p$. It is only when the periods become infinitely small or infinitely long that two further alternatives may theoretically be possible and these cases are of no practical interest.

§ 8. MATHEMATICAL ANALYSIS OF MOTION OF GALVANOMETER COIL

The method of reasoning which we have used for the case of pendulum-motion applies equally well to the case of the galvanometer coil. All we have to do is to substitute τ_2 for τ. The oscillations forming the beats are retarded by a time τ_2, and the beats themselves are retarded by

$$(\tau_2 - T_p . \delta\tau_2/\delta T_p),$$

no matter what the initial phase-difference. If, for simplicity, we assume that the free periods of the galvanometer and the pendulum are equal and that both instruments are critically damped, then we may write:

$$\tau_2 = \frac{T_p}{2\pi}\left(2\tan^{-1}\left(\frac{2u}{u^2-1}\right)+\frac{\pi}{2}\right)$$
$$= \frac{T_p}{2\pi}(2{\cdot}5\pi - 4\tan^{-1}u);$$

whence
$$d\tau_2/dT_p = 1{\cdot}25 - 2\pi^{-1}\tan^{-1}u - 2u/\pi(1+u^2)$$
$$= \tau_2/T_p - 2u/\pi(1+u^2).$$

The lag τ_b of the beats is then given by:

$$\tau_b = 2u . T_p/\pi(1+u^2),$$

which is double the lag that we found for the case of the pendulum. When there is no initial phase-difference between the original components, $T_p . d\tau_2/dT_p$ is the time by which the recorded beat-maximum is in advance of the oscillation at τ_2. If it is zero, the beat-maximum coincides with the oscillation at τ_2, making it the greatest in the beat. If $T_p . d\tau_2/dT_p$ is one half-period, the beat-maximum coincides with the earlier negative peak at $(\tau_2 - \frac{1}{2}T_p)$. This occurs when:

$$d\tau_2/dT_p = \tfrac{1}{2} = \tfrac{5}{4} - 2\pi^{-1}\tan^{-1}u - 2u/\pi(1+u^2),$$

or, writing $\tan^{-1}u = \psi$,
$$\tfrac{3}{4}\pi = 2\psi + \sin\psi,$$
$$\psi = 39^\circ{\cdot}9 \quad \text{or} \quad u = 0{\cdot}836.$$

If $T_p . d\tau_2/dT_p$ is one whole period the beat-maximum coincides with the positive peak at $(\tau_2 - T_p)$. This occurs when:

$$\pi/4 = 2\psi + \sin 2\psi,$$

i.e. when
$$\psi = 11^\circ{\cdot}4 \quad \text{or} \quad u = 0{\cdot}202.$$

The following other numerical values will help to explain the results already obtained graphically.

Table 2.

u	τ_2/T_p	$d\tau_2/dT_p$	τ_b/T_p
0	1·25	1·25	0
0·1	1·17	1·12	0·06
0·20	1·12	1·00	0·12
0·44	0·98	0·75	0·23
0·84	0·80	0·50	0·30
1·60	0·60	0·32	0·28
∞	0·25	0·25	0

In all of the curves showing galvanometer-motion the peak marked G is that which corresponds with the lag τ_2 behind the earth-maximum M (τ_2 being the Galitzin correction). In figure 3 the earth components have no initial phase-difference, so that the beat-maximum coincides with M and the corresponding beat-maximum C on the galvanometer curve is delayed by $0{\cdot}28T_p$, or it is in advance of G by $0{\cdot}32T_p$. The peak S therefore has the greatest amplitude, and the proper correction to apply is $(\tau_2 - \frac{1}{2}T_p)$, as suggested by Somville. A reversal of sign must, however, be given. A similar reasoning applied to figure 4 shows that the recorded beat-maximum C comes exactly one whole period in advance of G, making T the largest oscillation. In this case neither the Galitzin nor the Somville corrections apply, but a lag given by $(\tau_2 - T_p)$.

In figure 5 the initial phase-difference causes the beat-maximum of the earth-motion to occur at a time $0{\cdot}2T_p$ after M, i.e. at the point A. Table 2 shows us that there is a further delay of $0{\cdot}28T_p$ with the galvanometer, and this gives us the beat-maximum at C. The peak G is this time the maximum, and so the Galitzin formula is applicable. It should be noted that figure 3, for which T_p is the same, would also yield this information, the points marked A and B being chosen so that the time between them is $0{\cdot}28T_p$.

In figure 6 the retardation of the beats is $0{\cdot}06T_p$, and A and C correspond to the times of the beat-maxima. The oscillation of largest amplitude is the peak U. The appropriate correction therefore is $(\tau_2 - 1\frac{1}{2}T_p)$, which in this case means an advance, and the sign must be reversed.

As in the case of the pendulum, there is no means of deciding which is the proper correction to apply unless the initial phase-difference is known. It can be shown, however, that for any particular case the number of alternatives can be narrowed down to two. All cases of initial phase-difference can, as before, be covered by superimposing $\pm \frac{1}{4}T_p$ on the difference between the retardation of the oscillations and that of the beats. There is therefore an ambiguity which cannot be greater than $+\frac{1}{4}$ or less than $-\frac{1}{4}$ in the values of $d\tau_2/dT_p$. If then $d\tau_2/dT_p < 1{\cdot}25$ and $> 0{\cdot}5$, the Galitzin correction can never apply, for the recorded beat-maximum must be more than a quarter of a period in advance of the maximum oscillation given by τ_2. If $d\tau_2/dT_p > 1{\cdot}0$, neither the Galitzin nor the Somville corrections can apply. On the other hand, the correction $(\tau_2 - 1\frac{1}{2}T_p)$ does not fit if $d\tau_2/dT_p < 1{\cdot}0$, neither does the correction $(\tau_2 - T_p)$ if $d\tau_2/dT_p < 0{\cdot}5$. This information is summarized thus:

Limits of u	Alternative corrections
$\infty > u > 0{\cdot}84$	τ_2 ; $(\tau_2 - \frac{1}{2}T_p)$
$0{\cdot}84 > u > 0{\cdot}2$	$(\tau_2 - T_p)$; $(\tau_2 - \frac{1}{2}T_p)$
$0{\cdot}2 > u > 0$	$(\tau_2 - T_p)$; $(\tau_2 - 1\frac{1}{2}T_p)$

Those corrections which differ from τ_2 by an odd half-period imply a reversal of sign. Table 3 gives some numerical values for a case in which the instrumental period is 20 sec.

Table 3.

$\frac{T_p}{T} = u$	Earth period T_p	Retardation of registered maximum			
		Maximum of same sign as that of earth-movement		Maximum of opposite sign	
		True	Galitzin	True	Somville
	sec.	sec.	sec.	sec.	sec.
1·5	30	18·8	18·8	3·8	3·8
1·0	20	15·0	15·0	5·0	5·0
0·6	12	− 1·1	10·9	4·9	4·9
0·4	8	0·0	8·0	4·0	4·0
0·15	3	0·5	3·5	− 1·0	2·0

§ 9. CONCLUSIONS

Let us now reconsider how best we can interpret a record of the principal phase of an earthquake. We have on the record a group of waves of period T_p, which form a beat. In general the amplitude of one oscillation on the record will be greater than the amplitudes of neighbouring oscillations in the beat. This displacement is measured and is converted into actual earth-movement by applying the magnification-factor for sinusoidal waves. This procedure is sufficiently accurate for all practical purposes so long as the beats are reasonably long, but it is not strictly correct on account of the change in character of the beat-motion from the original form. The time, on the record, of the maximum displacement in the beat is measured and the next step is to arrive at the time of the corresponding oscillation in the earth-motion. For routine measurements it is scarcely worth while endeavouring to arrive at the initial phase-difference between the waves into which the beats could be resolved, and without this knowledge it is impossible to decide what shall be taken as the corresponding oscillation in the earth-motion.

In the case of direct registration, we can definitely say that the largest earth-displacement of the same sign as the recorded displacement occurred at a time τ in advance of the recorded time (τ being given by the usual formula). It must be remembered, however, that the oscillation of opposite sign which occurred at a time $(\tau - \frac{1}{2}T_p)$ in advance of the recorded time may have been greater. We can, at any rate, definitely conclude that no advantage will be obtained by changing from the present procedure of using the ordinary formula, equation (5), and adopting instead a correction which is one half-period less and which carries a reversal of sign. Whichever formula is adopted, it is very important that the distinction between the signs of the displacements in the case of a sudden impulse and in the case of sinusoidal motion be fully appreciated; as was pointed out in § 2, a sudden impulse is recorded with a reversal of sign.

It is not easy to decide the best course to adopt in the case of galvanometric registration. If we make the convention of always giving the time of the largest earth-displacement which has the same sign as the recorded maximum, then we must

apply Galitzin's correction τ_2 if $u > 0{\cdot}84$, but when $u < 0{\cdot}84$ we must reduce the Galitzin correction by one whole period. This is definitely better than retaining the Galitzin formula for all periods since that formula cannot apply when $u < 0{\cdot}84$, but to have two formulae in use may lead to confusion. There is a practical consideration which indicates that a better course is to adopt the Somville formula $(\tau_2 - \frac{1}{2}T_p)$. It has been shown that the range over which this formula cannot apply is that between $u = 0$ and $u = 0{\cdot}2$. Now most instruments are tuned to periods not exceeding 25 sec. This means that all earth-waves of periods greater than 5 sec. correspond to values of u in excess of $0{\cdot}2$. Since, during the surface-wave phase, the periods rarely, if ever, fall as low as 5 sec., there is no reason why the Somville formula should not give the correct result in as many cases as if the two alternatives were used. It is finally recommended, therefore, that for time-measurements of the maxima occurring in the principal phases of earthquakes the Somville formula be adopted as a phase-difference correction. The reversal of sign which this formula implies must, of course, be remembered. A reservation must be made if phase-difference corrections are to be applied to waves of such short periods as often occur in the case of microseisms.

§ 10. ACKNOWLEDGMENT

In conclusion I wish to thank Dr F. J. W. Whipple, Superintendent of Kew Observatory, for suggesting this investigation in the first place and also for helpful criticism.

DISCUSSION

Sir A. S. Eddington. The simple-harmonic solution represents a steady state, established from the beginning of time, with the pendulum and the ground oscillating in such a phase that they do not upset the steady condition. In that case there is no meaning in trying to pick out a single crest on the ground-wave as particularly responsible for a given crest on the pendulum-wave. (The strong damping would, however, make it far-fetched to associate crests too wide apart in time). The problem thus arises only if there is some variation from steady simple-harmonic motion—beats or a sudden pulse—so that the appropriate formula for correlating points on the two curves is necessarily the one given by Mr Scrase for the case of beats. If I have understood rightly, that is the essential point that his paper develops.

Mr T. Smith suggested that the identification of a particular crest with the disturbance was as arbitrary with a simple group of waves as with a simple-harmonic system. The association seemed to him to be essentially a matter of convention, and the criterion to be applied in judging between different conventions might well rest on practical experience of the convenience of different choices rather than on mathematical grounds alone. Thus with certain methods of computing tides it had been found convenient to associate component tides with the meridional passage of the sun or moon which took place about a day and a half before the tide rather than with another crossing which might equally well have been chosen.

AUTHOR's reply. Sir Arthur Eddington has summed up the essential points of the problem very clearly. The general solutions for a steady state of simple-harmonic motion show that one cannot regard any particular peak on the record as being produced by a given peak in the earth-motion. But when some irregularity is superimposed on the simple-harmonic motion, it is possible to interpret a corresponding irregularity on the record and to obtain a particular case of the general formula which will apply.

Mr Smith has remarked that since group-velocities are in question the problem remains indeterminate. I gather that he refers to the lag of the waves relative to that of the beats. It is true that this changes the form of the beats to some extent, but for practical purposes it is appropriate to consider the maximum displacement in the recorded beat as corresponding to the maximum displacement in the original beat. This procedure does not give an ambiguous result unless the positive and negative displacements in the middle of a beat are equal.

NOTE ON THE ELIMINATION OF THE β WAVE-LENGTH FROM THE CHARACTERISTIC RADIATION OF IRON

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Communicated by Dr G. W. C. Kaye, January 1, 1931. *Read and discussed February* 20, 1931.

ABSTRACT. A practical method of eliminating the β wave-length from the characteristic K-radiation of iron is described. The method is based upon the selective absorption produced by a thin film of pure manganese which is obtained in the required form by electrodeposition upon aluminium foil.

§ 1. INTRODUCTION

The characteristic K-radiation direct from an iron anticathode is very commonly used in X-ray analysis. The spectrum of an irradiated substance is complicated, however, by the presence of reflections due to wave-lengths other than the K_α component. The main difficulties arise from the K_β wave-length. In the case of substances of high crystal-symmetry, which give comparatively few lines, or of those obtainable in the form of single crystals, the lines from the different faces of which need not be superimposed, the extra spectra due to the β component can be readily recognized and allowance can be made for them. But in the majority of cases, especially if the power method of analysis be used, difficulty is experienced in determining definitely whether certain lines should be assigned to the α or to the β wave-lengths. In consequence, the elimination of the latter component from the radiation incident on a substance under examination is often very desirable. The object of the present paper is to describe a simple method of accomplishing this by the method of selective absorption. The removal of undesired wave-lengths could, of course, be secured once and for all by the use of a beam which had been rendered monochromatic by preliminary reflection from a standard crystal face. The disadvantage of this method lies in the considerable decrease in intensity which would follow upon a double reflection. Moreover, the efficacy with which a screen of nickel foil, in the analogous case of copper, removes the copper K_β wave-length by preferential absorption, would suggest that the extreme method of primary reflection is not necessary.

The appropriate absorbing material to be employed in the case of iron is manganese, since the wave-length of the absorption edge has a value less than that of the α and greater than that of the β wave-length. The discontinuity in the absorption/wave-length curve for manganese occurs, therefore, at a value which involves a much greater absorption of the iron β than of the α wave-length. It is desirable

that the screen should not reduce the intensity of the α component any more than is necessary. On that account, the use of screens made up of manganese compounds, such as the dioxide, is, as most workers will have found, inefficient in practice because of the additional absorption of the beam caused by the presence of the other constituents of the molecule. So far as the author is aware, the idea of using a thin film of manganese prepared for this purpose by electrodeposition is new, and, as the matter is of some practical importance to X-ray workers, it was considered worth while to outline the method.

§ 2. EXPERIMENTAL DETAILS

The films of manganese were first deposited on aluminium foil. Such foil can be obtained so thin that the absorption of radiation becomes very small. Also the presence of the aluminium backing has the advantageous effect of increasing the contrast of the lines on an X-ray photograph by absorbing most of the softer rays which would otherwise add to the continuous background. In consequence it was found that the use of the screen need increase the time of exposure by very little. If required, however, the manganese film can be fastened to a base of cellophane or some such material and the aluminium dissolved away. Alternatively, if an X-ray tube of the experimental demountable type be employed, the film may replace the usual aluminium window. It was found, too, that the thicker films could be stripped from the aluminium.

The arrangement of the electrolytic bath was as follows*. The anode consisted of a piece of platinum foil which was suspended in a strong solution of ammonium sulphate. This solution was separated from the rest of the bath by a porous pot. The catholyte contained per litre 250 gm. of pure manganese sulphate crystals and 100 gm. of ammonium sulphate. The cathode was a piece of thin aluminium foil which was smoothed out upon and attached by adhesive to a piece of glass. A very smooth surface appeared to be required in view of the weak throwing-power of the manganese ion. A preliminary cleaning of the surface by benzene and dilute acid was made. A certain amount of trouble was encountered in securing a deposit composed of crystal grains fine enough to give a smooth continuous surface. The manganese tended to deposit first as a fine layer with a metallic lustre, and then to increase further in thickness by growing large crystals. It was found, however, that the grain size varied critically with the current density and the temperature of the bath, and these could be adjusted to suit the type of deposit required. A current-density rather larger than usual, namely about 250 ma./cm.^{2}, with the electrolyte at 35° C. gave good results with the bath described, and a film sufficiently thick and uniform to produce an effective screen could be obtained after the current had been passed for about one hour. If the time of deposition was prolonged too far, the weakening of the bath appeared to have the effect of increasing again the grain-size of the layer. After being washed and dried with alcohol, the film was covered with a little varnish to prevent undue oxidation.

* A. J. Allmand and A. N. Campbell, *Proc. Far. Soc.* **20**, 379 (1925).

§ 3. RESULTS

The type of result obtained is illustrated by figures 1 and 2. These are microphotometer records of X-ray spectra photographed under exactly the same conditions except that a screen was interposed in the incident beam when the latter was being secured. The line *A* is a reflection of the iron K_α wave-length and *B* of the K_β. The reduction of the intensity of the β lines as a result of the screen is quite apparent. The exposure to the incident beam was adjusted so as to obtain

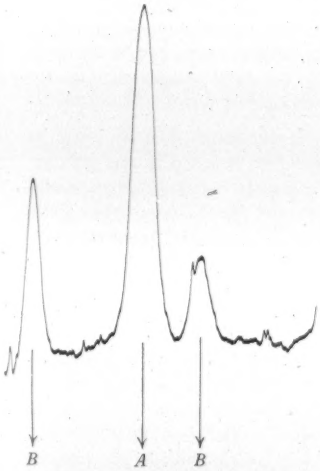

Fig 1. Spectral intensity without screen.

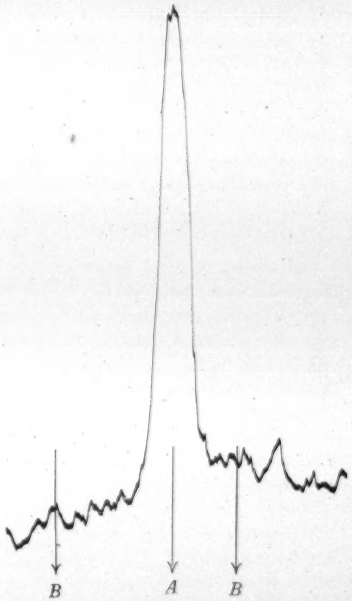

Fig. 2. Spectral intensity with screen.

as nearly as possible the same intensity of the α line in each case. The reduction of the intensity-ratio of β to α in figure 2 is thus more significant. It is possible, of course, by variation of the time of deposition to vary the efficacy of the absorbing screen. That thickness which is effective without unduly increasing the time of exposure necessary to secure a photograph can be selected by trial. It is the author's experience that such a layer will then eliminate the β lines from a photograph of normal exposure just as efficiently as nickel foil, in the better known case of copper, removes the β from the characteristic copper-radiation.

§ 4. ACKNOWLEDGMENTS

In conclusion the author expresses his thanks to Dr G. W. C. Kaye for his interest in the researches out of which the above work arose. He is very much indebted to Mr J. R. Clarkson, B.Sc., for extensive help in the preparation of the deposits.

DISCUSSION

Prof. H. R. Robinson. I should like to ask the author if he has made any estimates, by weighing or otherwise, of the effective thicknesses of his manganese films. He has referred to the well-known practice of filtering copper K-radiations with nickel foil; in my experience of this method, I have calculated the most suitable thickness of nickel from the available absorption data, and have subsequently found that very efficient filtration is in fact effected with foils appreciably thinner than those predicted by calculation. It seems possible that this may in part be due to the inadequacy of the absorption measurements in the immediate neighbourhood of the absorption discontinuity, and I should be interested to know if Mr Wood has had similar experiences with his manganese films.

Author's reply. I have found, in agreement with Prof. Robinson, that the thickness of the foils necessary to filter the β-radiation is less than that indicated by theory. The effect, however, may possibly be due to the fact that the films absorb the softer radiation from the incident X-ray beam and thereby remove part of the continuous background of the photographic plate. The enhanced contrast of the spectral lines would then permit of a less exposure to secure a given visibility.

DISPLACEMENTS OF CERTAIN LINES IN THE SPECTRA OF IONIZED OXYGEN (OII, OIII), NEON (NeII) AND ARGON (AII)

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ABSTRACT. In continuation of an investigation of the displacements which characterize certain spectral lines produced in a discharge tube at moderate gas pressure (of the order of a few cm.), the spectra OII, OIII, NeII and AII have been studied. As in the case of NII, which has been examined previously, many lines in each of the spectra are displaced to the red and for the majority of the lines the amount of displacement has been determined. The term shifts so obtained have been discussed in relation to the term schemes for each of the spectra. A comparison of the term shifts in the various spectra has been made and it is found that the 4*s* terms of OII, OIII, NII and NeII, and the 5*s* terms of AII (^3P family) all show roughly the same shift. Some typical shifted lines have been examined with a microphotometer and the curves are reproduced. The suggestion made in the previous paper that the shift is probably a Stark effect receives further support.

§ 1. INTRODUCTION

In a previous paper* the shifting of certain lines of the spectrum of ionized nitrogen, caused by the raising of the gas pressure in the discharge tube, was investigated. The general nature of the results was to show that the shift was related to the terms concerned in the production of the lines, and an examination of the cause of the shift led to the conclusion that it was in the main a Stark effect. The data obtained for NII were not sufficiently extensive to do more than indicate the general nature of the effect, and it was thought that a study of other spectra might reveal a systematic relation between the displacements of the various lines and the terms associated with the lines.

With this object in view the following spectra have been investigated: OII, NeII, AII, OIII and NIII. These spectra, together with NII, provide the material for several comparisons: (1) first ionized spectra of atoms of the first row of the periodic table, NII, OII, NeII; (2) single and double ionization, OII, OIII; (3) iso-electronic systems, NII and OIII; (4) successive spectra in a vertical column of the periodic table, NeII and AII.

The spectra have in the first instance been examined separately and in con siderable detail, the excitation conditions being adjusted so as to yield the best results for the particular spectrum concerned. Each spectrum has been observed under varying conditions of pressure and excitation, and varying exposures. It was found, in practice, that no great change in the electrical conditions was necessary, in passing from one gas to another, for the production of a suitable singly ionized

* *Proc. Phys. Soc.* **41**, 442 (1929).

spectrum. In the case of OIII, however, it was necessary to increase considerably the violence of the discharge. The comparison of shifts in different spectra is considered later.

Many of the shifts in OII* have been recorded by Fowler, as well as some in OIII†. All those in NeII, and all those in AII‡ below λ 3000 are, as far as the author is aware, recorded for the first time.

§ 2. EXPERIMENTAL

The arrangement for producing the spectra was the same as in the first investigation. The gas was excited in an H-type tube, ordinarily made of glass, of which the capillary was about 5 cm. long and of 1 mm. bore. The exciting mechanism consisted of a 12 in. induction coil, with mercury interrupter, a condenser being included in parallel and a spark gap in series. These two latter were adjusted as was necessary to produce the required spectrum.

As before, the shifts were obtained by increase in the pressure of the gas in the discharge tube, the high pressure being in the neighbourhood of 2 cm. of mercury. The displacements were measured relative to a comparison spectrum obtained when the gas pressure was low ($< 0{\cdot}1$ cm.). The same method was adopted of photographing overlapping regions so that the shifts in different parts of the spectra, photographed on different instruments, should be comparable with one another.

The region investigated for all the spectra was from λ 6800 to λ 2300, the instruments used being as follows:

(1) A glass prism instrument having a dispersion of about 30 A/mm. at λ 6600 and 5 A/mm. at λ 3800.

(2) A concave grating (Eagle mounting) with a dispersion of 5·5 A/mm. in the first order.

(3) A quartz Littrow spectrograph (Hilger's EI) having a dispersion of 10 A/mm. at λ 3800 and 2 A/mm. at λ 2300.

§ 3. NOTATION

For designating a term arising from a given electron configuration, the notation as used by Fowler in papers on CII§ and OIII‖ has been employed. For the simpler spectra, and particularly for the present purpose, it seems preferable to the more general notation recently suggested¶.

In AII three families of terms have been identified, and to distinguish between

* *Proc. R.S.* A, **110**, 476 (1926). † *Proc. R.S.* A, **117**, 317 (1928).

‡ Until recently I was under the impression that all the shifts in AII were newly discovered, but it has just come to my notice that most of the lines above λ 3000 which shift have been recorded by Eder and Valenta (*Denkschriften der Math.-Nat. Klasse der K. Akad. Wiss. Wien*, **44**). The shifts in their experiments resulted from the raising of the pressure of the gas in the discharge tube to 2 cm., which was roughly the same as that finally used in my argon tube. They did not, however, attempt any investigation of the shifts.

§ *Proc. R.S.* A, **120**, 312 (1928). ‖ *Proc. R.S.* A, **117**, 317 (1928).

¶ H. N. Russell, A. G. Shenstone and L. A. Turner, *Phys. Rev.* **33**, 900 (1929).

terms of different families arising from the same configuration of the series electron, the method used by de Bruin† has been adopted. This requires only that after the term as ordinarily expressed, e.g. $4p\ ^4D$, the family to which it belongs shall be indicated by addition as a postscript. That is, the term cited would be completely described as $4p\ ^4D\ (^3P)$ since it belongs to the family having a ^3P limit‡.

In OII only two families are known and for this spectrum the simple notation is used for the main family (^3P limit), e.g. $4s\ ^4P$; while the terms belonging to the ^1D family have been asterisked, e.g. $*4s\ ^2D$.

Table 1. Shifts in lines of OII.

λ	Int.	Classification	dλ	dν	λ	Int.	Classification	dλ	dν
§3830·45	(4)	$3p\ ^2P_2 - 4s\ ^2P_1$	—	—	3287·59	(9)	$3p\ ^1P_1 - 4s\ ^1P_1$	0·45	4·2
3821·68	(4)	$P_1 - P_1$	0·53	3·7	3277·69	(7)	$P_2 - P_2$	0·44	4·1
3803·14	(6)	$P_2 - P_2$	0·60	4·1					
3794·48	(3)	$P_1 - P_2$	0·56	3·9	‡3273·52	(7)	$*3p\ ^2F_4 - *4s\ ^2D_2$	0·45	4·2
					‡3270·98	(7)	$F_3 - D_2$	0·46	4·3
3777·60	(4)	$3p\ ^4S_2 - 4s\ ^4P_1$	0·59	4·1					
3762·63	(5)	$S_2 - P_2$	0·56	3·9	3139·77	(4)	$3p\ ^1D_2 - 4s\ ^1P_1$	0·41	4·2
3739·92	(6)	$S_2 - P_3$	0·58	4·1	3138·44	(8)	$D_3 - P_2$	0·37	3·8
					3134·82	(10)	$D_4 - P_3$	0·41	4·2
3735·94	(3)	$*3p\ ^2P_2 - *4s\ ^2D_{3/2}$	0·51	3·7	§3134·32	(3)	$D_1 - P_1$	—	—
§3729·34	(2)	$P_1 - D_{3/2}$	—	—	3129·44	(7)	$D_2 - P_2$	0·42	4·3
					‖3124·02	(2)	$D_1 - P_2$	—	—
3470·81	(8)	$3p\ ^2D_3 - 4s\ ^2P_2$	0·54	4·5	3122·62	(6)	$D_3 - P_3$	0·40	4·1
3470·42	(5)	$D_2 - P_1$			‖3113·71	(1)	$D_2 - P_3$	—	—
§3447·98	(5)	$D_2 - P_2$	—	—					
					2747·46	(6)	$3p\ ^2S_1 - 4s\ ^2P_1$	0·44	5·8
3409·84	(6)	$*3p\ ^2D_2 - *4s\ ^2D_{3/2}$	0·48	4·1	2733·34	(10)	$S_1 - P_2$	0·43	5·8
§3407·38	(7)	$D_3 - D_{3/2}$	—	—					
					2718·84	(2)	$†2p'\ ^2S_1 - 3p\ ^2P_1$	0·14	1·9
3306·60	(6)	$3p\ ^4P_2 - 4s\ ^4P_1$	0·41	3·8	2715·38	(3)	$S_1 - P_2$	0·15	2·0
3305·15	(6)	$P_3 - P_2$	0·46	4·2					
§3301·56	(3)	$P_1 - P_1$	—	—	2436·10	(4n)	—	0·25	4·2
3295·13	(4)	$P_2 - P_2$	0·45	4·1	2425·62	(5n)	—	0·24	4·1
3290·13	(5)	$P_1 - P_2$	0·43	4·0	2406·41	(3n)	—	0·22	3·8

* Based on ^1D state of core.
† Arises from the configuration sp^4. See figure 1.
‡ Classification by Russell.
§ High-pressure line confused.
‖ Shift observable but not measurable.

§ 4. OXYGEN

The spectrum of singly ionized oxygen, OII. The spectrum has been investigated very fully by A. Fowler§, and an interpretation of the experimental results on the basis of Hund's theory has been given by R. H. Fowler and Hartree‖. The analysis has been extended by Bowen¶ who identified the deepest terms thereby deter-

† *Proc. Acad. Amsterdam*, **33**, 198 (1930).

‡ It would be more logical to introduce the family designation between the electron symbol ($4p$) and the term type (^4D), but it seems to the author that in the method adopted the list of lines is more easily readable.

§ *Proc. R.S.* A, **110**, 476 (1926).

‖ *Proc. R.S.* A, **111**, 83 (1926).

¶ *Phys. Rev.* **29**, 242 (1927).

mining the ionization potential. Later, Russell* has identified many of the higher terms predicted by theory, and has also identified intercombinations, thus fixing definitely the values of the quartet terms which had previously been given an arbitrary starting-point by A. Fowler.

The wave-lengths and classifications are taken from those published by A. Fowler, the only change being in the notation.

In the case of OII, up to the present there have been identified two families of terms. The first, consisting of doublet and quartet terms, arises from the addition of an electron to a core in which the electrons outside the completed K shell have the configuration s^2p^3, the limiting value of the family being the ^3P term of OIII.

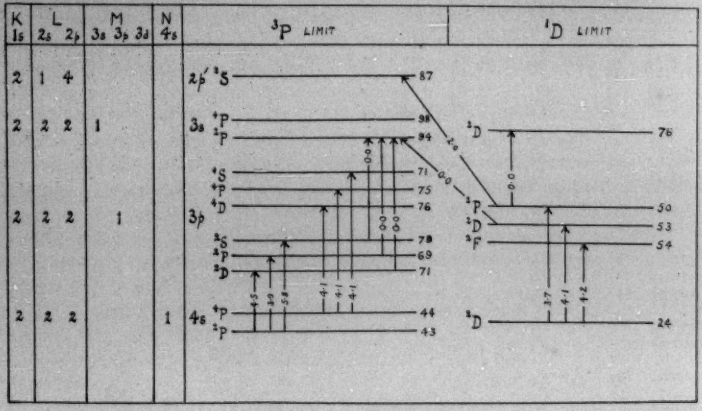


Fig. 1. Term scheme for OII.

This is usually referred to as the main family. The second is a family of doublets only and arises similarly but has as limit the ^1D term of OIII. A third system with limit ^1S is predicted by theory, but no terms have yet been definitely assigned to it. One term based on a different configuration of the electrons outside the K shell, viz. sp^4, is distinguished by a dash, e.g. $2p'\ ^2S$.

Table 1 contains the lines which have been observed to show large shifts, together with the measured shifts. In all cases the shift is to the red. With a few exceptions the lines have been recorded as unsteady lines by A. Fowler. The shifted lines are more or less symmetrical but there is a slight shading to the red. A portion of the spectrum is reproduced in the plate. Most of the shifts have been measured, and it will be observed that the lines of a given multiplet all show the same shift to the order of accuracy of the measurements.

The term scheme for OII and approximate term values (in thousands of cm.$^{-1}$) are shown in figure 1. The scheme is not at all complete but is sufficiently so

* *Phys. Rev.* **31**, 27 (1928).

for the present purpose. Transitions and corresponding shifts are indicated by arrows. The shift inserted is the mean of the shifts of the lines constituting the multiplet.

Combinations between terms each having total quantum number n equal to 3 are steady, while those between terms having n equal to 4 and those with n equal to 3 all show a pronounced shift. Moreover the doublet and quartet terms with ^3P limit show approximately the same shift, with the exception of $4s\,^2P$ when combining with $3p\,^2S$. The terms having limit ^1D are much smaller than the corresponding terms with limit ^3P, but they show much the same shift as the $4s \rightarrow 3p$ combinations of the main family.

In the previously investigated spectrum, NII, no transitions from a term of total quantum number 3 to one of total quantum number 2 fell within the observed range, and it was accordingly assumed that terms having n equal to 3 were stable. In OII, however, the combination $2p'\,^2S - 3p\,^2P$ is observable and is found to shift. Thus the absence of shift in lines associated with $3p$ to $3s$ transitions is probably due not to the absence of any effect on these terms but to the equality of the perturbations. If we assume that the term shift is really a property of the term under given conditions, it would appear from the diagram that if those terms are steady which have n equal to 2 the shift in terms with n equal to 3 is 2 frequency units and in terms with n equal to 4 it is 6 units. What is determined by experiment is the relative shift between two terms, and it must still be borne in mind that even this is determined by the exciting conditions, increasing as the pressure increases with a given discharge tube. Shifts were measured with the gas at various pressures and, for the same spark gap and condenser, the shift increased with the pressure as in the case of NII. For the shifts given in table 1 the pressure was about 1·0 cm., spark gap in series 3·0 mm. (1·5-cm. spheres) and condenser 0·005 μF. This was found the most suitable arrangement for obtaining the best results.

Most of the lines assigned by Russell* to terms involving total quantum numbers 4 and 5 fall in the observed region, but many of them are too weak, on the plates so far obtained, for decision whether they show a shift or not. There are a few strong lines, however, viz. λ 4087·16, λ 4253·98 and two groups, one consisting of 6 lines at λ 4280 and the other of 3 lines at λ 4290, which are associated with terms having n equal to 4 ($4f$) and yet do not shift to any observable extent. These lines are inherently somewhat diffuse, and this militates against the observations, but there seems little doubt that these lines do not show the same type of shift as shown by the $4s$ terms. This is surprising in the light of the results already obtained for NII, where the shift of the $4p$ and $4d$ terms was of the same order as that of the $4s$ terms. (Unfortunately no $4f$ terms in NII have yet been identified.) Lines involving Russell's $5f$ terms are not visible in the high-pressure spectrum, this being probably due to their becoming very diffuse.

The three unclassified lines which are given in the list are quite strong and shift in the same way as the others, but have so far evaded classification.

* *Loc. cit.*

It was thought that the anomaly of the $4s\ ^2P$ term in combination with $3p\ ^2S$ might be due to the latter term being affected to a lesser extent than the ^2P and ^2D terms arising from the $3p$ configuration. When each of these terms combines with $3s\ ^2P$, however, the resulting lines are all steady, and it would appear as if the behaviour of the $4s\ ^2P$ term in the $3p\ ^2S - 4s\ ^2P$ transition is really anomalous, as the $4d\ ^2P$ term in NII appears to be.

The spectrum of doubly ionized oxygen, OIII. It was necessary to use a longer spark gap and larger capacity for producing the doubly ionized spectrum of oxygen. In order to measure the shifts it was also necessary that a rather lower pressure should be used. In addition, a quartz tube of narrower bore was employed instead

Table 2. Shifts in lines of OIII.

λ	Int.	Classification	dλ	dv	λ	Int.	Classification	dλ	dv
†2713·40	(2)	$3d\ ^3P_0 - 4p\ ^3D_1$	—	—	†2453·54	(2n)	$3d\ ^3P_1 - 4p\ ^3P_0$	—	—
‡2708·87	(1)	$P_1 - D_1$	—	—	†2451·91	(2n)	$P_0 - P_1$	—	—
†2701·05	(3)	$P_1 - D_2$	—	—	†2448·21	(1n)	$P_1 - P_1$	—	—
‡2692·74	(1)	$P_2 - D_2$	—	—	†2441·72	(2n)	$P_1 - P_2$	—	—
†2677·81	(3n)	$P_2 - D_3$	—	—	‡2441·41	(00)	$P_2 - P_1$	—	—
					†2434·96	(2n)	$P_2 - P_2$	—	—
2609·59	(4)	$3d\ ^3P_0 - 4p\ ^3S_1$	0·23	3·4					
2605·41	(6)	$P_1 - S_1$	0·22	3·2					
2597·69	(8)	$P_2 - S_1$	0·25	3·7	2438·83	(5n)	$3d\ ^1P_1 - 4p\ ^1D_2$	0·15	2·6
					2422·84	(5n)	$3d\ ^1F_3 - 4p\ ^1D_2$	0·14	2·5
2558·06	(8)	$3p\ ^1P_1 - 4s\ ^1P_1$	0·25	3·8					
					2394·33	(5n)	$3d\ ^1P_1 - 4p\ ^3D_1$	0·19	3·3
†2549·62	(2)	*$3d\ ^3D_2 - 4p\ ^3D_1$	—	—	†2388·20	(1)	$P_1 - D_2$	—	—
†2547·45	(2)	$D_3 - D_3$	—	—					
2546·43	(4)	$D_1 - D_1$	0·24	3·6	2383·92	(6n)	$3d\ ^3F_3 - 4p\ ^3D_3$	0·19	3·4
2542·68	(5)	$D_2 - D_2$	0·20	3·0	2382·32	(7n)	$F_4 - D_3$	0·20	3·6
†2539·50	(2)	$D_1 - D_2$	—	—	2378·90	(4n)	$F_2 - D_1$	0·21	3·6
2534·08	(6n)	$D_3 - D_3$	0·18	2·8	†2372·82	(2n)	$F_2 - D_2$	—	—
‡2529·36	(1)	$D_2 - D_3$	—	—	†2372·21	(3n)	$F_3 - D_3$	—	—

* In Fowler's paper this term is misprinted $3p\ ^3D_2$.

† These lines could be seen to be displaced but were not measurable in the high-pressure spectrum. ‡ Not visible in high-pressure spectrum.

of the glass one used for the other spectra. Even so it was much more difficult to obtain plates suitable for measurement and the measured shifts are not considered as accurate as those in OII. The difficulty was the same as that experienced in a much lesser degree in the other spectra, viz. the diffuseness of the lines in the high-pressure spectrum and the continuous background always associated with the high-pressure spectrum.

Two fairly satisfactory plates were eventually obtained and the means of the shifts on the two plates are given in table 2*. The wave-lengths and classifications

* The values given in table 2 are not actually the measured shifts, but are the latter multiplied by 1·10. This plan has been adopted because the OII lines occurring on the plates used for measuring the OIII lines showed shifts equal to 1/1·10 of the values given in table 1. Hence to make the two sets of oxygen shifts comparable the OIII shifts were multiplied by 1·10. This question of comparing the shifts in different spectra is considered later in the paper.

are those of Fowler*. About half only of the observed shifts have been measured. In some groups the faintest lines were not visible in the high-pressure spectrum, but as no instance has yet been found in which the lines of a group behave differently these faint lines have been included in the list for completeness and to avoid any ambiguity.

The term scheme for OIII is represented in figure 2, and the term shifts have been inserted as in the previous diagram for OII. As in NII and OII lines arising from transitions from terms having n equal to 4 to those having n equal to 3 are shifted in the high-pressure spectrum, while lines associated only with terms for which $n = 3$ are steady. Unfortunately lines arising from $3 \rightarrow 2$ transitions are out of range.

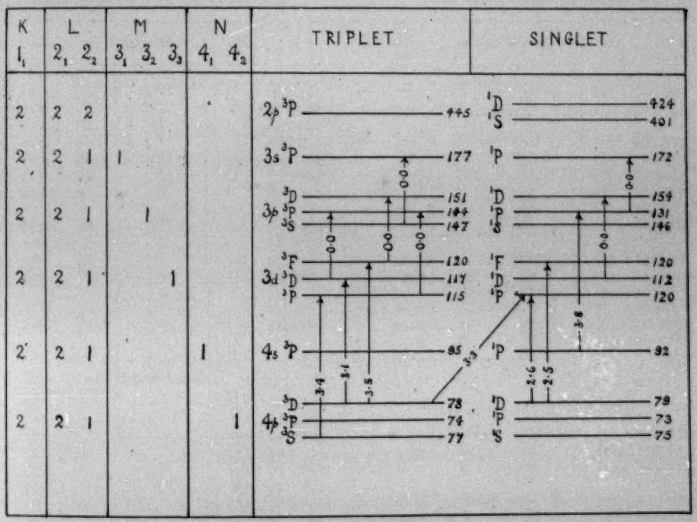


Fig. 2. Term scheme and shifts in OIII.

§ 5. NITROGEN

The spectrum of doubly ionized nitrogen, NIII. Some shifting lines on the plates taken for the investigation of NII were at first thought to be due to NIII but do not find a place in the list given by Freeman†, and their origin is not yet known. No large shifts in any of the lines which are known to be due to NIII have yet been observed, but the lines which might be expected to shift are weak and are not visible in the high-pressure spectra on the plates so far obtained.

* *Proc. R.S.* A, 117, 317 (1928).

† *Proc. R.S.* A, 121, 318 (1928).

§ 6. NEON

The spectrum of neutral neon, NeI. In view of the fact that many of the lines of the spectrum of neutral neon have been adopted as secondary standards* in the yellow-red region, it was considered appropriate to the investigation of the "pressure effect" to see whether any of the lines concerned were appreciably affected, when the pressure of the gas in a tube excited as in the foregoing experiments was increased. This was carried out with pressures up to 15 cm., this pressure being much higher than had been possible for nitrogen or oxygen.

Table 3. Shifts in lines of NeII.

Classified lines. Large shifts				
λ	Int.	Classification	dλ	dν
*3397·81	(1)	$3p\ ^4S_2 - 4s\ ^4P_3$	—	—
*3362·90	(1)	$S_2 - P_1$	—	—
*3072·68	(1)	$3p\ ^4D_1 - 4s\ ^4P_2$	—	—
*3059·15	(3)	$D_2 - P_2$	—	—
*3044·10	(2)	$D_1 - P_1$	—	—
†3039·62	(4)	$D_4 - P_3$	0·71	7·7
*3035·95	(2)	$D_3 - P_2$	—	—
*3030·82	(2)	$D_2 - P_1$	—	—
2809·51	(4)	$3p\ ^4P_2 - 4s\ ^4P_2$	0·54	6·9
2794·22	(3)	$P_1 - P_2$	0·58	7·4
2792·04	(4)	$P_3 - P_3$	0·57	7·4
2780·05	(2)	$P_2 - P_2$	0·60	7·7
2770·63	(1)	$P_1 - P_1$	0·55	7·2
2762·97	(3)	$P_3 - P_2$	0·59	7·7
2756·68	(3)	$P_2 - P_1$	0·58	7·6
Classified lines. Small shifts				
3194·58	(4)	$3p\ ^4D_2 - 3d\ ^4P_2$	0·21	2·1
3173·58	(3)	$D_3 - P_1$	0·19	1·9
3165·68	(2)	$D_3 - P_3$	0·22	2·2
3176·14	(3)	$3p\ ^4D_2 - 3d\ ^4F_2$	0·28	2·8
3118·00	(4)	$D_4 - F_5$	0·26	2·7
Classified lines. Small shifts (*cont.*)				
3054·70	(5)	$3p\ ^4P_1 - 3d\ ^4D_2$	0·19	2·0
3047·60	(6)	$P_2 - D_3$	0·20	2·1
3045·56	(3)	$P_1 - D_1$	0·18	1·9
3037·75	(3)	$P_2 - D_2$	0·19	2·0
3034·49	(5)	$P_3 - D_4$	0·19	2·0
3027·07	(4)	$P_2 - D_1$	0·17	1·9
3017·36	(3)	$P_3 - D_2$	0·16	1·8
2910·44	(2)	$3p\ ^4P_1 - 3d\ ^4P_1$	0·16	1·8
*2906·85	(1)	$P_1 - P_2$	—	—
2872·96	(2)	$P_2 - P_2$	0·13	1·6
2869·93	(1)	$P_3 - P_3$	0·16	1·9
2876·41	(3)	$3p\ ^4P_2 - 3d\ ^4F_2$	0·16	2·0
2858·02	(1)	$P_3 - F_2$	0·16	2·0
Unclassified lines				
3164·42	(3)	—	0·19	1·9
3197·15	(2)	—	0·23	2·4
3093·99	(4)	—	0·77	8·0
3092·91	(2)	—	0·27	2·8
3088·16	(3)	—	0·77	8·0
2967·20	(3)	—	0·67	7·6
2963·29	(2)	—	0·57	6·5
2933·71	(1)	—	0·16	1·8

* Shift visible but not measurable. † Slightly confused in high-pressure spectrum.

No shift of the type already found was observed, but at the higher pressures the lines became much broader and the broadening was asymmetrical, being greater to the red. At pressures of 1 cm. and less no pronounced broadening was observed. In the use of the lines as standards it is arranged to have the pressure very much lower than this so that there can be no doubt as to their freedom from "pressure effect."

The spectrum of singly ionized neon, NeII. By increase of the violence of the discharge the first ionized spectrum of neon was easily produced, and it was photo-

* *Trans. Int. Ast. Union*, 3, 10, 18 (1928).

graphed over the region λ 6800–λ 2300 although the examination has been less exhaustive than that of any of the other spectra. Owing to the fact that the lines were much better separated, a higher pressure than for nitrogen or oxygen could conveniently be used, not so much for obtaining better measures of the large shifts as for detecting smaller ones. The pressure used was 5·0 cm. and the series gap was increased to 0·6 cm.

In the region over which photographs have been taken only twelve lines showing large shifts have been measured, but there occurred many lines showing small but quite definite shifts, and, with the high pressure that it was possible to use, these were measurable. The shifting lines are given together with the measured shifts in table 3. All the shifts are to the red.

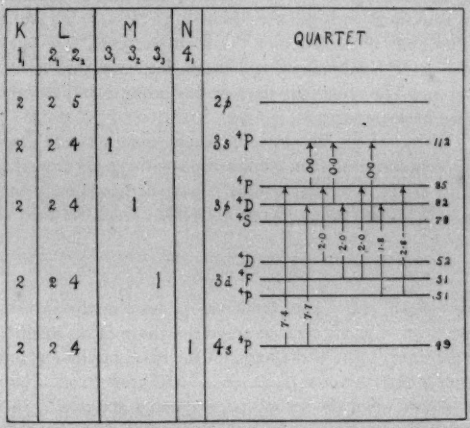


Fig. 3. Term scheme and shifts in NeII.

The wave-lengths are by L. Bloch and G. Dejardin* and the classifications are by de Bruin†. It will be observed that all the lines showing large shifts are, with four exceptions, associated with the quartet *P* term arising from the 4*s* level. The four remaining lines are as yet unclassified. The $4s\ ^4P$ term behaves then similarly to the corresponding terms in OII and NII. In the occurrence of many lines showing small shifts, the spectrum differs from OII and NII. These lines are all due to transitions between 3*d* terms and 3*p* terms, which in the two other spectra were steady. This may be partly due to the higher pressure used for neon making the detection of small shifts easier, but is certainly not due entirely to this cause.

* *Journ. d. Phys.* **7**, 129 (1926).

† *Proc. Acad. Amsterdam*, **31**, 2 (1928) and *Archives Néerlandaises des Sciences*, **2**, 132 (1928).

It should perhaps be explained here how the descriptions large and small are used. It will be observed by reference to table 3 that the so-called "small shifts" are about 2 cm.$^{-1}$, and this is actually of the same order as the "large shifts" in the NII spectrum. It will be seen later, however, that when produced in the same tube the "small" neon shifts are only about one-quarter of the nitrogen shifts, and are therefore really small compared to the NII shifts. Under these conditions the shifts in the lines of the $4s\ ^4P$ term in NeII, which in table 3 are rather more than 7 cm.$^{-1}$, are somewhat less than those of the $4s\ ^3P$ term in NII.

The term scheme and transitions in NeII are represented in figure 3. Only the quartet terms have yet been identified. It seems likely that some of the lines having large shifts, and not yet classified, belong to the $4s\ ^2P$ term, but no definite assignment has been attempted. Of the classified lines all but one of those showing large shifts are associated with the combination $3p\ ^4P - 4s\ ^4P$, and the shifts are equal within the limits of accuracy of the measurements. The other line arises from the $3p\ ^4D - 4s\ ^4P$ combination, and shows the same shift as the lines of the $3p\ ^4P - 4s\ ^4P$ group. The remaining lines in this group could be seen to shift but were too faint for measurement.

In the $3d \rightarrow 3p$ transitions the shifts are equal, except in the case of the $3d\ ^4F$ term, which shows a larger shift in combination with $3p\ ^4D$ than it does when in combination with $3p\ ^4P$. A similar double behaviour has already been noticed in the $4d\ ^3P$ term of NII, and the $4s\ ^4P$ term of OII.

§ 7. ARGON

Singly ionized argon, AII. The classifications used in the investigation of the shifts in the spectrum of singly ionized argon are those of de Bruin*. In the first paper cited he identified most of the terms of the main family (^3P limit), resulting from the addition to the core of a $3d$, $4s$, $4p$, $4d$ and $5s$ electron. The present work was almost completed when the second paper quoted appeared. In this many of the terms belonging to the families having limits ^1D and ^1S have been identified, resulting in the classification of a great many more lines. For the second analysis de Bruin made use of the very full list of wave-lengths recently published by Rosenthal†, and these have for the main part been used in the present paper.

Of the lines that have been found to shift, by far the greater number belong to the main family. Shifting lines belonging to each of the other two families have also been found, and the results obtained are much more complete than those for any of the other spectra investigated. The lines that have been observed to shift and the measured shifts are collected in table 4. The region investigated was from λ 6800 to λ 2300 although no large shifts have been observed above λ 4800, with the exception of three unclassified lines, λ 5847, λ 5929, λ 5819. Between λ 4800 and λ 3000 the list of lines published by Rosenthal agrees very well with the plates taken during the present work, but below λ 3000 the intensities of the lines on the

* *Proc. Acad. Amsterdam*, **31**, 771 (1928); **33**, 198 (1930).
† *Ann. d. Physik.* **4**, 49 (1930).

Table 4. List of AII lines showing shifts.

Table 4. List of AII lines showing shifts.

λ	Int.	Classification	dλ	dν	λ	Int.	Classification	dλ	dν
4721·62	(4)	$4p\ ^4S_2\ (^3P) - 5s\ ^4P_2\ (^3P)$	1·35	6·1	4420·94	(4)	$3d\ ^4D_1\ (^3P) - 4p\ ^4P_2\ (^3P)$	0·15	0·8
4703·36	(4)	$3d\ ^2P_1\ (^1D) - 5p\ ^2P_2\ (^1D)$	0·83	3·8	4352·20	(5)	$D_1 - P_1$	0·15	0·8
*4598·77	(5)	$3d\ ^2D_2\ (^3P) - 4p\ ^2P_2\ (^1D)$	0·51	2·4	4332·04	(5)	$D_2 - P_1$	0·16	0·8
*4474·77	(6)	$D_1 - P_1$	1·28	5·8					
					4099·47	(3)	$4p\ ^2P_1\ (^3P) - 3d\ ^2S_1\ (^1D)$	0·15	0·9
†4564·43	(5)	$4p\ ^4S_2\ (^3P) - 5s\ ^4P_1\ (^3P)$	—	—					
†4547·78	(5)	$4p\ ^2P_2\ (^3P) - 5s\ ^4P_2\ (^3P)$	—	—	4038·82	(4)	$3d\ ^4D_2\ (^3P) - 4p\ ^4D_4\ (^3P)$	0·14	0·9
					4013·85	(8)	$D_4 - D_4$	0·14	0·9
*4502·95	(5)	$4p\ ^2D_2\ (^3P) - 5s\ ^4P_3\ (^3P)$	0·59	2·9	3992·05	(4)	$D_2 - D_3$	0·15	1·0
					3968·35	(5)	$D_3 - D_3$	0·13	0·8
4498·55	(5)	$3d\ ^2P_2\ (^1D) - 5p\ ^2D_3\ (^1D)$	0·76	3·4	3944·26	(4)	$D_4 - D_3$	0·14	0·9
					3891·98	(4)	$D_3 - D_2$		
4448·88	(6)	$4p\ ^2D_3\ (^1D) - 3d\ ^2D_3\ (^1S)$	1·12	5·1	3891·40	(3)	$D_1 - D_1$	0·12	0·8
4440·09	(6)	$D_2 - D_3$	0·93	4·3					
4439·45	(3)	$D_2 - D_2$			4031·41	(2)	$4p\ ^2D_2\ (^3P) - 4d\ ^4D_1\ (^3P)$	—	—
					3988·18	(4)	$D_3 - D_3$	0·84	5·3
4433·83	(5)	$3d\ ^2D_3\ (^1D) - 5p\ ^2F_4\ (^1D)$	0·68	3·1	3958·39	(5)	$D_3 - D_2$	0·80	5·1
†4379·74	(8)	$4p\ ^2S_1\ (^3P) - 5s\ ^2P_1\ (^3P)$	—	—					
*4277·55	(8)	$4s\ ^2D_2\ (^1D) - 4p\ ^2P_2\ (^1D)$	0·46	2·5	3979·36	(7)	$4p\ ^4S_2\ (^3P) - 4d\ ^4P_1\ (^3P)$	0·77	4·9
*4237·23	(7)	$D_2 - P_2$	0·39	2·2	3932·55	(7)	$S_2 - P_2$	0·78	5·0
4275·19	(4)	$4p\ ^2P_1\ (^3P) - 5s\ ^2P_2\ (^3P)$	—	—					
4222·67	(5)	$P_2 - P_1$	0·90	5·1	3952·74	(6)	$4p\ ^4S_2\ (^3P) - 4d\ ^4F_2\ (^3P)$	0·79	5·0
4129·70	(4)	$P_1 - P_1$	—	—					
					3946·10	(7)	$4p\ ^2F_4\ (^1D) - 3d\ ^2D_3\ (^1S)$	0·83	5·3
4218·69	(5)	$4p\ ^2D_2\ (^3P) - 5s\ ^2P_2\ (^3P)$	0·94	5·3	3925·71	(3)	$F_3 - D_2$	0·80	5·2
*‡4201·58	(2)	$4p\ ^4D_1\ (^3P) - 5s\ ^4P_2\ (^3P)$	0·59	3·3	3911·58	(5)	$4p\ ^4D_1\ (^3P) - 4d\ ^4D_2\ (^3P)$	0·51	3·4
4179·31	(5)	$D_3 - P_2$	0·97	5·6	3900·63	(5)	$D_2 - D_3$	0·53	3·5
4156·11	(5)	$D_2 - P_2$	0·98	5·7	3880·34	(4)	$D_1 - D_1$	0·55	3·7
		$D_4 - P_3$	0·99	5·9	3872·15	(5)	$D_3 - D_3$	0·54	3·6
4103·91	(10)	$4p\ ^2D_3 - 5s\ ^2P_2$			†3844·75	(4)	$D_2 - D_4$	—	—
					3841·54	(3)	$D_2 - D_1$	0·51	3·5
4076·64	(5)	$^4D_1 - {}^4P_1$	1·01	6·1	3826·83	(6)	$D_3 - D_3$	0·58	4·0
4072·40	(7)	$D_3 - P_2$	0·98	5·9	†3799·39	(6)	$D_3 - D_2$	—	—
4033·83	(6)	$D_2 - P_1$	0·89	5·5	3780·84	(8)	$D_4 - D_4$	0·60	4·2
					3763·52	(5)	$D_4 - D_3$	0·58	4·1
*4131·73	(8)	$4s\ ^2D_2\ (^1D) - 4p\ ^2P_1\ (^1D)$	0·86	5·0					
					3825·70	(5)	$4p\ ^2D_2\ (^1D) - 4d\ ^2D_2\ (^1D)$	—	—
†4082·40	(6)	$4p\ ^2P_2\ (^3P) - 4d\ ^2D_3\ (^3P)$	—	—	3803·19	(6)	$D_2 - D_3$	0·88	6·1
		$4s\ ^4P_3\ (^3P) - 4p\ ^2D_3\ (^3P)$	—	—					

* Classification doubtful (or possibly two lines present).

† Shift visible but not measurable.

‡ The line corresponding to $3d\ ^2D_3\ (^1D) - 5p\ ^2D_3\ (^1D)$ should also occur here and the shift is consistent with this classification. If the line corresponding to de Bruin's classification is present it must be much weaker than $3d\ ^2D\ (^1D) - 5p\ ^2D_2\ (^1D)$.

Table 4 (*cont.*).

λ	Int.	Classification	dλ	dν	λ	Int.	Classification	dλ	dν
3809·49	(7)	$4p\ ^4P_2\ (^3P) - 5s\ ^4P_3\ (^3P)$	0·77	5·3	3535·33	(6)	$4p\ ^4P_1\ (^3P) - 4d\ ^4D_2\ (^3P)$	0·54	4·3
3770·54	(6)	$P_1 - P_2$	0·80	5·6	3514·39	(9)	$P_2 - D_3$	0·56	4·5
3765·27	(6)	$P_3 - P_3$	0·85	6·0	3509·78	(6)	$P_1 - D_1$	0·46	3·8
§3720·43	(5)	$P_2 - P_2$	—	—	3491·54	(8)	$P_3 - D_4$	} 0·50	4·2
3678·27	(5)	$P_3 - P_2$	0·76	5·6	3491·24	(6)	$P_2 - D_2$		
3622·15	(6)	$P_2 - P_1$	0·74	5·6	3476·74	(6)	$P_3 - D_3$	0·50	4·2
					3466·34	(5)	$P_2 - D_1$	0·40	3·4
†3754·06	(3)	$4p\ ^2P_1\ (^1D) - 4d\ ^2P_1\ (^1D)$	—	—	3454·10	(5)	$P_3 - D_2$	0·45	3·7
†3753·53	(4)	$4p\ ^2D_2\ (^1D) - 4d\ ^2P_2\ (^1D)$	—	—					
					3388·54	(7)	$4p\ ^2S_1\ (^3P) - 4d\ ^2P_2\ (^3P)$	0·52	4·6
3737·89	(6)	$4p\ ^2D_3\ (^1D) - 4d\ ^2F_4\ (^1D)$	0·89	6·4					
3718·21	(6)	$D_2 - F_3$	0·72	5·2	§3376·46	(7)	$4p\ ^2F_4\ (^1D) - 4d\ ^2F_4\ (^1D)$	—	—
					3350·94	(6)	$F_3 - F_3$	0·60	5·3
3660·44	(6)	$4p\ ^2P_2\ (^1D) - 4d\ ^2D_2\ (^1D)$	0·74	5·5					
					†3366·59	(4)	$4p\ ^2P_2\ (^3P) - 4d\ ^2P_1\ (^3P)$	—	—
3655·29	(6)	$4p\ ^2P_2\ (^3P) - 4d\ ^2F_3\ (^3P)$	0·70	5·2	§3307·24	(6)	$P_1 - P_1$	0·50	4·6
					§3293·66	(7)	$P_2 - P_2$	0·46	4·3
3650·90	(4)	$4p\ ^4P_1\ (^3P) - 5s\ ^2P_2\ (^3P)$	0·73	5·5	†3236·82	(4)	$P_1 - P_2$	—	—
3622·15	(6)	$P_2 - P_1$	0·74	5·6					
3603·91	(3)	$P_2 - P_2$	0·73	5·6	3281·72	(6)	$4p\ ^4P_1\ (^3P) - 4d\ ^4P_1\ (^3P)$	0·49	4·6
					3249·82	(7)	$P_1 - P_2$	0·54	5·0
3588·44	(10)	$4p\ ^4D_4\ (^3P) - 4d\ ^4F_3\ (^3P)$	0·72	5·6	3243·70	(7)	$P_2 - P_1$	0·52	4·9
3582·35	(8)	$D_2 - F_3$	0·70	5·4	3181·05	(7)	$P_3 - P_2$	0·54	5·2
†3581·62	(6)	$D_1 - F_2$	—	—	3169·68	(8)	$P_2 - P_3$	0·51	5·1
3576·62	(10)	$D_3 - F_4$	0·72	5·6	3139·02	(7)	$P_3 - P_3$	0·49	5·0
‖3548·51	(5)	$D_2 - F_2$	0·52	4·2					
3521·27	(5)	$D_4 - F_4$	0·65	5·2	¶3273·36	(4)	$4p\ ^2D_2\ (^3P) - 4d\ ^2P_1\ (^3P)$	—	—
3520·00	(6)	$D_3 - F_3$	0·62	5·0	3204·34	(5)	$D_2 - P_2$	0·46	4·5
3565·02	(5)	$4p\ ^4D_1\ (^3P) - 4d\ ^4P_2\ (^3P)$	0·60	4·8	†3263·60	(5)	$4p\ ^4P_1\ (^3P) - 4d\ ^4F_2\ (^3P)$	—	—
3421·64	(5)	$D_3 - P_3$	0·56	4·8	3194·25	(5)	$P_3 - F_2$	0·56	5·4
3370·97	(5)	$D_4 - P_3$	—	—					
					¶3163·38	(4)	$4p\ ^2S_1\ (^3P) - 4d\ ^2D_2\ (^3P)$	0·26	2·5
3561·04	(6)	$4p\ ^2F_4\ (^1D) - 4d\ ^2G_5\ (^1D)$	0·64	5·1					
3545·84	(8)	$F_3 - G_4$	0·72	5·7	*2979·05	(6)	$4s\ ^3P_1\ (^3P) - 4p\ ^2P_1\ (^1D)$	0·36	4·1
					*2891·61	(5)	$P_2 - P_1$	0·33	3·9
3559·53	(6)	$4p\ ^2D_3\ (^3P) - 4d\ ^2F_4\ (^3P)$	0·70	5·4					
3464·14	(6)	$D_3 - F_3$	0·60	5·0					

* Classification doubtful (or possibly two lines present). † Shift visible but not measurable.

‡ The line corresponding to $3d\ ^2D_3\ (^1D) - 5p\ ^2D_3\ (^1D)$ should also occur here and the shift is consistent with this classification. If the line corresponding to de Bruin's classification is present it must be much weaker than $3d\ ^2D\ (^1D) - 5p\ ^2D_2\ (^1D)$.

§ High-pressure line confused. ‖ This line always showed a definitely smaller shift than the other members of this group.

¶ Very diffuse [illegible]

Table 4 (*cont.*).

λ	Int.	Classification	dλ	dν	λ	Int.	Classification	dλ	dν
†2960·27	(4)	—	—	—	2708·28	(5)	—	0·23	3·2
†2931·49	(4)	—	—	—	†2708·12	(0)	—	—	—
2896·75	(4)	—	0·32	3·8	2647·29	(3)	$4p\ ^4S_2\ (^3P) - 6s\ ^4P_3\ (^3P)$	0·91	13·1
2775·07	(00)	—	0·22	2·8	2592·12	(0)	—	0·19	2·8
2769·74	(4)	—	0·31	4·0	2591·51	(2)	—	0·17	2·5
2767·40	(6)	—	0·36	4·7	2584·89	(3)	—	0·14	2·1
•2764·66	(3)	$4s\ ^4P_3\ (^3P) - 4p\ ^2F_3\ (^1D)$	0·52	6·8	2576·40	(2)	—	0·16	2·0
2744·82	(4)	—	0·30	4·0	2570·46	(2)	—	0·71	10·8
2732·53	(4)	—	0·38	5·1					

The following wave-lengths are by the author:

λ	Int.	ν	Classification	dλ	dν	λ	Int.	ν	Classification	dλ	dν
†2506·65	(3)	39881·9	—	—	—	†2427·52	(2)	41181·8	—	—	—
†2504·39	(3)	39917·9	—	—	—	†2427·24	(2)	41186·5	—	—	—
†2496·43	(3)	40045·1	—	—	—	†2426·20	(2)	41204·2	—	—	—
†2494·87	(3)	40070·7	—	—	—	2425·51	(5)	41215·9	—	0·17	2·9
†2488·88	(4)	40166·6	—	—	—	†2424·54	(1)	41232·4	—	—	—
2476·55	(3)	40366·6	—	0·12	1·9	†2424·32	(3)	41236·2	—	—	—
2476·07	(3)	40374·4	—	0·14	2·3	2423·98	(5)	41241·9	—	0·11	1·9
2472·96	(5)	40425·2	—	0·14	2·3	2423·55	(5)	41249·2	—	0·10	1·6
2471·91	(3)	40442·3	—	0·16	2·5	2419·98	(4)	41310·1	—	0·15	2·5
2468·70	(4)	40494·9	—	0·31	5·0	†2418·87	(5)	41329·0	—	—	—
†2464·59	(2)	40562·4	—	—	—	†2416·04	(4)	41377·5	—	—	—
†2464·27	(2)	40567·7	—	—	—	†2415·90	(4)	41379·9	—	—	—
†2463·00	(2)	40588·6	$4p\ ^2D_2\ (^3P) - 4d\ ^2P_2\ (^1D)$	—	—	†2415·67	(4)	41383·8	—	—	—
†2457·98	(2)	40671·5	—	—	—	2413·25	(6)	41425·3	—	0·14	2·3
2454·59	(4)	40727·7	—	0·15	2·4	2411·05	(5)	41463·1	—	0·15	2·5
2443·65	(3)	40910·0	—	0·45	7·4	2410·87	(5)	41466·2	—		
2441·25	(3)	40950·2	$4p\ ^2D_3\ (^3P) - 4d\ ^2D_3\ (^1D)$	0·43	7·2	2410·39	(3)	41474·4	—	0·13	2·3
2438·79	(7)	40991·5	—	0·15	2·5	2405·02	(4)	41567·0	—	0·15	2·6
2436·81	(5)	41024·8	—	0·17	2·9	†2404·59	(3)	41574·5	—	—	—
2432·79	(6)	41092·6	—	0·12	2·0	2399·22	(6)	41667·5	—	0·14	2·4
2431·57	(4)	41113·2	—	0·20	3·3	2395·67	(5)	41729·3	—	0·14	2·4

• Classification doubtful (or possibly two lines present).

† Shift visible but not measurable.

present plates show considerable divergence from those given by Rosenthal. Furthermore many of the shifting lines below λ 2500 are not given in Rosenthal's list, so that below this wave-length the values given in the table are determined from the plates taken for this investigation. Most of the shifting lines in this region are unclassified, but there is little doubt that they are due to argon; probably AII, possibly AIII.

§ 8. EFFECT OF INTENSITY ON THE MEASURED SHIFT

In the previously considered spectra NII, OII, and NeII, it was found that the measured shifts were not much affected by the length of exposure, but in the case of AII considerable variation, according to the exposure, was found in the measured shifts of a few strong lines, the shift increasing with increased exposure. It was also noticed that whereas in the other spectra all the lines of a multiplet showed the same shift, in a few multiplets in AII, notably among them $4p\,^4D - 4d\,^4F$, the lines showed different shifts, the strongest lines showing the largest shifts. By suitably arranging the length of exposure, however, the shifts were found to be more or less equal. The following list (table 5) shows how the shifts varied under different conditions.

Table 5.

λ	Int.	$d\lambda$ (i)	$d\lambda$ (ii)	Classification
3588·44	(10)	0·83	0·57	$4p\,^4D_4\,(^3P) - 4d\,^4F_5\,(^3P)$
3582·35	(8)	0·63	0·52	$D_2 - F_3$
3576·62	(10)	0·72	0·56	$D_3 - F_4$
3561·04	(6)	0·53	0·48	—
3559·53	(6)	0·59	0·58	—
3521·27	(5)	0·51	0·50	$D_4 - F_4$

(i) Measured shift—long exposure.
(ii) Measured shift—short exposure.

It will be observed that for λ 3521 the shifts are practically equal, whereas for the stronger lines of this group they are much greater in the long exposure.

The reason for this difference is undoubtedly to be found in the fact that these lines are broadened asymmetrically in the high-pressure spectrum, the intensity falling off more slowly to the red than to the violet. With moderate exposures this does not affect the measured shift, and the different members of a multiplet show approximately the same shift. As the exposure is increased a stage is reached at which the intensity maximum of the shifted line is fully exposed, and increased exposure beyond this results in a gradual movement of the centre of gravity of the line to the red, and it is the centre of gravity which is measured in order to determine the amount of shift.

Photographs with various exposures were taken (long exposures being necessary for the fainter lines), and it is thought that there is no appreciable error on account

of this intensity effect in the list of shifts given. It is also clear why no great difference was observed in the previous spectra, for the intensity was never so great as that of the strong argon lines, and in addition the shifted lines appeared more symmetrical, so that the effect would in any case be smaller.

§ 9. THE TERM SHIFTS IN AII

The term scheme for AII and approximate term values according to de Bruin* are given in figure 4. Nearly forty transitions involving shifts are indicated together with the amount of shift.

Several steady combinations have been included for completeness of description. Some of these have been asterisked to indicate that, although no definite shift was observable on the plates taken, the steadiness of the lines corresponding to these transitions is less certain than in the case of those which are not asterisked. This arises from the fact that the lines concerned occur in the red, or thereabouts, and as no large shifts were observed in this region, it was photographed only under low dispersion, so that a small shift would be less easily detected than in the violet. Moreover owing to the broadening which accompanies the use of a high gas pressure a small shift is on this account alone less evident. It is possible that these lines shift by as much as 0·4 cm.$^{-1}$, this being about the limit to be placed on their steadiness.

It will be observed that in the quartet system of the main family of terms (^3P limit) all transitions other than $4p \rightarrow 4s$ show shifts. The $4p \rightarrow 3d$ transitions show comparatively small shifts, while those from $4d$ and $5s$ terms to a lower state show large shifts. The $5s$ term exhibits a greater shift than the $4d$ terms, and to the probable accuracy of the measurements the three combinations involving the $5s$ term show the same amount of shift. ($4p\ ^4S - 5s\ ^4P$) is represented by only one line for which $d\nu = 6{\cdot}1$ cm.$^{-1}$. Of the terms arising from the $4d$ configuration $4d\ ^4P$ and $4d\ ^4F$ are affected more or less equally, but $4d\ ^4D$ shows quite definitely a smaller shift than the other two. The lines arising from $4d\ ^4D$ are several in number and there can be little doubt that this lower value is correct.

Only one line belonging to a $6s \rightarrow 4p$ transition has been measured, but the shift of 13·1 cm.$^{-1}$ is much greater than that for the $5s$ term and is not expected to be in error by more than 0·6 cm.$^{-1}$.

In the doublet system of the main family the shifts are rather smaller than the corresponding ones in the quartet system, but are of the same magnitude. No small shifts have been measured, but as was mentioned above the asterisked zero shifts may in fact be small ones. The doublet combinations corresponding to $3d\ ^4P - 4p\ ^4P$ and $3d\ ^4D - 4p\ ^4D$ are out of range.

For the same configuration of the series electron, the values of the terms based on the ^1D state of the core are much smaller than those of the terms based on the ^3P state. Thus $3d\ ^2S\ (^1D)$ is actually smaller than $4p\ ^2P\ (^3P)$ and the line λ 4099, $\{4p\ ^2P_1\ (^3P) - 3d\ ^2S_1\ (^1D)\}$, shows a small shift. Of the combinations involving terms of the ^1D family only the $4p \rightarrow 4s$ transitions are steady, as for the ^3P family,

* *Proc. Acad. Amsterdam*, **33**, 198 (1930).

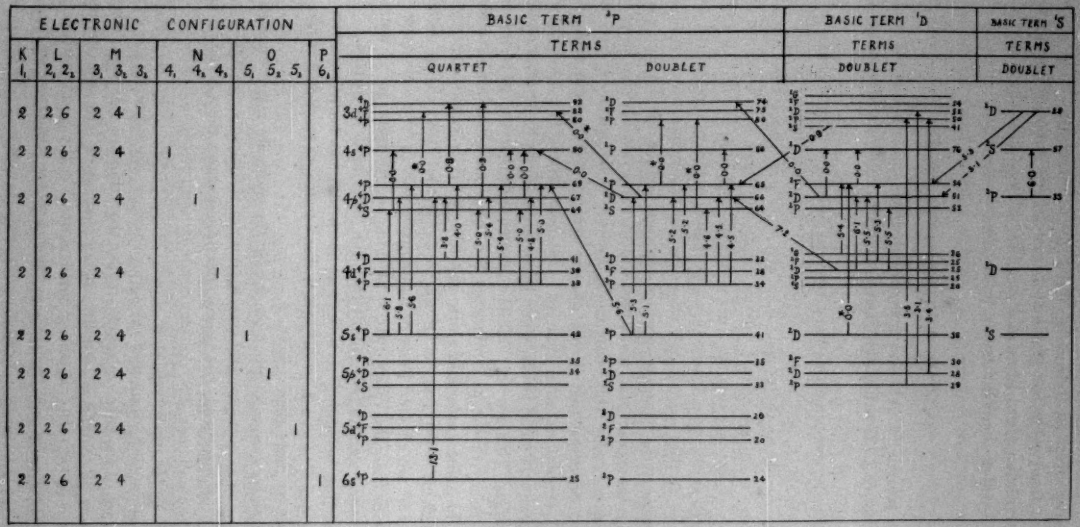


Fig. 4. Combinations and corresponding shifts in AII.

and the $4d \rightarrow 4p$ transitions show rather larger shifts than the corresponding ones in the main family. Several lines which arise from the $5p$ terms have been found to shift. The lower terms in this case are of the configuration $3d$ and the shifts are rather more than half as large as those for the $4d \rightarrow 4p$ transitions.

The lines assigned by de Bruin to a $5s \rightarrow 4p$ transition in the ^1D family are steady as far as can be ascertained, and it was at first thought that this was not likely to be the case for such lines, since the $5s \rightarrow 4p$ transitions in the main family showed large shifts. However a consideration of the results for the various spectra indicates that it is probably consistent with the other data. This will be referred to again later.

The observations in the case of the ^1S family are very few, but it is of interest to note that here the term values are still smaller than the corresponding ones of the ^1D family, and that the $4p\ ^2P\ (^1D) - 3d\ ^2D\ (^1S)$ combination shows a large shift. Of the lines arising from $4p\ ^2P\ (^1S) - 4s\ ^2S\ (^1S)$ one shows a shift of 6·0 cm.$^{-1}$, while the other shows apparently only a very small shift. As, however, there is some confusion in the latter case, the former observation has been assumed to represent the behaviour of the doublet.

In all of the transitions so far considered, the lines of any given multiplet have shown the same shift to the limits of accuracy of the observations, provided there is no great intensity difference between the lines. In the case of combinations with $4p\ ^2P\ (^1D)$ the following were the observed shifts (table 6).

Table 6.

λ	Int.	ν	Classification	$d\nu$
4598·77	(5)	21738·9	$3d\ ^2D_2\ (^3P) - 4p\ ^2P_2\ (^1D)$	2·4
4474·77	(6)	22341·3	$D_2 \quad P_1$	5·8
4277·55	(8)	23371·3	$4s\ ^2D_2\ (^1D) - 4p\ ^2P_2\ (^1D)$	2·5
4237·23	(7)	23593·7	$D_2 \quad P_2$	2·2
4131·73	(8)	24196·1	$D_2 \quad P_1$	5·0
2979·05	(6)	33557·9	$4s\ ^2P_1\ (^3P) - 4p\ ^2P_1\ (^1D)$	4·1
2891·61	(5)	34572·7	$P_2 \quad P_1$	3·9

In addition to the lines of the first two doublets showing different shifts, the lines of the third doublet show shifts which are equal to each other but different from any of the former. It will be noticed that in the first two doublets the shifts associated with $4p\ ^2P_1$ are roughly equal, as are those of the lines assigned to $4p\ ^2P_2$. The terms of this level, $4p\ ^2D$ and $4p\ ^2F$, do not, however, show any shift, and this seems to be the correct behaviour of terms of this level. It is probable therefore that the lines given below have been wrongly classified.

Oxygen group at $\lambda 3217$ *and CII doublet at* $\lambda 2837$. In addition to the spectra already considered there were observed shifts to the red in a group of three lines in the oxygen spectrum, suggested by Fowler* as being due to OI. The lines and the shifts of two of them are given in table 7.

The third line does not appear to shift.

* *Proc. R.S.* A, **110**, 476 (1926).

Table 7.

λ	Int.	$d\lambda$	dv
3218·04	(3)	0·48	4·7
3216·74	(2)	0·46	4·5
3216·01	(2)	—	—

The shifts are comparable with those in OII, being measured on the same plates. It will be seen that they are slightly larger than those associated with the $4s$ terms in OII. The spectrum of neutral oxygen, OI, has been much more fully investigated recently, but the above lines do not find a place in the scheme. It is possible that they are connected with a term of the ^1S family in OII.

A doublet due to singly ionized carbon occurring as impurity in the oxygen spectra also showed a shift to the red (table 8), the displacement being much smaller than in the oxygen groups.

Table 8.

λ	$d\lambda$	dv	Classification
2837·60	0·10	1·3	$2p'^2S_{\frac{1}{2}} - 3p\ P_{\frac{1}{2}}$
2836·71	0·12	1·5	$S_{\frac{1}{2}} - P_{\frac{3}{2}}$

Wave-lengths and classifications are by Fowler*. The corresponding doublet in OII has already been considered, the shift being 2·0 cm.$^{-1}$.

§ 10. COMPARISON OF THE SHIFTS IN DIFFERENT SPECTRA

It was found in the previous work on NII that several factors could affect the amount of shift if the primary exciting conditions were left the same. Hence to compare the shifts in the lines of different spectra it was necessary to ensure that the disturbing forces should be the same for the different atoms, and the best method seemed to be to excite the spectra at the same time in the same tube. It was not possible to produce a photograph with all the spectra considered appearing at once, so that various combinations of two spectra have been investigated. In some cases mixtures in more or less equal proportions of the gases concerned have been employed; in others, the occurrence of air as impurity has been responsible for the nitrogen element.

The following comparisons have been made: (*a*) Oxygen and nitrogen. The latter was present as impurity. (*b*) Oxygen and argon. In the first comparison roughly equal proportions of each gas were employed. In the second comparison the argon lines occur as impurity on oxygen plates. (*c*) Argon and nitrogen. A mixture of the two gases in a ratio of N : A : : 1 : 2 approximately. (*d*) Neon and nitrogen. Nitrogen lines due to air impurity. (*e*) Neon and argon. A mixture of the two gases in roughly equal proportions. Only one plate was taken and that not very suitable for measurement.

In this way it has been possible to correlate the term shifts in the various spectra, one assumption being made. It was found in the case of NII, and has been cor-

* *Proc. R.S.* A, **120**, 312 (1928).

roborated in several instances in the present work, that with changed exciting conditions the shifts of the lines are altered in amount, but that they are all changed in the same ratio. This has been assumed to hold for all the spectra in reducing them to a common basis. For simplicity, it has further been arranged to make the comparison taking one term only to represent a particular spectrum.

Thus in (*a*) the $4s\ ^4P$ term in OII is compared directly with the $4s\ ^3P$ term in NII. The representative argon term is $5s\ ^4P$, but no shifting lines involving this term were observable on the photographs of the mixed gases, so that the comparison in (*b*) is made by calculation of what the shift in $5s\ ^4P$ would be under the conditions yielding the shifts in (i), and comparison of this with the observed shift of $4s\ ^4P$ in OII under the same conditions. The results obtained were as shown in table 9.

Table 9.

(*a*) *Oxygen and nitrogen.*

NII			OII		
λ	classification	dv	λ	classification	dv
3838	$3p\ ^3P_2 - 4s\ ^3P_2$	4·2	3277	$3p\ ^4P_2 - 4s\ ^4P_3$	4·1
4227	$3p\ ^1P_1 - 4s\ ^1P_1$	3·8	3287	$P_3 - P_3$	4·2
			3290	$P_1 - P_2$	4·0
	Mean...	4·0		Mean...	4·1

$$\therefore \frac{\text{OII}\ (4s\ ^4P)}{\text{NII}\ (4s\ ^3P)} = \frac{4{\cdot}1}{4{\cdot}0} = 1{\cdot}03.$$

(*b*) *Oxygen and argon.*

1st comparison

	AII	dv			OII	
λ	classification	(i)*	(ii)†	(i/ii)	λ	dv (i)
3561	$4p\ ^2F_4\ (^1D) - 4d\ ^2G_5\ (^1D)$	3·8	5·1	0·75	3277	5·0
3559	$4p\ ^2D_3\ (^3P) - 4d\ ^2F_4\ (^3P)$	4·4	5·4	0·81	3287	4·5
3545	$4p\ ^2F_3\ (^1D) - 4d\ ^2G_4\ (^1D)$	4·1	5·7	0·72	3290	4·7
		Mean ratio...		0·76	Mean...	4·7

Then if dv in $5s\ ^4P$ in AII (table 4 and figure 4) is taken as 5·7 (adopted mean), under conditions (i) it would be $5{\cdot}7 \times 0{\cdot}76$.

$$\therefore \frac{\text{AII}\ (5s\ ^4P)}{\text{OII}\ (4s\ ^4P)} = \frac{5{\cdot}7 \times 0{\cdot}76}{4{\cdot}7} = 0{\cdot}92.$$

2nd comparison

	AII	dv			OII	
λ	classification	(i)	(ii)	(i/ii)	λ	dv (i)
3588	$4p\ ^4D_4\ (^3P) - 4d\ ^4F_5\ (^3P)$	4·2	5·6	0·75	3277	5·0
3576	$D_3 - F_4$	4·2	5·6	0·75	3287	4·5
3561	$4p\ ^2F_4\ (^1D) - 4d\ ^2G_5\ (^1D)$	3·9	5·1	0·76	3290	4·6
3559	$4p\ ^2D_3\ (^3P) - 4d\ ^2F_4\ (^3P)$	4·6	5·4	0·85		
		Mean ratio...		0·78	Mean...	4·7

$$\therefore \frac{\text{AII}\ (5s\ ^4P)}{\text{OII}\ (4s\ ^4P)} = \frac{5{\cdot}7 \times 0{\cdot}78}{4{\cdot}7} = 0{\cdot}95.$$

Mean of the two comparisons: $\dfrac{\text{AII}\ (5s\ ^4P)}{\text{OII}\ (4s\ ^4P)} = 0{\cdot}94.$

* The values given in column (i) are the shifts measured on the photographs of the mixtures of the various gases.

† The values given in column (ii) are taken from the complete list of argon lines given in table 4.

(*c*) *Argon and nitrogen.*

λ	AII classification	dv (i)	dv (ii)	dv (i/ii)	λ	NII classification	dv (i)
3979	$4p\ ^4S_2\ (^3P) - 4d\ ^4P_1\ (^3P)$	3·7	4·9	0·76	3838	$3p\ ^3P_2 - 4s\ ^3P_2$	4·0
3952	$4p\ ^4S_2\ (^3P) - 4d\ ^4F_2\ (^3P)$	3·8	5·0	0·76	3830	$P_1 - P_2$	4·0
3946	$4p\ ^2F_4\ (^1D) - 3d\ ^2D_3\ (^1S)$	4·1	5·3	0·77	—	—	—
3932	$4p\ ^4S_2\ (^3P) - 4d\ ^4P_2\ (^3P)$	3·4	5·0	0·68	3609	$3p\ ^3S_1 - 4s\ ^3P_1$	4·0
3925	$4p\ ^2F_3\ (^1D) - 3d\ ^2D_3\ (^1S)$	4·0	5·2	0·77	3593	$S_1 - P_2$	3·9
3868	$4p\ ^4S_2\ (^3P) - 4d\ ^4P_3\ (^3P)$	3·2	5·0	0·64			
3809	$4p\ ^4P_2\ (^3P) - 5s\ ^4P_3\ (^3P)$	3·3	5·3	0·62			
		Mean ratio...		0·71			Mean... 4·0

$$\therefore \frac{\text{AII}\ (5s\ ^4P)}{\text{NII}\ (4s\ ^3P)} = \frac{5{\cdot}7 \times 0{\cdot}71}{4{\cdot}0} = 1{\cdot}01.$$

(*d*) *Neon and nitrogen.*

NeII	NII		
	λ	classification	dv
Mean shift for the $4s\ ^4P$ group as given in list of NeII shifts	3838	$3p\ ^3P_2 - 4s\ ^3P_2$	9·3
	3006	$3p\ ^1D_2 - 4s\ ^1P_1$	9·2
= 7·4		Mean...	9·3

$$\therefore \frac{\text{NeII}\ (4s\ ^4P)}{\text{NII}\ (4s\ ^3P)} = \frac{7{\cdot}4}{9{\cdot}3} = 0{\cdot}80.$$

(*e*) *Neon and argon.*

λ	AII	dv (i)	dv (ii)	dv (i/ii)	λ	NeII classification	dv (i)
3780	$4p\ ^4D_4\ (^3P) - 4d\ ^4D_4\ (^3P)$	6·7	4·2	1·60	2792	$3p\ ^4P_3 - 4s\ ^4P_3$	6·7
3476	$4p\ ^4P_3\ (^3P) - 4d\ ^4D_3\ (^3P)$	6·5	4·2	1·55			
		Mean ratio...		1·57			

$$\therefore \frac{\text{NeII}\ (4s\ ^4P)}{\text{AII}\ (5s\ ^4P)} = \frac{6{\cdot}7}{1{\cdot}57 \times 5{\cdot}7} = 0{\cdot}75.$$

The ratios are represented, to the accuracy of the determinations, by the following arrangement:

	$\frac{\text{OII}}{(4s\ ^4P)}$	:	$\frac{\text{NII}}{(4s\ ^3P)}$	:	$\frac{\text{AII}}{(5s\ ^4P)}$	:	$\frac{\text{NeII}}{(4s\ ^4P)}$	:	$\frac{\text{OIII}^*}{(4s\ ^1P)}$
	1·00		0·96 ± 0·05		0·96 ± 0·05		0·77 ± 0·04		0·93 ± 0·05
or	4·1		3·9 ± 0·2		3·9 ± 0·2		3·2 ± 0·2		3·8 ± 0·2

if we take OII ($4s\ ^4P$) = 4·1 as the basis of the comparison.

In this comparison the following terms have been taken as steady:

OII	NII	AII	NeII	OIII
$3p\ ^4S$	$3p\ ^3S$	$4p\ ^4P$	$3p\ ^4P$	$3p\ ^1P$

The calculated values of the ratios are seen to agree very well with the observed:

	$\frac{\text{OII}}{\text{NII}}$	$\frac{\text{AII}}{\text{NII}}$	$\frac{\text{AII}}{\text{OII}}$	$\frac{\text{NeII}}{\text{NII}}$	$\frac{\text{NeII}}{\text{AII}}$
Observed	1·03	1·01	0·94	0·80	0·75
Calculated from the above arrangement	1·04	1·00	0·96	0·80	0·80

The adopted arrangement seems therefore to be justified, the agreement actually being better than can safely be expected from the measurements. Hence the possible error is given in the comparison as 5 per cent. although in some cases it may be much smaller if a large number of shifts is concerned.

* The ratio of OIII to OII was determined in the investigation of the oxygen plates.

It appears, then, that for the same electron configuration ($4s$) the spectra OII, NII, OIII show very nearly the same shift, while NeII shows about four-fifths of this shift. In AII $5s\ ^4P$ is affected to roughly the same extent as the $4s$ terms of the other spectra, and, as the terms of AII are very similar to those of NeII, provided that for a given term of total quantum number n in the latter case we take the term having total quantum number $(n + 1)$ in the former, this is what might be expected.

By use of the comparison just made the term values and corresponding shifts for all the terms considered are given in table 10. The terms with zero shift have been adopted as steady. In AII, for the quartet terms of the main family, two sets of shifts are given according to whether the $4p$ terms or the $3d$ terms are assumed steady. The latter (column (ii)) is probably more correct but the former is given because in the other spectra it has been necessary to assume the analogous terms ($3p$) to be steady.

It has been seen that for the $4s$ configuration (in the case of AII the $5s$) the spectra considered all show the effect to more or less the same extent. While a detailed study of the data does not reveal an exact correspondence between the other terms, it is of interest to consider one or two comparisons.

Firstly, the iso-electronic system giving rise to NII and OIII. The term schemes for these two are identical, the difference between the spectra being in the actual term values, which are roughly in the ratio OIII : NII : : 9 : 4; i.e. in the ratio of the squares of the effective nuclear charges. The values are given in table 10.

It will be observed that for three of the four terms for which shift data have been obtained, the term shifts in OIII and NII are roughly equal, but in the remaining case ($4p\ ^1D$) the two shifts are considerably different. It may be that the $4p\ ^1D$ term in one of the spectra has not been correctly identified.

Secondly, it may be noticed that for successive ionization of the same atom (OII and OIII being the corresponding spectra) the terms corresponding to the same configuration of the series electron are shifted to approximately the same extent. Here, as in the first comparison, the term values OIII : OII are in a ratio of about 2 : 1 if we consider the main family in OII, and considerably more than this if we are concerned with the family having the ^1D limit. In the third place we may compare the shifts in the spectra of successive members of a column of the periodic table, i.e. NeII and AII. It has been shown by de Bruin* how close is the connexion between the two spectra when the argon term has total quantum number greater by 1 than the neon term with which it is compared. We notice in table 10 that in AII the term value of $5s\ ^4P$ is approximately the same as $4d\ ^4P$, and similarly, in NeII $4s\ ^4P$ is roughly the same as $3d\ ^4P$. In the extent to which they shift, however, there is a complete difference, for whereas in AII the terms mentioned show approximately the same shift, in NeII the ratio of the shift of $4s\ ^4P$ to that of $3d\ ^4P$ is more than three to one. In showing a small shift the $3d$ terms of NeII fall intermediate between the $3d$ terms of OII, NII, and OIII, and the $4d$ terms of AII.

* *Proc. Acad. Amsterdam*, **31**, 593 (1928).

Table 10.

OII

^3P limit			^1D limit		
Term	Value (cm.$^{-1}$)	$d\nu$ (cm.$^{-1}$)	Term	Value (cm.$^{-1}$)	$d\nu$ (cm.$^{-1}$)
$2p'\ ^2S_1$	87311	−2·0	—	—	—
$3p\ ^2S_1$	79079	0·0	$3p\ ^2P_1$	50541	0·0
$3p\ ^2P_1$	68851	0·0	—	—	—
$3p\ ^2D_2$	71499	0·0	$4s\ ^2D_2$	23734	4·0
$3p\ ^4S_2$	70859	0·0	—	—	—
$3p\ ^4P_1$	74675	0·0	—	—	—
$3p\ ^4D_1$	76290	0·0	—	—	—
$4s\ ^2P_1$	42692	5·8	—	—	—
		4·2	—	—	—
$4s\ ^4P_1$	44395	4·1	—	—	—

OIII

			NII		
$3d\ ^3F_3$	119935	0·0	$3d\ ^3F_3$	52275	0·0
$3d\ ^3D_2$	117316	0·0	$3d\ ^3D_2$	51384	0·0
$3d\ ^3P_1$	115011	0·0	$3d\ ^3P_1$	49937	0·0
$3d\ ^1D_2$	111816	0·0			
$4p\ ^3S_1$	76641	3·4	$4s\ ^3P_1$	42254	3·9
$4p\ ^3D_2$	77999	3·3	$4p\ ^3S_1$	35314	3·2
$4s\ ^1P_1$	91712?	3·8	$4p\ ^3D_2$	36081	2·8
$4p\ ^1D_1$	78870	2·6	$4s\ ^1P_1$	40987	3·9
			$4p\ ^1D_2$	36153	3·9
	NeII		$4p\ ^3P_1$	35658	2·2
$3p\ ^4P_1$	85033	0·0	$4d\ ^3F_2$	29107	2·8
$3p\ ^4D_1$	81791	0·0	$4d\ ^3D_2$	28580	3·9
$3p\ ^4S_2$	78678	0·0	$4d\ ^3P$	28095	5·5
$3d\ ^4D_1$	52208	0·9			4·1
$3d\ ^4F_2$	50459	0·9	$4p\ ^1S_0$	36677	1·8
? $3d\ ^4P_1$	50684	0·8	$4d\ ^1D_2$	28920	3·7
		1·1	$5s\ ^3P_1$	24589	9·2
$4s\ ^4P_1$	48951	3·2	$5s\ ^1P_1$	24019	10·3

AII

^3P limit				^1D limit		
Term	Value (cm.$^{-1}$)	$d\nu$ (i) (cm.$^{-1}$)	$d\nu$ (ii) (cm.$^{-1}$)	Term	Value (cm.$^{-1}$)	$d\nu$ (cm.$^{-1}$)
$3d\ ^4D_3$	92273	−0·6	0·0	$4s\ ^2D_2$	76134	0·0
$3d\ ^4F_4$	82037	−0·6	0·0	$4p\ ^2F_3$	54354	0·0
$4s\ ^4P_2$	89668	0·0	0·6	$4p\ ^2D_2$	51407	0·0
$4p\ ^4P_2$	69403	0·0	0·6	$4d\ ^2G_4$	26160	3·1
$4p\ ^4D_3$	67081	0·0	0·6	$4d\ ^2F_3$	24520	3·7
$4d\ ^4D_3$	40957	2·7	3·3	$4d\ ^2D_2$	25230	4·0
$4d\ ^4F_4$	39130	3·7	4·3			
$4d\ ^4P_2$	38284	3·4	4·0	$5p\ ^2F_3$	29893	2·1
				$5p\ ^2D_2$	28133	2·3
$5s\ ^4P_2$	42533	3·9	4·5	$5p\ ^2P_1$	28664	2·6
$6s\ ^4P_2$	25617	9·0	9·6			
					^1S limit	
$3d\ ^2D_2$	74280	0·0	—			
$3d\ ^2F_3$	74607	0·0	—	$4s\ ^2S_1$	57447	0·0
$4p\ ^2P_1$	65048	0·0	—	$3d\ ^2D_2$	28888	3·6
$4p\ ^2D_2$	65361	0·0	—	$4p\ ^2P_1$	32421	4·1
$4p\ ^2S_1$	63665	0·0	—	—	—	—
$4d\ ^2F_3$	37165	3·6	—	—	—	—
$4d\ ^2P_1$	34820	3·1	—	—	—	—
$5s\ ^2P_1$	40840	3·6	—	—	—	—

§ 11. RELATION BETWEEN TERM SHIFT AND TERM VALUE

It has been pointed out that in OII the shift in $4s\ ^2D$ (^1D family) is the same as that in $4s\ ^4P$ (^3P family), although the values of these terms are quite different. Similarly it has been noticed that in the iso-electronic systems, NII and OIII, corresponding terms show the same amount of shift, although the term values are different. This would suggest that it is the configuration of the series electron that chiefly determines the magnitude of the shifts. In the ^1D family of AII, however, the lines assigned to the transition $5s\ ^2D$ to $4p\ ^2F$ are steady, i.e. $5s\ ^2D$ is steady. On the foregoing evidence this was not expected, since the $5s$ terms in the main family are appreciably affected. As the lines concerned are quite strong, and seem to be correctly classified, an attempt was made to see if the results were reconcilable.

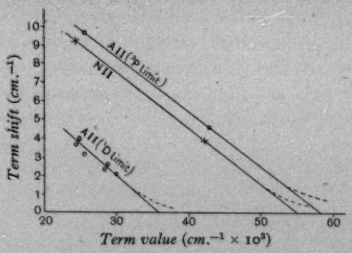


Fig. 5. Relation between term shift and term value.

For the $4s$ and $5s$ triplet terms of NII the term shift was plotted against the term value, figure 5. These were joined by a straight line, and the line was produced to cut the frequency axis. It intersects the latter at about ν55,000, which is roughly the value of the terms in NII which are assumed to have zero shifts. The points representing the results for the $5s$ and $6s$ terms of the main family of AII lie very near the NII line, and on production of the line joining these points it intersects the frequency axis at ν58,000, at which term value the shift is certainly small, although not quite zero.

If now the observations for the terms of the ^1D family of AII are plotted, and the best straight line is drawn through them, it is roughly parallel to the others and cuts the axis at ν36,000 approximately, which is less than the term value of $5s\ ^2D$ (^1D) (ν38,000). That is to say, if we can attach any importance to the approximately linear relationship between term shifts and term values, which is indicated by the results for the main family of AII and for NII, then the deduction is that in the ^1D family a term having a value ν38,000 should show at any rate a very small if not zero shift, and this is what has been found experimentally.

The probable courses of the curves in the neighbourhood of the axis are shown dotted: it is unlikely that the curves would intersect the axis abruptly. It must be borne in mind that the shift is probably determined only partly by the term value, and the graphical representation indicates only the general behaviour.

A consideration of the inter-combination shifts between the ^1D family and the main family suggests that the term shifts for the terms of the ^1D family given in table 10 may be low by about 1·0 cm.$^{-1}$ in comparison with those in the main family. Any correction on this account would increase all the shifts of the ^1D family by the same amount, and would have the effect of shifting the ^1D curve in figure 5 parallel to itself, but would not affect the inference drawn with regard to the behaviour of $5s\ ^2D\ (^1D)$.

The curves would indicate therefore that there is a more or less regular variation of the term shift with term value. In addition, they show that in the case of AII, for a given term value, the terms of the ^1D family are less perturbed than those of the main family, and as the shift appears to be mainly a Stark effect, it means that the ^1D family is more stable in the presence of a disturbing electric force than is the main family.

The shifts for terms with the same total quantum number but different azimuthal quantum numbers in NII and AII, when plotted in the above way, do not suggest any regularity between term shift and term value, but it is probable that other factors are responsible for the irregularities.

§ 12. EXAMINATION OF THE SHIFTED LINES WITH A MICROPHOTOMETER

Since the completion of the foregoing work, it has been possible to examine the nature of the lines with a microphotometer (Cambridge Instrument Co.) and the lower part of the plate illustrates some typical intensity curves.

In the first place it will be noticed that the shifted lines are not symmetrical although in the oxygen groups the asymmetry is not very pronounced. The NeII line λ2809·5 is likewise fairly symmetrical. λ6357 in NII seems rather less symmetrical, and λ3781 in AII is still less symmetrical. This line probably represents the least symmetrical of the lines measured.

An endeavour was made in measuring the lines to estimate the centre of gravity of the shifted line, and it seems probable that no appreciable error has been introduced on account of the asymmetry of the shifted lines.

By comparing a photometer record of the low pressure spectrum with a record of the same spectrum under high pressure it can be seen that almost all the broadening is on the long wave-length side. This is seen better in the reproductions of the actual spectra in the upper part of the plate.

§ 13. DISCUSSION OF RESULTS

The displacement of lines associated with the higher terms of a spectrum under certain conditions of excitation has been noticed in several spectra, but it is believed that the present paper and the former one on NII represent the first systematic attempt to investigate the effect, at least as far as the spectra of ionized gases are concerned. The foregoing results show that the "pressure effect" observed first in NII is quite general and affects each of the spectra investigated in much the same

way. K. Asagoe has observed similar shifts in the spectra of chlorine*, bromine and iodine† when excited at relatively high pressures. He did not, however, attempt any correlation of the shifts. More recently‡ he has investigated the Stark effect for many of the lines of these spectra. The results show that the shift caused by raising the pressure of the gas in the discharge tube is in nearly every case of the same type as the Stark displacement.

This strengthens the conclusion arrived at by the author with regard to the shifts in NII, and indicates that although the "pressure effect" is probably not a very pure Stark effect, owing to the variation of the disturbing electric force, it may prove useful in cases where the Stark effect is not determinable in the ordinary way. From the scarcity of Stark effect data for the spectra of ionized gases it seems that these spectra are not easy of investigation by the ordinary methods.

In a recent paper C. J. Humphreys records having observed§ a change of wave-length of many of the arc lines of krypton and xenon when the conditions were such as to excite also the spark spectra, and he attributes the displacement to Stark effect. The lines were under these conditions noticeably diffuse and asymmetrical, with their centres of gravity displaced in the direction of increasing wave-length. With uncondensed discharges, exciting only the spectra of the neutral atoms, the lines are recorded as being exceedingly sharp and perfectly reproducible. This agrees with the present observations on the spectrum of neutral neon.

The spectra of ionized nitrogen, NII and NIII, have quite recently been investigated by K. Asagoe‖ by the same method that I employed; the chief differences being in the use of air instead of nitrogen, and in the use of higher pressures (7 cm. and 1 atmosphere). As a consequence of the use of these higher pressures his shifts are somewhat larger than those that I¶ recorded but are otherwise in general agreement. He finds, however, that although for a pressure of 7 cm. his term shifts are all larger than mine in the same ratio (as I had found in my experiments for two different pressures), at a pressure of 1 atmosphere the ratio is different for different terms, being greater for $4s\ ^3P$ than for $4p\ ^3D$ and $4p\ ^3P$. This is important in reducing shifts in different terms to a common basis as has been done in the present paper; but as the pressures and resulting shifts have, in the present case, been kept below the stage at which a departure from the constant ratio has been observed by Asagoe, it is not likely that the adoption of the same reduction factor for all the terms of a spectrum has introduced any error.

In NIII Asagoe finds that lines of the $3p \rightarrow 3s$ transitions show a shift to the violet, but even at the higher of the pressures used the displacements are quite small. No large shifts to the red such as have been observed in NII are recorded. By a consideration of the shifts of some previously unclassified lines in NII it was possible to identify three new singlet terms. The hope was then entertained that

* *Mem. Coll. Sci. Kyoto. Univ.* A, **10**, 15 (1926).
† *Jap. Journ. Phys.* **4**, 85 (1927).
‡ *Sci. Pap. Inst. Phys. Chem. Res. Tokyo*, **11**, 243 (1929).
§ *Bur. Standards Journ. Res.* **5**, 1041 (1930).
‖ *Sci. Rep. Phys. Inst. Univ. Tokyo*, **1**, 47 (1930).
¶ In my experiments much larger shifts than those published were obtained but were not considered to be susceptible of sufficiently accurate measurement.

the shift might prove useful in the analysis of spectra. The data for a spectrum of fair complexity, such as AII, although indicating which lines arise from steady and which from perturbed terms, do not assist very much further in the analysis of the spectrum, chiefly owing to the fact that the shifts are so much of the same magnitude. The shifts nevertheless afford a good criterion as to whether the lines assigned to a particular combination are all associated with the same pair of terms.

§ 14. DESCRIPTION OF PLATE

The upper part shows typical shifts in four of the spectra investigated, viz. AII, OII, NeII and NII. In each case there are two spectra in juxtaposition, the low pressure spectrum serving as a standard of reference for detecting the shifted lines. The lower part of the plate comprises microphotometer records of some of the lines investigated. In the OII record, a curve of the high pressure spectrum is arranged above the curve of the same part of the spectrum at low pressure. The vertical lines are drawn in the positions where the maxima of the lower spectrum would occur if the two were superposed. It can be thereby seen which lines are shifted, and the nature of the shift. Below are records of three other shifted lines. In each of the AII and NII records there occurs a steady line which, it will be observed, is practically symmetrical. The faint vertical line in the NeII record is irrelevant.

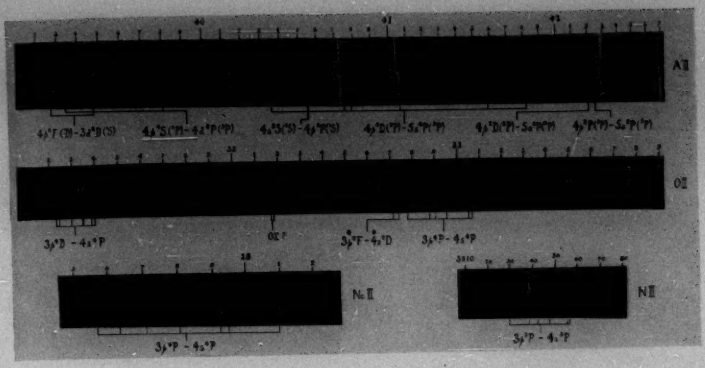
§ 15. ACKNOWLEDGMENT

The author has much pleasure in recording his deep appreciation of the interest which Prof. A. Fowler has shown throughout the investigation.

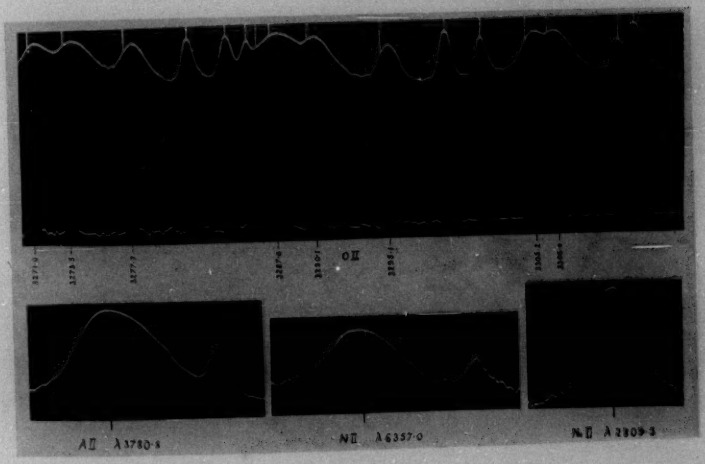
DISCUSSION

Mr M. C. Johnson. From the point of view of studies of intermolecular forces it is very important that experimental spectroscopists should investigate carefully, as Mr Pretty has done in this paper, the possible distortion of lines by Stark effect. Such work may make possible a decision as to the value of theories of the disturbance of lines by intermolecular fields, such those put forward by Holtsmark and his followers. In comparing those Stark-effect theories with experiment I have generally been forced to conclude that the concentration of free positive and negative charges is too small for the amount by which the line is actually shifted or broadened. Similarly, in the broadened lines of star spectra the Stark effect is qualitatively satisfying but quantitatively inadequate, except perhaps as the modified theory of Stark disturbance suggested by Sir Arthur Eddington in his *Internal Constitution of the Stars*, where he gives reasons for expecting from transitory fields a larger Stark effect than that measured in steady electric fields.

(*a*)



(*b*)



PRACTICAL INVESTIGATIONS OF THE EARTH RESISTIVITY METHOD OF GEOPHYSICAL SURVEYING

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ABSTRACT. The paper deals with the methods of geophysical survey which depend on the measurement of the electrical resistivity of the earth. The theory is explained for the case of a single horizontal stratum, and practical tests are described and discussed.

§ 1. INTRODUCTION

GEOPHYSICAL methods have aroused a considerable amount of interest recently in view of the work which has been done all over the world with various types of apparatus. These methods are all based on measurements of variation in some property of the earth such as its density, elasticity, magnetic permeability, and electrical conductivity; some of them have given satisfactory results in the search for mineral bodies. They may be classified as gravimetric, seismic, magnetic and electrical, and the earth resistivity method is one of the simplest of the electrical type. It is the purpose of this paper to explain the theory underlying this method in the simple case of a single horizontal stratum, and to show both theoretically and practically how it can be applied.

§ 2. THE METHOD

The earth resistivity method is based on measurements of the apparent specific resistance of the earth. The presence of any body in the earth, the specific resistance of which differs from that of the surrounding material, will affect the apparent specific resistance to an extent which is dependent on a number of different factors. The method thus affords a means of detecting the presence of such bodies in the earth. Whether the results can be used to determine the exact position of the bodies remains to be seen.

§ 3. THEORY OF THE METHOD

(*a*) *Fundamental theory.* A method of measuring specific resistance of homogeneous earth is described in a United States Bureau of Standards paper entitled "A Method of Measuring Earth Resistivity" by F. Wenner. Four electrodes are driven into the earth in a straight line at equal intervals *a* as shown in figure 1, *a*

I and a current I amperes is passed between the two outer electrodes, and the potential
V difference V between the two inner electrodes is measured.

ρ Then if a is the electrode-interval, and ρ is the specific resistance of the earth,

$$\rho = 2\pi a V/I \qquad \ldots\ldots(1).$$

If a is expressed in inches, ρ will be given in ohms/in.[3]. If a is in centimetres, ρ will be in ohms/cm.[3]. The fraction V/I may be termed a resistance and designated
R R. Then formula (1) may be written

$$\rho = 2\pi a R \qquad \ldots\ldots(2).$$

Since it is only under special conditions that the earth involved in the measurements can be considered homogeneous, the value of ρ obtained by means of these formulae is only an apparent specific resistance, and will be affected by the extent of the non-homogeneity and the electrode separation.

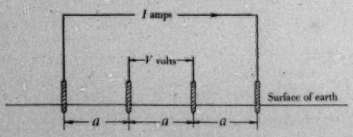

Fig. 1.

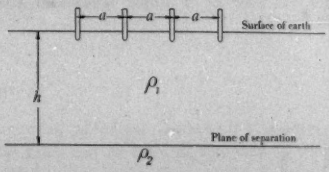

Fig. 2.

(*b*) *Single horizontal stratum.* One of the simplest cases met with is that of a single horizontal underlying stratum, and the presence of this stratum will affect the value of the apparent specific resistance obtained by the application of equation (2) to an extent which will be dependent on the specific resistance of the surface layer, the specific resistance of the stratum, the depth to the stratum, and the electrode-separation. It is desirable to have some relationship between these various quantities which can be used in such a way that when the values of the apparent specific resistance, the resistance of the surface, and the electrode-interval are known, the depth of the stratum can be calculated. If, as in figure 2, it is assumed
h that there is a single horizontal stratum at a depth h below the horizontal surface
ρ_1 of the earth, that the specific resistance of the surface layer is ρ_1, and that the
ρ_2 specific resistance of the underlying stratum is ρ_2, it can be shown* that the value
ρ_a of the apparent specific resistance ρ_a, as obtained by the application of equation (2) to the resistance obtained on measurement, is related to the actual specific resistances, depth, and electrode separation by an expression of the form

$$\rho_a/\rho_1 = 1 + 4F \qquad \ldots\ldots(3).$$

F In this expression F is a function representing the sum of an infinite series and may be written

$$F = \sum_{n=1}^{n=\infty} \left\{ \frac{k^n}{\sqrt{[1+(2nh/a)^2]}} - \frac{k^n}{\sqrt{[4+(2nh/a)^2]}} \right\} \qquad \ldots\ldots(4),$$

k in which k has the value,

$$k = (\rho_2 - \rho_1)/(\rho_2 + \rho_1) \qquad \ldots\ldots(5).$$

* E. Lancaster-Jones, *Mining Magazine*, **43**, 19 (1930).

In practice ρ_a and a are definitely known, and ρ_1 may be determined by careful measurements with very small electrode-separations, and thus there are two unknown quantities k and h, the latter of which it is desired to ascertain.

It is possible to calculate theoretically the value of the ratio ρ_a/ρ_1 for definite values of k, h, ρ_1 and a. Consider the quantity k equation (5). This may be written as

$$(1 - \rho_1/\rho_2)/(1 + \rho_1/\rho_2)$$

and is therefore dependent only on the ratio ρ_1/ρ_2. Thus k may have any value between $+1$ and -1; the values of ρ_1/ρ_2 corresponding to various values of k are given in table 1. Positive values of k correspond with the condition that the lower stratum has the higher resistivity, negative values of k occurring when the resistivity of the lower stratum is less than that of the surface layer.

Table 1.

k	ρ_1/ρ_2	k	ρ_1/ρ_2
1·0	1/∞	−1·0	∞
0·9	1/19	−0·9	19
0·8	1/9	−0·8	9
0·7	1/5·67	−0·7	5·67
0·6	1/4	−0·6	4
0·5	1/3	−0·5	3
0·4	1/2·33	−0·4	2·33
0·3	1/1·85	−0·3	1·85
0·2	1/1·5	−0·2	1·5
0·1	1/1·33	−0·1	1·33
0	1/1	0	1

For given values of the electrode-interval a and of k, a curve can be drawn showing the relation between ρ_a/ρ_1, and h. By giving k the values in table 1, a series of curves can be drawn for one given electrode-interval. When k has a positive value, however, it is more convenient to express the results in terms of conductivity by plotting the values of σ_a/σ_1 instead of ρ_a/ρ_1, σ_a being the apparent conductivity as measured $= 1/\rho_a$, and σ_1 the conductivity of the surface layer $= 1/\rho_1$. It is obviously as easy to obtain these curves as to obtain those showing ρ_a/ρ_1 since $\sigma_a/\sigma_1 = \rho_1/\rho_a$. Typical curves of these types are shown in figures 3 and 4, which are for electrode-intervals of 150 and 400 ft. respectively. The sets of curves marked A correspond with positive values of k and show the variation of the conductivity ratio σ_a/σ_1, while those marked B are for negative values of k and show the variation of the resistivity ratio ρ_a/ρ_1. Similar sets of curves can be drawn for any desired electrode separation.

It is now necessary to devise some means in which the theoretical results can be applied to the values of the apparent specific resistance actually obtained in practice so as to give the values of the depth h.

§4. METHOD OF INTERPRETING THE RESULTS

(*a*) *Empirical method.* The method of interpreting the results that has been used up to the present is an empirical one, and does not appear to be sound as it is not confirmed by any theory. Suppose a series of tests are made at one point, the centre of the electrode-system being kept at that point while the electrode-interval is varied. Then a series of values for the apparent specific resistance ρ_a will be obtained. If a curve of apparent specific resistance against electrode interval be plotted, a sudden change in curvature is taken as indicating the presence of a body which has a conductivity different from that of the surface medium, and it has been assumed that this body lies below the surface at a depth equal to the electrode-interval at which the change in curvature occurs. This interpretation is based on tests carried out by Messrs Gish and Rooney over a filled-in ravine and also on a lake. In the curves representing their observations, they obtained certain changes in curvature at points equal to the depth of the ravine or lake, and concluded from this that the body of earth included in tests of this type is limited substantially in all directions from the line joining the potential electrodes to a distance equal to the electrode-interval.

Most of the users of the method have adopted this way of interpreting their results, but as most of the tests appear to have been carried out in regions where the substructure of the earth was known, they cannot truly be called impartial tests. From theoretical considerations the occurrence of sudden changes of curvature in the curves representing results obtained in the case where there are horizontal strata, over one another, would appear to be unlikely,

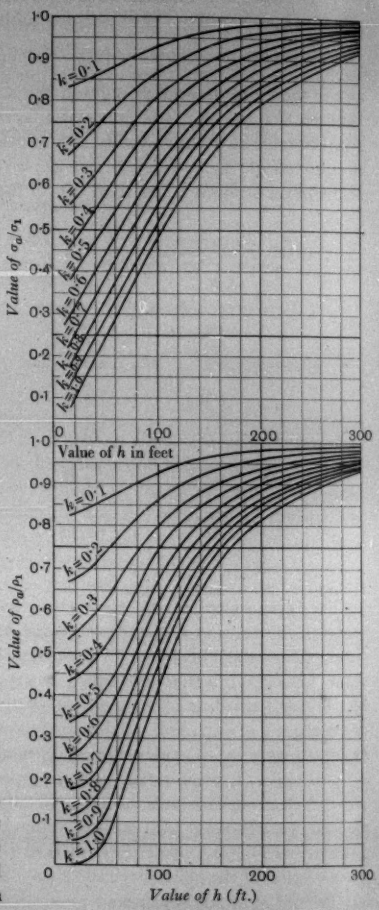

Fig. 3. Curves A 150 (upper) and B 150 (lower). Electrode interval 150 ft.

and if it is found, the explanation must be sought in some other type of geological formation.

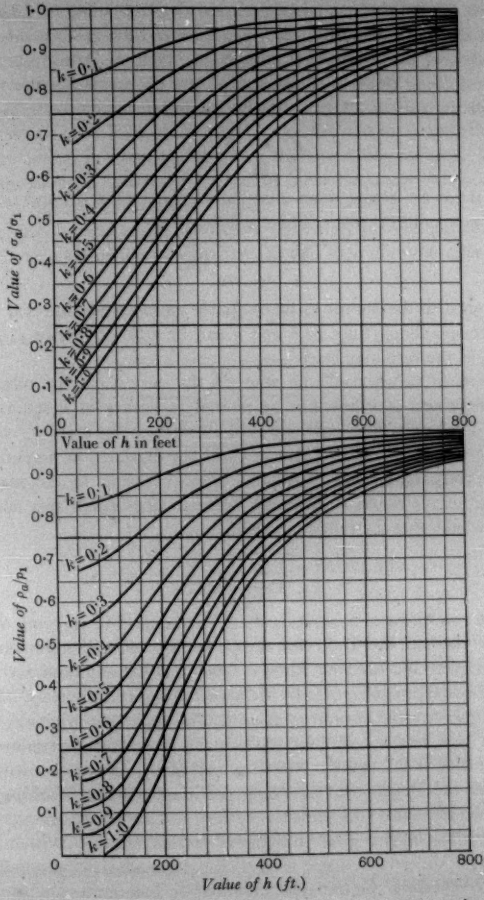

Fig. 4. Curves A 400 (upper) and B 400 (lower). Electrode interval 400 ft.

(*b*) *Theoretical method.* The author has suggested that the theoretical curves obtained as described above can be used to determine the depth of a horizontal stratum, as follows. Resistivity measurements are first made with one or two small

electrode intervals, the electrodes in each case being disposed along the traverse in positions symmetrical about the selected station. The object of these first measurements is to enable a fairly accurate determination to be made of the resistivity of the surface stratum ρ_1. Then further tests are carried out at larger electrode intervals, 150, 200, 250 ft. and so on. In these latter tests the values of the apparent specific resistances are, say ρ', ρ'', ρ''' .... Then the values of the ratio ρ_a/ρ_1 for the various electrode-intervals will be

ρ', ρ'', ρ'''

$$\frac{\rho'}{\rho_1} \cdot \frac{\rho''}{\rho_1} \cdot \frac{\rho'''}{\rho_1} \cdots$$

If these ratios are greater than unity ρ_2 is greater than ρ_1, and the reciprocals of these ratios should be taken. These will be

$$\frac{\sigma'}{\sigma_1} \cdot \frac{\sigma''}{\sigma_1} \cdot \frac{\sigma'''}{\sigma_1} \text{ etc.}$$

$\sigma', \sigma'', \sigma'''$

σ_1

On reference to the curves in figure 3, for an electrode-interval of 150 ft., a series of corresponding values of h and k can be read off, since $\rho_a/\rho_1 = \rho'/\rho_1$ or $\sigma_a/\sigma_1 = \sigma'/\sigma_1$ as the case may be.

Similar sets of values could be read off the curves for the other electrode-intervals. These sets of values can be plotted in curves of h against k. In the theoretically ideal case these curves would all intersect in a point, corresponding to the actual values of h and k. This can hardly be expected in practice, but under conditions corresponding with reasonable approximation to the mathematically ideal case the curves would all intersect within a fairly small area and the centre of this area would give the required value.

§ 5. APPARATUS

There are two forms of apparatus which can be used in carrying out measurements by this method, the first being the potentiometer-milliammeter equipment, originally designed and used by Messrs. Gish and Rooney, and the second, the Megger earth-tester, manufactured by Messrs Evershed and Vignoles, Ltd., London.

(*a*) *Potentiometer-milliammeter equipment.* The apparatus originally used for the measurement was designed by Messrs Gish and Rooney and employed by them for measurements of the specific resistance of earth in connexion with investigations made for the Department of Terrestrial Magnetism at Washington. It consisted of a potentiometer, milliammeter, battery and set of reversing commutators. A diagram of connexions is given in figure 5. In this figure A is the milliammeter, B the battery, G the galvanometer used with potentiometer, P the potentiometer, C_1 and C_2 the reversing commutators mounted on a common spindle so that the reversals are synchronized.

It was immediately recognized that the employment of direct current would introduce a number of errors such as those due to electrolysis, back e.m.f., and stray currents, and it was necessary to use alternating current in the soil. On the other hand it was more convenient to use direct-current instruments for the actual

measurements, particularly as direct current could be obtained from a portable source such as a battery. It was these considerations which led to the inclusion of the reversing commutator C_1 shown in figure 5, to convert the direct current from the battery to alternating current after it has passed through the milliammeter; and to the inclusion of C_2 to rectify the current produced by the potential difference between the two inner spikes, so that balance can be obtained on the potentiometer.

The use of these commutators introduces a correction-factor into the result if the correct value of R is to be obtained, since the current is definitely switched off for a short period in every half-cycle of the alternating current. The milliammeter reads the current I passing between the outer electrodes, while the potentiometer gives the potential difference V between the two inner spikes. The ratio V/I after modification by the correction-factor caused by the commutators gives the resistance R. An application of equation (2) will then give the apparent specific resistance.

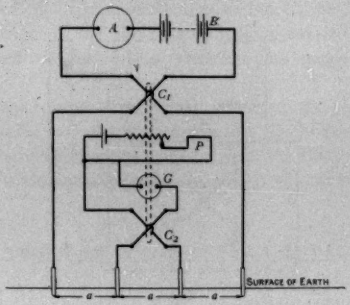

Fig. 5.

The four spikes or electrodes will each have a resistance to earth, and if the ground is dry or sandy these resistances may easily be of the order of several thousands of ohms each. The resistances of the two outer electrodes are included in the main current circuit and reduce the testing current and the voltage drop between the two inner spikes in the same proportion. They do not therefore affect the value of the ratio V/I. If these resistances are so high that they appreciably reduce the current flow, this may be overcome by increase of the battery voltage. The two inner electrodes are connected to the potentiometer, and since no current is taken from them when balance has been obtained, their resistances to earth do not affect the value V measured on the potentiometer. If the resistances of these electrodes are large they may reduce the sensitivity of the potentiometer, but in most cases this will not be serious.

The main drawback to this apparatus is its bulk, as several instruments and a battery are required in addition to the electrodes, connecting cables, etc.

(*b*) *Megger earth-tester*. The Megger earth-tester is a combined ohmmeter and generator of specialized type, so designed that alternating current is supplied to

the soil section of the testing circuit to overcome difficulties due to electrolytic back e.m.f., and direct current to the measuring part of the testing circuit to allow of the use of the direct-reading moving-coils ohmmeter with its large working force and uniformly divided scale.

The ohmmeter embodies two coils mounted at a fixed angle to one another on a common axle and swinging in the field of a permanent magnet; the axle carries a pointer moving over a scale marked in ohms. A current proportional to the total current flowing in the testing circuit passes through the current coil, while the potential coil carries a current proportional to the potential drop across the resistance under test. The coils are so wound that the resulting forces oppose one another. The final position of the moving coils, and hence that of the pointer, depends on the ratio of the potential drop to the total current, and the instrument is therefore a true ohmmeter giving readings in ohms, which are independent of the applied voltage.

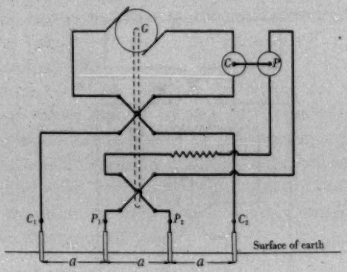

Fig. 6.

The connexions are as shown in figure 6. Direct current from the generator G passes through the current coil C of the ohmmeter to a rotating current-reverser, and to the terminals C_1 and C_2 of the instrument, which are connected to the two outer electrodes. The potential coil P of the ohmmeter obtains its supply from the terminals P_1 and P_2, which are connected to the two inner electrodes. Since this supply is taken from the soil section of the current circuit, and is therefore alternating, it must be made uni-directional before it passes through the potential coil. A commutator mounted on the same shaft as the main current-reverser, and synchronized with it, is therefore interposed as a rectifier between the terminals P_1 and P_2 and the potential coil. In this manner the current and potential coils of the ohmmeter are both supplied with direct current, and the soil section of the testing circuit is supplied with alternating current.

The instrument is so calibrated that the value of the resistance R in equation (2) is read directly on the scale, so that the apparent specific resistance can be at once determined. The resistances of the outer or current electrodes will not affect the result, for the reason already explained above in connexion with the potentiometer-milliammeter equipment. The resistances of the potential electrodes will, however,

have an effect, as current is taken from them and any large resistances at these points will cause the instrument readings to be too low. In calibrating the instrument a certain value is allowed for the total electrode-resistance and a simple correction can be applied to the resistance obtained to give the correct value. Thus if R_a be the resistance read on scale in specific resistance test, p the total electrode resistance for which allowance is made in calibration, l the total internal resistance of potential circuit of instrument, p_1, p_2 the actual potential electrode resistances, and R the correct value of resistance, then R is given by the formula

R_a, p
l
p_1, p_2
R

$$R = R_a\,(l + p_1 + p_2)/(l + p) \qquad \text{.....(6).}$$

The values of l and p are given on the instruction card supplied with the instrument. The resistances of the potential electrodes may be measured with the instrument itself by connexion of the terminals P_1 and C_1 to one another and to one potential electrode, the second potential electrode being connected to the P_2 terminal, and one of the current electrodes to the C_2 terminal. The instrument then reads the resistance of the electrode directly on the scale. The Megger earth-tester at present used for this class of work has four ranges of 0–3, 0–30, 0–300 and 0–3000 ohms.

In all surveys which have been carried out up to the present time with both the potentiometer-milliammeter equipment and the Megger earth-tester, it has been alleged that there are discrepancies between the values obtained by the two sets of apparatus. Fundamentally there is no reason why the results obtained should not agree.

§ 6. ARRANGEMENTS FOR EXPERIMENTAL SURVEY

(*a*) *Objects*. In view of the comparatively wide divergence between practice and the results to be expected according to the theory outlined above, and of the need for information on a number of further points, Messrs Evershed and Vignoles, Ltd., instructed the author to carry out an experimental survey using both the Megger earth-tester and a potentiometer-milliammeter equipment. The main objects of the survey were as follows: (i) to test the theory and method described above, (ii) to test the method of interpretation, and (iii) to check the results obtained with the Megger earth-tester against those obtained with the potentiometer-milliammeter equipment.

(*b*) *Apparatus*. As one of the objects of the survey was to obtain comparative tests of the potentiometer-milliammeter equipment and the Megger earth-tester, it was necessary to take both sets of apparatus. The potentiometer, galvanometer, and standard cell were kindly lent by the Cambridge Instrument Co. Ltd. The milliammeter was made up by Messrs Evershed and Vignoles, Ltd., and had four ranges of 0–100 microamps, 0–1, 0–10, and 0–100 milliamps.

The reversing commutators which were used with the potentiometer-milliammeter equipment were similar to those employed in the Megger earth-tester, although they had to be slightly modified before they were found to be completely satisfactory for this type of apparatus. The Megger earth-tester was of the four-range

type mentioned above. A diagram of connexions of the complete equipment is given in figure 7. Various switches were included so that connexions could be changed rapidly from one set of apparatus to the other, and also so that the leads could be reversed and the electrode resistances measured. The apparatus was all mounted on a table and carried in a van.

The electrodes which were used were tubular and were fitted with mild steel caps and driving points. The connecting cables were 4 mm. motor-ignition flexible, and ten coils each of 100 yards were taken. The exact positions of the stations selected for test were obtained by means of compass bearings made on land marks.

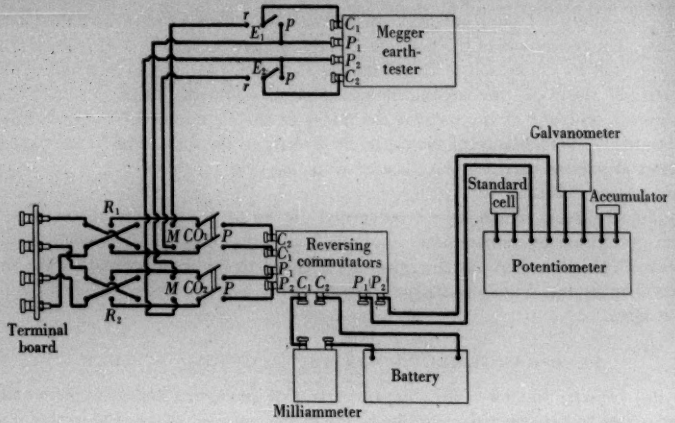

Fig. 7.

(*c*) *Site*. To investigate the various matters in any satisfactory way it was necessary to obtain a site which would approach as nearly as possible to the ideal case which had been investigated mathematically in the manner indicated above. The Director of the Geological Survey of Great Britain was therefore approached, and asked if he could suggest a site where the surface was approximately level, and where there was an underlying stratum practically horizontal, the electrical resistance being different from that of the surface material. The Director of the Geological Survey went into the matter very thoroughly and suggested a number of suitable sites. The one finally chosen was on Cleeve Hill Common near Cheltenham, Gloucestershire, and permission to carry out the tests was kindly given by the Board of Conservators of Cleeve Common. On the Common, which is in the Cotswolds, the surface material is limestone, the depth varying from 50 to 266 feet. Under this is either sand or clay. On the Common it was found possible to select sites which were practically level and sufficiently large for the tests. The results which were obtained at two stations termed *A* and *B* are given below. At both these stations the ground was practically flat.

§7. EXPERIMENTAL RESULTS OF SURVEY

(*a*) *Station A.* At Station *A* tests were made with the electrodes on a north-south line only, and results were obtained at electrode-intervals varying from 20 to 500 ft. These results are plotted in the form of curves of the apparent specific

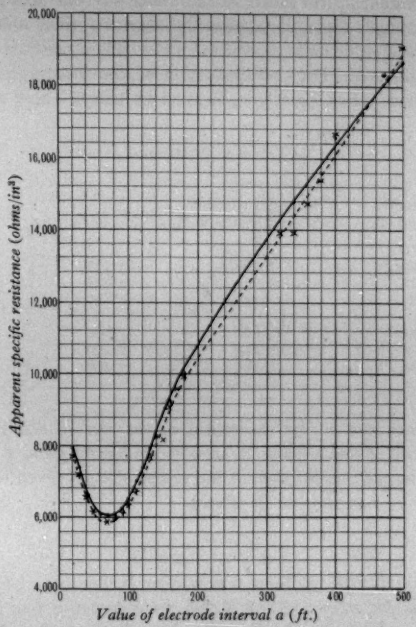

Fig. 8. Station *A*. Megger earth-tester: experimental points •; curve ————; Potentiometer equipment: experimental points × ; curve --------.

resistance against electrode separation in figure 8. The full line represents the results obtained with the Megger earth-tester, and the dotted line those obtained with the potentiometer equipment. It will be noted immediately that the agreement between the two sets of apparatus is very close, and at no point is the difference greater than the possible instrumental and observational errors. Further, with the exception of a dip at the first part of the curve which is due undoubtedly to surface variations, there are no sudden changes in curvature. Although it is known that the depth of the limestone was between 100 and 200 ft., there is no change in curvature at this part of the curve. There is in the apparent specific resistance a steady increase which indicates that the resistance of the underlying stratum is

higher than that of the surface material. To obtain a value for the average specific resistance of the surface material an average of the values obtained at electrode intervals up to 70 ft. has been taken. This average value is 6703 ohms/in.[3].

Next, from the experimental curve it is necessary to read off the values of the apparent specific resistance at a number of electrode-intervals, and to determine the value of the ratio ρ_a/ρ_1 for each interval. Since the resistance is increasing with the electrode-interval, the ratio ρ_a/ρ_1 will be greater than unity, a fact which indicates that the specific resistance of the underlying stratum is higher than that of the surface material. For the application of the theory outlined earlier the reciprocals of these ratios, namely σ_a/σ_1 must be taken. These ratios are set out in table 2.

Table 2. Station *A*.

Electrode-separation (ft.)	Apparent specific resistance (ohms/in.[3])	Ratio ρ_a/ρ_1	Ratio σ_a/σ_1
150	8,960	1·338	0·748
200	10,740	1·601	0·625
250	12,320	1·840	0·544
300	13,860	2·068	0·483
350	15,220	2·270	0·441
400	16,480	2·460	0·407

With reference now to the theoretical curves worked out for these various electrode-intervals, of which examples are given in figures 3 and 4, a series of values of h and k can be read off for each of the values of σ_a/σ_1 given in this table. The values thus obtained are set out in table 3.

Table 3. Station *A*.

Value of k	Values of h in feet					
	150 ft. σ_a/σ_1 =0·748	200 ft. σ_a/σ_1 =0·625	250 ft. σ_a/σ_1 =0·544	300 ft. σ_a/σ_1 =0·483	350 ft. σ_a/σ_1 =0·441	400 ft. σ_a/σ_1 =0·407
1·0	180	183	194	204	214	226
0·9	168	170	178	184	193	201
0·8	157	156	161	165	168	173
0·7	144	140	142	142	144	144
0·6	131	124	122	118	116	114
0·5	117	104	98	88	80	67
0·4	99	83	67	45	—	—
0·3	78	51	—	—	—	—
0·2	46	—	—	—	—	—
0·1	—	—	—	—	—	—

Inspection of table 3 shows that the values for $k = 0{\cdot}7$ agree very closely. The average value of h for this value of k is 142 ft. To obtain the values more closely the figures in table 3 are plotted in the form of curves in figure 9, in which values of h are plotted as abscissae and values of k as ordinates. The six curves intersect in a small area and the centre of the area gives values 142 ft. of h and 0·702 of k. Thus it appears that the depth of the limestone at this point is 142 ft., and so far as it is possible to judge from the geological map this is approximately correct. At this station the method of interpretation suggested by and based upon the theoretical investigation appears to give entirely satisfactory results.

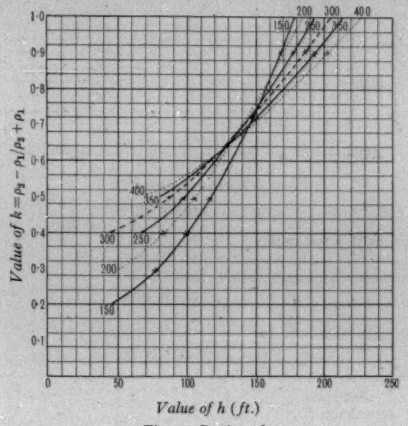

Fig. 9. Station *A*.

(*b*) *Station B*. At station *B*, tests were carried out with the electrodes on both the north-south and east-west lines, but while the results obtained on the north-south line are for electrode-separations varying from 20 to 400 ft., those on the east-west line were only obtained for electrode-intervals varying from 20 to 240 ft. The results are all plotted in the form of curves of apparent specific resistance against electrode-interval in figure 10.

It is noticeable that the results on the east-west line appear to be giving a slightly higher value than those on the north-south line. At this station again, as at station *A*, the discrepancy between the Megger earth-tester and the potentiometer-milliammeter equipment is very small, and is again within the limit of observational and experimental errors.

Adopting a similar procedure as for station *A* we obtain the value of ρ_1 as 45,700 ohms/in.^{3}. This value is very much higher than at station *A*, although the surface material is limestone in both cases. The probable explanation of the difference is that at station *A* the stratum under the limestone was clay, which would

keep the limestone waterlogged and thus of a low resistance, while at station *B*, the stratum was sand which would permit water to filter away, leaving the limestone comparatively dry and thus of a higher resistance.

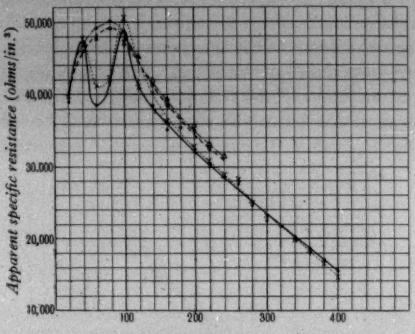

Fig. 10. Station *B*.

		Points	*Curve*
Megger earth-tester results	N—S line	•	———
,, ,, ,,	E—W line	⊙	- - - - -
Potentiometer equipment results	N—S line	×	.....
,, ,, ,,	E—W line	△	—·—·—

Table 4. Station *B*.

Electrode separation (ft.)	ρ_a	ρ_a/ρ_1
150	36,800	0·805
200	32,000	0·700
250	27,400	0·600
300	23,000	0·503
350	19,400	0·424
400	15,500	0·339

The experimental curve in this case shows a steady decrease in apparent specific resistance, indicating that the resistance of the underlying stratum is lower than that of the surface material. This condition, again, differs from that at station *A*, where there was a steady increase in resistance and this difference also is due to the presence of clay under the limestone at station *A* and sand at station *B*. With the exception of surface variations at small electrode intervals, there are no sudden changes in curvature. As for station *A*, a series of values of the apparent specific resistance at a number of electrode-intervals was read off the curve and the values of ρ_a/ρ_1 calculated for each. These values are given in table 4. For each of the values

of ρ_a/ρ_1 in the last column, a series of values of h and k was read off from the theoretical curves corresponding to the electrode separation, and the values are given in table 5 and plotted as curves in figure 11. The curve for 400 ft. should

Table 5. Station *B*.

Value of k	Value of h in feet					
	150 ft. $\rho_a/\rho_1 = \cdot 805$	200 ft. $\rho_a/\rho_1 = \cdot 700$	250 ft. $\rho_a/\rho_1 = \cdot 600$	300 ft. $\rho_a/\rho_1 = \cdot 503$	350 ft. $\rho_a/\rho_1 = \cdot 424$	400 ft. $\rho_a/\rho_1 = \cdot 339$
− 1·0	195	206	218	226	237	240
− 0·9	185	196	206	214	220	219
− 0·8	174·5	184	193	198	200	196
− 0·7	164	172	179	180	185	166
− 0·6	152	158	161	158	150	130
− 0·5	140	142	140	130	112	—
− 0·4	123	121	112	90	—	—
− 0·3	103	94	70	—	—	—
− 0·2	74	44	—	—	—	—
− 0·1	—	—	—	—	—	—

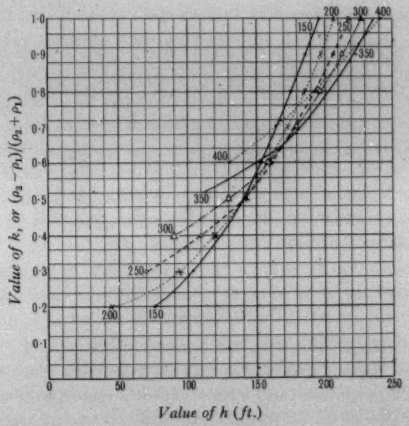

Fig. 11. Station *B*.

really be ignored, as at a distance of about 500 ft. from the station the ground was beginning to slope in a northward direction. The centre of the area of intersection of the remaining curves gives values of 156 ft. for h and $-0{\cdot}6$ for k. The value of h again agrees closely with that estimated from the geological map. Thus the method of interpretation based on the theory seems again to give entirely satisfactory results.

§8. CONCLUSIONS

From the results of the tests the following conclusions can be drawn:

(i) There is no fundamental disagreement between the values of resistance obtained by the use of the Megger earth-tester and of the potentiometer equipment. Such differences as occur do not exceed 5 per cent. except in very isolated cases and can reasonably be attributed to instrumental errors. One item which contributes to this instrumental error is the error in the correction-factor for the commutators, which cannot be considered to be less than 3 per cent. There is of course no theoretical reason for expecting any difference between the values obtained with the two sets of apparatus.

(ii) In the curves of experimental results there are no sudden changes in curvature, with the exception, of course, of those due to the surface variations. This seems to confirm the theory, and also to discredit the method of interpretation which relies on the appearance of sudden changes in curvature in the curves.

(iii) The application to the experimental results obtained at stations *A* and *B* of the method of interpretation based on the theory worked out above gives satisfactory results. There does not appear to be any reason why perfectly satisfactory theories should not be developed for various cases of earth structure, and methods of interpretation evolved which are based on these theories. The author is proceeding with the investigation of a rather more complicated type of earth structure with a view to developing further methods of interpretation analogous to that described above.

§9. ACKNOWLEDGMENT

The author's thanks are due to Messrs Evershed and Vignoles, Ltd., under whose auspices the experimental survey was carried out, and for their permission to publish the information contained in this paper.

DISCUSSION

Prof. A. O. Rankine. Mr Tagg has presented this paper in a very clear way. I am glad that the Physical Society has decided to publish this paper belonging to the field of applied geophysics, and hope that it will continue to provide a channel of publication for similar papers of sufficient merit. It was in 1923, I think, that the Society published two important papers by Mr Lancaster Jones and Capt. Shaw on the use of the Eötvös torsion balance, and it has been interesting to note that there have since been many demands from all over the world for the particular part of the *Proceedings* containing them. I am not suggesting increase of sales as the main reason why papers should be accepted. In respect of contributions on practical geophysics it is rather a question of securing that they should become subject to criticism by physicists, and not escape it by being presented to a geological or mining society or journal.

Mr J. H. AWBERY. In reducing the results of the observations, it is necessary to divide each of the quantities ρ', ρ'' ... by ρ_1, the surface resistivity. Mr Tagg states that this was obtained by averaging the apparent resistivities to an electrode separation of 70 ft. Can he state the effect of taking some other estimate for ρ_1, and, in particular, is it possible that some process of successive approximation could be used to improve the final accuracy? In other words, if it had been found that the surface stratum extended downwards to about 142 ft., would there be an advantage in selecting some electrode distance related to this figure, and carrying the calculation through afresh? In the second place, I would ask for the values of ρ_2 found at the two stations. We have k in each case, which gives ρ_1/ρ_2 for each station, but it would be interesting to show ρ_2 explicitly, having regard to the value of the underlying stratum. I should also like to associate myself with Prof. Rankine's remarks on the value to the Physical Society of papers on applied physics. These can be quite as scientific, in the true sense, as papers on pure physics.

Mr T. SMITH remarked that the observer had to start from an assumed value of the depth, and in general this would be a wrong value. Was it possible to find any criterion for determining the sign of the error, and so ensuring a move in the right direction at the next trial?

Mr S. WHITEHEAD. I should like to ask the author whether he could give a rough estimate of the greatest depth of the plane of separation for fairly accurate results, presuming the lower stratum to have a resistivity some few times that of the upper, which is assumed of the order of 10,000 ohms/cm. or less. The method described would be of great value in surveying the electrical properties of the ground with a view to application to the linear flow of alternating current therein. When alternating current flows along a conductor such as an overhead line or cable and returns through the earth, the e.m.f. in the earth near the conductor does not vary very greatly with resistivity, but in regions remote from the conductor the e.m.f. varies rapidly with resistivity. This problem is of considerable practical importance and has been studied for some years by international committees (the C.C.I. and C.M.I.). The results of experiments in Germany, Sweden and America on the distribution of a linear flow of alternating current have not shown correlation with continuous-current tests or tests employing apparatus similar to the author's, where skin effect is neglected and the current distribution is similar to that for a continuous current. In some English tests some degree of correlation was observed, the conditions being more advantageous. The discrepancies appear to have been due to an incomplete interpretation of results so that only resistivities near the surface were obtained, which were not important in the linear flow tests. Since only a few linear flow tests (in which the distribution of current over a wide tract is observed) can be carried out, owing to labour and expense, it would be of great value if tests such as the author's could be used to determine the resistivities of different layers to a sufficient depth. If this were possible without inconveniently increasing the depth of the probes or the separation of the electrodes, then the

linear flow distribution at different frequencies could be deduced, special tests being avoided, while those made could then be interpreted more scientifically.

AUTHOR'S reply. Before replying to the various comments which have been made on my paper I should like to thank the Physical Society for accepting it, and I quite agree with Prof. Rankine that papers of this type should be criticized, if necessary very severely, by physicists.

Mr Awbery asks what would be the effect of taking some value for ρ_1 other than that which was obtained by averaging the first part of the experimental curves. If an incorrect value were taken, the area of intersection of the final curves would be very considerably increased. I do not think that a process of successive approximation would be of any use in improving the final accuracy. There is no definite relation between any electrode separation and the depth of the stratum, since the effect of the underlying stratum on the result obtained for any given electrode separation will depend not only on the depth of the stratum but on its resistivity as compared with that of the surface layer.

The values of ρ_2 can be obtained from the values of k, obtained experimentally, table 1, and the values of ρ_1. Thus at station A, k is equal to 0·7 and from table 1 this gives a value for ρ_1/ρ_2 of 1/5·67. The value of ρ_1 is 6700, and thus ρ_2 will be 38,000 ohms/in.^{3}.

Again, at station B the value of k is $-0{\cdot}6$, and this corresponds to a value for ρ_1/ρ_2 of 4. As ρ_1 is 45,700 ohms per inch cube, this means that ρ_2 is 11,925 ohms per inch cube.

In reply to Mr T. Smith I would mention that the observer does not start from an assumed value of the depth. This is one of the main points in favour of the method. The observer is totally unaware of the correct depth and deduces his result by a mathematical process. There is no criterion for giving an algebraic sign of the error.

Mr S. Whitehead's remarks are very interesting, and I would first of all mention that the maximum depth of the plane of separation for accurate results depends only on the sensitivity of the apparatus which is employed. The greater the depth of this plane of separation, the greater must be the maximum electrode separation used, in order to obtain accurate results. The greater the electrode separation, the lower will be the resistance measured, thus necessitating the use of more sensitive apparatus. In the case of the experimental survey described in the paper, the maximum electrode separation used was 500 ft., and at this value the resistance measured was about 0·5 ohm, but we had high-resistance soil either at the surface or in the underlying stratum keeping the resistance up. We could, with the apparatus at our disposal, have worked to a depth of about 500 ft. More sensitive apparatus is being developed so that greater depths can be employed.

I am very interested in the application of resistivity methods which is described by Mr Whitehead and I agree that the method I described could be used. I have read a report which introduces this method of earth resistivity in connexion with the impedance of overhead power lines, and I thought at the time that an independent determination of the earth resistivity would be of assistance. I believe that in-

dependent measurements were made in a method similar to the one I have described, but the results were taken as giving the surface resistivity and the effects of underlying strata were ignored.

Another point which must be borne in mind is that if tests are carried out at high frequencies, errors will be introduced in the result owing partly to the inductance of the earth and partly to the effect of inductance and capacity between the connecting leads. All our tests were carried out at 50 ~, and a variation of the frequency between 20 and 80 ~ did not produce any effect on the results. I believe that the skin effect mentioned by Mr Whitehead is due to the presence of the overhead conductor and so could not be taken into account in any independent resistivity tests.

With regard to the question of several layers, the method I have described might be useful for giving average resistivities of some kind, but the actual interpretation in terms of depth and resistivities would be a very difficult one. The theory of multilayers has been worked out by Dr Hummel of Göttingen University*, but the result is very complicated. No method has been devised whereby this very complex theory could be applied to practical tests.

* See *Zeitschrift für Geophysik*, 5, Nos. 5 and 6 (1929).

A PHOTOELECTRIC SPECTROPHOTOMETER FOR MEASURING THE AMOUNT OF ATMOSPHERIC OZONE

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ABSTRACT. A special instrument is described in detail which has been designed in order to allow measurements of the amount of ozone in the atmosphere to be made easily and rapidly under nearly all conditions. The instrument follows the usual practice of measuring the absorption by the ozone of solar ultra-violet energy; from this measurement the amount of ozone can be deduced. A double quartz monochromator isolates certain pairs of wave-lengths, and the relative energy in the two wave-lengths of a pair is measured by allowing them to fall alternately on to a photoelectric cell, the current from which is amplified by thermionic valves. This allows great sensitivity to be obtained so that very small amounts of light can be accurately measured. For measuring the amount of ozone a pair of wave-lengths, one of which is strongly absorbed by ozone while the other is not, is selected. It is shown how the amount of ozone can still be measured when the sky is cloudy if a second pair of wave-lengths, both unabsorbed by ozone, be also measured. The results of tests which show that the accuracy is ample for meteorological requirements are given.

§ 1. INTRODUCTION

One of the most interesting results which was obtained from the early measurements* of the amount of ozone in the atmosphere was the close connexion which was found between the amount of ozone and the type of atmospheric pressure-distribution. An attempt was made to study this relationship in N.W. Europe during 1926–27†, and six special spectrographs were built for the purpose and distributed to stations scattered over that area. With these instruments spectra were taken of direct sunlight, from which the amount of ozone could be calculated, the plates being returned to be developed and measured at Oxford. While much useful information was gathered from this series of observations, it was naturally found that the delay caused by the return and measurement of the plates, and especially the fact that no observations could be made on cloudy days, were very serious disadvantages. Some of the most interesting meteorological conditions are always associated with much cloud, so that it was very difficult to get any ozone measurements in these cases. As it seems possible that fuller knowledge of the connexion between the amount of ozone and the pressure-distribution might lead to a better understanding of the nature and behaviour of cyclones and anticyclones, it seemed very desirable to design an instrument which should give direct readings

* *Proc. R.S.* A, **110**, 660 (1926).

† *Proc. R.S.* A, **114**, 521 (1927); **122**, 456 (1929). The distribution of ozone over the world is discussed in *Proc. R.S.* A, **129**, 411 (1930).

of the amount of ozone without the use of a photographic plate and which could be used even when the light was very weak, so that observations could be taken when the sun was low or by using the light which had passed through clouds. This latter requisite involves difficulties other than the strength of the light available; these will be dealt with later. A very brief sketch of the instrument and the principle involved was given at the Society's Discussion on Photoelectric Cells, but as the instrument has now been in regular use for several months and has proved very satisfactory, it may be useful to give a detailed account of it.

§ 2. GENERAL PRINCIPLE

By means of a double quartz spectroscope, two narrow bands in the ultra-violet region are isolated. The wave-lengths of these bands are chosen so that the longer one is almost outside the great ozone absorption band in the region 3200 ÅU. to 2200 ÅU., while the shorter one is well within it. By measuring the ratios of the energies received from the sun in these two bands, and knowing certain constants

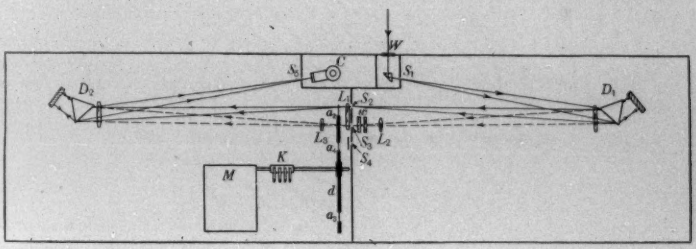

Fig. 1.

which will be discussed later, we can calculate the amount of ozone through which the sunlight has passed. To determine this ratio the two wave-lengths are allowed to fall alternately, by means of a rotating sector wheel, on to a sodium photoelectric cell. The strength of the brighter beam (that of longer wave-length) can be reduced in a known ratio by an adjustable optical wedge. The cell is connected to the first grid of a four-stage low-frequency valve amplifier, figure 3. With such an arrangement only fluctuations in the current are amplified, and if it should happen that the current passed by the cell was exactly the same whichever wave-length was falling on it, then the steady current so produced would not be amplified and there would be no current in the output circuit. The optical wedge is adjusted so that this condition is obtained, and the position of the wedge then allows the ratio of the intensities of the two wave-lengths to be read off when certain constants of the instrument are known. It will be seen that the total intensity of the light does not affect the setting of the wedge, which is determined by the ratio of the intensities only; this is a most important condition since the total light may fluctuate greatly in a short time owing to varying thickness of cloud etc. while the ratio of the intensities of the wave-lengths remains constant.

An alternating-current instrument might be used to read the current flowing in the output circuit, or this current might be rectified by a valve or metal rectifier and a direct-current instrument employed. There are, however, several objections to either course. In the first place, with the high amplification used there must be valve-noise currents in the output circuit amounting to a few μA, so that if rectifiers are used the current can never be reduced to zero for any setting of the optical wedge; also any vibration of the instrument will increase this current owing to microphonic effects, so that its minimum value will not be constant. Secondly, alternating-current instruments are generally less sensitive than direct-current instruments, and if rectifiers are used they are apt to have approximately square-law characteristics for very small currents, so that they are unsuitable for showing

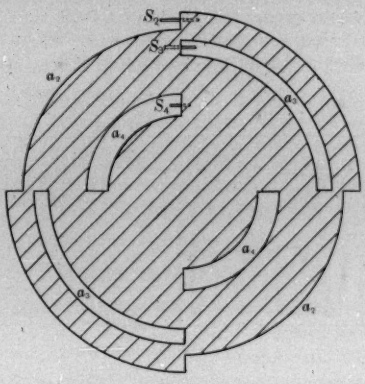

Fig. 2.

when the current is zero within a few hundredths of a microampere. In the actual method used there is mounted on the shaft carrying the sector wheel a commutator which reverses the direction of the current in the output circuit at the right times, thus converting the alternating current into a pulsating unidirectional current which is read on a d.c. microammeter. The strength of this current is directly proportional to the difference in intensity between the two beams, and is in one direction if one beam be the stronger and in the other direction if the other beam be the stronger. This is most important for quick and accurate setting of the wedge.

There is one serious disadvantage of using a commutator with an amplifier giving such large amplification as that used here, since, owing to the fact that the output circuit must be either broken or short-circuited at each reversal, a disturbance is set up and in some way which we have not yet been able to ascertain causes the galvanometer to be rather unsteady. This makes it difficult to tell when the current is exactly zero unless a very heavily damped galvanometer be used which makes observation slow. At the Discussion on Photoelectric Cells, I suggested that

if instead of an ordinary commutator, one on the principle shown in figure 4 were used, there would never be any break or short-circuit, and the unsteadiness might be greatly reduced. In practice the commutator is made by taking a dynamo commutator with about 50 segments and connecting adjacent segments together by a suitable resistance. Two segments at the opposite ends of a diameter are connected to two slip-rings through which the current from the amplifier is led in. It has not

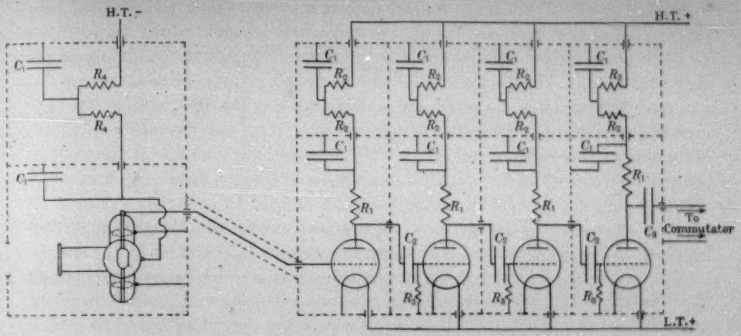

Fig. 3. Electrical system. Broken lines indicate screening.
$C_1 = 2\,\mu F$; $C_2 = 0{\cdot}01\,\mu F$; $C_3 = 2\,\mu F$; $R_1 = 0{\cdot}1\,M\Omega$; $R_2 = 0{\cdot}005\,M\Omega$; $R_3 = 5\,M\Omega$; $R_4 = 0{\cdot}1\,M\Omega$.

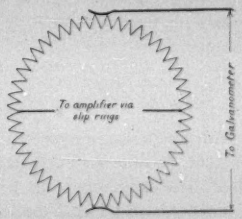

Fig. 4.

yet been possible to fit such a commutator to the spectrophotometer, but tests made by Dr Perfect and Mr Thomas at the National Physical Laboratory on photoelectric measurements with an amplifier similar to that used here show that the unsteadiness is much reduced. The chief effect of fitting such a commutator to the instrument will be to allow readings to be taken more quickly, while observations made when there is little light available will be materially improved in accuracy, so that measurements may be made when the sun is somewhat lower than the altitudes which at present permit of observations.

The amount of amplification that can usefully be employed is fixed by the unsteadiness referred to above and the galvanometer which it is convenient to use.

When the amplification is so great that the pointer becomes markedly unsteady, further amplification is useless. We have worked with a microammeter which was somewhat heavily damped, reading 6 μA for a full-scale deflection, and with this the four-valve amplifier is all that is wanted. Direct measurements show that the current amplification obtained is about 10^8.

Since the intensity of daylight in the region of 3110 Å.U., where the measurements are made, is very small compared to that in the longer wave-lengths, if a single spectroscope were used the light of longer wave-lengths which was scattered by the lens and prism surfaces would be an appreciable part of that falling on the photoelectric cell. For this reason a double spectroscope must be used, so that this scattered light is again dispersed and a negligible amount falls on the cell. The general arrangement of the instrument is seen in figure 1, and the relative positions of the slits and the sector-wheel in figure 2. The radiation passes into the instrument through a window W to the first slit S_1 and thence to the first dispersing system D_1. Three slits S_2, S_3, S_4 isolate three narrow bands at 3110 Å.U., 3265 Å.U. and 4435 Å.U. (S_4 is for measuring the transparency of the atmosphere for wave-lengths unaffected by ozone as described below). The dispersing system D_2 is similar to D_1, and recombines on slit S_5 radiations of the proper wave-lengths which have passed through S_2, S_3 and S_4, but disperses radiation of other wave-lengths which may have passed these slits, so that it will not fall on S_5. Two narrow optical wedges w of neutral gelatine between quartz plates serve to reduce by an accurately known amount the intensity of the radiation which has passed S_3. Immediately behind S_5 is the sodium photoelectric cell C. The sector-wheel d revolves close to S_2, S_3 and S_4 and admits light from S_2 and S_3 alternately (or from S_3 and S_4 if required). K is the commutator and M the driving motor.

§ 3. THEORETICAL BASIS

As was stated before, the instrument was designed to work with either the direct light from the sun, the light from the blue sky overhead or the light from a thinly clouded sky overhead. Each of these conditions must be considered separately and for simplicity the following notation will be used throughout:

x — x is the equivalent vertical thickness in cm. of the ozone present in the atmosphere reduced to a layer of pure gas at 0° C. and 760 mm. of mercury;

α, α' — α, α' the absorption coefficients of ozone per cm. of pure gas at 0° C. and 760 mm. for the wave-lengths 3110 Å.U. and 3265 Å.U. ($\alpha = 1{\cdot}275$, $\alpha' = 0{\cdot}122$);

I_0, I_0', I_0'' — I_0, I_0', I_0'' the intensities of the wave-lengths 3110 Å.U., 3265 Å.U. and 4435 Å.U. as received from the sun on the outside of the atmosphere;

I, I', I'' — I, I', I'' the intensities of the same wave-lengths as received at the earth's surface;

K — K the constant of the optical wedge used for the wave-length 3265 Å.U.;

Z — Z the apparent zenith distance of the sun at the place of observation;

ζ — ζ the zenith distance of the sun at the place where the sun's ray which reaches the observer cuts the ozone layer; and

β, β', β'' — β, β', β'' the extinction coefficients of the atmosphere due to scattering by pure

air and small particles, for the wave-lengths 3110 Å.U., 3265 Å.U. and 4435 Å.U. respectively. (For average conditions at Oxford $\beta = 0{\cdot}44$, $\beta' = 0{\cdot}36$, $\beta'' = 0{\cdot}11$.)

(*a*) *Measurements with direct sunlight.* We have shown* that the amount of ozone in the atmosphere is related by the formula

$$x = \{\log (I_0/I_0') - \log (I/I') - (\beta - \beta') \sec Z\}/(\alpha - \alpha') \sec \zeta$$

to the intensity of direct sunlight received at the earth's surface. The value of $\log (I_0/I_0')$, and similarly of $\log (I_0'/I_0'')$, can be found from a series of observations

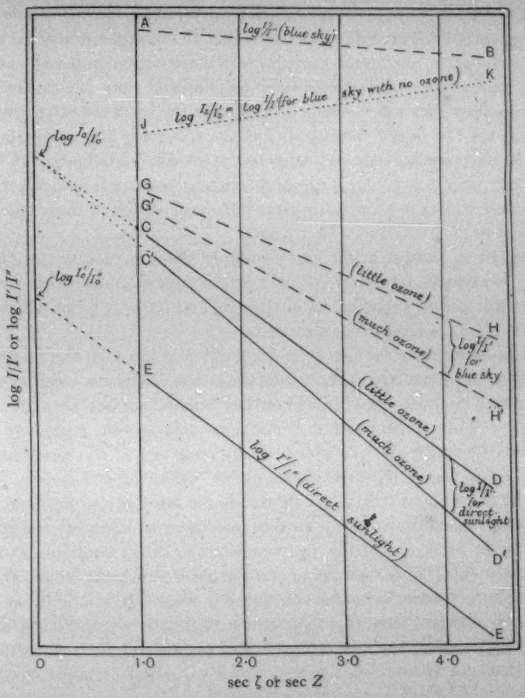

Fig. 5.

extending throughout the day, and having different values of sec ζ, or sec Z, when the atmospheric conditions are remaining uniform. Since $\log (I/I')$ is a linear function of sec ζ†, the observed values of $\log (I/I')$ should lie on a straight line when plotted against sec ζ (see *CD*, figure 5), and by extrapolation of this line the value of $\log (I_0/I_0')$ may be found. The value of $\log (I_0'/I_0'')$ is found in a similar way.

* *Proc. R.S.* A, **110**, 668 (1926).

† See § 5 for details.

The value of $(\beta - \beta')$ will depend on the clearness of the atmosphere, but the variations will generally be small since the wave-lengths have been chosen as near together as other circumstances will permit. In our older, photographic measurements this value was assumed to remain constant and it was known that a very small error must result. In the new photoelectric instrument $\log (I'/I'')$ is measured also and since the corresponding wave-lengths are outside the ozone absorption band* this allows us to determine $(\beta' - \beta'')$ when we have found the value of $\log (I_0'/I_0'')$, which we assume to remain constant. We can calculate the value of $(\beta - \beta')$ from $(\beta' - \beta'')$ if we know how β varies with the wave-length. Two formulae for this have been proposed. Formula (1) supposes that the scattering may be treated as made up of two parts, one due to particles which are large compared to the wave-length and which therefore scatter all wave-lengths alike, and one due to air molecules and small particles which scatter according to the inverse-fourth power of the wave-length. (2) The second formula divides the scattering into two parts of which one is due to air molecules only and varies as the inverse-fourth power of the wave-length and the other to particles in the air which scatter according to $\lambda^{-1\cdot 27}$. For our present purpose it does not make any great difference which of these two formulae we use except in the case of very hazy days.

With regard to changes in the emission from the sun, neither $\log (I_0/I_0')$ nor $\log (I_0'/I_0'')$ will remain absolutely constant, but it may be shown that variation in these values will lead to wrong values of $(\beta - \beta')$ and $(\beta' - \beta'')$, but will cause very little error in the amount of ozone deduced.

(*b*) *Measurements using the light from the zenith blue sky.* All the measurements of the height of the ozone layer indicate that the average height is about 45 to 50 km. At these heights the pressure will be about 10^{-3} of the surface pressure, and it is evident that, as MM. Cabannes and Dufay have pointed out, nearly all the light received from the zenith sky will have been scattered out of the direct solar beam by the atmosphere *below* the ozone layer except when the sun is very low. The absorption by the ozone will therefore be exactly the same as that in the direct solar beam, i.e. proportional to $\alpha x \sec \zeta$. In this case there is obviously no fixed value corresponding to $\log (I_0/I_0')$ in the measurements on direct sunlight. It is found, however, that if $\log (I/I')$ for the light from the clear zenith sky be plotted against $\sec \zeta$ the points lie on a straight line (see figure 5, lines GH and $G'H'$) though the line is naturally different from that for direct sunlight. It is easy to calculate from this observed line, GH, and the amount of ozone (found from observations on direct sunlight) another line JK, figure 5, which will also be straight, representing the values of $\log (I/I')$ which would have been obtained from the measurements on the blue zenith sky if there had been no ozone present. This line will be the same for all clear days and may be used to determine the amount of ozone according to the formula

$$x = \{\log (I_z/I_z') - (\log I/I')\}/(\alpha - \alpha') \sec \zeta,$$

where $\log (I_z/I_z')$ is the value of $\log (I/I')$ given by the line referred to above for the

* Actually 3265 Å.U. is slightly absorbed by ozone and a correction is applied according to the approximately known amount of ozone present.

particular value of sec ζ at the time of observation. On days when the sky is hazy the values of log (I/I') for the zenith sky may be corrected by means of the values of log (I'/I'') in the same way as for a cloudy sky (see below).

The light received from the clear zenith sky must be composed of light scattered from the direct solar beam by the atmosphere (1) below the ozone layer, (2) within the ozone layer, and (3) above the ozone layer. As indicated above, (1) may be expected to be predominant when the sun is high, but, as Dr Götz has recently pointed out, when the sun is low the light of short wave-lengths may be so reduced in passing through the ozone layer and by scattering that (2) and (3) become important. This seems to be the explanation of the fact that when observations on the clear

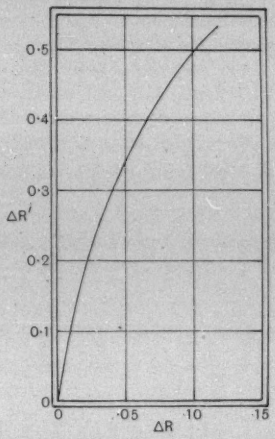

Fig. 6.

zenith sky are continued till the sun is nearly setting, it is found that while log (I/I') decreases at first, it afterwards becomes constant and finally increases again. Naturally the method of calculating the ozone breaks down when (1) ceases to be the only important part. An example of this is given by the last readings of October 5, shown in table 1.

(*c*) *Measurements of light from a thinly clouded sky.* The light illuminating the upper surface of a cloud layer will be partly direct sunlight and partly light from the sky, and these two have a very different spectral composition. The light received by an observer under the cloud layer, being a mixture of the lights from these two sources, will have a spectral composition intermediate between them, so that it is not possible to calculate the amount of ozone from measurements of log (I/I') alone. Since, however, clouds appear to scatter or absorb in the same proportion light of all of the three wave-lengths used by us, we may correct the observed values

Table 1.

17. vii. 30	Time	10 : 05	10 : 22	10 : 30	10 : 35	10 : 40	11 : 39	11 : 53	11 : 59	16 : 48
	Sec ζ	1·295	1·264	1·247	1·239	1·230	1·174	1·170	1·168	2·04
	Ozone	0·279	0·276	0·277	0·278	0·277	0·283	0·285	(0·288)	0·281
3. x. 30	Time	09 : 45	11 : 10	11 : 14	11 : 24	11 : 28	11 : 32	11 : 45	11 : 50	11 : 59
	Sec ζ	2·105	1·795	1·785	1·770	1·770	1·770	1·765	1·760	1·755
	Ozone	0·191	0·193	0·194	0·196	0·195	0·196	0·196	0·195	0·197
6. x. 30	Time	09 : 08	09 : 12	09 : 18	09 : 24	09 : 28	09 : 32	09 : 39	11 : 39	11 : 44
	Sec ζ	2·48	2·43	2·38	2·33	2·29	2·26	2·21	1·82	1·82
	Ozone	0·218	(0·221)	0·218	(0·222)	0·218	(0·223)	0·221	*0·217*‡	*0·221*‡
6. x. 30 (*cont.*)	Time	11 : 50	11 : 51	11 : 54	12 : 32	13 : 02	15 : 45	—	—	—
	Sec ζ	1·82	1·82	1·82	1·84	1·90	2·64	—	—	—
	Ozone	*0·220*‡	0·217	(0·219)	0·218	(0·218)	0·220	—	—	—
7. x. 30	Time	11 : 49	11 : 54	12 : 00	12 : 06	12 : 12	12 : 30	—	—	—
	Sec ζ	1·83	1·83	1·83	1·83	1·835	1·865	—	—	—
	Ozone	*0·198*†	*0·197**	*0·198**	0·200	*0·199*‡	*0·200*‡	—	—	—
9. x. 30	Time	09 : 30	09 : 37	11 : 37	12 : 40	12 : 45	12 : 58	13 : 05	14 : 37	14 : 44
	Sec ζ	2·345	2·295	1·870	1·912	1·922	1·955	1·980	2·58	2·64
	Ozone	(0·244)	0·242	0·241	(0·242)	0·240	(0·239)	0·238	0·242	0·240
10. x. 30	Time	09 : 22	09 : 27	10 : 48	10 : 52	10 : 56	11 : 23	11 : 46	11 : 48	11 : 52
	Sec ζ	2·45	2·41	1·955	1·955	1·945	1·900	1·88	1·88	1·88
	Ozone	(0·221)	0·219	*0·227*†	*0·218**	*0·215**	*0·207**	(0·216)	*0·216**	*0·208*‡
10. x. 30 (*cont.*)	Time	11 : 56	12 : 00	12 : 03	12 : 06	12 : 10	13 : 44	13 : 54	—	—
	Sec ζ	1·88	1·88	1·88	1·88	1·88	2·175	2·225	—	—
	Ozone	*0·211*‡	*0·216*§	*0·222*§	*0·225*§	*0·226*§	0·212	(0·215)	—	—
4. xi. 30	Time	09 : 12	09 : 54	09 : 58	11 : 09	11 : 20	11 : 52	12 : 34	12 : 38	13 : 20
	Sec ζ	3·61	3·01	2·965	2·55	2·52	2·50	2·565	2·59	2·81
	Ozone	0·257	0·256	(0·250)	0·249	0·249	0·244	0·242	(0·244)	0·241
26. xi. 30	Time	11 : 20	11 : 24	11 : 34	11 : 38	12 : 00	12 : 05	12 : 09	12 : 13	12 : 16
	Sec ζ	3·23	3·23	3·19	3·19	3·18	3·18	3·19	3·19	3·21
	Ozone	*0·223*†	*0·223*†	0·220	(0·219)	*0·221**	*0·222**	*0·221**	*0·221**	*0·220**
5. i. 31	Time	10 : 44	11 : 35	14 : 14	14 : 26	14 : 34	14 : 37	14 : 42	14 : 46	14 : 52
	Sec ζ	3·88	3·50	4·42	4·70	4·88	4·98	5·14	5·24	5·45
	Ozone	(0·222)	(0·219)	(0·221)	(0·218)	(0·217)	(0·216)	(0·214)	(0·213)	(0·211)
5. i. 31 (*cont.*)	Time	14 : 56	15 : 00	15 : 04	—	—	—	—	—	—
	Sec ζ	5·59	5·75	5·89	—	—	—	—	—	—
	Ozone	(0·209)	(0·206)	(0·206)‖	—	—	—	—	—	—

Plain figures denote observations on direct sunlight. Figures in brackets are observations on the zenith blue sky.
Figures in italics are observations on the cloudy zenith sky.

* = Observations through thin white cloud.
† = Observations through moderate A.Cu. cloud.
‡ = Observations through moderately thick grey cloud.
§ = Observations through heavy grey cloud.
‖ Observations between 10.48 and 11.23 a.m. on October 10 were made on rapidly changing cloud and were very variable, so that no good mean value could be obtained. This probably accounts for the large variation shown by these readings, which is almost the greatest that has been found.

of $\log(I/I')$ by means of the observed values of $\log(I'/I'')$ in the following way and then calculate the amount of ozone as for observations made on blue sky.

From observations on clear days we can fix the lines AB and JK of figure 5. Let the difference between the values of $\log(I/I')$ for blue sky and cloudy sky at any one time be (ΔR) and let the corresponding values of $\log(I'/I'')$ be $(\Delta R')$. If observations are made on partially cloudy days a number of pairs of values of (ΔR) and $(\Delta R')$ may be obtained for approximately equal values of $\sec\zeta$ but with different types and thicknesses of cloud. If for each pair (ΔR) be plotted against $(\Delta R')$ the points are found to lie approximately on a curve such as that shown in figure 6. Then on a cloudy day, from a measurement of $\log(I'/I'')$ and the line AB of figure 5, we obtain $(\Delta R')$ and from a curve such as figure 6 we can read off (ΔR) and so correct the reading of $\log(I/I')$ to the value it would have had if the observations had been made on the clear blue sky. The ozone value is then calculated as for clear blue sky observations. As might be expected, such a curve of corrections does not hold exactly in all cases, but so long as the cloud is not very thick it appears that the errors introduced will seldom exceed 0·01 cm. of ozone.

A practical difficulty arises in using the light from a cloudy sky since it is necessary to make determinations of $\log(I/I')$ for exactly the same conditions as for $\log(I'/I'')$, and with certain types of cloud such as alto-cumulus or rapidly drifting fracto-cumulus the changes may be so rapid that this is almost impossible. In any case it is desirable to take at least three measurements each of $\log(I/I')$ and $\log(I'/I'')$ alternately and to use the mean values.

§4. DETAILED DESCRIPTION OF THE INSTRUMENT

In order that the instrument shall not be unduly heavy it is built of duralumin, but it still weighs about 50 kg. It is constructed as a double box with central diaphragm which makes it very rigid. The optical parts are mounted on one side of this double box and the amplifier occupies the other side. The whole instrument stands on three small legs so that it cannot be subject to any twisting forces which might distort it. The external dimensions of the instrument are 137 cm. long by 25 cm. wide by 30 cm. high. At present the focal length of the collimator lenses is 46 cm. and their diameter 5 cm. If other instruments are made it is intended to reduce the focal length while keeping the same diameter and to replace the mirrors by 15° reflecting prisms. This will reduce the length and weight of the instrument while increasing the light-gathering power.

The dispersion in the plane of the three central slits is about 23 Å.U. per mm. at 3200 Å.U. In fixing the slit-widths it is necessary to make a compromise, since if the slits are wide more light will be available, but if they are too wide the variation of α over the range of wave-lengths passed by S_2 will introduce appreciable errors unless a correction (depending on the value of $x\sec\zeta$) is applied*. The actual slit-

* The percentage error in ozone due to a finite slit-width, D, is proportional to $C^2D^2\,x\sec\zeta$, where C is the change of α with λ. For the slit-width adopted the error amounts to 1 per cent. when $x\sec\zeta = 1{\cdot}36$, and is therefore negligible in most cases.

value could be obtained. This probably accounts for the longer ro were made on rapidly changing cloud and were very variable, so that no good mean ...ger variation shown by these readings, which is almost the greatest that has been found.

widths adopted were $S_1 = 0{\cdot}22$ mm., $S_2 = 0{\cdot}62$ mm., $S_3 = 1{\cdot}20$ mm., $S_4 = 0{\cdot}50$ mm., $S_5 = 2{\cdot}5$ mm. Since the change of atmospheric absorption-coefficient with wave-length is much smaller for 3265 Å.U. than for 3110 Å.U., S_3 can be made wider than S_2. The width of S_4 is immaterial since there is always ample light at that wave-length. S_5 need not be very narrow since it has merely to prevent stray light of other wave-lengths from falling on to the photoelectric cell.

Selection of wave-lengths. This again is a compromise. The shorter the wave-length set on S_2 the larger will be the value of α, but the smaller will be the amount of light available, and therefore the smaller the percentage accuracy with which it can be measured. It can be shown that, apart from variations of $(\beta - \beta')$, the greatest accuracy would be obtained if S_2 were set at about 3175 Å.U., but then the value of α would become rather small in relation to β and errors in the measurement of $(\beta - \beta')$ might lead to errors in the calculated quantity of ozone. Again, in order that $(\alpha - \alpha')$ may be large, S_3 should be set at a much longer wave-length than S_2, but $(\beta - \beta')$ also is thereby made large while it should preferably be kept small. The exact setting of S_4 is of no great importance so long as it is within the region where the sodium cell is sensitive. The value of 3110 Å.U. for S_2 was chosen because there is a bright band in the solar spectrum here.

Optical details. The two halves of the double spectroscope must be in line so that the light travels straight through from one to the other, and the slits S_2, S_3 and S_4 must be at right angles to this line; the focal plane of both spectroscopes will, however, be inclined to it. To get over this difficulty the wave-length 3110 Å.U. is focussed accurately on S_2 for both spectroscopes. The wave-lengths 3265 Å.U. and 4435 Å.U. would then come to a focus behind the slits S_3 and S_4, so that additional lenses must be inserted in the paths of these rays to bring them to a focus on the slits. Figure 7 shows the detail of the central part of the instrument, the lettering being the same as in figure 1.

In order that all the light passing through S_2 and S_3 shall fall within the prism of the second spectroscope, a lens L_1 is placed immediately behind these slits and projects an image of the prism of the first spectroscope on to that of the second. As the slit S_4 is some distance from S_3 it is found best to add a small prismatic lens behind this slit for the same purpose.

A shutter (not shown in the figures), which can be operated from the outside, shuts off either S_2 or S_4 as desired. One of two neutral screens of different density can be inserted in the path of 4435 Å.U. when required by another lever also worked from outside. This is necessary since the value of $\log (I'/I'')$ varies over a very wide range between the value for blue sky with high sun and the value for direct sunlight when the sun is low. The optical wedge is operated by means of a graduated dial on the top of the instrument and readings are taken from this dial.

The sector-wheel and commutator are driven by a gramophone motor since an electric motor would be liable to disturb the amplifier. The frequency of the alternations is about 30 ~. The brushes can be swung in order to get the best setting for the reversal of current.

Since the instrument cannot easily be carried by one person, and moreover has

to be taken out of doors, it is conveniently mounted on a trolley in which the necessary batteries also are carried. It thus forms a self-contained unit which can easily be moved about to any suitable position. With direct sunlight a quartz reflecting prism and lens are used to throw an image on to the slit. A ground quartz plate is generally inserted in front of the slit to even out the illumination and to reduce its intensity, since the energy is far too great for convenience except when the sun is low.

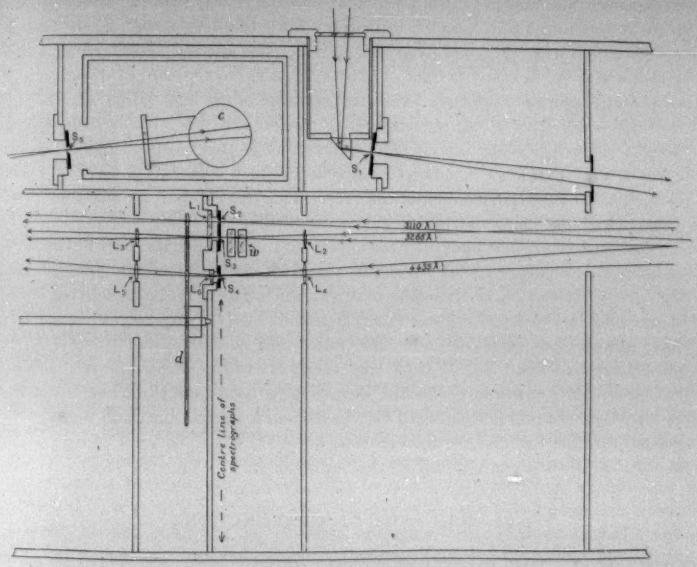

Fig. 7.

The present instrument is sufficiently sensitive to allow measurements to be made on the light from the clear zenith sky even when the sun is nearly on the horizon. One cannot calculate the ozone from such readings in the usual way, however, on account of the effect discussed at the end of § 3 (*b*). Observations on direct sunlight may be made until sec Z reaches about 7 or 8.

If an image of the full moon be thrown on to the slit with a quartz lens an appreciable deflection can be obtained on the galvanometer and very rough measurements of the amount of ozone can be made in this way when the moon is high.

§ 5. CALIBRATION OF THE INSTRUMENT

(1) *Wedge constant.* The constant of the optical wedge is given by the formula

$$K = d^{-1} \log (i_0/i),$$

i_0, i where i_0 and i are the intensities of light passing through two places on the wedge at
d a distance d cm. apart in the direction of the gradient of the wedge, when the wedge is uniformly illuminated. Fortunately it is exceedingly easy with the present instrument to determine this constant accurately. Both the slits S_2 and S_4 are blocked temporarily so that light only passes through S_3. The galvanometer reading is then a measure of the intensity of light of 3265 Å.U. which reaches the photoelectric cell. A metal gauze which transmits a known percentage of the light falling on it is held over the window admitting light to the instrument and the galvanometer reading noted for a certain wedge setting. The metal gauze is then removed and the wedge adjusted till the same galvanometer reading is obtained. The shift of the wedge then corresponds to the transmission of the gauze. This test can best be made on a uniformly clear day, using the light from the zenith sky.

(2) *Determination of* $\log (I_0/I_0')$ *and* $\log (I_0'/I_0'')$. This value can only be found by measurement of $\log (I/I')$ for a series of values of sec ζ and extrapolation to find $\log (I_0/I_0')$. As part of the atmospheric absorption is due to ozone high in the atmosphere and part to scattering low down, a plot of $\log (I/I')$ against either sec Z or sec ζ will not give quite a straight line, but it can be shown that if $\log (I/I')$ be plotted against values of sec ζ which correspond not to the true height of the ozone but to a slightly lower level (for the wave-lengths used here to 40 km. instead of 50 km.) then the points should fall very closely on a straight line, and when extrapolated this line will give the correct value of $\log (I_0/I_0')$. The value of $\log (I_0'/I_0'')$ is obtained in the same way but by the use of sec Z.

§ 6. ACCURACY OF MEASUREMENT

The possible errors of measurement can be divided into two classes, (*a*) those due to instrumental imperfections, and (*b*) those due to atmospheric causes.

(*a*) *Instrumental.* The values of $\log (I_0/I_0')$ and $\log (I_0'/I_0'')$ found from a set of observations on one day depend on the constancy of the atmosphere during the observations, and the values found on different days will naturally differ somewhat for this reason. In so far as the changes are fortuitous, the mean value can be obtained with any required accuracy by means of observations taken on a sufficiently large number of days. If, on the other hand, there is a regular change throughout the day such as a diurnal variation of the amount of ozone, the values will be systematically wrong so that the error cannot be reduced by taking a large number of observations. Apart from such a systematic error, the value of $\log (I_0/I_0')$ should easily be obtained correct to within 0·005. As the variations of $\log (I'/I'')$ are much greater than those of $\log (I/I')$, the value of $\log (I_0'/I_0'')$ cannot be obtained with as great accuracy, but as an error in this quantity has little effect on the calculated ozone value, no appreciable error in the ozone content will be introduced. The error in the ozone value

due to an error in $\log (I_0/I_0')$ is inversely proportional to $\sec \zeta$. Under the most unfavourable conditions the error in ozone due to non-systematic error in $\log (I_0/I_0')$ will not exceed 0·005 cm.

Gelatine wedges, such as we use here, cannot be made with a density gradient which is strictly uniform at different parts of the wedge, but the error in the ozone from this cause is not likely to exceed 0·003 cm., and, if necessary, the wedge can be calibrated along its length.

The error in setting the wedge so that the current in the output circuit is zero, is governed by the unsteadiness of the galvanometer which cannot be entirely eliminated. It will naturally be inversely proportional to the intensity of the light available. When using direct sunlight, the percentage error in ozone due to this cause increases about 4·4 times for each increase of 1·0 in sec Z. The error increases less rapidly with increasing sec Z when observations are made on the clear zenith sky. With an artificial light which could be kept constant and was roughly equivalent in intensity to that received from the clear sky when sec Z is about 8, it was found that the probable error of setting was under 0·005. Thus, for measurements when the sun is higher and more light is available, the error from this cause is quite negligible.

(*b*) *Atmospheric.* In observations on direct sunlight the effect of changes in the transparency of the atmosphere can be accurately allowed for and the chief errors should be those inherent in the instrument itself. In those on the light from the clear zenith sky the effect of haze will be similar to the effect of cloud and can be allowed for in the same way, but there is always some doubt whether a curve such as that in figure 6 will hold accurately under all conditions. In order to see what actual agreement would be obtained between observations taken at fairly close intervals of time, when the amount of ozone would probably not have changed much, the series of observations shown in table 1 were made. It will be seen that the agreement is very satisfactory, and that even when observations are made on a cloudy sky the error (the results from clear-sky observations being taken as correct) is nearly always under 0·01 cm. of ozone. For the purpose of studying the distribution of ozone in different meteorological conditions an error in reading of 1 per cent. is unimportant, so that it is seen that the new instrument allows observations of the amount of ozone to be made in all cases when the sun is more than about 12° above the horizon when the sky is lightly clouded, and when the sun is much lower with clear skies. If, therefore, it is found possible to arrange for a number of these instruments to be made and daily observations to be taken at a dozen or more stations distributed over N.W. Europe, the results which will be obtained in the course of a year or two should give us a thorough knowledge of the variation of ozone with pressure conditions. Whether or not this will lead to any knowledge of the formation and subsequent behaviour of cyclones and anticyclones it is naturally impossible to say at present, but in view of the closeness of the relationship which has already been found, and the value of any such knowledge if it could be obtained, it seems most highly desirable that the necessary instruments should be made and the observations carried out.

§ 7. ACKNOWLEDGMENTS

Even though the instrument was constructed in this laboratory, the cost of the materials and optical parts both for the final instrument and the preliminary investigations was very appreciable and I am greatly indebted to the Council of the Royal Society for a grant to cover these expenses.

DISCUSSION

Prof. A. O. RANKINE asked what was the frequency of commutation? Was it lower than that of the amplifier noises, so that the former could be amplified without the latter?

Mr J. GUILD. It is a great advantage to be able to obviate, in the manner described by the author, the tedious reduction of spectrum photographs involved in the older method. Would there be much difficulty in adapting the apparatus to the comparison at any desired pair of wave-lengths instead of only at a fixed pair as required for its present purpose?

Has it been experimentally verified that the relative sensitivity of the photoelectric cell for the two wave-lengths in question is independent of the actual intensity of the radiation? This would, of course, be the case with vacuum cells, in which, for each wave-length, the photoelectric current is proportional to the intensity, but it does not appear to be necessarily the case with gas-filled cells, such as are here used, if the current/intensity curves for the two wave-lengths are appreciably curved within the range of intensities at which the measurements are made.

Sir A. S. EDDINGTON. I should like to ask how rapidly the amount of ozone in the atmosphere changes. If it changes in a few hours, that must create a difficulty in using observations of the sun at different altitudes for standardization purposes, especially if there is a systematic variation according to the time of the day. There is a current belief that the ozone tends to disappear in the night, and a few years ago it was suggested that in high latitudes during the long polar night it might disappear altogether—to the immense advantage of astrophysics—but I understand that the belief is quite erroneous.

AUTHOR'S reply. Replying to Mr Guild, I do not see any reason why an instrument should not be made on the principle of the one now described, but with the slits capable of adjustment to different wave-lengths. There might be some small mechanical difficulties, but it is not beyond the powers of the British instrument-maker to overcome them. With regard to the second part of the question, I have not actually tested the relative sensitivity of the cell to different wave-lengths with different intensities of illumination, but it is very unlikely that there would be any change.

With regard to the President's question, it is found that the amount of ozone present may, at times, change by 10 per cent. or more during a day, and for this reason it is necessary to choose suitable days for the calibration of the instrument

and also to use the mean of the results from a number of sets of observations. It is not yet certain, but it is probable, that there is little difference in the amount of ozone present by day and by night; it is definitely known that there is not a markedly smaller amount at night. The amount of ozone during the polar night also is almost certainly above the average for the whole world.

In answer to Prof. Rankine, the frequency of the shutter admitting the two wave-lengths alternately is about 30 per second.

THE TUBE EFFECT IN SOUND-VELOCITY MEASUREMENTS

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ABSTRACT. The modifications of Kirchhoff's formula required to take account of the finite thermal conductivity of the tube, slip between the gas and the walls, temperature-discontinuity between the gas and the walls, and absorption of energy by the walls are calculated and are found to be negligible. The effect of roughness of the walls is discussed, and the conclusion is drawn that the large tube-effects often found in practice are due to irregular motion of the gas. It is argued that the methods of correcting for the tube-effect used hitherto are unreliable. The effects of oscillation of the piston and of the gas behind the piston are investigated and it is shown that these may become appreciable in some forms of apparatus. The yielding of a tube of elliptical cross-section is calculated and is shown to have a negligible effect.

§ 1. INTRODUCTION

The marked disagreement between the specific heats of gases as found by the sound-velocity method and those predicted from spectroscopic data, and found by the author by a constant-flow method, led him to investigate theoretically a number of factors affecting the sound-velocity in a tube*. The correction for the effect of the tube is the most uncertain part of the sound-velocity method, and may amount to a considerable percentage of the result, especially when this is expressed as a specific heat; for a change of 1 per cent. in the velocity means a change of 7 per cent. in the specific heat at constant volume in the case of air, and much more in the case of a gas of high specific heat. Very little theoretical work has been done on the subject since the papers of Kirchhoff[1] in 1868 and of Thiesen[2] in 1907. On the other hand, a great deal of experimental work has been done, and with very discordant results; the only point of agreement being the inadequacy of Kirchhoff's formula to represent the results quantitatively. It must

* As an example of this disagreement, which is in all cases particularly great at high temperatures, we may take the case of oxygen. Values of C_v for this gas, as found by Partington and Shilling, and as calculated from the vibration frequency of the molecule, are tabulated below.

Temperature	0° C.	500° C.	1000° C.
P. and S.	5·04	5·18	5·34 cals./mol.
Theoretical	5·00	6·00	6·52 ,, ,,

For nitrogen and carbon monoxide the divergences are in the same direction. The author's values for oxygen and nitrogen are in agreement with the theoretical values but are not given here as they are to be published elsewhere shortly, and the determinations are not quite finished.

be remembered that the tube effect can be determined directly for the case of air at room temperature only, for only in this case can the velocity in the free gas be measured. By the use of helium, argon, and neon, which can now be obtained fairly cheaply, the effect could be determined with certainty over a wide temperature range, for there is no reason to suppose that the specific heats of these gases vary from the value $\frac{3}{2}R$ except by small calculable amounts due to deviations from the perfect gas laws. The effect of varying viscosity and thermal conductivity might thus be investigated. Up to the present, however, this has not been done. The less certain methods which are the only ones available for other gases are discussed later.

§ 2. THE VELOCITY IN THE FREE GAS

There seems to be no reason to doubt that, if the frequency and intensity of the sound be not too high, the velocity in the free gas is given by $\sqrt{(\gamma p/\rho)}$ with small corrections for imperfection of the gas, where p and ρ are the pressure and density respectively. With high-frequency sound, however, the effects of viscosity and thermal conduction become appreciable even in the free gas, and there is the additional possibility that the rate of transfer of the internal energy between molecules may be too slow for the equipartition of energy to be maintained[3]. Thus Pierce[4] found experimentally that the velocity in air had a high value at about 40,000 vibrations per second, and that in carbon dioxide it started to rise at a frequency of about 10^5. It does not seem impossible that in gases whose molecules are more complicated the energy-exchanges might be slow enough to produce effects at lower frequencies. p, ρ

§ 3. THE VELOCITY IN AN INFINITELY LONG CYLINDRICAL TUBE

Experiment shows that the velocity of sound in gas contained in a tube is less than that in the free gas. It was early suggested that this was due to the finite viscosity and thermal conductivity of the gas, which will have much more effect in the neighbourhood of the walls of the tube than in the free gas. Helmholtz calculated the effect of viscosity and Kirchhoff[1] the effects of viscosity and heat conduction. The latter obtained the following expression:

$$\text{Velocity in tube} = a\{1 - \beta/2R\sqrt{(\pi n)}\} \qquad \ldots\ldots(1),$$

where

$$\beta = \sqrt{\mu} + (a/b - b/a)\sqrt{\nu} \qquad \ldots\ldots(2) \qquad \beta$$

$$= \sqrt{(\eta/\rho)}\{1 + \tfrac{1}{2}.\sqrt{(9\gamma - 5)}.(\gamma - 1)/\sqrt{\gamma}\} \qquad \ldots\ldots(3),$$

if we put $k/\eta c_v = \frac{1}{4}(9\gamma - 5)$.

a, b are the adiabatic and isothermal velocities, a, b

R is the tube radius, R

n is the frequency, n

η is the viscosity, η

ρ is the density, ρ

k is the thermal conductivity, k

$\mu = \eta/\rho$ and $\nu = k/\rho c_v$. μ, ν

The assumptions which he made were: (*a*) That the gas in contact with the walls is at rest. (*b*) That the gas in contact with the walls is at the same temperature as the walls. (*c*) That the temperature of the walls is constant. (*d*) That there are no irregularities in the walls of the tube of sufficient size to produce appreciable irregular motion in the gas.

Kirchhoff expressed assumption (*d*) by saying that the walls were assumed to be "perfectly smooth." This expression seems to have given rise to some misunderstanding amongst subsequent writers, for we come across such statements as "The friction and heat exchange with the walls of the tube are not accounted for in Kirchhoff's formula"[5]. Assumptions (*a*) and (*b*) are equivalent to an assumption that the walls are what is called in text-books of mechanics "perfectly rough," i.e. that the coefficient of friction with the walls is infinite and that the heat transfer between the walls and the gas in immediate contact with them is perfect. Kirchhoff himself states that "If the smooth surface of the tube is made rough, the effect of viscosity as well as that of heat conduction must increase." Since he has already assumed that they are producing the maximum effect under the conditions postulated, this statement can only be true if the roughness is sufficient to upset assumption (*d*)—i.e. to produce irregular motion.

Before carrying the theoretical discussion any further let us see how far the results of experiments agree with Kirchhoff's formula. There is an extraordinary diversity of evidence amongst the work of the numerous experimenters, but the general results may be summarized as follows: (1) The formula is qualitatively correct in that the effect of the tube increases as the radius or the frequency decrease. (2) Though one or two experimenters have considered the formula to be quantitatively correct, the majority state that the actual effect of the tube is considerably greater than that given by Kirchhoff's formula. (3) The effect on the velocity depends largely upon the nature of the inner surface of the tube, being specially great if this is rough so that assumption (*d*) is upset. (4) The effect is not always inversely proportional to the radius. Partington has, however, pointed out that it may be difficult to insure that the tubes of different radii used have exactly the same type of surface. (5) The effect is not always inversely proportional to the square root of the frequency with a given tube. Thus Seebeck found that it varied as $n^{-\frac{1}{2}}$. The evidence is summarized by Partington and Shilling, with many references, in *Specific Heats of Gases*, p. 53.

It is evident then that one or more of the assumptions (*a*), (*b*), (*c*) and (*d*) are not correct. As a matter of fact, in practice, none of them are correct. Let us consider them one by one.

(*a*) The assumption that the gas in contact with the walls is at rest involves the suppositions that the walls are at rest, and that there is no relative motion between the walls and the layer of gas immediately in contact with them: unless, of course, we suppose that the two motions cancel out, which is unlikely.

The effect of the yielding of the tube has been calculated by Lamb. The frequency of the simplest mode of radial vibration of a circular tube is given by $\sqrt{(H/\rho_t)}/2\pi R$, where R, H, and ρ_t are the radius, Young's modulus, and density.

R, H, ρ_t

This works out at about 50,000 per second for a glass tube of radius 2 cm. Thus the frequency of the sounds used in practice are much lower than the resonance frequency for this vibration, the increase in diameter of the tube is almost exactly in phase with the pressure, and the inertia of the tube can be neglected. The effect on the velocity of the sound is consequently equivalent to a change in the elasticity of the gas, and is given by

$$\frac{\text{Velocity in actual tube}}{\text{Velocity in rigid tube}} = 1 - \frac{\gamma p R}{H\tau} \qquad \text{.....(4),}$$

where τ is the wall thickness. For a glass tube of radius 2 cm. and wall thickness τ
2 mm., containing air at atmospheric pressure, the correction amounts to about 3 parts in 100,000 and so is negligible.

It is known that when a gas flows through a tube it behaves as if there was a slight slip between the gas next the wall of the tube and the wall, this effect being especially prominent at low pressures. Maxwell[6] gave a formula for the extent of this slip in terms of the fraction of the molecules striking the boundary which are "specularly reflected," and showed that there will be slight slip even if this fraction is zero. The phenomenon has been experimentally investigated by Kundt[7] and by Knudsen[8]. The effect of slip on the velocity of sound in a tube is worked out in the appendix, where it is shown that the only alteration in Kirchhoff's formula is a change in the viscosity term of β, equation (2), so that we now have

$$\beta_1 = \frac{\sqrt{\mu}}{1 + \pi\{(2-f)/f\}\sqrt{(2n\eta/p)}} + \left(\frac{a}{b} - \frac{b}{a}\right)\sqrt{\nu} \qquad \text{.....(5),}$$

where f is the fraction of the molecules which are diffusely reflected. Thus the f
effect of slip is to diminish the effect of the tube on the velocity, i.e. to increase the velocity. Since f is not far from unity, we find that for air at one atmosphere and a frequency of 1000 $\sim$, β is only altered by about 2 parts in 1000; which means about 2 parts in 100,000 of the velocity, if the tube correction is 1 per cent.

Finally it remains to enquire whether there could be any apparent motion, relative to the wall, of the gas next the wall, in a direction normal to the surface. Lately many experiments have been done showing that when plane waves of sound are reflected from a solid surface there is a loss of energy in the reflected beam, particularly if the solid be porous. This may be due to scattering, to motion of the solid surface, or to viscous absorption of energy by the air in the pores of the solid; but whatever the mechanism, the effect, so far as the body of the air is concerned, is that of a motion of the layer of air next the surface such that the displacement towards the solid is a quarter of a period out of phase with the pressure at the surface, i.e. such that there is a velocity towards the wall proportional to the excess pressure at any instant. In the appendix, an expression is derived giving the effect of such motion at the walls on the velocity of sound. It is shown that we must add two extra terms to Kirchhoff's equation, (1), thus:

$$\text{Velocity of sound} = a\left\{1 - \frac{\beta}{2R\sqrt{(\pi n)}} + \frac{\nu\chi'}{a^2 R} - \frac{1}{8}\left(\frac{\gamma\chi'}{\pi n R}\right)^2\right\} \qquad \text{.....(6),}$$

χ'

where the radial velocity at the walls $= \chi'(p - p_0)/p_0$. The ratio of the radial amplitude at the walls to the longitudinal amplitude at the centre of the tube is easily shown to be $\chi'\gamma/a$. From this we find that if the term in $\nu\chi'$ is to produce a change of 1 part in 1000 in the velocity, the ratio must be about 500 : 1 which is obviously impossible. For the term in χ'^2 to produce such an error, with a tube of radius 2 cm. and a frequency of 3000 $\sim$ (as in Partington's experiments), the ratio need be only 1 : 20, which might be possible with a porous tube. It is shown in the appendix, however, that the amplitude of a progressive wave decreases, as it passes along the tube, proportionally to $\exp(-\gamma\chi' x/aR)$, which would mean in this case that the amplitude would diminish to one-twelfth for every metre of its path. Thus, even with only half a metre between the source of sound and the reflecting stop, the reflected waves would only have one-twelfth of the amplitude of the direct waves near the source; and resonance would be practically undetectable. So that if it is possible to use a given tube we may be sure that the error due to absorption of energy by the tube is less than 1 in 1000.

(*b*) It is known that when heat is being transferred from a gas to a solid surface there often appears to be small but finite temperature-jump at the boundary. This phenomenon is especially evident at low pressures and corresponds to the phenomenon of slip mentioned above, though it is considerably more difficult to deal with theoretically. It obviously invalidates Kirchhoff's assumption (*b*). In the appendix the effect of this on Kirchhoff's formula is shown to be merely an alteration in the conductivity term in β, so that

$$\beta = \sqrt{\mu} + (a/b - b/a)\sqrt{\nu}/\{1 + 3\sqrt{(\pi n k \rho c_v)}/\zeta\} \qquad \ldots\ldots(7),$$

where ζ is the rate of heat-transfer across unit area of the boundary if the temperature-jump is one degree. It follows from this that the greater the temperature-jump (i.e. the smaller ζ) the less is the effect of the tube on the velocity, and the greater is the velocity. This we should have anticipated; since the temperature-jump at the boundary means that the compressions and expansions will be more nearly adiabatic than would otherwise be the case.

If we substitute in this expression Knudsen's formula* for the temperature-jump, we get $\zeta = (k/1{\cdot}9\,\lambda)\,.\,2g/(2 - g)$ where g is an accommodation coefficient analogous to the f in equation (5), and λ is the mean free path of the molecules of the gas. Hence

$$\beta = \sqrt{\mu} + (a/b - b/a)\sqrt{\nu}/\{1 + 2{\cdot}8\,\lambda\,(2 - g)/g\,.\,\sqrt{(\pi n \rho c_v/k)}\} \qquad \ldots\ldots(7a).$$

For air at room temperature, and a frequency of 3000 $\sim$, we find that the conductivity term in β is decreased by about 1 per cent., so that the effect on the sound-velocity is negligible.

(*c*) Kirchhoff's assumption that the walls of the tube are at a constant temperature is equivalent to assuming that they have an infinite thermal conductivity or an infinite specific heat. Actually, rapidly damped temperature-waves will pass from the inner surface of the tube for a short distance into the thickness of the wall, so that the temperature of the inner surface does not remain constant. In

* See e.g. Knudsen, *Ann. der Phys.* **34**, 593 (1911) or Hercus and Laby, *Phil. Mag.* **3**, 1061 (1927).

the appendix it is shown that the effect of this is to decrease the conductivity term in β thus:

$$\beta = \sqrt{\mu} + (a/b - b/a)\sqrt{\nu}/\{1 + \sqrt{(k\rho C_p/\kappa\rho_r C)}\} \quad \ldots\ldots(8),$$

where k, ρ, and C_p are the thermal conductivity, density, and specific heat at constant pressure per gm. of the gas, and κ, ρ_r, and C are the corresponding quantities for the tube. k, ρ, C_p κ, ρ_r, C

For glass and air under atmospheric pressure the correction to β is of the order 1 : 600, so that the effect on the velocity will be of the order 1 : 60,000 (I assume a tube correction of 1 per cent.) and is negligible.

We thus see that the invalidity of Kirchhoff's three assumptions (*a*), (*b*) and (*c*) (see p. 342) cannot appreciably affect the accuracy of his formula, and cannot account for the experimental results. In fact the effects of slip, bad heat-transfer between gas and tube, and finite conductivity of the tube not only are negligible but go in the wrong direction, for nearly all experimenters agree that the effect of the tube is greater than that predicted by Kirchhoff's formula. We are thus driven to suppose that the cause of the variations lies in the irregular motion of the gas or in some cause unknown. Partington and Shilling claim [9] that Stürm's experiments [10] and their own show that the velocity is dependent on the material of the tube; but this argument suffers from the same objection that Partington and Shilling have themselves pointed out when discussing experiments on the effect of variation of the tube radius: the conductivity was not the only variable in these experiments, for this could not be altered without the surface being altered as well. That irregular motion of a sort not accounted for in Kirchhoff's theory does occur in resonance tubes appears to be shown by the recent experiments of Andrade [11] and Pringle [12], who watched individual dust particles in a Kundt tube. From the sinusoidal tracks of particles shown in the photograph which accompanies Andrade's letter to *Nature* [11], it is evident that some of the particles had steady radial velocities equal to about one-sixth of their maximum vibrational velocity: though whether this indicated a motion of the gas, or a motion of the particle relative to the gas, I do not know. This must result in extra dissipation of energy and might well also affect the sound velocity. It is well known that when a stream of gas is flowing through a tube there is a certain velocity above which the steady streamline motion becomes unstable, and eddy motion sets in at the slightest disturbance. It does not seem unreasonable to suppose that the same might hold for the oscillatory motion in a resonance tube*; though owing to the very different velocity-distribution across the tube in the two cases, the critical velocity would probably be quite different. Slight irregularities in the tube might well cause such eddy motion to set in; and, indeed, the observed dependence of the velocity on the surface shows that the effect is aggravated, if not caused, by irregularities in the surface. It is possible that the effect might be avoided by the use of much smaller sound-intensities, and, if necessary, of an amplifier to detect resonance. The maximum air velocity might thus be reduced below the critical value. We can form an estimate of the size of the irregularity required by noting that at a distance

* Since this paper was read Prof. Andrade has published a second letter to *Nature*, 127 (1931) in which he shows photographs of such vortices in a sounding tube.

of $2\pi\sqrt{(2\mu/n)}$ from the wall the amplitude of the oscillations is only about 0·002 of its value at the centre of the tube[13]. For air, and a frequency of 3000 ~, this distance is about 0·25 mm., and it does not seem that irregularities of a grain-size less than this could produce much effect, since they would be situated in comparatively still air*. This does not necessarily mean that the irregularities must project more than this distance into the tube in order to produce an effect, for if they were of considerable area their effect could penetrate beyond it. The point is that mere roughness should not produce such a large effect as a slightly wavy surface. In this connexion it is worthy of note that Cornish and Eastman[14], two of the very few observers who have confirmed Kirchhoff's formula, used metal tubes which, though rougher than glass tubes, would be less prone to waviness.

Let us now consider the bearing of the above discussion on the determination of specific heats. The use of a tube to contain the gas can hardly be avoided except for air at room-temperature unless we use very high frequencies, which would introduce other errors. Hence a tube correction is necessary; and we can get an idea of its importance from the facts that in the experiments of Partington and Shilling[15] on air, oxygen and nitrogen, it varied from about 0·07 to 3·6 per cent. of the velocity, i.e. from 0·5 to 25 per cent. of C_v.

Three methods, other than the use of Kirchhoff's formula, have been tried for determining the correction. Of these the first is to use tubes of two or more different diameters and to extrapolate to the case of infinite diameter, either by assuming that the tube correction varies inversely as the diameter as in Kirchhoff's formula, or empirically, using several tubes. The uncertainty of the true dependence of the correction on the diameter and the fact that the perfection of the surface will vary from tube to tube make this method quite unreliable.

The second method is to use only one tube but several frequencies, and to extrapolate to infinite frequency, either by assuming that the correction varies inversely as the square root of the frequency, or empirically. Here again the trouble is that the correction does not always obey the above rule; and extrapolation under such circumstances is uncertain.

The third method is to abandon that part of Kirchhoff's formula which deals with R and n, but still assume that the correction is proportional to β. The correction is determined experimentally for air at room-temperature by making use of the determinations of the velocity in free air. For air at other temperatures and for other gases the above assumption is made. So far as the author is aware, no justification has been brought forward for this assumption. Seeing that the correction for air at room-temperature for one of Partington and Shilling's tubes was 14 times, and for another tube was only 1·05 times that given by Kirchhoff's formula, the retention of even a part of this formula requires justification; especially when, in one case, the correction amounted to about 25 per cent. on the specific heat! If we assume that the true correction for a given gas is known at 0° C., and if the value of γ for the gas does not vary much with temperature, then the procedure is equivalent to assuming that the correction is proportional to η/ρ for all temperatures. Now

* See, however, the remarks of Dr Richardson and the author's reply in the discussion at the end of this paper.

several experimenters have stated that the effect of the tube is not inversely proportional to $\sqrt{n}$; for instance Seebeck[16] found it varied as $n^{-\frac{1}{2}}$. If this be so, and if, as has been shown above, the only properties of the tube which can affect the velocity are its size and shape (I include roughness in this); both being properties requiring lengths only to define them, the theory of dimensions shows that the correction cannot be proportional to η/ρ, as this would require the time dimensions to be wrong. Even if, with a given value of γ, the correction were proportional to η/ρ it is difficult to see why the effect of the tube should be divided between the viscosity effect and the conductivity effect in the same ratio as that given in Kirchhoff's formula, in spite of the fact that the correction is sometimes 14 times too large. In other words, even if the dependence on η/ρ were correct, it is improbable that the dependence on γ would be so.

Before leaving the subject of the tube correction, we may point out that where, as in Dixon's experiments, the group-velocity is measured in place of the wave-velocity, the magnitude of the correction will depend upon the way in which the velocity varies with the frequency. The group-velocity is given by

$$U = V - \lambda dV/d\lambda \qquad \text{.....(9),}$$

or

$$U = V + ndV/dn \quad \text{if the difference is small} \qquad \text{.....(10).}$$

V is here the wave-velocity. Application of this formula gives at once the result that if the effect of the tube varies inversely as $\sqrt{n}$, then the correction to be added to the observed group-velocity is one-half of that which would be required for the wave-velocity at about the same frequencies. If the tube effect varied as $n^{-\frac{1}{2}}$ the correction would also be half that required for the wave-velocity, but would have to be subtracted instead of added. The fact that Dixon found that the velocity was less in the tube than in free air apparently shows that, in his case at least, the tube effect could not have varied according to a higher inverse power of n than n^{-1}. The above formula for the group-velocity applies, however, only to a sound whose components lie within a narrow frequency range. A pulse would change its form as it progressed, and no very definite conclusion can be drawn.

It appears, then, that much systematic experimental work will have to be done before great faith can be placed in the results of sound-velocity determinations in tubes.

§ 4. THE VELOCITY IN A TUBE OF FINITE LENGTH

Having discussed the factors which must be taken into account in any method of measuring sound-velocity in a tube, we will now consider a few of the—less important—effects due to the ends of the tube. It is fairly obvious that the ends will not appreciably affect the velocity of a progressive wave in the tube except in their immediate neighbourhood. They will not, therefore, alter the distances between adjacent nodes in the middle of the tube, though they will determine the positions of these nodes. It follows that the dust-figure methods are unaffected by end-errors, though end-corrections are necessary with certain resonance methods.

Thiesen[17] has shown that in the use of his method, in which the whole length of the resonator and the frequency are measured, an extra term depending on the

l length of the resonator must be introduced into Kirchhoff's expression. Thus if l be the length of the resonator,

$$\text{Velocity} = a\{1 - \sqrt{\mu}/2R\sqrt{(\pi n)} - (1/R + 2/l)(a/b + b/a)\sqrt{\nu}/2\sqrt{(\pi n)}\} \quad \ldots(11).$$

It will be seen that the extra correction becomes negligible for the long tubes used by most experimenters; and in any case the equation is of little more use than Kirchhoff's equation, to which it reduces when $l = \infty$.

Of more interest is it to enquire whether vibration of the end of the tube can affect the results. It is shown in the appendix that only that component of the motion of the stop which is in phase with the gas pressure (i.e. the component which absorbs no energy from the gas) can affect the positions of the nodes. If D'' be the velocity-amplitude, per unit pressure-amplitude in the gas, of this component of the stop's motion, then the position of the nodes and antinodes are given by

D''

$$\tan 4\pi x/\lambda = 2\sqrt{(\rho E)}\, D'' \quad \ldots\ldots(12),$$

x, E where x is the distance from the stop, while $E = \gamma p$ and is the elasticity of the gas. Thus, providing that D'' remains constant, the distances between the nodes are equal to the half-wave-length in the gas, and no error is introduced. In some experiments, however (e.g. those of Partington and Shilling), the stop is moved along to obtain the various resonance points, so that the free length of the rod or tube which supports it varies during the experiment. It is shown in the appendix that if we neglect the mass of the piston itself, D'' is given by

$$D'' = \frac{1}{2\pi n\rho'} \cdot \frac{\mu^{-1}\sinh(2l/\mu) - 2\pi\lambda'^{-1}\sin(4\pi l/\lambda')}{\cosh(2l/\mu) + \cos(4\pi l/\lambda')} \cdot \frac{A_p}{A_r} \quad \ldots\ldots(13),$$

ρ', l, λ' where ρ' is the density of the piston-rod, l is its free length; λ' is the wave-length

μ of the sound in the rod, μ is the distance which waves must travel along the rod before they are reduced to e^{-1} times their original amplitude (i.e. an inverse

A_p, A_r measure of the damping), and A_p, A_r are the areas of cross-section of the piston and rod respectively.

If λ'/μ be small, as it probably is in practice, this can be simplified and we get

$$\tan\left(\frac{4\pi x}{\lambda}\right) = \sqrt{\left(\frac{\rho E}{\rho' H}\right)} \cdot \frac{A_p}{A_r} \cdot \frac{(\lambda' l/\pi\mu^2) - \sin(4\pi l/\lambda')}{l^2/\mu^2 + \cos^2(2\pi l/\lambda')} \quad \ldots\ldots(14),$$

H where H is Young's modulus for the piston-rod. The fraction on the right is shown plotted against l/λ' in figure 1 for the case in which $\lambda'/\mu = \frac{1}{8}$.

It is difficult to form an estimate of the value of λ'/μ in practice; but whilst the extreme peaks of the curve are greatly dependent on μ, being higher as μ is greater, the remainder of the curve is almost independent of μ. Since the errors are small, $\tan(4\pi x/\lambda)$ is nearly equal to $4\pi x/\lambda$ for the first node, and we can regard the graph as giving directly the displacement of the nodes for various positions of the piston. For air and a silica rod, if $A_p/A_r = 30$ the ordinates of the above curve must be multiplied by about 3×10^{-5} to give the ratio of the displacement to the wave-length. Also if the frequency is 3000, λ' is about 170 cm., so that the above graph corresponds to the range $l = 0$ cm. to $l = 170$ cm. The conditions mentioned

are roughly those holding in Partington and Shilling's experiments with silica tubes. It is evident, then, that except near the resonance points for the piston-rod, the errors in setting are of the order of 0·01 mm. and can be neglected. If a reading were taken near a resonance point, however, an appreciable error might result.

Another vibratory system sometimes coupled to that of the gas in the resonance tube is the gas which is contained in the tube behind the piston. If the latter is made smaller than the bore of the tube, as it often is to avoid its rubbing against the tube, there will be an interchange of energy between the systems. In the appendix it is shown that the proportional error introduced by this effect is approximately equal to

$$-\alpha/\{1-\mathscr{R}+B\sin 2\theta+(\mathscr{R}-1/\mathscr{R}-B^2/\mathscr{R})\sin^2\theta\} \quad \text{.....(15)},$$

where α is the fractional difference between the sound-velocities on the two sides α

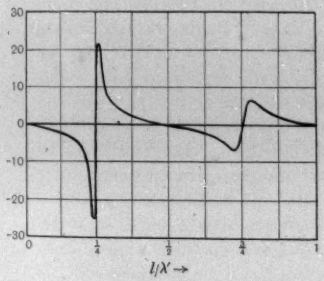

Fig. 1.

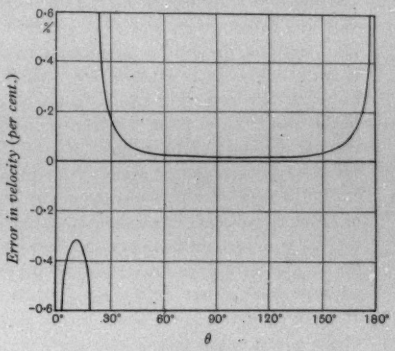

Fig. 2.

of the piston, due to the presence of the piston-rod, $\mathscr{R}$ is the ratio of the cross-sectional areas of the gas-spaces in front of the piston and behind it, $B=2\pi^2R^2/\lambda C$, C is the acoustic conductivity of the annular space between the piston and the walls of the tube, and θ can be taken as equal to $(2\pi L/\lambda - N\pi)$, where L is the total length of the tube. $\mathscr{R}$ / B, C / θ, L

If $\mathscr{R}$ is nearly unity (that is, if the piston-rod is much smaller than the tube) and $B^2/\mathscr{R}$ is much greater than unity, we can write the above expression

$$\alpha/B\,(B\sin^2\theta-\sin 2\theta) \quad \text{.....(16)}.$$

To get a rough estimate of the value of α we will assume that the effect of the tube and piston-rod on the sound-velocity is proportional directly to the combined perimeters of their cross-sections and inversely to the area of the cross-section of the gas-space between them. This would follow from the simple theory of the effect given in Rayleigh's book on sound [18]. If R and r are the radii of the inner surface of the tube and of the piston-rod respectively, and if the effect of the tube alone R, r

v_0, δ is given by $v = v_0(1 - \delta)$, the effect of the tube and piston-rod together should be given by

$$v' = v_0\{1 - R/(R - r).\delta\}, \qquad \text{.....(17).}$$

v' where v' is the sound-velocity in the space between the tube and piston-rod.

Thus $$\alpha = r\delta/(R - r) \qquad \text{.....(18).}$$

We also have $$\mathscr{R} = R^2/(R^2 - r^2) \qquad \text{.....(19).}$$

In Partington and Shilling's silica apparatus $R = 2$ cm., $r = 0{\cdot}5$ cm. and $\delta = 0{\cdot}009$, so that $\alpha = 0{\cdot}003$ for air at room temperature, and $\mathscr{R} = 16/15$.

It is difficult to calculate a value for the acoustic conductance of the space between the piston and the wall, so it was determined experimentally by construction of an accurate model of the piston and tube walls. This was filled with a solution of an electrolyte, two large electrodes were inserted some distance away on either side of the piston, and the electrical resistance between them was measured with and without the piston. The specific resistance of the solution was measured also; and this, divided by the increase in cell-resistance due to the piston, gave a quantity numerically equal to the acoustic conductivity of the gap. The result came out as 1·5 cm. For a wave-length of 10 cm., used by Partington and Shilling in the case of air at room temperature, this makes B equal to 5 nearly. Thus we can use expression (16) in place of (15). In figure 2 the percentage error in the velocity has been plotted against θ for these values of α and B.

An increase of π in θ corresponds to change of tube-length or of wave-length of about $2\frac{1}{2}$ per cent. for a 2-metre tube and a wave-length of 10 cm. It will be seen that the error is about 0·015 per cent. over about half of the range but that it is greater than 0·3 per cent. over about a sixth of the range. It would be difficult to predict in advance on what part of the range θ would lie for any given wave-length and tube. It is not claimed that the above theory gives anything but the order of magnitude of the effect; but it does at least show that the effect has to be reckoned with. It could be diminished by allowance of as small a gap as possible between the piston and the walls, or by the use of a piston of considerable thickness.

§ 5. THE VELOCITY IN A TUBE OF IMPERFECT SHAPE

We have already mentioned the possible effect of small irregularities in the tube wall. When there are variations in the bore of the tube extending over lengths greater than the radius of the tube it is possible to make an approximate calculation of their effect if no irregular motion of the gas is produced. Lord Rayleigh has shown in his book on sound [19] that in the case of a tube which departs slightly from the cylindrical shape, the deviations of the nodes from their normal positions are given by

$$\Delta l = \int_0^l \cos\frac{2\pi m x}{l}\frac{\Delta s}{S_0}\,dx \qquad \text{.....(20),}$$

l, m, S_0, Δs where l is the length from one end of the resounding tube to the mth node; S_0 is the mean cross-sectional area; and Δs is the difference between the actual area at any position and the mean.

To take an example, let us suppose that for a distance of $\lambda/4$ on one side of a given node the diameter of the tube is 1 per cent. greater than the mean. Then that node will be displaced by a distance

$$\int_{\lambda/4}^{\lambda/2} \cos\frac{4\pi x}{\lambda} \cdot \frac{1}{50}\,dx, \text{ which} = \frac{\lambda}{100\pi} \qquad \text{.....(21).}$$

Deviations greater than 1 per cent. in the diameter may be expected in glass and silica tubes. Errors due to this, however, will tend to cancel out if many nodes are measured and the mean distance between them obtained in the best way. This will not be the case, however, if the mean is obtained by taking the distances of all the nodes from the first node, dividing each by the appropriate number of half-wavelengths, and taking the mean, for in this case an error in the position of the first node would be reproduced *in toto* in the mean.

The last effect to be discussed is that of the yielding of a tube whose cross-section is not circular. As has been stated above, Lamb showed that the yielding of circular tubes produced a negligible effect; it is not immediately evident, however, that the same would apply to an elliptical tube, since bending as well as stretching would take place. In experiments where the tube is coiled round in a spiral, as in those of Dixon, the cross-section is almost certain to be slightly elliptical. Dixon showed experimentally that there was no measurable difference between the sound-velocities in a leaden tube when straight and when coiled. It was thought worth while to show mathematically that this will be so with all tubes. It is shown in the appendix that the fractional effect on the velocity is given by

$$-\frac{\gamma P}{H}\left(\frac{b}{t}\right)^3 \epsilon \qquad \text{.....(22),}$$

where H is the Young's modulus for the tube material, b is the semi-axis major of the ellipse, t is the wall thickness and *H*, *b* *t*

$$\epsilon = 2f^{-1}\{12\pi^{-1}\beta^3 E f^4 - 3/8 \cdot \beta f^2(1-f^2)(5+3f^2) + 3/64 \cdot (1-f^2)^2(7+5f^2)\} \quad \text{...(23),} \qquad \epsilon$$

where f is the ratio of the minor to the major axis of the ellipse; β is a certain function of f whose value is given in Timoshenko's *Elasticity*[20] and E is an elliptic integral of the second kind giving the ratio of half the perimeter of the ellipse to the major axis and depending upon f. Values of these are tabulated below for a few values of f. *f*, *β* *E*

Table

f	β	E	ϵ
1·0	0	$\pi/2$	0
0·9	·057	1·492	·014
0·8	·133	1·417	·054
0·6	·391	1·278	·204

If we put $\gamma = 1{\cdot}4$, $p = 10^6$ dynes/cm.^{2}, $H = 6 \times 10^{11}$ dynes/cm.^{2}, we get

Fractional effect on velocity $= -3{\cdot}2 \times 10^{-8}.(b/t)^3$ if $f = 0{\cdot}9$

$-12{\cdot}4 \times 10^{-8}\,(b/t)^3$ if $f = 0{\cdot}8$

$-4{\cdot}6 \times 10^{-7}\,(b/t)^3$ if $f = 0{\cdot}6$.

It will be seen that even if we suppose f to be 0·6 and b/t to be 10, the effect is only about 1 part in 2000; and these conditions are extreme. In practice, then, we can at ordinary pressures neglect the effect of the yielding of the tube whether it be of circular or elliptical cross-section.

MATHEMATICAL APPENDIX

(i) *Kirchhoff's wave equations.* For the case of waves symmetrical about an axis, and assuming that the three quantities u, s, and $\theta \propto e^{mx+ht}$, Kirchhoff has shown that

m, x, h, t

$$u = AQ - A_1 m\,(h/\lambda_1 - \nu)\,Q_1 - A_2 m\,(h/\lambda_2 - \nu)\,Q_2 \qquad \ldots\ldots(24),$$

$$s = -A.\frac{m}{h/\mu - m^2}\frac{dQ}{dr} - A_1\left(\frac{h}{\lambda_1} - \nu\right)\frac{dQ_1}{dr} - A_2\left(\frac{h}{\lambda_2} - \nu\right)\frac{dQ_2}{dr} \qquad \ldots(25),$$

$$\theta = A_1 Q_1 + A_2 Q_2 \qquad \ldots\ldots(26),$$

u, s where u and s are the longitudinal and radial velocity components respectively, and

θ θ is proportional to the temperature-difference between the actual state and the

δT undisturbed state, and is defined by $\theta = \delta T.\alpha/(\gamma - 1)$, where δT is this temperature-

α difference, and α is defined by $p/\rho = p_0/\rho_0.(1 + \alpha.\delta T)$, so that for a perfect gas

μ, ν $\theta = T^{-1}\,\delta T/(\gamma - 1)$. μ and ν are equal to η/ρ and $k/c_v\rho$ respectively, and are thus

λ_1, λ_2 measures of the viscosity and thermal conductivity. λ_1 and λ_2 are the roots of the equation

$$h^2 - \{a^2 + h\,(4\mu/3 + \nu)\}\,\lambda + \nu/h.\{b^2 + h.4\mu/3\}\,\lambda^2 = 0 \qquad \ldots\ldots(27),$$

a, b where a and b are defined by $\sqrt{(p_0/\rho_0.\gamma)}$ and $\sqrt{(p_0/\rho_0)}$ respectively, so that for a perfect gas they are equal to the adiabatic and isothermal sound-velocities respectively.

Q, Q_1, Q_2 Q, Q_1, and Q_2 are functions of r which satisfy the following equations:

$$\left.\begin{aligned} d^2Q/dr^2 + 1/r.dQ/dr - (h/\mu - m^2)\;Q &= 0 \\ d^2Q_1/dr^2 + 1/r.dQ_1/dr - (\lambda_1 - m^2)\,Q_1 &= 0 \\ d^2Q_2/dr^2 + 1/r.dQ_2/dr - (\lambda_2 - m^2)\,Q_2 &= 0 \end{aligned}\right\} \qquad \ldots\ldots(28).$$

A, A_1, A_2 A, A_1, and A_2 are constants to be determined by the boundary conditions. Apparently, the only approximations which have been made in deducing the above equations are the assumption of small amplitude, and that u, s, and θ vary as e^{mx+ht}. Lord Rayleigh[21] has shown that in a Kundt's tube there are actually continuous circulations between the nodes and antinodes, but these depend upon small quantities of the second order, which for our present purpose we can neglect.

Kirchhoff determined the velocity of sound in the tube on his assumptions by putting u, s, and θ all equal to zero when $r = R$ and eliminating the A's from the resulting equations. We shall allow for various factors by putting the appropriate values for u, s, and θ when $r = R$ and then proceeding as before. In order to avoid complications we shall not allow for all the factors at once, but shall take them one by one. Since it will appear that the corrections thus introduced are all very small, we may suppose that any combination terms which would appear if we allowed for all the factors at once will be still smaller and may be neglected.

(ii) *The effect of slip.* To allow for this we will make use of Maxwell's expression and put $u = -\omega\,(\partial u/\partial r)_{r=R}$ at the boundary, where

$$\omega = \eta\sqrt{(\pi/2\,p_0\rho_0)}\,.(2-f)/f \qquad \text{.....(29),} \qquad \omega$$

f being the fraction of the molecules which is diffusely reflected on striking the wall. f

Substituting in equation (24) we get, when $r = R$,

$$A\,(Q + \omega\,dQ/dr) - A_1 m\,(h/\lambda_1 - \nu)\,(Q_1 + \omega\,dQ_1/dr) - A_2 m\,(h/\lambda_2 - \nu)\,(Q_2 + dQ_2/dr) = 0 \qquad \text{.....(30).}$$

Putting $s = 0$, $\theta = 0$ when $r = R$ in equations (25) and (26), and eliminating the A's, we get

$$\begin{vmatrix} 0 & Q_1 & Q_2 \\ -(Q + \omega\,dQ/dr) & m\,(h/\lambda_1 - \nu)\,(Q_1 + \omega\,dQ_1/dr) & m\,(h/\lambda_2 - \nu)\,(Q_2 + \omega\,dQ_2/dr) \\ \dfrac{m}{h/\mu - m^2}\dfrac{dQ}{dr} & \left(\dfrac{h}{\lambda_1} - \nu\right)\dfrac{dQ_1}{dr} & \left(\dfrac{h}{\lambda_2} - \nu\right)\dfrac{dQ_2}{dr} \end{vmatrix} = 0 \qquad \text{.....(31).}$$

After multiplying out and dividing through by QQ_1Q_2, we now approximate, as did Kirchhoff, by putting

$$d\log Q/dr = \sqrt{(h/\mu)};\quad d\log Q_1/dr = r\,(\lambda_1 - m^2)/2;$$
$$d\log Q_2/dr = \sqrt{\lambda_2};\quad \lambda_1 = h^2/a^2;\quad \lambda_2 = ha^2/\nu b^2$$

and in the term containing $\sqrt{\mu}$, $m = h/a$. See Kirchhoff's paper for the justification of these. We thus get

$$m^2 = h^2/a^2\,.(1 + 2\delta/R\sqrt{h}) \qquad \text{.....(32),}$$

where $$\delta = \sqrt{\mu}\,\{1 + \omega\sqrt{(h/\mu)}\}^{-1} + (a/b - b/a)\sqrt{\nu}. \qquad \delta$$

Putting $h = 2\pi n i$, separating out the real and imaginary parts and neglecting the square of the quantity $\omega\sqrt{(\pi n/\mu)}$ which with actual gases is small, we get

$$m = \pm\left[\frac{\sqrt{(\pi n)}}{aR}\left\{\sqrt{\mu} + \left(\frac{a}{b} - \frac{b}{a}\right)\sqrt{\nu}\right\} + i\left\{\frac{2\pi n}{a} + \frac{\sqrt{(\pi n)}}{aR}\left(\frac{\sqrt{\mu}}{1 + 2\omega\sqrt{(\pi n/\mu)}} + \left(\frac{a}{b} - \frac{b}{a}\right)\sqrt{\nu}\right)\right\}\right] \text{...(33),}$$

so that if $m = m' + im''$, we have m', m''

$$\text{Sound-velocity} = 2\pi n/m'' = a\,\{1 - \beta_1/2R\sqrt{(\pi n)}\},$$

where $$\beta_1 = \frac{\sqrt{\mu}}{1 + \pi\,\{(2-f)/f\}\sqrt{(2n\eta/p)}} + \left(\frac{a}{b} - \frac{b}{a}\right)\sqrt{\nu} \qquad \text{.....(34).} \qquad \beta_1$$

(iii) *The effect of radial motion at the walls.* This includes both the yielding of the tube walls, and, as explained on page 343, the absorption of energy by the walls. Let us suppose that at the walls $s = \chi\,(p - p_0)/p_0$, where χ is complex. Let σ be the condensation at any point, so that $\rho = \rho_0\,(1 + \sigma)$. Then it follows from the definitions of σ and θ that χ σ

$$p - p_0 = p_0\sigma + p_0\,(\gamma - 1)\,\theta.$$

Therefore $(p-p_0)/p_0 = \sigma + (\gamma - 1)\theta$

$= \gamma\theta - \nu\Delta\theta/h,$

where $\Delta\theta = \partial^2\theta/\partial x^2 + \partial^2\theta/\partial y^2 + \partial^2\theta/\partial z^2,$

therefore $(p-p_0)/p_0 = \gamma\theta - (A_1\lambda_1 Q_1 + A_2\lambda_2 Q_2).\nu/h$

$= A_1(\gamma - \nu\lambda_1/h)\,Q_1 + A_2(\gamma - \nu\lambda_2/h)\,Q_2$ .....(35).

See Kirchhoff's paper for the derivations of the relations on which these rest.

Hence, putting $s = \chi(p-p_0)/p_0$ when $r = R$ in equation (25), and $u = 0$ and $\theta = 0$ when $r = R$ in equations (24) and (26), and eliminating the A's, we get

$$\begin{vmatrix} 0 & Q_1 & Q_2 \\ \dfrac{m}{h/\mu - m^2}\dfrac{dQ}{dr} & \left(\dfrac{h}{\lambda_1} - \nu\right)\dfrac{dQ_1}{dr} + \chi\left(\gamma - \dfrac{\nu\lambda_1}{h}\right)Q_1 & \left(\dfrac{h}{\lambda_2} - \nu\right)\dfrac{dQ_2}{dr} + \chi\left(\gamma - \dfrac{\nu\lambda_2}{h}\right)Q_2 \\ Q & -m(h/\lambda_1 - \nu)\,Q_1 & -m(h/\lambda_2 - \nu)\,Q_2 \end{vmatrix} = 0$$

.....(36).

This after a little simplification and division by Q_1Q_2Q gives

$$\frac{m^2 h}{h/\mu - m^2}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)\frac{d\log Q}{dr} + \left(\frac{h}{\lambda_1} - \nu\right)\frac{d\log Q_1}{dr} - \left(\frac{h}{\lambda_2} - \nu\right)\frac{d\log Q_2}{dr} - \chi\frac{\nu}{h}(\lambda_1 - \lambda_2) = 0 \quad \text{.....(37),}$$

and with the same approximations as before, this gives:

$$m^2 = \frac{h^2}{a^2}\left[1 + \frac{2}{R}\left\{\frac{\beta}{\sqrt{h}} + \chi\left(\frac{1}{h}\frac{a^2}{b^2} - \frac{\nu}{a^2}\right)\right\}\right].$$

χ', χ'' Putting $h = 2\pi ni$, so that $\sqrt{(1/h)} = (1-i)/2\sqrt{(\pi n)}$ and $\chi = \chi' + i\chi''$, we get

$$m^2 = -\frac{4\pi^2 n^2}{a^2}\left[1 + \frac{1}{R}\left\{\frac{\beta}{\sqrt{(\pi n)}} + \frac{\gamma\chi''}{\pi n} - \frac{2\nu\chi'}{a^2}\right\} - \frac{i}{R}\left\{\frac{\beta}{\sqrt{(\pi n)}} + \frac{\gamma\chi'}{\pi n} + \frac{2\nu\chi''}{a^2}\right\}\right].$$

Now if x and y are small

$$\sqrt{(-1 - x + iy)} = (-\tfrac{1}{2}y + \tfrac{1}{4}xy) + i(1 + x + \tfrac{1}{4}y^2) \text{ nearly,}$$

so that if we neglect squares of β and ν and the cubes, etc. of χ' and χ'', and if $m = m' + im''$, we get

$$\left.\begin{aligned} m' &= \frac{\sqrt{(\pi n)}}{aR}\beta + \frac{\gamma}{aR}\chi' + \frac{2\pi n}{a^3 R}\nu\chi'' - \frac{\gamma^2}{2aR^2\pi n}\chi'\chi'' \\ m'' &= \frac{2\pi n}{a} + \frac{\sqrt{(\pi n)}}{aR}\beta + \frac{\gamma}{aR}\chi'' - \frac{2\pi n}{a^3 R}\nu\chi' + \frac{\gamma^2}{4aR^2\pi n}(\chi'^2 - \chi''^2) \end{aligned}\right\} \quad \text{...(38).}$$

$\therefore$ Sound-velocity $= 2\pi n/m''$

$$= a\left\{1 - \frac{\beta}{2R\sqrt{(\pi n)}} - \frac{\gamma}{2\pi nR}\chi'' + \frac{\nu\chi'}{a^2 R} + \left(\frac{\gamma}{R\pi n}\right)^2\frac{2\chi''^2 - \chi'^2}{8}\right\}$$

.....(39).

The term in χ'' is the correction for the elastic yielding of the tube, and if we put $\chi'' = 2\pi nR^2 p_0/H\tau$ it becomes $-\gamma pR/H\tau$, which is identical with the expression

obtained by Lamb. We have already seen (p. 343) that it can be neglected. Hence we may also neglect the term in χ''^2, and write

$$\text{Sound-velocity} = a\left\{1 - \frac{\beta}{2R\sqrt{(\pi n)}} + \frac{\nu\chi'}{a^2R} - \left(\frac{\gamma}{R\pi n}\right)^2 \frac{\chi'^2}{8}\right\} \quad \text{.....(40).}$$

The amplitude of a progressive wave diminishes as it goes along the tube proportionally to $e^{-m'z}$; and since the terms in χ'' can be shown to be negligible we may write

$$m' = \{\beta\sqrt{(\pi n)} + \gamma\chi'\}/aR \quad \text{.....(41).}$$

(iv) *The effects of the temperature discontinuity* at the surface of the tube, and of the *finite conductivity* of the tube wall may conveniently be treated together. Let ϕ be the temperature, reckoned from the mean, at a point in the material of the tube at a distance z from the inner surface, and let κ, ρ_t, and c be the thermal conductivity, the density, and specific heat per gramme of the tube material. At the frequencies used in practice the changes of temperature due to the vibrations of the gas will only penetrate a short distance into the tube wall, and we shall not introduce any appreciable error if we consider the surface to be plane instead of cylindrical. We then have

ϕ

z, κ, ρ_t, c

$$\frac{\partial^2\phi}{\partial z^2} = \frac{\rho_t c}{\kappa}\frac{\partial\phi}{\partial t}.$$

We may assume that $\phi \propto e^{ht}$, and we get as the solution for which $\phi = 0$ when $z = \infty$, and $\phi = \phi_0 = \Phi_0 e^{ht}$ when $z = 0$

ϕ_0, Φ_0

$$\phi = \phi_0 . \exp\{-\sqrt{(\pi n\rho_t c/\kappa)}\, z\} \quad \text{.....(42),}$$

or, taking the real part,

$$\phi = \Phi_0 . \exp\{-\sqrt{(\pi n\rho_t c/\kappa)}\, z\} . \cos 2\pi n\{t - \sqrt{(\rho_t c/4\pi n\kappa)}\, z\} \quad \text{.....(43).}$$

This represents a system of waves whose amplitude decreases to $1/e$ times its initial value in a distance which, for glass, is about 7×10^{-4} cm.; so that our assumption of a plane surface instead of a cylindrical was quite justifiable.

The rate at which heat crosses unit area of the inner surface of the tube is given by

$$-\kappa\,(\partial\phi/\partial z)_{z=0} = \sqrt{(h\rho_t c\kappa)}\,\phi_0,$$

and this must be equal to

$$\zeta\,(\delta T - \phi_0) = \zeta\,\{(\gamma - 1)\,\theta/\alpha - \phi_0\},$$

where ζ is the thermal permeability of the interface; and also to

ζ

$$-k.\partial T/\partial r = -k\,(\gamma - 1)/\alpha . \{A_1 . dQ_1/dr + A_2 . dQ_2/dr\}.$$

Eliminating ϕ_0 between these three equal quantities, we get

$$-k\left(A_1\frac{dQ_1}{dr} + A_2\frac{dQ_2}{dr}\right) = \frac{\sqrt{(\rho_t c\kappa h)}}{\sqrt{(\rho_t c\kappa h)} + \zeta} . \zeta\theta$$
$$= \psi\,(A_1Q_1 + A_2Q_2)$$

by equation (26), where

$$\psi^{-1} = (\rho_t c\kappa h)^{-1} + \zeta^{-1}.$$

Hence when $r = R$

$$A_1\left\{Q_1 + \frac{k}{\psi}\frac{dQ_1}{dr}\right\} + A_2\left\{Q_2 + \frac{k}{\psi}\frac{dQ_2}{dr}\right\} = 0 \quad \text{.....(44).}$$

Putting $u = 0$ and $s = 0$ when $r = R$ in equations (24) and (25), and eliminating the A's, we have

$$\begin{vmatrix} 0 & Q_1 + \frac{k}{\psi}\frac{dQ_1}{dr} & Q_2 + \frac{k}{\psi}\frac{dQ_2}{dr} \\ Q & -m\left(\frac{h}{\lambda_1} - \nu\right)Q_1 & -m\left(\frac{h}{\lambda_2} - \nu\right)Q_2 \\ -\frac{m}{h/\mu - m^2}\frac{dQ}{dr} & -\left(\frac{h}{\lambda_1} - \nu\right)\frac{dQ_1}{dr} & -\left(\frac{h}{\lambda_2} - \nu\right)\frac{dQ_2}{dr} \end{vmatrix} = 0 \qquad \text{.....(45).}$$

Simplifying this, dividing by QQ_1Q_2 and making the usual approximations, we get

$$m^2 = h^2/a^2.(1 + 2\epsilon/R\sqrt{h}),$$

ϵ where

$$\epsilon = \sqrt{\mu} + \frac{(a/b - b/a)\sqrt{\nu}}{1 + (ka/\psi b)\sqrt{(h/\nu)}}.$$

Putting $h = 2\pi ni$ so that $\sqrt{h} = \sqrt{(\pi n)}.(1 + i)$ and $m = m' + im''$, we finally get

$$\epsilon = \sqrt{\mu} + \frac{(a/b - b/a)\sqrt{\nu}}{1 + \sqrt{(\rho c_p k/\rho_t c\kappa)} + \sqrt{(\pi nkc_p\rho)}.(1 + i)/\zeta},$$

and

$$m = \pm\{\sqrt{(\pi n)}\,\epsilon/aR + i\,(2\pi n/a + \sqrt{(\pi n)}\,\epsilon/aR)\},$$

from which we get

$$\left.\begin{aligned} m' &= \frac{\sqrt{(\pi n)}}{aR}\left\{\sqrt{\mu} + \frac{(a/b - b/a)\sqrt{\nu}}{1 + \omega_1 + \omega_2}\right\} \\ m'' &= \frac{2\pi n}{a} + \frac{\sqrt{(\pi n)}}{aR}\left\{\sqrt{\mu} + \frac{(a/b - b/a)\sqrt{\nu}}{1 + \omega_1 + 3\omega_2}\right\} \end{aligned}\right\} \qquad \text{.....(46),}$$

ω_1 where

$$\omega_1 = \sqrt{(k\rho c_p/\kappa\rho_t c)},$$

ω_2 and

$$\omega_2 = \sqrt{(\pi nk\rho c_p)}/\zeta,$$

so that

$$\text{Sound-velocity} = 2\pi n/m'' = a\,\{1 - \beta_2/2R\sqrt{(\pi n)}\} \qquad \text{.....(47),}$$

β_2 where

$$\beta_2 = \sqrt{\mu} + \frac{(a/b - b/a)\sqrt{\nu}}{1 + \omega_1 + 3\omega_2}.$$

The expressions (7) and (8) on pp. 344, 345, are obtained by putting $\omega_1 = 0$ and $\omega_2 = 0$ respectively.

x (v) *The effect of vibrations of the stop.* Let x denote the distance of any point along the tube from the stop. Let the particle-velocity of the gas in the tube be given by

u

$$u = c_1 e^{mx+ht} + c_2 e^{-mx+ht} \qquad \text{.....(48),}$$

c_1, c_2, m, h where c_1, c_2 and m may be complex, and $h = 2\pi ni$.

Then if the stop vibrates so that its velocity is given by

u_0, c

$$u_0 = ce^{ht} \qquad \text{.....(49),}$$

we must have

$$c_1 + c_2 = c,$$

and

$$u = \{c_1 e^{mx} + (c - c_1)\,e^{-mx}\}\,e^{ht} \qquad \text{.....(50).}$$

The motion of the stop is produced by the changes of pressure on it. Let it be given by

$$u_0 = Dp_0 \qquad \text{.....(51),}$$

D, p_0

where D may be complex.

Now if p be the excess pressure in the gas over the mean, due to the vibrations, we have

p

$$p = -(Em/h)\{c_1 e^{mx} - (c - c_1)\, e^{-mx}\}\, e^{ht},$$

and when $x = 0$

$$p_0 = -(Em/h)(2c - c_1)\, e^{ht} \qquad \text{.....(52),}$$

where E is the elasticity of the gas.

E

From equations (49), (51) and (52) we get

$$c = -(DEm/h)(2c - c_1),$$

so that

$$c_1(1 + h/DEm)(1 - h/DEm)^{-1} = -c(1 + 2DEm/h)$$

nearly, if DEm/h is small.

Thus, substituting in equation (50), we get

$$u = c_1\{e^{mx} - (1 + 2DEm/h)\, e^{-mx}\}\, e^{ht} \qquad \text{.....(53).}$$

If we neglect the damping effect of the tube on the gas vibrations, we can put $m = 2\pi i/\lambda$. Also $h = 2\pi n i$, so that

$$h/m = n\lambda = a = \sqrt{(E/\rho)};$$

and

$$Em/h = \sqrt{(E\rho)}.$$

Let us also put $D = D' + iD''$, and take the real part of equation (53). We thus get for the velocity-amplitude at any point

$$|u| = 2c_1\sqrt{[E\rho\,(D'^2 + D''^2)} + \{\sin 2\pi x/\lambda - 2\sqrt{(E\rho)}.D'' \cos 2\pi x/\lambda\}\{1 + 2\sqrt{(E\rho)}\, D'\} \sin 2\pi x/\lambda] \quad \text{...(54),}$$

and this gives for the positions of the nodes and antinodes

$$\tan 4\pi x/\lambda = 2\sqrt{(E\rho)}.D'' \qquad \text{.....(55).}$$

Turning our attention now to the piston-rod, let us denote by l the distance between the point at which it is clamped and the piston. Then if H and ρ_1 be the Young's modulus and density of the rod, and we now measure distances from the clamped point, we can assume for the motion of the rod (if we neglect the mass of the piston)

l

H, ρ_1

$$u_1 = B(e^{m_1 x} - e^{-m_1 x})\, e^{ht} \qquad \text{.....(56),}$$

m_1

and also

$$p_1 = -(l\rho_1 B/m_1)(e^{m_1 x} + e^{-m_1 x})\, e^{ht} \qquad \text{.....(57).}$$

p_1

Since the damping of the motion of the rod will be found to be important, we cannot now assume that m_1 is entirely imaginary, but write $m_1 = m_1' + im_1''$. At the piston we have $x = l$, and $p_1 = p_0 . A_p/A_r$, where A_p and A_r are the areas of cross-section of the piston and rod respectively, and p_0 is the gas pressure. Also $u_1 = u_0$ and hence

A_p, A_r

p_0, u_0

$$D' + iD'' = u_0/p_0 = -\frac{m_1' + im_1''}{h\rho_1}\,\frac{e^{m_1 l} - e^{-m_1 l}}{e^{m_1 l} + e^{-m_1 l}}\,A_p/A_r \qquad \text{.....(58).}$$

Putting $h = 2\pi ni$; separating real and imaginary parts, and equating the latter, we get

$$D'' = \frac{1}{2\pi n\rho_1} \cdot \frac{A_p}{A_r} \cdot \frac{m_1' \sinh 2m_1' l - m_1'' \sin 2m_1'' l}{\cosh 2m_1' l + \cos 2m_1'' l} \qquad \text{.....(59).}$$

Putting $2\pi n/m'' = \sqrt{(H/\rho)}$; $m'' = 2\pi/\lambda'$; $m' = 1/\mu$,

and (since $m_1' l'$ is small)

$$\sinh 2m_1' l = 2m_1' l$$

and

$$\cosh 2m_1' l = 1 + 2m_1'^2 l^2;$$

we get

$$D'' = \frac{1}{\sqrt{(\rho_1 H)}} \cdot \frac{A_p}{A_r} \cdot \frac{(\lambda' l/\pi\mu^2) - \sin(4\pi l/\lambda')}{(l^2/\mu^2) + \cos^2(2\pi l/\lambda')} \qquad \text{.....(60).}$$

Combining equations (55) and (60) we get for the positions of the nodes and anti-nodes in the gas

$$\tan(4\pi x/\lambda) = \sqrt{\left(\frac{\rho E}{\rho_1 H}\right)} \cdot \frac{A_p}{A_r} \cdot \frac{(\lambda' l/\pi\mu^2) - \sin(4\pi l/\lambda')}{(l^2/\mu^2) + \cos^2(2\pi l/\lambda')} \qquad \text{.....(61).}$$

(vi) *The effect of the gap between the piston and tube wall.* Irons[22] has recently published two papers on the effect of constrictions in a Kundt's tube, in which he shows both theoretically and experimentally, that if an aperture of acoustical con-

c, *s*, l_1, l_2 ductance *c* divides a tube of cross-sectional area *s* into two parts, and if l_1 and l_2 be the distances from the aperture to the nodes on either side of it, then

$$\cot(2\pi l_1/\lambda) + \cot(2\pi l_2/\lambda) = 2\pi s/\lambda c \qquad \text{.....(62).}$$

If the length of the tube, or the position of the aperture, is adjusted for resonance, the distances of the ends of the tube from the aperture will be connected by this equation.

If, owing to the presence of the piston-rod on one side, the cross-sectional areas and the wave-lengths are different on the two sides, we must write, when the piston is adjusted for resonance,

$$s_1^{-1} \cdot \cot(2\pi l/\lambda_1) + s_2^{-1} \cdot \cot\{2\pi(L - l)/\lambda_2\} = 2\pi/\lambda_1 c \qquad \text{.....(63),}$$

L, *l* where *L* is the total length of the tube, *l* is the length of the portion not containing the piston-rod. If the piston-rod end of the tube is open instead of being closed by a gland, we merely substitute a minus sign for the plus in the above equation. This equation, if solved for *l*, gives the various resonance positions of the piston. I have not obtained an exact solution, but we can get an approximate solution as follows:

L' Let $\lambda_2 = \lambda_1(1 - \alpha)$, and $L' = L + (L - l)\alpha$; and $\mathscr{R} = s_1/s_2$. Equation (63) now becomes

B $$\cot(2\pi l/\lambda_1) + \mathscr{R} \cot\{2\pi(L' - l)/\lambda_1\} = 2\pi s_1/\lambda_1 c = B \text{ (say)} \qquad \text{.....(64).}$$

We can think of L' as the effective length of the tube; i.e. the length which would give resonance with the same value of *l*, if the velocity were the same in both parts of the tube. It is a quantity which varies slowly with the position of the piston, for α may be of the order 0·002. L' is equal to *L* when the piston is right out, and in other positions

$$L' - L = \alpha(L - l) \qquad \text{.....(65).}$$

Hence the shift in the position of the nodes relative to the piston, due to the sound-velocity behind the piston not being equal to that in front (i.e. due to the effective tube length being L' instead of L), is given by

$$\alpha\,(L-l).\,dl/dL' \text{ approximately} \qquad \text{.....(66).}$$

Where dl/dL' is obtained by differentiating equation (64) not from (65).

Now one of the nodes is at the source of the sound when the piston is adjusted for resonance, and hence the above expression is equal to the shift in the resonance position of the piston due to the inequality of the sound-velocities on the two sides of the piston.

If these velocities were equal, equation (62) would hold for the resonance positions, and it is readily seen that in this case the distance between successive positions is exactly equal to λ. From this it follows that, in the actual case, the error in measuring this distance is equal to

$$\alpha\,(L-l_1)\,(dl/dL')_1 - \alpha\,(L-l_2)\,(dl/dL')_2 \qquad \text{.....(66).}$$

Now $l_1 - l_2$ is nearly equal to λ_1; and it will shortly be shown that $(dl/dL')_1$ and $(dl/dL')_2$ are very nearly equal. Thus the error in λ is

$$-\alpha.\,dl/dL',$$

and the fractional error is $\quad -\alpha\lambda_1.\,dl/dL' \qquad \text{.....(67).}$

Differentiating equation (64) we get

$$[s_2^{-1}.\operatorname{cosec}^2\{2\pi\,(L'-l)/\lambda_1\} - s_1^{-1}.\operatorname{cosec}^2(2\pi l/\lambda_1)]\,dl - [s_2^{-1}.\operatorname{cosec}^2\{2\pi\,(L'-l)/\lambda_1\}]\,dL' = 0.$$

Hence
$$dL'/dl = 1 - \mathscr{R}^{-1}.\{1+\cot^2(2\pi l/\lambda_1)\}\sin^2\{2\pi\,(L'-l)/\lambda_1\}$$
$$= 1 - \mathscr{R}^{-1}.\{1+(B-\mathscr{R}\cot\theta)^2\}\sin^2\theta,$$

where
$$\theta = 2\pi\,(L'-l)/\lambda_1,$$

and
$$dL'/dl = \tfrac{1}{2}\,(2-\mathscr{R}-1/\mathscr{R}-B^2/\mathscr{R}) - \tfrac{1}{2}\,(\mathscr{R}-1/\mathscr{R}-B^2/\mathscr{R})\cos 2\theta + B\sin 2\theta \qquad \text{.....(68).}$$

If the piston-rod is small compared with the tube, so that $\mathscr{R}$ is not far from unity; and if $B^2/\mathscr{R}$ is considerably greater than unity, we can write this

$$dL'/dl = B\,\{\sin 2\theta - B\sin^2\theta\} \qquad \text{.....(69).}$$

Now
$$\theta = 2\pi\,(L'-l)/\lambda_1 = 2\pi\,(1+\alpha)\,(L-l)/\lambda_1,$$

and since α is of the order 0·002 (in the case of Partington and Shilling's silica apparatus), and $l = \tfrac{1}{2}N\lambda_1$ approximately at the resonance positions, θ will be approximately equal to $2\pi\,(1+\alpha)\,L/\lambda_1 - \pi N$; and dL'/dl will be nearly the same for all the resonance positions and roughly will be dependent on L/λ_1 only. This will be a good approximation for the conditions when the effect of the coupling is small, whilst if the tube length is such that the effect is large we must apply $\theta = 2\pi\,(L'-l)/\lambda_1$ separately for each position of the piston in the tube. On these assumptions, we get

$$\text{Proportional error in velocity} = \alpha/B\,(B\sin^2\theta - \sin 2\theta) \qquad \text{.....(70).}$$

(vii) *The yielding of a tube of elliptical cross-section.* Rayleigh[23] gives for the frequency of vibration of an inextensible elastic circular ring a formula which for the lowest mode, becomes

$$n = (3t/\pi R^2)\sqrt{(H/60\rho)} \qquad \text{.....(71),}$$

after the appropriate substitutions have been made (and a misprint corrected). Here
t, ρ, H t is the wall thickness, ρ the density of the material, and H the Young's modulus. For a silica tube of diameter 2·5 cm., and wall thickness 3 mm. this gives a frequency of about 8000 ~. We shall assume that the sound vibrations have a considerably lower frequency than this, so that the displacements of the tube wall will have the same values as they would have if the same pressures were applied statically. Consider a thin ring of the tube contained between two planes perpendicular to the axis
δx of the tube and at a distance δx apart. Let the minor and major axes of the ellipse
a, b be $2a$ and $2b$ respectively. We shall suppose the wall to be thin compared with a
ϕ or b and we shall define positions on the ellipse by means of the eccentric angle ϕ,
M_ϕ reckoned as zero at an extremity of the minor axis. Let $M_\phi.\delta x$ be the bending
p moment at the position ϕ; and p be the excess pressure on the inside due to the sound waves.

The resultant of this pressure on one quarter-arc of the ellipse must be identical with that which the same pressure would exert on the arc joining the extremities of the quadrant, by a well known theorem in hydrostatics. Hence, taking moments about an extremity of the minor axis, of the forces on the quadrant we get

$$M_\phi = M_0 + \tfrac{1}{2}p\,(b^2 - a^2)\sin^2\phi \qquad \text{.....(72).}$$

U Let $U.\delta x$ be the total potential energy of bending of the ring.

Then $$U = \oint \tfrac{1}{2}M^2/P\,ds$$

round the ellipse; where $P = Ht^3/12$ and H and t are the Young's modulus and thickness of the tube walls.

Therefore $$U = \frac{2}{P}\int_0^l M^2 ds,$$

l where l is the length of a quadrant,

$$= \frac{2M_0^2 l}{P} + \frac{2p\,(b^2 - a^2)}{P}\int_0^{\pi/2} \{M_0 \sin^2\phi + \tfrac{1}{4}p\,(b^2 - a^2)\sin^4\phi\}.b\sqrt{(1 - e^2\sin^2\phi)}.d\phi,$$

(where $e^2 = 1 - a^2/b^2$)

$$= 2P^{-1}.\{M_0^2 l + M_0 p e^2 b^3.\tfrac{1}{4}\pi\,(1 - \tfrac{3}{8}e^2) + (3\pi/64)\,p^2 e^4 b^5\}$$

nearly, if e is small. If $a/b = 4/5$ the error is less than 2 per cent.

Therefore

$$U = 2p^2 b^5 P^{-1}.\{\beta^2 E f^4 - \beta f^2.\tfrac{1}{4}\pi\,(1 - f^2)\,(\tfrac{5}{8} + \tfrac{3}{8}f^2) + (3\pi/64)\,(1 - f^2)^2\,(7/12 - 5f^2/12)\},$$

where M_0 has been given its value $\beta p a^2$*; f has been written for $a/b = \sqrt{(1 - e^2)}$ and
E E is an elliptic integral of the second kind giving the ratio l to b.
δs Now U must also be equal to $\frac{1}{2}p.\delta s$, where δs is the increase in area.

Therefore $$\delta s/s = 2U/ps$$

$$= (4b^3 p/P\pi f).\{\text{——}\},$$

since $$s = \pi b^2 f.$$

* See Timoshenko's *Elasticity*, p. 242.

If we consider a slice of gas extending across the tube and of volume $s.\delta x$ when undisplaced, its volume when displaced is $(s + \delta s)(1 + \partial\xi/\partial x)\,\delta x$, which is nearly equal to $s.\delta x\,(1 + \partial\xi/\delta x + \delta s/s)$.

Hence if E is the elasticity of the gas, we have

$$p = -E.\delta v/v = -E\,[\partial\xi/\partial x + (4b^3/P\pi f).p\,\{\text{——}\}],$$

and hence

$$p = -E\,(\partial\xi/\partial x)\,[1 + (4b^3/P\pi f).\{\text{——}\}]^{-1},$$

i.e. the elasticity has apparently been decreased by the fraction

$$(4b^3/P\pi f)\,\{\text{——}\}.$$

Hence the sound-velocity is decreased by a fraction

$$(\gamma p\epsilon/H).(b/t)^3,$$

where

$$\epsilon = 2f^{-1}\{12\pi^{-1}\beta^2 Ef^4 - \tfrac{3}{8}\beta f^2(1 - f^2)(5 + 3f^2) + 3/64.(1 - f^2)^2(7 + 5f^2)\} \quad \text{.....(73).}$$

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DISCUSSION

Dr E. G. Richardson. With regard to the possibility of irregular motion, and its influence on the propagation of sound in a tube, I would like to point out that owing to the annular effect the amplitude of vibration of the particles may be greater near the walls of the tube than at the centre. The position of the peak amplitude is in fact determined by the very factor quoted by Mr Henry: i.e. the distance of the maximum amplitude from the walls is proportional to $\sqrt{(\mu/n)}$. The excrescences on the surface referred to by the author are therefore likely to project into a region where there is a steep gradient of particle-velocity, instead of into comparatively still air. Will not this fact alone, by encouraging irregular motion, invalidate the premises of the Kirchhoff formula?

Author's reply. I am much indebted to Dr Richardson for pointing out that the velocity-gradient near the walls may be much greater than that given by the simple theory on which my statement on p. 345 was based. The effect he describes should very much reduce the size of the irregularities required to produce unordered motion. Thus if we make use of the theory given in the paper of Richardson and Tyler* and assume a frequency of 3000 ~ in air we find that the velocity has a maximum amplitude 1·07 times that at the centre of the tube at a distance of only about 0·1 mm. from the wall. The theory would probably be upset by the closed ends of the tubes used in practice, but the gap between the piston and the walls at one end would probably accentuate the effect near that end at least. It would be interesting to see whether the form of the piston and telephone receiver affect the tube-effect in this way.

Mr W. Lucas referred to the sentence following equation (21) in which the author states that diameter-errors will cancel out if the nodal distances be measured in the best way. Would the author state what is the best way?

Author's reply. The way I had in mind is to find the positions of, say, twenty nodes, to take the mean of the distances between the first and eleventh, the second and twelfth, the third and thirteenth, and so on; and to divide this by ten. This method gives equal weight to all the observations, instead of giving one observation the same weight as all the rest put together.

* *Proc. Phys. Soc.* **42**, 1 (1929).

DEMONSTRATION

Some elementary experiments concerned with sound-waves in tubes. *Demonstration given on March* 6, 1931, by ERIC J. IRONS, Ph.D.

The following experiments and slides were shown:

1. Mechanical lantern slide to demonstrate the formation and properties of stationary waves. The slide consists of two chemically fixed photographic plates upon each of which a sine curve is drawn. Means are provided whereby the curves move in opposite directions and the resultant displacements for various positions of the waves are registered on the board upon which the slide is focussed.

2. Five lantern slides* illustrating the formation of dust figures in a Kundt's tube excited by a rod.

3. Determination of the end-correction of a tube. Dust figures give an ocular demonstration of the fact that an antinode is not formed exactly at the open end of a Kundt's tube and enable an estimate of the end-correction to be made.†

4. Demonstration of the effect of a Quincke filter on the sound emitted from a valve-maintained tube. If a side branch tuned to a note of a particular frequency is fixed in a conduit down which sound-waves are passing, the energy associated with that frequency is absorbed in the branch.

* *Phil. Mag.* 7, 523 (1929).

† *Phil. Mag.* 6, 580 (1928).

REVIEWS OF BOOKS

Tables Annuelles de Constantes et Données Numériques. Vol. **7**, Pt 2. Pp. xv + 950. (Paris, Gauthier-Villars et Cie.)

Earlier parts of this series of scientific tables have been favourably mentioned in these columns previously. A complete set comprises seven volumes, which must be bought as a whole, and for which the price of subscription is 84 dollars.

The present volume 7 gives data for the years 1925–26. Notes and descriptive portions belonging to the various tables are given in French and English. The matter dealt with includes subjects of an electrical nature such as ionization, thermions, radioactivity, isotopes, but also organic chemistry and subjects such as the properties of materials, which border on engineering. The book is very clearly printed and well produced. It is interesting to note that an index to volumes 1 to 5 is to appear, a very unexpected but acceptable feature in a French book.

J. E. C.

Standard Four-Figure Mathematical Tables, by L. M. Milne-Thomson, M.A. and L. J. Comrie, M.A., Ph.D. Pp. xvi + 245. (London: Macmillan and Co., Ltd.) 10*s.* 6*d.*

This is a very useful collection of mathematical tables and formulae which are of very frequent application. Several features of interest may be mentioned.

Although the logarithms are given to only four figures, differences are printed at the side of a column, and to secure greater accuracy than that obtained by the usual method of interpolation, the following device is used. A dot if placed high after the fourth figure indicates that + 3 occurs in the next place, and if placed low may be taken to represent − 3 in the fifth place. Thus when several members are multiplied together a little additional accuracy is obtained.

A good feature of the tables is that on some pages are given no fewer than ten functions such as $\sin x$, $\cot x$, $\log \sin x$, $\log \cos x$, etc. Natural logarithms, hyperbolic functions, gamma functions, roots and reciprocals are included. The latter part of the book is devoted to lists of integrals, series, mensuration formulae and other similar information. One has rather a suspicion that a reader using some of the more advanced tables would require logarithms of greater accuracy than that given by four figures, but nevertheless the book may be strongly recommended.

J. E. C.

National Physical Laboratory Collected Researches. Vol. **21**, 1929. Pp. iv + 448. (London: H.M. Stationery Office.) 22*s.* 6*d.*

National Physical Laboratory Collected Researches. Vol. **22**, 1930. Pp. iv + 417. (London: H.M. Stationery Office.) 20*s.*

The twenty-first and twenty-second volumes of the collected researches of the National Physical Laboratory show an impressive record of work put out by the members of the Electricity Department. As is to be expected, the papers in the main are concerned with precision methods of measurement and with instrument design.

Volume 21 comprises some twenty-one papers of very varied character. Selection were invidious but, if mention is to be made of any special papers, those by Dr Dye on a cathode-ray tube method for the harmonic comparison of frequencies, on a magnetometer for the

measurement of the earth's vertical magnetic intensity, on the piezo-electric quartz resonator and its equivalent electrical circuit, and the paper written in collaboration with Dr Hartshorn on a primary standard of mutual inductance, may be noted as exhibiting those qualities of finish and thoroughness which are always to be found associated with Dr Dye's work. Seven of the papers in this volume are concerned with radio problems and include papers on the polarization of radio waves, the cause and elimination of night errors in radio direction-finding, and investigations on wireless waves arriving from the upper atmosphere.

Volume 22 also contains twenty-one papers of which six are directly concerned with wireless problems. Two papers deal with condenser losses and condenser-resistance measurements at radio frequencies and four are concerned with cable problems; the remainder are difficult to classify, but mention may be made of Dr Hartshorn's paper on the measurement of the dielectric constants of liquids, or of Mr Vigoureux's paper on the valve-maintained quartz oscillator.

The two volumes constitute a record of achievement in precision measurements which is altogether encouraging.

A. F.

The Observatories Year Book, 1928 (M.O. 320). Air Ministry Meteorological Office, published by the authority of the Meteorological Committee. Pp. 450. (London: H.M. Stationery Office.) £3. 3*s*. 0*d*.

There is always a satisfaction in perusing a year book of the observatories under the British meteorological service, for it is clearly the work of physicists, not merely of routine observers; we are not left in doubt over the methods of reduction, or over the conditions of exposure at the station, and we have therefore the materials for a satisfactory comparison with corresponding data at other observatories. Thus the distinctions between occasions on which fog, mist and haze are reported are made clear, and we have a table of the objects used in measuring visibility.

The traditional method of estimating base-line values of magnetograph records was to adopt individual readings of the absolute instruments under favourable conditions, and so to trust a comparatively inaccurate instrument liable to change and give no weight to the one that was undisturbed. Now the base-line values adopted are obtained from a curve drawn smoothly through points given by the values deduced from the absolute observations, due allowance being made for discontinuities in the records. This method is far superior in that it tends to eliminate non-systematic errors in the absolute instruments; but unless these measurements are of very reliable pattern some recognition of the magnetographs might perhaps be made by adopting a fraction, say three-fourths, of the change thus obtained.

G. T. W.

Magnetic Phenomena, by SAMUEL ROBINSON WILLIAMS, Ph.D., D.Sc. Pp. xxii+230. (London: McGraw Hill Publishing Co.) 15*s*.

The outline of this book differs very considerably from that of the orthodox text-book. "The pages are intended as a guide to the beginner," but the word *beginner* must be interpreted with caution, for, although the treatment is quite elementary, the appeal made is to an inquiring and mature mind. It is not a school book, but a college class book designed to stimulate the spirit of research and inquiry among its readers.

The few pages of mathematics that one meets are of a very simple type, and the book emphasizes in a very healthy manner the importance of physical rather than mathematical thinking. The way in which the author handles his subject is best shown by the titles of his chapters. "Magneto-mechanics," "magneto-acoustics," "magneto-electrics," "magneto-

thermics," "magneto-optics," "cosmical magnetism" and "magnetic theories and experimental facts" are the chapter headings of the book taken in order. Throughout, clear accounts are given of the essentials of the experimental methods employed, with a full bibliography of references to original papers.

If a very mild criticism may be hinted, it is that one would like to see fuller reference to English work and English instruments—for example in the chapter dealing with cosmical magnetism, where "current" methods for the measurements of both H and V are hardly sufficiently stressed. The chapter on magnetic theories, although as it stands it is an admirable short summary, could be expanded with considerable advantage. The book fills a niche and may be warmly commended.

A. F.

Physics, Part II, Sound, by W. J. R. CALVERT. Pp. viii + 140. (London: John Murray.) 3*s*.

Physics, Part III, Light, by W. J. R. CALVERT. Pp. viii + 202. (London: John Murray.) 3*s*.

An Introduction to Science, by P. E. ANDREWS, B.A., B.Sc., and H. G. LAMBERT, B.Sc., A.I.C. (London: Longmans, Green and Co., Ltd.) 2*s*. 6*d*.

There are signs of the approach of certain long-overdue reforms in the teaching of elementary physics. With the growth of the examination system, when candidates for school examinations are to be reckoned by thousands and by tens of thousands, the tendency to sacrifice teaching to examination necessities becomes yearly more pronounced. Questions are asked which are designed not so much to test the candidates' knowledge and to bring out their power of original thought as to provide material for answers which may be marked at a rate high enough to ensure that the results may be made public within a few weeks of the examination. And text-books follow suit in their treatment of their subjects.

It is tragic, but almost inevitable. Almost, we say, for authors are to be found bold enough to pursue the better path, and publishers with enterprise sufficient to make their work public.

The volumes before us are excellent examples of their kind. Mr Calvert's books on *Sound* and *Light* provide sufficient weapons for the attack on the numerical and algebraical exercises beloved of the average examiner to ensure that the candidate for a school certificate examination will do himself justice. At the same time the treatment is thoroughly lively, is in touch with reality, and is designed to encourage physical thinking rather than wooden juggling with mathematical symbols.

Messrs Andrews' and Lambert's book is more elementary in character, providing a year's course in science for the beginner. The text covers a first course in heat, light, sound, magnetism and electricity, the teaching being conducted for the most part by well-planned and simple experiments designed to make the learner think.

All three volumes can be unreservedly commended.

A. F.

Proceedings of the Institution of Mechanical Engineers. Vol. **1**, January–May, 1930. Pp. viii + 851. (London: Institute of Mechanical Engineers.)

This volume contains the third Thomas Lowe Gray lecture given by Engineer Vice-Admiral Skelton on "Progress in Marine Engineering," and reports of four other addresses. Of these Mr William Taylor's lecture on "Science and Works Management" calls for special attention, a plea being here put forward for the inclusion of this as a subject in the curriculum of an engineer's training.

The reports of three of the Institution's research committees are given in this volume: (1) The first report of the Welding Research Committee. This work has for its object the study of modern methods of welding metals, and deals chiefly with the application of welded joints to pressure vessels. (2) The sixth and final report of the Steam Nozzles Research Committee. (3) The fourth report of the Wire Ropes Research Committee.

Professor Coker has made a further contribution to his work on photoelasticity in a paper on stresses in cams, rollers, and wheels. An account is given by Messrs Barklie and Davies of their investigations into the effects of surface conditions on the fatigue resistance of metals. It is to be noted here that the practice of the electro-deposition of nickel on steel for the purposes of building up worn parts of machinery and of providing wear-resisting surfaces has become fairly common, but if care is not taken under certain conditions an electro-deposit may lead to a weakening of the machine part.

Considerable space is given to steam generation, and there is in this connexion much valuable information on the use of pulverized fuel. The results of tests carried out on the first "Wood" steam generator are interesting. This boiler is the first radiant-heat boiler to be constructed in this country for the exclusive combustion of pulverized fuel, and its design embodies the important feature of surrounding the combustion chamber with a wall of vertical water-tubes. The results of tests show a remarkable performance from the standpoint of evaporation. The boiler, on account of its lightness and its flexibility, should have an important application not only to steam generation in power stations but on board ship.

A valuable contribution to the technique of testing large steam power units is made by Mr H. L. Gray, who has given an account of some carefully conducted trials made by him at Barton Power Station; in this it is to be noted that there is no economy gained by the use of pulverized fuel unless cheaper coal is available.

There is also an interesting paper on the design, construction, and results of a boiler operating under a pressure of 600 lbs./in.^{2}.

In the memoirs of this volume we regret to note the death at the untimely age of 47 of Dr Paul Telford Petrie of Manchester, who acted as reporter of the Steam Nozzles Research Committee.

G. A. W.

Photo-Chemistry, by D. W. G. Style, Ph.D. Pp. vii + 96. (London: Methuen and Co., Ltd.) 2*s*. 6*d*.

This is the latest publication in the very useful series of monographs on physics edited by Dr Worsnop. No attempt is made within the limits of this half-crown volume to present an account of the mass of chemical data that has been accumulated on this subject, but the author has been most successful in giving an outline of the main achievements and theories, especially from the physical standpoint. It bears throughout the impress of a writer who is himself an enthusiastic worker in this field and Professor Allmand fittingly contributes a preface. The book should prove valuable to students of physics and chemistry; it provides an excellent introduction for those intending to carry out research work in this field.

D. C. J.

The Plant in Relation to Water, by N. A. Maximov; authorized English translation edited, with notes, by R. H. Yapp. Pp. 451. (London: George Allen and Unwin.) 21*s*. net.

This volume, which is essentially a study of the physiological bases of drought resistance, is the most comprehensive and critical work on the subject in the English language. An immense mass of literature exists which is grouped around this topic, and physiologists are much indebted to Professor Maximov for the pains at which he has been to present a clear and ordered summary thereof.

The book is concerned primarily, as we have said, with a physiological topic, but there is in this particular branch of botany so much that is of interest to the physicist, so much in which the assistance of the physicist is to be desired, that no apology is necessary for introducing the work to the notice of students of physical science.

Naturally in such a subject the mechanism of the transport of water by the plant is a matter of primary importance; and the physical theories that have, from time to time, been devised to explain the laws of this transmission urgently demand the criticism of students trained in the facts and theories of physical science and not likely to make dynamical mistakes in discussing the phenomena: for these, vital though they may be, must be explained in terms of theories which, at least, do not outrage dynamical laws.

Moreover, the instruments employed and the methods of measurement are of compelling interest to physicists, especially to those versed in instrument design. For example, despite the care and skill that have been expended on such a problem as the measurement of relative transpiration, it is not too much to say that no completely satisfactory method has yet been devised which shall give for this important quantity figures sufficiently safe in themselves from criticism to serve as a basis for purely physiological arguments.

Atmometers by means of which transpiration values may be compared; instruments for the measurement of transpiration; self-recording balances for the determination of the relation between transpiration and absorption; porometers by which stomatal movements may be followed and compared; these are but a selection from a number of instruments which, in spite of the care which has been lavished on their design in the past, will well repay the attention of the physicist who is interested in the problems of vegetable life. And he will find no better nor more reliable guide within moderate compass to the use and interpretation of the instruments already employed than the volume under review.

A bibliography of some six hundred entries adds much to the value of the book, which in its English dress owes a great deal to the labour spent on it by the English editor, the late Professor Yapp. It is a melancholy reflection that Professor Yapp, though the final proofs passed through his hands, did not live to see the appearance of the completed work.

A. F.

Colloid Science applied to Biology. A general discussion held by the Faraday Society, 1930. Pp. 197, including figures, plates, and full bibliographies. Price 12*s*. 6*d*.

We start with a definition of life. Professor A. V. Hill in the first paper presented at the conference, the proceedings of which are now under review, writes of life in the physico-chemical sense as a "self-perpetuating and generally periodic complex of events, occurring indeed in a medium of matter, depending on the properties of matter, but as distinct from matter as music is from the air in which it is propagated." The Faraday Society did not meet to discuss life, but the physico-chemical states of that matter on which life, as we know it, depends.

Nearly 100 years have passed since the knowledge won by microscopic observations led biologists to the far-reaching conclusion that the simplest living unit, classically called a cell, is essentially an aqueous mass of proteinaceous slime to which the name protoplasm was later given. For example, a certain microscopic viscous entity, now called amoeba, was found to grow and reproduce and was, therefore, shown to be a living unit. Associated with these major attributes of growth and reproduction are the phenomena of the continual exchanges of matter and energy between the organism and its environment and the apparently purposeful responses of the former. Clearly, if the myriad-celled organisms, like elephants, or the giant sequoias of California, are to be satisfactorily conceived of as physico-

chemical systems, the unit of their structure, the cell, must form the basis of such a conception.

Even if it is granted that some influence that is remote from the matter of the organism is necessary to vitalize amoeba and other living units —a view by no means universally held—there are still legitimate physico-chemical problems, related to protoplasm, the elucidation of which will be of much profit to the biologist. Explicit in the introductory remarks of Sir F. Gowland Hopkins, P.R.S., who was in the chair on the second day of the conference, is his belief that for an adequate description of the periodic complex of events during the life of any organism it will be necessary to determine the physical states of matter in which these events occur. The P.R.S. insists, however, that there is, in organic chemistry, an independent field of importance equal to that of physical chemistry in relation to specific syntheses on which assimilation, and so, also, the formation of the structural elements of organisms, are dependent. Both he and Sir William Hardy, who was invited by Professor T. M. Lowry (President of the Faraday Society), to take the chair on the first day of the session, believe that many obscure biological phenomena will be classified in the future as our knowledge of molecular orientation at interfaces increases.

The analytical study of protoplasm is as fundamental to biology as that of the atom is to physics. The specialized study called cytology aims at describing the structures that are visible under the microscope, and their behaviour and functions. Protoplasm is micro-heterogeneous. It is differentiated into nucleus, with its chromosomes, nucleolus, nuclear sap, etc., and cytoplasm with its plastids, mitochondria, golgi apparatus and other micro-structures. The meeting was periodically reminded of the possession by protoplasm of a visible structure. But beyond the scope of the cytologist is the ultramicroscopic structure that is revealed by the arts and arguments of chemistry, physics, and physiology. For examples, the presence of very thin membranes, possibly of unimolecular thickness only, with amazing properties, can be argued: and differences of electrical potential between different parts of a given cell may be deduced: and invisible systems possessing specific chemical activity, called enzymes, can be proved to exist in protoplasm.

Chemical analysis has shown that cytological structures are chiefly composed of proteins, either free or physically associated, or possibly even chemically combined, with lipoids or with nucleic acid. Sterols, phosphatides, carbohydrates and other organic substances are now known to be invariably present in small quantities, as are also mineral salts and water. The idea was advanced towards the end of last century that protoplasm consists of a giant molecule with unstable side-chains. There was never any evidence for this view, and it is now considered that it would be just as accurate to say that a structure, such as Westminster Abbey, or a functioning machine, such as a Rolls Royce car, are respectively composed of giant molecules. The micro-architecture of a cell, and also cell activities, are now attributed to a complex physical association of an unique kind, in an aqueous saline medium, of the various organic compounds found by analysis in protoplasm. What is the nature of this structure, and how is the machine-like precision of its work governed? Most biologists and biochemists, but not all, find it helpful to construct pictures of cell structures; in these, membranes and active surfaces are prominent. From time to time the picture are altered; something is added, something erased. That made by Professor R. A. Peters, and framed in the book under review, deserves careful study, as do the remarks of those who were present at the private view of it. How far any such pictures represent the real state of things cannot be determined, however, until much more purely physico-chemical work is done.

Although it was clearly recognized immediately after Thomas Graham's pioneer work, in the middle of last century, that the proteins and other organic compounds exist in protoplasm in the colloidal state, the chemist, as the P.R.S. reminds us, paid no regard to slimes until towards the close of the century; they were only fit for the sink. Thus the science of colloidics, as Professor Wo. Ostwald, who was present at the symposium, calls

it, is still in its infancy. The subject matter of part I of this discussion shows, quite obviously, that the physical work that is proceeding on protein-water-electrolyte* systems may well be forming a background for a truer picture of the structures in and the properties of living matter than any as yet in existence. Comparing, however, the complexity of protoplasm —in which there are many different kinds of proteins, other organic substances, an array of ions, and water (and even water, according to Gortner, may be present in two different states)—with the relative simplicity of protein-water-electrolyte systems, we at once see that the systematic intellectual construction of a whole, in which there are so many variables, by summation of its parts forms a task of such immensity that a life of centuries is assumed for the infant science of colloidics in its relation to biology.

M. T.

* A big advance in recent years (see Prof. Svedberg's contribution to the general discussion after Prof. Pauli's paper) is that well-defined mineral-free proteins can now be separated from vegetable and animal material. The physical constants characterizing these proteins are not changed by re-crystallization.

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THE GENERATION OF CURRENT PULSES OF RECTANGULAR WAVE-FORM

BY A. J. MADDOCK, M.SC., A.INST.P.

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ABSTRACT. A circuit for the production of single current-pulses or for use as a heavy-duty time switch is described in which there is practically no limit to current-carrying capacity. Such pulses of current can be obtained when desired by the simple depression of a key. It is shown that two distinct wave-shapes are available in different branches of the circuit and that by suitable choice of inductance, resistance and capacity the current time wave in one branch should be of rectangular form. Two separate timing-circuits are described, one for periods up to about 2×10^{-2} sec. and the other for use up to 16 sec. or longer. Suggested uses and extensions of the circuit are given.

§ 1. INTRODUCTION

Of the various methods which have been employed for applying a voltage to a circuit for a short interval of time or for obtaining single pulses of current, many are not suitable for currents greater than about 0·5 amp. owing to sparking which may occur at the contact devices and give an increased time of current flow, whilst some mechanical devices, owing to variations in contact, may not be sufficiently accurate.

The circuit to be described was designed to eliminate the current limit referred to and for this purpose a hot-cathode mercury-vapour triode, known as a thyratron, is used as the contact-making and breaking device while, for the obtaining of accurate timing in the duration of current flow, use is made of an electrical oscillatory circuit. The circuit in fact was primarily to constitute a time switch capable of passing from a few milliamperes up to many amperes of current. One of the conditions therefore is that at "make" the current shall rise instantaneously to its full value set by other circuit conditions, remain steady for the duration of flow and fall to zero instantaneously at the point of "break"; in other words, the circuit is to produce single current-pulses of rectangular wave-form.

For a full description of thyratrons the original papers of Hull[1, 2, 3] should be consulted; it will suffice here to say that in operation the mercury is ionized so that the tube becomes practically a space-charge-free device with a voltage drop between cathode and anode of 12–15 volts which is almost constant for any current up to a limit set by the cathode dimensions. Tubes have already been made capable of passing 100 amp. or more. Current flow through the tube can be prevented by a suitable negative bias on the grid, but once the discharge has been initiated the grid has no further controlling action, its effect being neutralized by the positive ions of mercury which collect around it; the discharge can be stopped only by removal of the applied anode voltage or the application to the anode of a momentary negative voltage of sufficient duration to allow of the tube deionizing.

§ 2. CIRCUIT ARRANGEMENT AND ACTION

The circuit diagram is given in figure 1 and can be analysed into three divisions: (1) The main thyratron A with its load resistance R_A and battery B_1. (2) The secondary thyratron B, load R_B and battery B_1. (3) The timing circuit LR_1R_2C in the grid circuits of the valves. It is in the main circuit $AR_A\ B_1$ that the current pulses are generated.

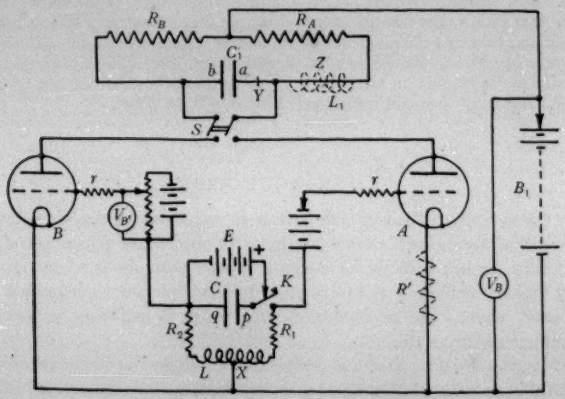

Fig. 1. Circuit diagram for short time intervals.

Deferring for the moment consideration of the timing action, we can consider the action of divisions (1) and (2). Suppose that thyratron A has been released and that current is flowing through it, then if we suppose the battery voltage of B_1 to be 220 volts and the drop across A 15 volts, the plate a of condenser C_1 is at a potential of + 15 volts and plate b at + 220 volts, a difference of 205 volts. (All voltages are referred to the common negative ends of the filaments.) If now

thyratron B is released, plate b is connected to the cathode of A through valve B. Thus plate a, and hence the anode of A, is 205 volts negative with respect to plate b, so that to the anode of A we have, in effect, applied a momentary negative voltage of 205 − 15 (the tube drop of B), i.e. 190 volts; the discharge through A is therefore stopped owing to the negative potential of the anode. Condenser C_1 promptly discharges through B and R_A and charges up in the opposite direction, so that plate a is ultimately at + 220 volts, but, provided the resistance of the load R_A is sufficiently high, it is possible for the tube A to deionize before the voltage of plate a and the anode are sufficiently positive for the discharge to recommence: the functions of condenser C_1 and thyratron B are thus to stop the current flow through A by applying a momentary negative voltage to its anode[3].

The timing circuit, constituting division (3), has to release thyratron A when desired and after the required interval of time to release thyratron B, which then stops current flow through A as already explained. The condenser C is charged by the battery E and when desired the key K is depressed so that the condenser is discharged through the inductance L and resistances $R_1 R_2$, which are of such values as to cause the discharge to be of a damped oscillatory nature. Since the mid-point of the inductance is connected to the cathodes, during the first quarter-cycle negative bias on the grid of thyratron A is annulled by the positive voltage of the plate p of the condenser, so that thyratron A is released and current can flow in its anode circuit; at the same time the normal negative bias of B is increased by the voltage between X and plate q. During the next quarter-cycle, when the voltages are reversed, thyratron B is able to start as soon as the positive voltage between X and q, opposed by the steady bias applied to the grid, is equal to the critical bias*. (A study of figure 2 should make the action clear.) It is arranged that the damping be sufficiently great to ensure that when next p is positive even at the peak of the wave, the resultant bias on the grid of A shall not be reduced to the critical bias: thus thyratron A does not restart, and only the single pulse of current has passed from cathode to anode. Current continues to flow in thyratron B until the switch S is opened; a double-pole switch is necessary to open the anode circuit of A also, otherwise the charged condenser C_1 is liable to release A when B is stopped. C_1 is automatically discharged through R_A and R_B when S is open, and the circuit is now ready to be employed again.

It will be seen that the steady bias of A is not at all critical, and provided the first positive peak of the oscillatory wave is able to reduce it below the critical bias, whilst the second peak is unable to do so, we have the essentials for thyratron A starting once only. It is the conditions obtaining on thyratron B, together with the timing circuit, that determine the time of current flow in A, and in order that the grid voltage of B may not slowly approach the critical bias but undergo a definite

* The critical bias is the bias at which the grid just loses control and anode current commences to flow: it is a function of the anode voltage, a larger anode potential requiring a greater negative bias. The relation is almost linear except for low values of anode and grid potentials. The temperature of the vapour, by altering the pressure within the arc space, also modifies the critical bias and voltage across the tube when current is flowing, and for this reason similar conditions should always be aimed at in the operation of the circuit, especially with regard to the stopping thyratron.

sudden change*, it is arranged for the steady bias to be only slightly in excess of the critical bias, so that B starts soon after the reversal of polarity in the oscillatory circuit: i.e. soon after a quarter-period when the slope of the voltage wave is a maximum.

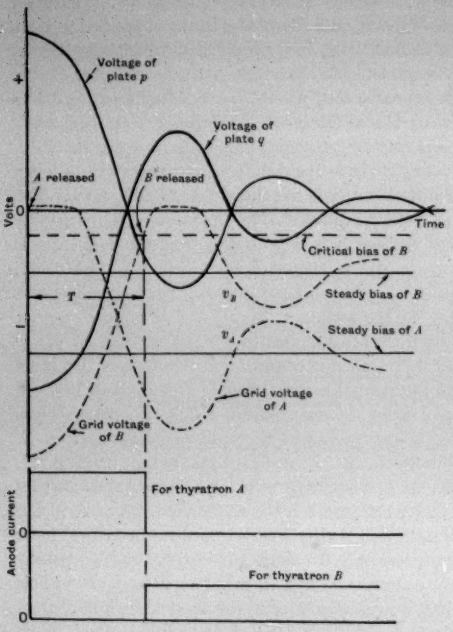

Fig. 2.

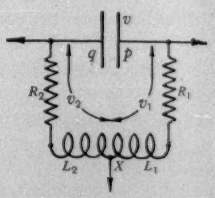

Fig. 3. R_1 and R_2 are the combined resistances of the chokes and any extra damping resistances inserted.

C, E
L, R, v

If the condenser of capacity C charged to a voltage E is discharged through the inductance L and total resistance of the circuit R, then the voltage v across the condenser at any instant is given by

t

$$v = (En/p)\, e^{-kt} \cos(pt - \phi) \qquad \text{.....(1),}$$

n, k, p

where $n^2 = 1/LC$, $k = R/2L$ and $p = \sqrt{(n^2 - k^2)}$, $(n^2 > k^2)$.

If the inductance and resistance were equally divided by point X the voltage either

* Owing to temperature changes noted on page 373 (footnote) it is advisable that a sudden change of bias should occur rather than that it should gradually approach the critical bias, in which latter case changes which occur with temperature considerably affect the bias at which the discharge is initiated.

side would be $v/2$, but as the inductances are not likely to be exactly equal we can facilitate the calculation of the voltages as follows (see figure 3):

$$v/v_2 = (v_1 + v_2)/v_2 = 1 + (L_1\,\partial i/\partial t + R_1 i)/(L_2\,\partial i/\partial t + R_2 i)$$
$$= 1 + L_1/L_2 + i\,(R_1 - R_2 L_1/L_2)/(L_2\,\partial i/\partial t + R_2 i),$$

where v_1 and v_2 are the voltages at any instant across the two halves of the circuit. If we make R_1 equal to $R_2 L_1/L_2$ the last term is always zero and then v_1, v_2

$$v_2 = vL_2/(L_1 + L_2) \qquad \ldots\ldots(2).$$

Hence, knowing the inductance values and arranging the resistances as above, we can express the oscillatory voltage on one side in terms of the condenser voltage as given by equation (1). Probably the best and simplest way of arriving at the time of current flow in thyratron A is to draw the oscillatory voltage curve v_2 for the plate q of the condenser on a large scale and to move the time axis up and down in accordance with various selected values of grid bias, in effect obtaining a curve similar to v_B of figure 2. The circuit is arranged so that damping resistances may be inserted if required to increase the natural resistances of the inductances and to satisfy the above relation.

The timing circuit will, of course, operate satisfactorily with an iron-cored choke coil, but it is then not an easy matter to obtain the voltage curve, since the inductance varies both with frequency and with a.c. voltage; for these reasons air-cored chokes and non-inductive condensers are used. An indication of the condenser values required with a 20-henry choke coil (centre-tapped) for various required times of current flow are given in table 1; the time interval is calculated as being one quarter-period of a simple-harmonic oscillation, but owing to damping and the fact that in practice the steady bias must be slightly greater than the critical bias, both factors operate to give longer time intervals than those indicated.

Table 1. Approximate values of capacity for various times.

Time (sec. $\times 10^{-3}$)	L (henries)	C (μF)
1	20	0·02
2	20	0·08
5	20	0·5
10	20	2·0
20	20	8·0

The circuit has operated very satisfactorily at all these values and it should be possible to obtain times shorter than 10^{-3} sec. by the use of a smaller inductance. Resistances r, figure 1, of 0·25 megohms are placed in each grid lead to prevent large electron-currents from flowing in the grid circuits when the grids tend to become positive.

§ 3. EXPERIMENTAL RESULTS

For the purpose of observing the shape of the current time curves in the main thyratron circuit a cathode-ray oscillograph was used with which the transient could be seen visually, but in order to obtain a diagram of the wave-shape the timing circuit as described was removed and the grids driven by a.c. at 50 ~ from a centre-tapped transformer. A continuous series of pulses was obtained in each circuit instead of only the one used normally; sketches only were made of the wave-shape on the screen and are given herewith.

Owing to the charge and discharge currents through the stopping condenser C_1 the current pulses will be of different shape in the anode and cathode leads of thyratron A. We shall consider first of all the wave-shape in the anode lead, i.e. the wave-shape of the current flowing through the resistance R_A.

(*a*) *Current in anode resistance.* With a pure resistive load the wave-shape is as shown in figure 4 (*a*), the current commencing suddenly, having a flat top and at the end a peak and exponential decay due to the discharge and charge in the reverse direction of condenser C_1. The sketches are drawn approximately to the same scale, and the "true time" of current flow, as it may be called, was 0·01 sec. in all cases, this being the time between the release of thyratrons A and B.

This peak at the end was an undesirable feature and to reduce it a choke of low d.c. resistance was placed in series with condenser C_1 in position Y. Results with various values of inductance are given in figure 4 (*b*) (*c*) (*d*) (*e*) whence it will be seen that the peak has been removed, and for some classes of work the wave-shape of figure 4 (*d*) might be suitable. Another method of eliminating the peak was to place a condenser across the stopping-thyratron B with the choke as before, for which it was found that a lower value could be used, see figure 4 (*f*): this method is not advisable as high peak currents are set up by the discharge of this auxiliary condenser directly through thyratron B, with possible damage to the cathode. The wave-shapes with different values of inductance in the common d.c. supply lead are given in figure 4 (*g*) (*h*) (*i*) (*j*), (*h*) being fairly symmetrical and flat topped. Various other attempts were made to annul or modify the condenser current, including the use of a double circuit of two thyratrons in series of which one was used purely for stopping the current flow; but even in this case condenser current (though of a smaller magnitude) did still flow through the load since this formed, in effect, a parallel circuit.

Although with the methods tried we have been unable to obtain rectangular pulses in the anode resistance, yet for certain work the waves obtainable therein may be useful and it can be seen how the contour may be modified if desired.

The circuit was originally intended as a short-time switch for measuring total emission of oxide-coated cathodes in thermionic valves. The valve under test was to constitute the load, being inserted in place of the resistance R_A in figure 1 so as to afford, together with the voltage applied by the battery B_1, the only limit to the current. The nearest approach to sudden application of voltage for a short period was obtained by the circuit conditions that gave the current curve of figure 4 (*d*).

A ballistic galvanometer can be used to measure the total quantity passed, and thence the value of the current can be obtained from a knowledge of the time of application of the battery voltage. It is possible to calibrate the galvanometer to read current directly without the time interval being calculated, but as this interval can easily be found from equations (1) and (2) the method described above seems preferable. One of the chief drawbacks to a thermionic valve constituting the load is that since different valves have different effective resistances, when these are inserted in turn as the load the time of charging of condenser C_1 will be variable, and in some cases may be so short as not to allow thyratron A to deionize; insertion of an extra anode resistance is a method of overcoming this difficulty, though a considerable voltage will be lost across the resistance for large currents.

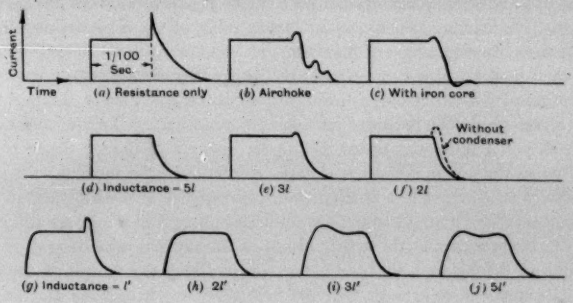

Fig. 4.

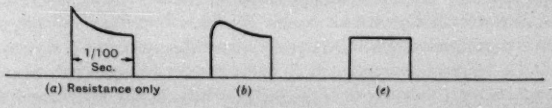

Fig. 5.

(*b*) *Current in cathode lead.* We now consider the conditions in the cathode lead of thyratron A. Current starts and stops suddenly with resistive loads, and the only additional current is that which charges condenser C_1 to the voltage drop across the anode resistance when A is released; this gives a wave-form as shown at figure 5 (*a*): the insertion of a choke at Y rounds off the peak and gives a wave of the shape shown in figure 5 (*b*). The advantage of working in the cathode lead is that the wave has perpendicular ends, though it must be borne in mind that an anode resistance must always be used to limit the current.

It is much regretted that circumstances have not permitted of the author's carrying out further experiments on the wave-shape in the cathode lead, but from the following consideration it will be seen that rectangular pulses should be obtained if the circuit as given in figure 1 were modified by the insertion of an inductance L_1

at Z. When A is released inductance L_1 and resistance R_A are in parallel with condenser C_1; hence the inductive growth of current in the cathode lead due to L_1 and the capacitative charging current of condenser C_1 can, together, be made to give a steady current in the cathode lead from the commencement of flow, the strength of this current being equal to the final strength reached. I.e. the pulse will be flat-topped, figure 5 (*c*). It is easy to show that for this condition to be realized R_A must equal R_B (R_A now including the d.c. resistance of inductance L_1) and

$$L_1 = C_1 R_A^2 \qquad \text{.....(3).}$$

Thus C_1 and R_A may have practically any value and the wave-shape may still be rectangular provided a suitable value of choke is inserted in the circuit to satisfy equation (3). The obvious arrangement is to make R_A small to avoid a large voltage drop across it with correspondingly increased value of C_1. The limiting factor is again the time of deionization of thyratron A.

We shall now attempt to fix limits for the various circuit values imposed by consideration of this deionization-time. Although the grid bias of A when the arc current ceases gradually becomes increasingly negative as the positive ions are neutralized, yet if ions are present in the arc space a discharge could be again initiated when the anode voltage is slightly greater than the normal voltage drop which obtains when current is flowing. For convenience of working and to obtain a general idea of the circuit conditions it will be assumed that if ions are still present the arc can recommence at the same voltage as the normal tube drop of E_c volts. If the time of deionization is T_0 sec., the aim in fixing the values of the circuit components must be to arrange that the voltage between cathode and anode of thyratron A shall not, owing to the charging up of condenser C_1, reach a value E_c volts within the time T_0. A further simplification is to assume that the tube drop of thyratron B when passing current is also E_c; these simplifications are justifiable since we are concerned only with setting a limit to the values of C_1, R_A, etc., so that thyratron A is released once only. If E_b is the battery voltage, then when B starts the p.d. across the condenser is $-(E_b - E_c)$, which finally becomes $+(E_b - E_c)$ when the condenser has discharged and been charged in the reverse direction. Thyratron A can start again, if positive ions are present, when the p.d. across C_1 is zero; this takes place, in the limit which we are considering, in time T_0 after B is released. The problem is therefore the same as that presented by a condenser initially with zero charge and finally charged to a value double that which it has at time T_0. The above considerations hold for the circuit however it may be modified to alter the wave-shape. With the choke L_1 inserted to make the form rectangular it will be noticed that, when B is released, condenser C_1, inductance L_1 and resistance R_A are all in series, and since $L_1 = C_1 R_A^2$ the current tends to be of an oscillatory nature. It can be shown that at time T_0

$$(\sqrt{3}/4) \exp(T_0/2C_1R_A) = \cos(T_0 \sqrt{3}/2C_1R_A - \pi/6) \qquad \text{.....(4).}$$

T_0 increases with increase of arc current due to the greater number of positive ions, but for any one value of current it is to be noticed that, on this simplified working,

T_0 is independent of the battery voltage, the explanation being that although with a higher battery voltage the condenser charges up more rapidly initially, a lower negative voltage was originally applied to the anode of thyratron A when B was released.

The minimum values of C_1 and R_A for which thyratron A is just not released a second time, having been determined experimentally for a given set of circuit conditions, T_0 can be calculated from equation (4). If now it is desired to use a different value of R_A, the values of C_1 and L_1 can be found for the same current for which T_0 will be the same. For example, for a certain current, with the thyratron used it was found that T_0 was approximately 200 microseconds. For R_A to equal 50 ohms, substitution in (4) gave a value for C_1 of 3·1 μF with a consequent value 0·0078 H. of L_1 as determined by (3) to keep the current pulse flat-topped. These are the limiting values and slightly greater ones would be chosen in practice. It will be seen that heavy currents can be passed when the circuit is used in this manner with a moderate anode voltage supply.

If the circuit in the cathode lead into which we wish to inject the current (i) has a resistance R', then the negative voltage applied across thyratron A when B starts is

$$-R_A i + E_c = -[E_b - (E_c + R'i)] + E_c.$$

If this is less than E_c, the discharge in A is stopped: i.e. for A to be stopped at all

$$E_c > -[E_b - (E_c + R'i)] + E_c,$$

whence

$$E_b > E_c,$$

since

$$i = (E_b - E_c)/(R_A + R').$$

Thus the minimum battery voltage to be used is equal to the normal tube drop, no matter what values R_A and R' may have or what the current value is. In practice it must be greater than this, for E_b must always be greater than the value required to start the arc in A when its grid bias is reduced below the critical bias by the timing circuit. It should be noted that there must always be a resistance R_A to give a voltage drop across the condenser for stopping A.

By working in the cathode lead we have therefore achieved the object of obtaining a single pulse of current of rectangular form and, further, the circuit is more flexible than when the current in the anode resistance is used, for it has been shown that neither the voltage of the anode supply nor the value of the resistance in the cathode lead affect the time of deionization, except in so far as they determine the magnitude of the current. If, therefore, R_A, L_1 and C_1 were kept the same and set for the largest current, then a large range of current values less than the maximum could be obtained by alteration of the battery voltage E_b without trouble arising over variation of the time of deionization with the arc current. Thus under these conditions the circuit is well adapted to the measuring of total emission of radio valves inserted in the cathode lead and constituting the resistance R'. Other suggested uses are the testing of fuses, cut-outs, circuit-breakers, overload relays, etc., when a known extra current could be applied to them for a known length of time, it being thus possible to determine the time delay in their action and the percentage overload carried

during short time intervals. The circuit, in fact, constitutes a heavy duty time switch and can be used wherever such an arrangement is required. It is possible, if desired, to obtain two or even three pulses of current all of equal magnitude through A by reducing the damping of the oscillatory timing circuit and decreasing the steady bias of A towards the critical bias.

§ 4. EXTENSION OF THE CIRCUIT

The circuit described above is essentially designed to obtain single pulses of current of rectangular wave-shape of any magnitude and duration. By suitable design almost any shape can be given to the current wave, including a practically sinusoidal form. Prince[4] has constructed a self-oscillatory circuit in the anode circuits of the thyratrons so that when these are fed with d.c. an alternating current is obtained in the output circuit; this can, if required, be stepped up or down, rectified and smoothed, forming a complete inverter circuit or "d.c. transformer." The grids can be driven by any alternating voltage instead of that derived from the damped oscillatory circuit described above, and it occurred to the writer that the grids might be controlled by a tuning fork maintained in vibration by the output, the power developed in the anode circuit being used to control the frequency of a wireless transmitter; greater power is obtainable from these two valves alone than by many stages of amplification with vacuum valves or by the use of fork-controlled flashing neon tubes[5]. However, the frequency with which the thyratrons may be released and stopped has a limit set by the time required for deionization of the space between cathode and anode, but harmonics of this fundamental could be used for higher frequencies.

§ 5. CIRCUIT FOR LONGER TIME INTERVALS

In the circuit of figure 1, for times greater than about 0·02 sec. ($L = 20$ H., $C = 8\mu$F), the size of inductance and condenser become unwieldy and some other timing device must be employed. A suggested circuit is shown in figure 6 which differs from figure 1 only in respect of the method of releasing thyratron A and then B after the required interval; according to figure 6 a condenser is charged through a high resistance and then its partial discharge through a neon tube is permitted in the usual fashion[6]. The action of the circuit is as follows: on closure of the switch K the battery B_2 commences to charge condenser C through the high resistance R_1. The current at the start is a maximum, and flowing through the resistance R_2 from p to q it causes an e.m.f. to be developed across that resistance in opposition to the steady bias on the grid of thyratron A, which is thus released at the moment when switch K is closed. When C is charged to the striking voltage of the neon tube N it discharges through this latter and resistance R_3, thus decreasing the bias of thyratron B, releasing it and stopping A as in the original circuit. The discharge current also flows through R_2 from q to p, increasing the bias of A. When the voltage at which the neon tube discharge can no longer be maintained is reached, the battery commences to charge condenser C again but, as the latter was not fully discharged, the

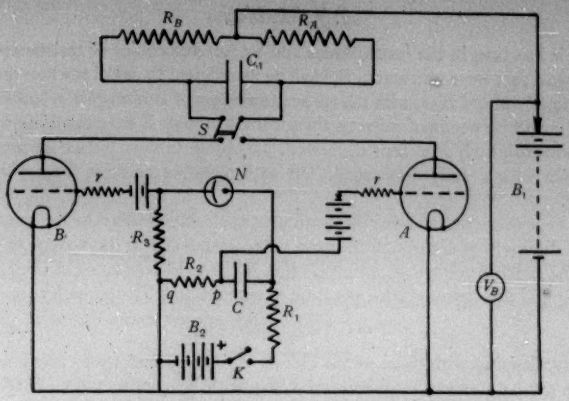

Fig. 6. Circuit diagram for long time-intervals.

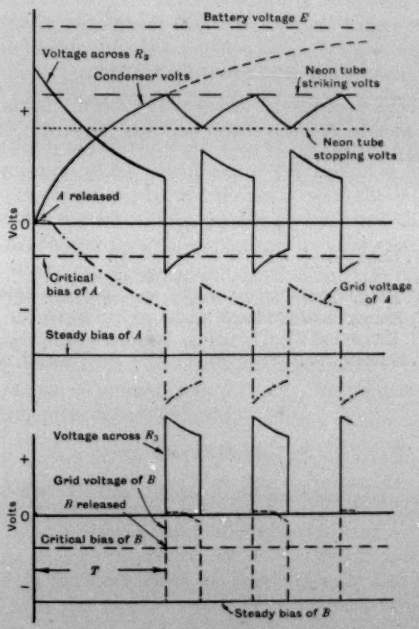

Fig. 7.

current is less than in the first instance and by suitable choice of resistances it can be arranged that the e.m.f. across R_2 shall be insufficient to annul the bias on thyratron A to the critical bias, with the consequence that A is not again released. Thus one pulse will be obtained through thyratron A though B may continue with the neon tube flashing if K is kept depressed. To obtain further pulses we must start over again with C fully discharged. The action should be clear from a study of figure 7.

The time of current flow in A is the time from closing switch K to the commencement of discharge of C through the neon tube. Hence if E is the voltage of battery B_2, V the striking voltage of the neon tube, R the total resistance of the charging circuit and C the capacity, then the time of current flow T is given by

$$T = CR \log_e [E/(E - V)] \qquad \text{.....(5).}$$

Times overlapping with those of the circuit of figure 1 and up to many seconds' duration can be obtained by this method, e.g. with E equal to 180 v., V to 140 v., R to 0·1 megohms, and C to 0·1 μF, $T \fallingdotseq 0{\cdot}015$ sec.; and with E equal to 160 v., R to 2 megohms, C to 4 μF, $T = 16{\cdot}6$ sec.

§6. ACKNOWLEDGMENT

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AN IMPROVED METHOD FOR THE COMPARISON OF SMALL MAGNETIC SUSCEPTIBILITIES

By R. A. FEREDAY

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ABSTRACT. In continuation of an earlier investigation, it is shown that a magnetic field, specially suitable for determinations of relative susceptibility by an improved non-uniform field method, can be produced by an electromagnet whose pole-pieces are respectively plane and spherically concave.

§ 1. INTRODUCTORY

In an earlier communication* (subsequently referred to as "the previous paper"), an account was given of a new method, of the non-uniform field type, for the comparison of small magnetic susceptibilities. A theory was developed which indicated that between two suitably shaped pole-pieces of an electromagnet a field could be produced having such a configuration that, over a wide region, the product of field-strength by its gradient along the axis of symmetry would be constant, so that a feebly magnetic body placed on this axis would experience a force which would be directed along the axis and constant at all parts of it. Equations were developed theoretically to determine the shape of the necessary pole-pieces; however, as is to be expected in a calculation of this nature, the field-distribution obtained in practice departs somewhat from that predicted by theory.

In order to be able to realize more accurately a field-distribution of the required configuration, it has been necessary to make empirical corrections to the results of the theoretical investigation, and the present paper gives an account of experiments made with this object. These experiments are described in some detail, in the hope that the results may be of use to others faced with the problem of designing an apparatus for susceptibility measurements.

It should be emphasized that the expression "magnetic force" is used to denote the actual force exerted on a weakly magnetic body, this being proportional to the product of the field-strength and field-gradient.

§ 2. EXPERIMENTAL

In the experiments to be described, the field was explored by direct measurement of the force exerted on a small paramagnetic body. Arrangements were made so that the force could be determined at different points along the axis of symmetry of the pole-pieces, or along any vertical line passing through this axis. The actual determination of the force exerted on the body was made with a torsion balance similar in principle to that used for susceptibility-measurements and described in the previous paper.

* *Proc. Phys. Soc.* **42**, 251 (1930).

The torsion balance. This was very similar to that illustrated in figure 4 of the previous paper, but of somewhat simpler construction. The two arms were made of equal length, and observations of deflections made directly on the specimen, which consisted of a small quantity of a strongly paramagnetic salt contained in a glass bulb, of diameter about 2·5 mm., fused to one end of the torsion balance arm.

In the case of the balance previously described, a small correction had to be applied for a deflection due to the magnetic force exerted on the arm of the balance itself. In the present case, an improvement has been effected: the torsion balance arm has been made of a glass whose susceptibility is so small that this effect is negligible.

Ordinary soda glass, such as that used for the construction of the earlier balance, is diamagnetic, having a mass-susceptibility of about $-0{\cdot}2 \times 10^{-6}$. On the other hand, paramagnetic glasses also are available; for the present work some bottle glass having a susceptibility of about $+2{\cdot}0 \times 10^{-6}$ was obtained. This was finely powdered, mixed with some finely powdered soda glass, and fused up in a furnace. The resulting product, which had a susceptibility of $+0{\cdot}8 \times 10^{-6}$, was again powdered, and fused up with more soda glass. In order to secure thorough mixing of the components, the resulting melt was again finely powdered, well mixed, and fused up. By this procedure a glass of extremely small susceptibility (less than 10^{-8}) was obtained. This was fused and drawn into thin rods by insertion into the crucible when the contents were in a viscous state of a stout platinum wire, which was slowly withdrawn together with the adhering glass. In this way, by suitable choice of the temperature and speed of withdrawal of the wire, rods several feet long and of any desired diameter could be produced. One of these rods was selected, and from it the arm of the torsion balance was constructed.

The susceptibility of such a complex mixture may be expected to show a temperature-variation, but since in the present investigation all observations were made at room temperatures no appreciable susceptibility-variation occurred.

The torsion balance support. In order to plot out curves of the force-variation along and perpendicular to the axis of symmetry, it was necessary to be able to move the balance in either of these directions. To this end the T-tube was mounted in clamps in which it could be moved vertically (for curves along a perpendicular to the axis) and the clamps were supported from a carriage which could traverse the magnet horizontally (for curves of distribution along the axis).

A stout brass bar *A*, figure 1, was bolted across the top of the magnet (cf. figure 3 of the previous paper), and on this was supported a piece of channel brass *B*, the upper edges of which were filed down carefully and rounded off to form a pair of rails lying parallel to the axis of the magnet. On these rails could be moved a brass trolley consisting of a brass framework carried on a pair of rollers *R*, in which grooves were turned so that the carriage would roll accurately on the rails. Two brass clamps *C* were rigidly attached to the framework, and held the torsion-balance T-tube *T*. A heavy lead weight *W* balanced the whole and kept the carriage steady on the rails.

Method of observation. The force on the specimen was evaluated by a direct observation, by means of a travelling microscope provided with an eyepiece scale, of

the deflection of the bulb when the magnetizing current was switched off. In order to follow the bulb as it was shifted to different positions along the magnetic axis, or along a perpendicular line, the microscope was arranged to travel horizontally when observations of the force-distribution along the axis were to be made, and vertically for observations along the perpendicular direction. The distance of the specimen from a fixed point was first observed by means of the vernier scale of the travelling microscope, the current was then switched off, and the resulting deflection was measured with the eyepiece scale. In the case of curves of force-distribution along the axis, the mark with respect to which the position of the specimen was determined was a small projection from the centre of one pole-piece; in the case of the perpendicular curves the scale illustrated in figure 3, and fully described later was employed.

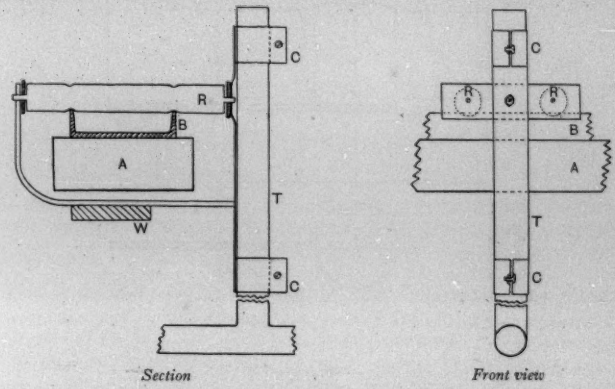

Fig. 1. Diagrammatic view of torsion balance carriage.

Special arrangements were necessary to ensure accurate setting of the specimen. Figure 2 shows the device used to set the specimen into the vertical plane through the axis of the magnet. The two arms *ab*, *a'b'*, projected horizontally to equal distances from the centres of the poles, and a length of thread was stretched between *a* and *a'* so that it ran parallel to the axis of symmetry of the magnetic system. The specimen was adjusted to a distance from this thread equal to the length *ab*, by means of a depth gauge supported horizontally. This ensured its being on the axis.

Two pieces of card, graduated in millimetres, were attached to the pole-pieces so as to lie in a vertical diametral plane, figure 3. These served as a check on the accurate centring of the specimen, since when situated on the axis it was midway between them.

When observations of the distribution of force along a vertical line perpendicular to the axis were to be made, the microscope had first to be set up to travel in a direction accurately perpendicular to the axis. To this end it was so arranged that

on being displaced vertically, it focussed on the upper and lower scales successively at the same graduations, i.e. at the same distance from the face of the pole-piece. The effect of a small error in this adjustment on the results is discussed in § 3. When

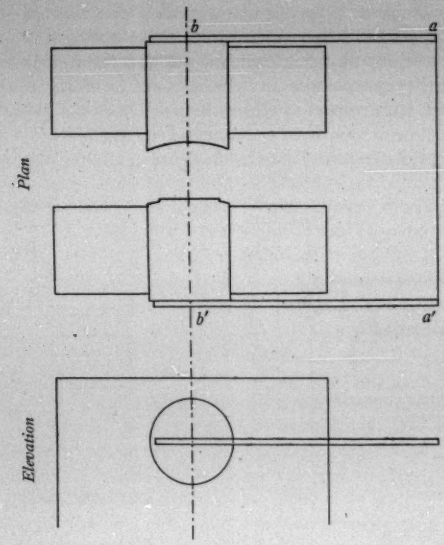

Fig. 2.

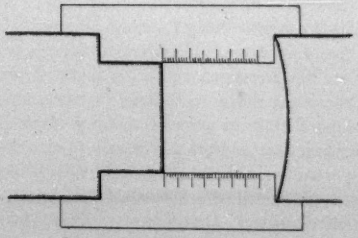

Fig. 3.

observations for these "perpendicular" distribution-curves were being made, the distance of the specimen from the axis was evaluated by an observation of its vertical distance from the side of one of these cards.

§ 3. RESULTS

For the first tests a pair of pole-pieces was constructed to the design of equation (8) of the previous paper, the following values being substituted for the constants of that equation: $a = +2{\cdot}0$ and $-1{\cdot}6$ for the two pole-pieces respectively; $b = 0{\cdot}04$. These pole-pieces are shown in section, in full line, in figure 4. As in the case of the pole-pieces described previously, the smaller one was elongated in order to reduce any effect of the outer part (*ab*) of the pole-face on the field in the gap. The force-distribution along the axis is shown by the curve of figure 5, and it will be seen that the theoretically uniform distribution is not too well realized.

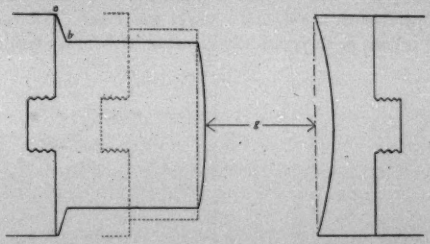

Fig. 4. Pole-pieces in section. Theoretically designed pole-pieces shown in full line; empirically corrected left hand pole-piece in dotted line.

Effect of annular pole-face. The calculations on which the design of the pole-pieces is based rest on the assumption that the field in the gap is due entirely to a distribution of magnetism over the faces of the pole-pieces. As, however, one pole-piece is of smaller diameter than the pole itself, there will be an annular area of the pole exposed, as at *ab* in figure 4. This will modify the field configuration in the gap, and in order to reduce this effect the smaller pole-piece was elongated, in the manner already described. In order that an idea of the magnitude of the effect of this part of the pole-face might be obtained, the following experiment was performed. A pair of pole-pieces was set up, consisting of a concave one of the full diameter of the pole-core and a plain cylindrical pole-piece of diameter 43 mm. and length 18·4 mm. at an available gap* of 34·8 mm. A force-distribution curve along the axis was taken, and the cylindrical pole-piece was then removed and faced off until its length was reduced to 15·3 mm. It was then replaced in the magnet, and the poles were brought together until the original gap was restored. A new force-distribution curve was then taken. The two curves are reproduced in figure 6 as (*a*) and (*b*) respectively. The apparent difference in magnitude of the force in the two cases is due to a change in sensitivity of the torsion balance caused by the disturbance due to the shifting of the poles on adjustment of the gap, but it is seen quite definitely that the shape of each curve is essentially the same. It therefore follows that the main geo-

* The term "available gap" is used as a convenient designation for the distance *g* of figure 4.

metrical factor deciding the shape of the force-distribution curves is the shape of the opposite faces of the pole-pieces, the approach of the annular shoulder by 3 mm. scarcely affecting matters. Further, with gaps of the order 30 mm., the contribution to the force in the gap, of the annular face of the pole at 18 mm. distance from the pole-piece face, is negligible. The distance 30 mm. was therefore standardized as the approximate thickness of the smaller pole-piece in succeeding experiments.

Effect of bored poles. The poles of the magnet were pierced with axial holes 11 mm. in diameter. In order to investigate any possible effect of this on the magnetization of the pole-pieces, and hence on the force-distribution curves, a test was made in which the bore was filled with a closely fitting soft-iron rod. Immediately on the completion of the observations of curve 6 (*b*), the bore behind the cylindrical pole-piece was filled with such a rod, and a curve was taken. The curve is shown as figure 6 (*c*), and is seen to be practically identical with figure 6 (*b*).

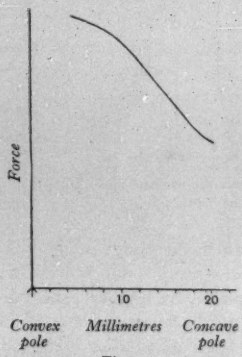

Fig. 5.

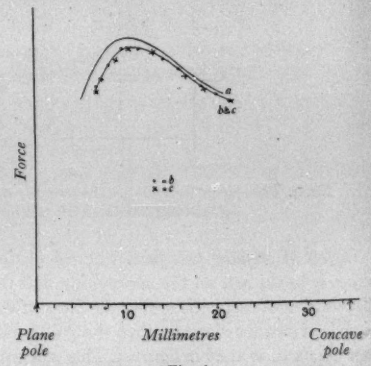

Fig. 6.

Hence the conclusion reached is that the nature of the force-distribution curve along the axis, for a given magnetizing current, depends only on the shape of the faces of the pole-pieces. The shape of the curves was shown to be independent of the magnetizing current also; the experiments are described later.

Improvement of field configuration. In the attempt to attain a more uniform force-distribution by modification of the pole-pieces, there are three principal ways in which an alteration can be made: (1) change of curvature of one or both pole-piece faces; (2) change of diameter of one or both pole-pieces; (3) change of distance between pole-pieces.

In order to simplify matters the concave pole-piece was left in position and all experiments were directed to modification of the convex one. A number of modified pole-pieces were tried, and it became apparent that the effect of a small change in any one of the above variables on the shape of the force-distribution curve could be almost exactly reproduced by a corresponding change in either of the others.

Thus a study of a progressive change in one variable gave force-distribution curves of all likely types. Such a study was made by observation of the force-distribution between the concave pole-piece and a series of plane pole-pieces, situated at a constant distance from it and having different diameters. Actually one pole-piece was used, and starting at the full diameter of the poles, it was turned to successively smaller diameters. The thickness of this pole-piece was 19 mm., and the available gap 33 mm. The force-distribution curves are reproduced in figure 7, and it will be seen that for one diameter of the pole-piece, namely 51·2 mm., curve (*e*), the force is uniform within close limits over a considerable region.

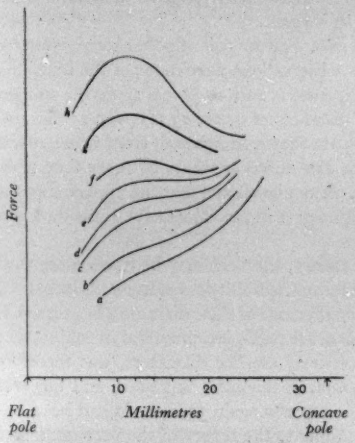

Fig. 7. Force-distribution along axis for the concave pole-piece of figure 4, and flat cylindrical pole-pieces of the following diameters: (*a*) 59·5 mm., (*b*) 57·1 mm., (*c*) 54·0 mm., (*d*) 52·4 mm., (*e*) 51·2 mm., (*f*) 49·6 mm., (*g*) 46·4 mm., (*h*) 43·0 mm.

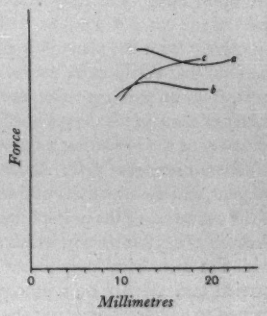

Fig. 8.

Change of curvature of pole-piece face. As has been mentioned above, it was found that a change of curvature of the pole-piece faces was equivalent to a change of diameter. To check this conclusion the face of the plane pole-piece was rounded off to a slight convexity in the case of the pole-pieces giving the distribution of curve 7 (*e*). The resulting force-distribution curve, figure 8 (*a*), is intermediate in shape between 7 (*f*) and 7 (*g*), so that the effect of the rounding is equivalent to a reduction of the diameter by about 2 mm.

Change of interpole gap. Curves (*c*) and (*b*) of figure 8 are force-distribution curves for the pole-pieces giving the distribution of figure 7 (*e*), but with the interpole gap slightly increased and decreased for the two curves respectively. It is again

seen that the effects are equivalent to those of a change in diameter; in this case there is also a change in magnitude of the force due to the change in field strength with the change in gap.

Variation with magnetizing current. As was explained in the previous paper, the pole-pieces were not magnetized to saturation. It therefore became of importance to determine whether the shape of the force-distribution curves depended on the degree of magnetization of the pole-pieces. A number of observations were made, and these showed that over wide ranges of magnetizing current the shape of the curves remained unaltered. Unfortunately, it was not possible, in the case of the magnet used, to reach saturation, but it is hoped to investigate the nature of the force-distribution at saturation on another magnet.

Variation of force perpendicular to the axis. For the pair of pole-pieces giving the most uniform distribution along the axis, observations were made of the force-distribution along a line perpendicular to the axis, at various points along the straight part of the curve 7 (*e*). The results, for distances of 9, 11, 13, 15, and 17 mm. respectively from the plane pole-piece face are shown in figure 9, from which it will be seen that the force at 2 mm. from the axis differs nowhere by more than 1 per cent. from its uniform value on the axis. It can be shown that the apparent asymmetry of some of the curves is due to a slight error in the adjustment of the direction of travel of the observing microscope.

More convergent field. According to theory, the field may be made more convergent, and the sensitivity of the method for susceptibility-measurements increased, if the curvature of the pole-pieces and the difference of their diameters be increased. A series of experiments in which a more concave pole-piece was used in conjunction with flat cylindrical pole-pieces of successively smaller diameters, was therefore carried out. As before, a pole-piece 19 mm. in thickness was used, and this was turned to smaller diameters; the available gap was again 33 mm. It had been observed that the pole-pieces designed according to the theory of the previous paper were almost exactly spherical in curvature, and for the purpose of these observations the concave pole-piece was therefore turned spherically concave, with a radius of curvation of 6 mm. The concave pole-piece of figure 4 is almost exactly a part of a sphere of radius 10 cm.

The force-distribution curves are shown in figure 10, and they are seen to resemble closely those of figure 7. It is obvious from a comparison with figure 7 (*f*) that curve 10 (*d*) corresponds to a pole-piece of slightly less than the optimum diameter.

Ageing. In some of the earlier experiments there seemed to be some evidence of an ageing effect, viz. a change with time in the shape of the force-distribution curve for a given pair of pole-pieces, and it was thought that a freshly machined pole-piece, if not annealed, might take some time to settle to a steady state. Some careful experiments indicated, however, that no appreciable ageing could be detected in a period of several days following machining.

§4. CONCLUSIONS

Although the pole-pieces designed in accordance with the theory give a force-distribution which is only approximately uniform, it is clear that a very slight modification of their shape is necessary to obtain a distribution which is uniform over a considerable region. The amount of modification necessary can be seen by reference to figure 4, in which the pole-pieces designed theoretically are shown in full lines, and those giving the nearly uniform distribution of curve 7 (*e*) in dotted lines. The observations indicate that the same result could be obtained by the use of the pole-pieces drawn in full lines, the interpole gap being slightly decreased.

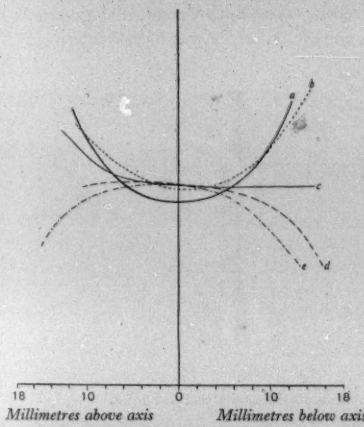

Fig. 9. Force-distribution perpendicular to the axis at various parts of the gap. Distances from plane pole-piece face: (*a*) 9 mm., (*b*) 11 mm., (*c*) 13 mm., (*d*) 15 mm., (*e*) 17 mm.

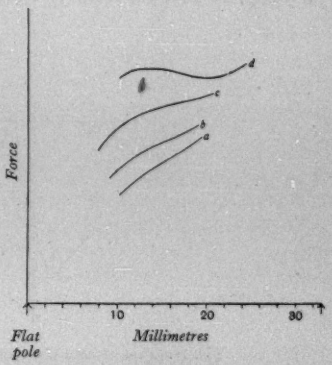

Fig. 10. Force-distribution along axis for more concave pole-piece and flat cylindrical pole-pieces of the following diameters: (*a*) 59·9 mm., (*b*) 57·1 mm., (*c*) 52·7 mm., (*d*) 49·4 mm.

The curves of figure 9 show that the force changes slowly and symmetrically as the point of observation passes away from the axis of symmetry. The field-configuration is therefore suitable for use in comparing the susceptibilities of moderately large specimens.

The concave pole-pieces designed in accordance with theory are practically indistinguishable from portions of spheres, and hence these force-distributions may be realized by means of the very simple pole-piece combination of a concave spherical pole-piece and a plane one. By the use of a more concave pole-piece, with the appropriate plane, the force on the specimen is increased somewhat, and the region of uniformity does not decrease in extent.

It is of interest to make a comparison between the new method and the more generally used Curie method for susceptibility-measurements. Curie* gives a curve showing the force-distribution $H_y \cdot \partial H_y/\partial x$ which shows two very sharply defined maxima, for forces in opposite directions. A similar curve is given by Owen† and is of the type reproduced in figure 11. It is seen that neither of the two maxima can be said to have a region of uniform force; the force increases to a sharp peak, on either side of which it falls off asymmetrically. The advantage of a force-distribution of the type of figure 7 (*e*) is obvious.

For the pole-pieces shown in full line in figure 4, theory indicates that

$$\partial H/\partial x = 0{\cdot}16\, H$$

at the centre of the gap. For the empirically corrected pole-pieces shown in dotted lines, the value found by experiment was about 0·10. With a magnetizing current of 5 amp. the field was about 3500 gauss, so that $H.\partial H/\partial x$ was of the order 10^6.

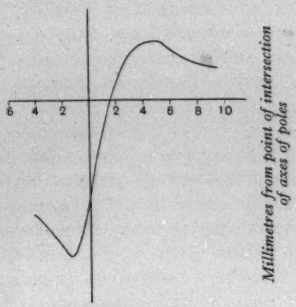

Fig. 11. Type of force-distribution curve obtained with inclined poles.

Some ratios of gradient to field-strength obtained by the Curie method are: 0·188‡, 0·145§, 0·43‖. Since the field-strengths are comparable in all cases, these ratios serve as a measure of the sensitivity of the apparatus. The value of 0·10 obtained in the case of the present apparatus, although below that obtainable by the Curie method, does not indicate a serious lack of sensitivity. A greater sensitivity has already been obtained, figure 10, but the value of the gradient to field-ratio in this case has not yet been determined. It would be possible to increase the sensitivity by working at the peak of a curve such as figure 7 (*h*). This would involve some sacrifice of force uniformity, but the uniformity attained would still be better than with a distribution of the type of figure 11.

It will be obvious that the new method should be particularly applicable to the determination of the principal susceptibilities of crystals, since a crystal could be set to the region of uniform force for the three successive measurements with a

* Curie, *Ann. Ch. Ph.* **7**, 5, 289 (1895), figure 1. † Owen, *Ann. d. Phys.* **37**, 657 (1912).
‡ Curie, *loc. cit.* § Hayes, *Phys. Rev.* **3**, 295 (1914). ‖ Owen, *loc. cit.*

reasonable certainty that each measurement would be made under the same conditions. Further, the method is inherently more suitable for comparisons of susceptibilities of magnetically anisotropic bodies than a method involving the use of a non-uniform field of *force*.

§ 5. ACKNOWLEDGMENTS

This work has been carried out in the Physical Laboratories at East London College, with the aid of a grant from H.M. Department of Scientific and Industrial Research. I have pleasure in expressing my thanks to Prof. C. H. Lees (at whose suggestion it was originally commenced) and to Prof. H. R. Robinson, for their advice and interest in the research, and for the facilities which they have placed at my disposal.

EDGE TONES

By E. G. RICHARDSON, B.A., Ph.D., D.Sc.

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ABSTRACT. If a fluid leaves an orifice as a jet and strikes an edge, two vortex streets are formed on each side of the wake, and maintain the jet in pendulation at a definite frequency. The tones so produced are examined from theoretical and experimental standpoints, and relations connecting the frequency with (*a*) velocity, (*b*) distance from orifice to edge, (*c*) width of orifice, (*d*) form of the orifice, are tested. It is shown that all the features of the phenomena can be explained in terms of the hydrodynamics of a viscous fluid, without postulating compressibility in addition.

§ 1. INTRODUCTION

When a fluid debouches from a circular orifice or slit into stagnant fluid a surface of discontinuity, as Helmholtz called it, is formed between the swiftly moving and the stationary fluid. Some of the latter is drawn into the jet and accelerated, with the result that the surfaces of discontinuity tend to spread out in the form of a conical or a plane surface as the fluid leaves the slit. The exact form of this spreading will depend upon the previous history of the jet, particularly upon the shape of the boundaries of the orifice and the method of approach. Generally there will be a vena contracta, just as in the case of a liquid projected into the air from an orifice; in which case the surface will not open out until the fluid has travelled a certain distance from the orifice. In this condition the jet is sensitive, for a very slight alteration in the pressure will cause a large change in the length of the jet as measured to the point where it opens out; this is the mechanism of the sensitive jet or flame. Experiments indicate that from the point where this mixing commences the mixing-zone increases linearly with distance along the jet, leading to plane or conical surfaces of discontinuity as the case may be.

Along these surfaces there are considerable shearing forces which give rise to the production of vortices. In the absence of periodic disturbance of the orifice itself, or of the formation of a stable vortex system, there is nothing to give rise to a tone in the jet. It is suggested that, in the production of an edge tone by the pushing of a sharp wedge into the stream of air issuing from a linear slit, disturbances of certain frequencies are encouraged and further stabilized by the eddies produced by friction on the solid wedge and may, with the vortices of opposite sense produced along the planes of discontinuity, give rise to two stable avenues of alternate and definitely spaced vortices of the Benard-Kármán type, one on each side of the wedge. These secondary eddies were noticed by Schmidtke* in photographs of the motion. The wave-length of this system determines the frequency of the edge-tone heard.

* *Ann. d. Phys.* **60**, 715 (1919).

§ 2. CALCULATION OF VELOCITY POTENTIAL AND OF FREQUENCY OF EDGE TONES

The axis of x being taken along the edge, and that of y through a pair of vortices (dotted line in figure 1), if h be the lateral and l the longitudinal spacing of each vortex-avenue, the complex velocity potential z at any point, due to the double street of vortices (assumed to be all of equal strength K), is to be calculated, where $z = x + iy$. x, y h, l z

The velocity-potential at z due to an isolated vortex at z_0 is w where w

$$w = (iK/2\pi) \log (z - z_0)^*.$$

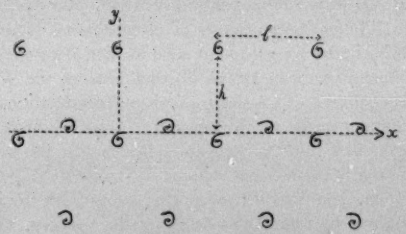


Fig. 1. Double street of vortices.

The corresponding expression for the two infinite series is given by

$$\begin{aligned} w &= u + iv \qquad u, v \\ &= \frac{iK}{2\pi}\left[\sum_{j=-\infty}^{j=+\infty} \log (z + jl - hi) + \sum_{j=-\infty}^{j=+\infty} \log (z + jl)\right. \\ &\qquad \left. - \sum_{j=-\infty}^{j=+\infty} \log \left(z + jl + \frac{l}{z}\right) - \sum_{j=-\infty}^{j=+\infty} \log \left(z + jl + \frac{l}{z} + hi\right)\right] \\ &= \frac{iK}{2\pi} \log \prod_{j=0}^{j=\infty} \left[1 - \left(\frac{z - hi}{jl}\right)^2\right]\left[1 - \left(\frac{z}{jl}\right)^2\right]\left[1 - \left(\frac{z}{jl - l/z}\right)^2\right]\left[1 - \left(\frac{z + hi}{jl - l/z}\right)^2\right] \\ &= \frac{iK}{2\pi} \log \frac{\sin \left\{\frac{\pi}{l}(z - hi)\right\} \sin \frac{\pi}{l} z}{\cos \left\{\frac{\pi}{l}(z + hi)\right\} \cos \frac{\pi}{l} z}. \end{aligned}$$

If z be taken at a point $(x, 0)$ along the dividing line of the two streets,

$$v = \frac{\cosh \frac{\pi h}{l}\left(1 + \cos \frac{2\pi}{l} x\right) - \sinh \frac{\pi h}{l} \sin \frac{2\pi}{l} x}{\cosh \frac{\pi h}{l}\left(1 + \cos \frac{2\pi}{l} x\right) + \sinh \frac{\pi h}{l} \sin \frac{2\pi}{l} x}$$

after real and imaginary parts have been separated and the expressions have been cleared of constant terms.

* See e.g. Lamb, *Hydrodynamics*, p. 665.

The above expression for the velocity perpendicular to the stream and imposed upon it by the two systems of vortices shows a simple harmonic variation of v with distance forward of the edge. It indicates, in fact, a stationary velocity-wave of which the maxima occur at distances l apart.

But the whole of the vortex system is moving with a velocity u from left to right; consequently a wave-motion will be imposed upon the jet as it emerges from the slit, and its wave-length will be equal to l, the longitudinal spacing of the vortex system. The frequency n of the edge tone will then be given by the simple formula

n

$$u = nl.$$

What may be imagined to happen is something like this: the fluid leaving the slit spreads out along two planes of discontinuity which pass the edge at a definite distance h from it. Along these surfaces of discontinuity vortices tend to form, while vortices rotating in the opposite sense tend to form along the edge itself. These roll up into two Benard-Kármán streets on each side of the edge with discrete vortices at a definite spacing of h/l, determined by considerations of stability. The stability of such a street in the neighbourhood of a semi-infinite wall has not yet been completely examined, but the analysis indicates that the system will tend to leave the wall and rapidly assume that value of h/l, namely 0·28, which is proper to a free Kármán system. The fluid subsequently issuing from the jet oscillates with a frequency determined by l and the velocity of the vortices. The usual formula given for edge-tone production is:

$$U/na = \text{a constant}.$$

u To fit this to the earlier formula we must assume (*a*) the velocity u of the vortex
U system to be a constant fraction of that of the jet U. (*b*) The longitudinal spacing,
l, a l, to be a constant fraction of the distance from slit to edge, a.

§ 3. EXPERIMENTAL INVESTIGATION OF VELOCITY-DISTRIBUTION

The main purpose of this paper is to examine how far these assumptions are justified. As there seemed to be no data on the distribution of velocity and turbulence in a jet emerging from a linear slit, an investigation of this question was first made.* Air was admitted from a reservoir under pressure into a wooden box measuring 40 × 20 × 16 cm. from which it emerged into the atmosphere through a vertical slit 6 cm. long, whose width could be adjusted by means of a micrometer head. This part of the apparatus was substantially the same as that used by Benton†. The jaws of the slit were thick and bevelled at 45° from inside. The velocity and amplitude of velocity-fluctuation at various points in the median plane through the slit were determined with a hot wire held vertically in a fork. The velocity-determinations were made from a knowledge of the steady change of resistance of the

* Tollmien has examined the spreading of jets theoretically and Benton traced out the surfaces of discontinuity by traversing the wedge across the slit till the edge tone ceased to be audible.

† *Proc. Phys. Soc.* **36**, 109 (1926). This paper contains a bibliography on edge tones.

wire, which had previously been calibrated for this purpose in a wind-channel. The amplitude of the velocity-fluctuations, taken as a measure of the vortex-strength, was found by coupling the hot wire through a transformer to a string galvanometer. The response of the latter, corrected for the average velocity in which the hot wire was placed, was taken as a measure of the vorticity, by the extent of the disturbance shown in the mean velocity. The pressure inside the box was measured with a Chattock gauge.

A characteristic plot of velocity and velocity-fluctuation is shown in figure 2 for a slit 0·09 cm. wide at a pressure of 8 mm. of water. It will be observed that the two surfaces spread from a vertex 1·2 mm. from the slit at an angle of 21°. For orifices wider than about 0·05 cm. this angle is changed but little, as the pressure is altered. Some values of this angle of spreading are shown in table 1.

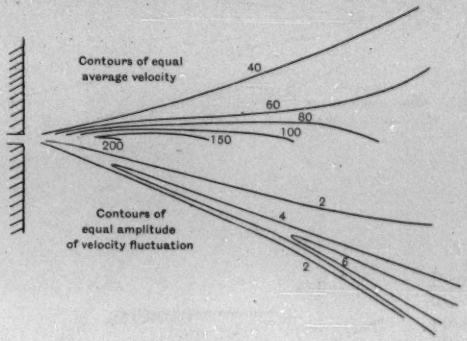


Fig. 2.

The results indicate that the angle θ of spread increases as the width d of the slit increases, reaching an ultimate value of approximately 20° for wide slits at high velocities of efflux. θ, d

Table 1

Width d of slit (mm.)	Pressure p (mm. of water)	Vertex at (mm.)	Angle θ (degrees)
0·9	0·65	10	10
	2·5	3·7	20
	8·0	1·2	21
	14·0	0	18
0·25	1·0	6·5	6
	2·5	3·2	12
0·5	1·0	7	16
	2·5	3·2	22

The next points to be tested experimentally are those involved in the two assumptions above: (1) the dependence of θ on the introduction of the wedge; (2) the

dependence of h/l on the introduction of the wedge. Figure 3 is a plot of velocity and velocity-fluctuation for the same jet as that to which figure 2 refers, with an edge of angle 15°, at 1 cm. from the slit. The secondary vortex-row is seen on each side of the edge arising from the tip. The streets broaden out somewhat after leaving the edge, besides being deflected. Figures 4 and 5, show the surfaces of discontinuity for wedges of angle 30° and 5° respectively. It will be observed that the

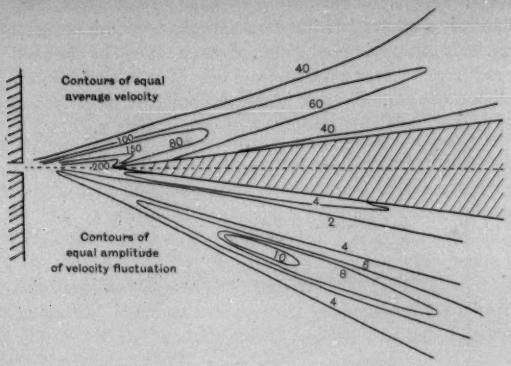


Fig. 3.

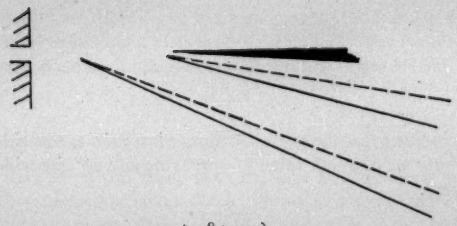


Fig. 4. Paths of vortices.

widths of the avenues are but slightly affected by the change of angle of wedge, but that they are deflected out from the tip to a greater extent as the angle of the wedge is increased.

That the vortex system is practically independent of the angle of the wedge is borne out by frequency-measurements. Stroboscopic examination of the response of the string galvanometer gave the values of frequency, for different wedges, shown in figures 6 and 7. The jump of pitch indicated by the short vertical lines should be noticed. With these wide-angle wedges the tones are more feeble, probably owing

to loss of energy in friction, as the jet has to turn a sharp corner. Blunting of the nose of the wedge has a similar effect. Weerth records no change of frequency when wedges of 15°, 47° and 138° are used.

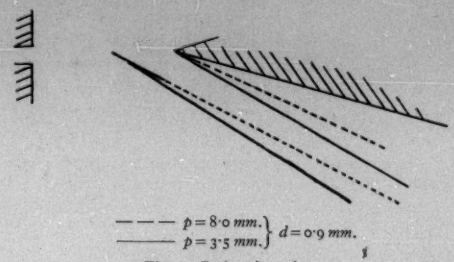


Fig. 5. Paths of vortices.

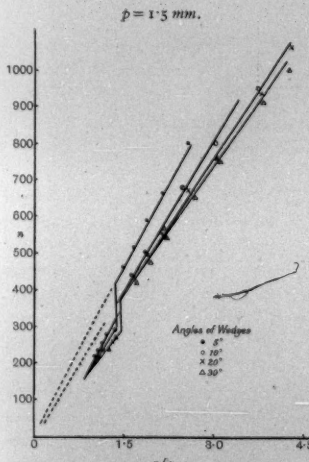


Fig. 6. Effect of angle of wedge.

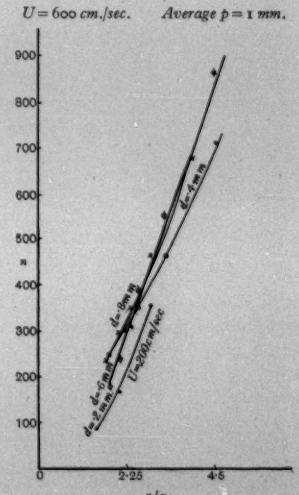


Fig. 7. Effect of width of slit.

The formula

$$U/na = \text{a constant}$$

depends on two assumptions: (*a*) equality between *a* and the "wave-length" of the vortex system; (*b*) proportionality between the velocity of the vortex system and the mean velocity of the jet.

The first of these assumptions may readily be verified by means of a photograph of the motion of a jet of coloured fluid emerging from an orifice and impinging on an edge. An apparatus for this purpose, designed by Dr R. S. Clay and kindly lent to me by him, is shown in figure 8. Water entering by the pipe on the right flows towards the left of the figure between two glass plates about 5 mm. apart. Ink drawn in from the series of capillary tubes serves to render the pendulation of the water visible as it leaves the slit and strikes the edge. Part of a cinematograph film of the motion is reproduced on the plate; the part of the apparatus which appears on the film is marked by the dotted line on figure 8. By following the series, a disturbance may be traced as it grows from the slit to the edge. In these pictures the wave-length of the disturbance is equal to the height of the mouth; in other cases photographed the wave-length was one-half or one-third of this distance. The secondary vortices coming from friction along the edge fall into line with the main vortices due to the pendulation of the jet, on either side of the edge.

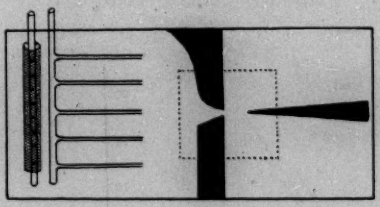


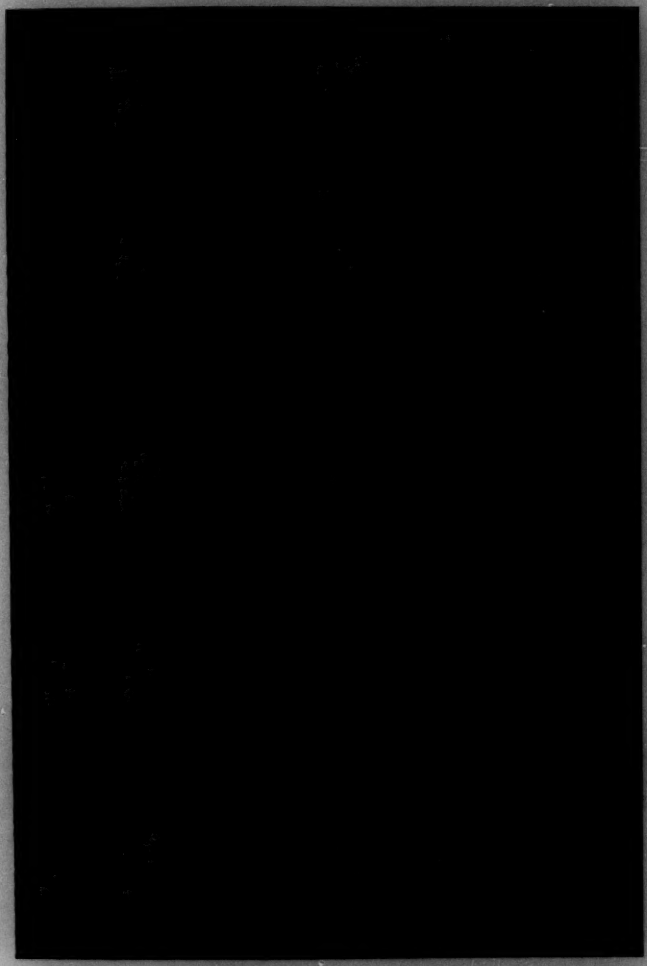
Fig. 8. Dr Clay's apparatus for demonstrating edge tones.

The wave-length of the vortex system can be measured by means also of a pair of hot wires, a method employed by Tyler* to measure the longitudinal spacing of the Benard-Kármán system of vortices behind a cylinder. For this purpose one hot wire was held in a fixed position by a bracket round the wedge, while the other was gripped by a traverse so that its fork could be run along the system longitudinally; further, each wire was capable of lateral adjustment on its fork. The current through each was led to a separate primary of a transformer, the two secondaries being then connected in series with the string galvanometer. The line of maximum velocity-fluctuation past the wedge was first fixed by the separate use of the wires, and the two wires were then placed in this path. As the movable wire was traversed, places of maximum deflection of the string galvanometer indicated that the disturbances were in phase, and therefore a wave-length apart. Results obtained by this method are given in the table 2. In most of the instances, the wave-length of the disturbance was half the distance from the slit to the edge.

With regard to the relationship between the velocity of the vortex system and the mean velocity of efflux from a linear slit, Rayleigh† has given a formula which is pertinent. For a jet of width $2b$ travelling with velocity U in stagnant fluid, the

b, U

* *J. Sci. Inst.* **6**, 310 (1929). † *Scientific Papers*, **1**, 368.



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relation between the velocity u of a disturbance of frequency n along the jet, so disposed that the sinuosities on either side are parallel, is given in the form u, n

$$(n + kU)^2 \tanh kb + n^2 = 0,$$

where k is the cycle-frequency $2\pi n/u$. k

The amplitude h of a wave of initial amplitude H travelling along the axis of x then becomes h, H

$$H \exp.(\pm \mu kUt).\cos k\,[Ut/(1 + \coth kb) - x],$$

where $\mu = \sqrt{(\coth kb)/(1 + \coth kb)}$. The wave-velocity u, viz. $U/(1 + \coth kb)$, tends to the limit $U/2$ for wide jets. The line on figure 9 shows the variation of u/U with kb. It is to be noted that in Raleigh's theory the frequency is quite arbitrary. μ, u

Table 2

p (mm.)	U (cm./sec.)	n	1 (cm.)*	a (cm.)
1·5	120	210	0·325	1·0
			0·325	
			0·30	
			0·29	
2·5	200	195	0·50	1·0
			0·49	
			0·47	
2·5	210	165	0·78	1·5
			0·74	
			0·70	

Since $u = nl$, we should expect the "constant" U/nl to be greater for narrow jets, more especially when the frequency is low. This is in fact the case, not only for edge tones of all kinds but also in the allied case of Aeolian tones, where the "constant" U/nd is greater for thin wires at low frequencies†, apparently from the same cause, for it makes no difference to the theory whether the stratum of fluid is moving rapidly past still fluid, or whether the main body of the fluid is in motion relative to the stagnant wake of an obstacle.

To test this formula of Rayleigh's against experiment, values of the frequency of the edge tone at various values of a, the distance of slit from edge, and of d, the slit width, were obtained and plotted on figure 10, the pressure being kept constant. The corresponding velocity of efflux U, which as a matter of fact varied little with the slit width, was obtained by means of a hot wire in the middle of the jet as it emerged. Having regard to the wave-length of the vortex system relative to a, the slopes of these lines in figure 10 together with those in figures 6 and 7 give the corresponding value of u, since $u = nl$. Points so obtained are shown as crosses on figure 9. a, d

It is to be noted that Rayleigh's theory takes no account of the dragging of stag-

* The numbers in this column give the distances between successive vortex pairs at any instant, passing outward from the slit.

† *Proc. Phys. Soc.* **36**, 163 (1924).

nant air by the jet and consequent spreading. When this spreading takes place in linear fashion from the orifice itself, as it does at higher pressures, l is an exact sub-multiple of a, and the edge-tone constant jU/na is given by the reciprocal of the
j ordinates in figure 9 (i.e. U/u), j being any integer.

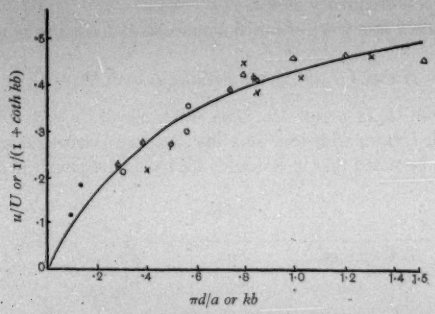

Fig. 9. Variation of wave-velocity with width of jet.

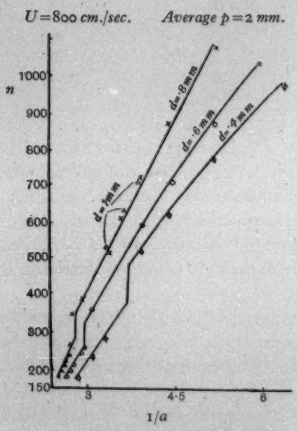

Fig. 10. Effect of width of orifice.

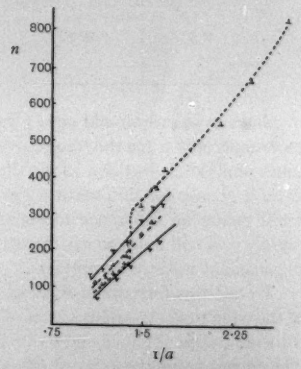

Fig. 11. Effect of thickness of jaws.
$p=3$ mm., thin jaws ⊥, thick jaws ⊤.
$p=2$ mm., thin jaws △, thick jaws ▽.

It is interesting to note in parenthesis that another formula given by Rayleigh
r for a cylindrical jet emerging at velocity U from a circular orifice of radius r gives us the wave-velocity corresponding to this case, so that

$$u = U/(1+\eta^2)$$

η where

$$\eta = \log 8/kr + \pi^{-\frac{1}{2}}\,\Gamma'(\tfrac{1}{2}).$$

The value of u is U, when $kr = 0$, and decreases as kr increases. The expression is valid only for small values of kr, so that the other extreme value cannot be deduced from it. This formula is applicable to the case of the bird-call where the jet emerges from a circular hole in a plate and impinges on another equal hole in a parallel plate a short distance away, and to the Galton whistle in which this second plate is replaced by a short pipe co-axial with the hole. If the orifice is partially closed by a circular disc we have the case of the annular jet, data for which have previously been obtained by the author and E. Tyler*.

§ 4. VORTICES FROM A HOLE IN A THICK PLATE

The system of vortex rings from a hole in a plate can be stabilized without the intervention of the edge if the plate be made thick; in this case the thickness of the plate acts as the determinant length in the edge-tone formula. The tones of such a system are, however, weak. As bearing upon the question of the influence of the spreading on the edge-tone wave-length, some measurements of frequency were made with the linear edge and slit, but with thick jaws to the slit, figure 11. The jet in this case spreads out less readily, and the velocity of the jet is reduced. Consequently the graph for the thick jaws lies below those for the thin ones at corresponding pressures.

§ 5. SUMMARY AND CONCLUSIONS

The fluid leaving a slit spreads out from a vertex a little beyond the orifice, dragging in stagnant fluid as it goes. Along the surfaces of discontinuity thus set up eddies are formed. Another set of secondary eddies leaves the tip of any edge inserted in the jet, and the two streets of discrete vortices so formed pass down the stream. The width of these at the tip of the edge is h or a' sin θ, where a' is the distance of the edge from the vertex and θ is the angle of spread. The ratio of the lateral spacing h to the longitudinal spacing l of the vortices in a Kármán street in a free stream is 0·28, so that $h = 0{\cdot}28\,l$. In practice $\theta = 20°$, whence $h = 0{\cdot}34a'$. Assuming that h/l is little changed by the proximity of the edge, and comparing the above formulae, we see that l will be nearly equal to a. The vortex system thus stabilized will produce a velocity potential along the issuing jet which will tend to make undulations of the same wave-length, which in turn will grow in amplitude as they pass along to the edge, where they will supply new vortices to the alternate procession, maintaining the original wave-length. a'

An alternative theory due to Krüger and examined by Schmidtke may be mentioned. In this theory, an embryo vortex leaving the slit and travelling with the stream starts off a new vortex from the slit when it hits the edge. A wave of compression then travels back with the velocity of sound to start the new disturbance from the jet. At the suggestion of Mr A. M. Cassie a crucial test of this theory was staged. In the apparatus of figure 8 an additional slit was made in the barrier, about

* *Phil. Mag.* **2**, 436 (1926).

5 cm. to the right of the existing one, but was not provided with an edge. The left-hand jet being stabilized to rhythmic pendulation by its edge, the wave of compression should have affected the right-hand jet equally. However, no definite periodic motion of this jet was noted at any velocity. This test, negative though it is, seems to dispose of the compression-wave theory.

§ 6. ACKNOWLEDGMENTS

In conclusion, I wish to express my best thanks to Prof. E. N. da C. Andrade, in whose laboratory this work was done, for his interest in the problem and for placing every facility at my disposal.

DISCUSSION

Dr L. Simons. The hiss of escaping steam from a safety valve is to my ear associated with a definite musical pitch which, I have noticed, apparently rises as I approach the source and remains constant if I am stationary. If I have not misjudged the phenomenon, is this associated with a regular vortex system as described by Dr Richardson? What would be the cause of the steady rise in pitch on my comparatively slow motion towards the source?

Author's reply. I can only suggest that the phenomenon described by Dr Simons is a reflection tone associated with the stationary-wave pattern between the listener and a neighbouring wall. The vortex system from a simple jet like the safety valve will have no edge to stabilize it; consequently the sound emitted is a mixture of frequencies, a high-pitched noise in fact, each component having nodes at different planes between the listener and the source. Movement towards the source will therefore cause a change in the quality of the note, which under some circumstances may sound like a rise in pitch. The same phenomenon may be observed at a large waterfall, owing to reflection from the rock behind, particularly if one can walk behind the waterfall towards the rock.

THE METHOD OF FORMATION OF SAND FIGURES ON A VIBRATING PLATE

By Professor E. N. da C. ANDRADE and D. H. SMITH,
University of London, University College

Received May 5, 1931.

ABSTRACT. The conditions governing the movement of small particles on a vibrating surface are investigated experimentally and theoretically, close agreement being obtained. An arrangement by means of which the formation of the Chladni sand figures can be studied in detail, and projected on a screen, is described.

In connexion with a discourse delivered by one of us at the Royal Institution, an apparatus was constructed to enable Chladni figures to be projected epidiascopically. For this purpose a square steel plate, cut from a piece of Brown and Sharp ground stock, was mounted centrally on a pillar in the way usual for Chladni plates, but, to throw the plate into vibration, an electromagnet fed with alternating current was employed instead of the usual bow. The magnet *M*, taken from a loud-speaker unit, was fixed rigidly to the end of a pivoted arm, as shown in figure 1, the inclination of the arm being controlled by a screw *C* kept in contact with a

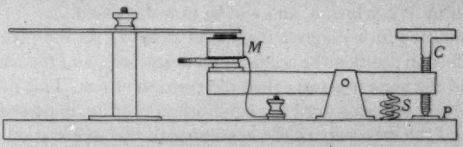

Fig. 1. Diagram of apparatus.

plate *P* by a spring *S*. The whole was mounted on a wooden board provided with levelling screws. Current to the magnet was provided by a valve-oscillator set, with suitable amplification, which gave a good output over a range of frequency from 100 to 8000 ~. The plate being levelled and strewn with sand, a range of beautiful figures is easily produced by progressive increase of the frequency, and careful tuning as soon as the plate shows signs of resonant response. With the help of an epidiascope the formation of the patterns of nodal lines, and the passage from one pattern to another, can be made visible to a large audience, which may be of interest to the many exponents of wave-mechanics who use the free modes of a vibrating plate as an analogy. This method of producing the figures is to be preferred over the older bowing method, not only because the vibration can be maintained at a fixed frequency and constant amplitude over a long period, but also because the vibration

is really free, which is not strictly the case with bowing, as was pointed out by Lord Rayleigh*.

This apparatus seemed to offer a good opportunity for investigating the question as to exactly why the sand moves to the nodes, which does not appear to have ever received serious attention. The only explicit reference which we have been able to trace is in Winkelmann†, who says: "Neither in the original publications nor in the textbooks is there any satisfactory statement as to how the sand figures are mechanically formed. For if we consider that a particular part of the plate bounded by nodal lines is alternately convex and concave upwards, and that the impulse of the particle of sand which is thrown upwards in one case has a component directed outward to the node, and in the other case inward to the loop, we must conclude that what occurs, on the average, is a hopping of the particle at a fixed and given spot: we must therefore consider the process more carefully." He then supposes that two cases can occur, either that the particle always falls on a surface convex upwards, or else alternately on a convex and concave surface, the third case (presumably that in which the particle always falls on a concave surface) not occurring, "weil die konkave Fläche allein keinen Impuls erteilt." He therefore states that the particle is always thrown outwards on the whole, since the first case moves it out, and the second leaves it stationary. While this appears plausible at first sight, as far as it goes, it is not really satisfactory, for it assumes implicitly that, in the second case, the particle is always thrown outwards from the convex surface by just the same amount (or at any rate not less) as it is thrown inwards when it falls on the concave surface. This is certainly not necessarily so. The point is examined at the end of the present paper. The process which leads to the particle leaving the plate is not indicated by Winkelmann, and will be now discussed.

The mechanism which suggests itself is as follows. As the plate moves down from the equilibrium position the acceleration is upwards, and the reaction between the particle and the plate is greater than the reaction at rest. This holds true while the plate is moving upwards until the equilibrium position is passed, but between this position and the extremity of the upward path the acceleration of the plate is downwards, and the reaction is diminished. If, before it reaches the highest point, the acceleration of the part of the plate considered becomes g, the particle will cease to be in contact with it, and will proceed upwards with an initial velocity equal to that of the plate at this moment. The subsequent path of the particle will be the parabola described by a projectile thrown upwards with this velocity, supposing that it be heavy enough for the turbulent motion of the air above the plate not to affect it.

If this is what actually happens, we should expect the sand to move on the plate until it reaches, not a nodal line, but a line at which the maximum acceleration of the plate is just g. Of course, if the motion of the plate is very vigorous, such lines

* "In certain cases deviations occur, which are usually attributed to irregularities in the plate. It must however be remembered that the vibrations excited by a bow are not, strictly speaking, free, and that their periods are therefore liable to certain modifications," etc. *Sound*, **1**, 367, 368 (1894).

† *Handbuch der Physik*, **2**, 385 (1909).

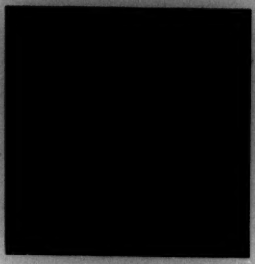

Fig. 2.

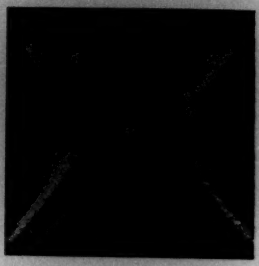

Fig. 3.

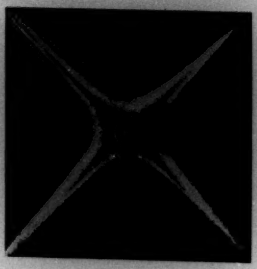

Fig. 4.

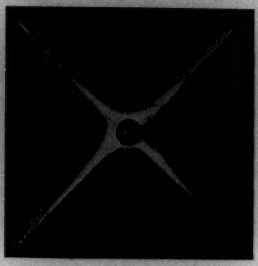

Fig. 5.

Fig. 6.

Fig. 7.

will lie close to the nodal line on either side, but with a moderate motion of the plate they may be very far from it. This was easily shown to be the case with the apparatus of figure 1. Figures 2, 3, 4 and 5 show the appearance obtained with four different intensities of vibration. With a vibration just exceeding in energy that at which no movement of the sand takes place we have, after the plate has been left in steady vibration for some minutes, the state shown in figure 2: a stronger vibration, steadily maintained, leads to figure 3: figures 4 and 5 correspond to cases where the vibration is still more vigorous. The double lines can still be distinguished in figure 4; in figure 5 the sand is too thick for them to show separately.

To investigate the question quantitatively it is necessary to measure the maximum acceleration of the plate at the spot where the sand grains just do not move, that is, at the extreme boundary of the sand pattern. If the motion is represented by

$$s = a \sin nt,$$

this acceleration is n^2a. n can be ascertained by measurement of the frequency. To this end the magnet current was passed through a string galvanometer, and the motion of the string was recorded photographically on a film on which a standard time-marking was recorded also. The amplitude at a given place near the edge of the plate was taken as follows: a microscope is focussed on a bright spot on the edge, which is drawn out into a line when the plate is thrown into vibration, and the length of this line is measured with a calibrated eyepiece thread micrometer. With suitable illumination chance irregularities on the edge of the plate furnished bright points without any special preparation.

The first experiments were carried out with a square plate giving the diagonal cross figure shown in the plate. The sand used was washed successively with hydrochloric acid, water and ether and then strongly heated. The value for the maximum acceleration at the sand boundary came out to be 1480 cm./sec.^{2}. It was noticed, however, that a small chip of sealing-wax which happened to get on to the plate moved at a spot where the sand was at rest, and this led to the conclusion that sand is not a suitable substance for these measurements, as, apparently, it tends to stick to the plate. Particles were therefore prepared by crushing a piece of ordinary soda glass in a mortar, the fragments so prepared being of the order of half a millimetre across. With crushed glass the preliminary measurements on the square plate gave 1230 cm./sec.^{2}.

Our square plate was not very suitable for accurate measurement, as the frequency corresponding to the diagonal cross was 463 $\sim$, and consequently the amplitude was very small, of the order of a few μ, and difficult to measure with precision. For final measurements a long steel bar was therefore selected, whose frequency at the first overtone was 124·05 $\sim$. This was supported on knife edges, distant 0·224 of the length from the free end, so that the bar vibrated with a node at the mid-point; it was maintained in vibration by a magnet placed beneath, as with the square plate. Crushed glass was used as the particles. Figure 6 shows the appearance with a given intensity of vibration, the groups of particles about the three nodes having sharp boundaries: figure 7 shows the central node on an enlarged scale. Measure-

ments made with this bar gave astonishingly good results, as will be seen from the table, from which it appears that the mean acceleration is 978 cm./sec.^{2}.

The amplitude given is the mean of ten readings in each case.

Table. Measurements with steel bar.

Frequency 124·05 ~.

Observation	Amplitude		Acceleration (cm./sec.^{2})
	(Microhead divisions)	(cm. × 10^{-3})	
1	66·8	1·629	990
2	65·0	1·585	962
3	66·5	1·622	984
4	65·8	1·604	974

As, when the plate is convex upward, there may be a slight tendency for the grain to slip outwards (although the tendency to slip inwards at the other extreme position would seem to counterbalance this), a rough calculation was made to see the magnitude of this possible effect. The coefficient of friction, measured roughly by tilting of the bar until the particles just slid on it, was taken as 0·5. It turned out that the particles will not slip until the acceleration of the plate has reduced the reaction of the particle to 0·3 M dynes, instead of the normal 981 M dynes, where M is the mass of the particle, or this effect in any case cannot exceed 0·03 per cent. of the main effect.

To see if sand really does show a sluggishness of motion as compared with glass particles an experiment was carried out with the washed sand. The value of the critical acceleration as measured with the bar came out to be 1608 cm./sec.^{2}. Why the sand shows this sticking effect is not known to us. It may be that, in spite of the washing, it remains hygroscopic, and adheres to the bar by a film of moisture.

It is well known that lycopodium and other light powders do not move to the nodes, but form agitated heaps at the antinodes, and this was shown by Faraday to be due to the dust being carried by the turbulent motion of the air above the plate. It has often been stated that, *in vacuo*, lycopodium powder would behave in the same way as sand, but, as far as we can tell, the experiment has never been tried, nor is it remarkable, in view of the difficulty of maintaining a plate in vibration *in vacuo* before the valve technique was developed. The apparatus being to hand, it seemed worth while to test the point. The plate was placed in a large vacuum desiccator, the top of which was closed by a thick slab of plate glass, and the pressure was reduced by a Hyvac pump to a value of something less than 0·5 mm. of mercury, as judged by a discharge tube. At this pressure lycopodium powder gave a pattern very similar to that given by sand, as tested by the diagonal cross figure. It was noticeable, however, that it required a greater intensity of vibration to produce a given spacing with lycopodium than with sand. A rough measurement gave, as the critical acceleration, 4000 gm./sec.^{2}. Sand appeared to behave in exactly the same way *in vacuo* as in air.

We can now consider in a little more detail the movement of a particle on the plate. Let the displacement of a particular point in the surface of the plate be s where

$$s = a \sin nt, \qquad s, a, n, t$$

and let s_g be the displacement at the time t_g when the acceleration downwards is g. s_g, t_g

Then
$$n^2 s_g = g \quad \text{and} \quad s_g \equiv g/n^2 \qquad \text{.....(1)},$$

while
$$\dot{s}_g = na \cos nt_g$$
$$= na\,(1 - g^2/n^4a^2)^{\frac{1}{2}},$$
$$t_g = n^{-1} \sin^{-1}(g/n^2a),$$

and hence the time t' measured from the moment when the particle leaves the plate is t'

$$t' = t - n^{-1} \sin^{-1} p$$

if we put
$$g/n^2a = p. \qquad p$$

The position of the particle at any time t is therefore given by

$$x = g/n^2 + na\,(1 - p^2)^{\frac{1}{2}}\,(t - n^{-1} \sin^{-1} p) - \tfrac{1}{2} g\,(t - n^{-1} \sin^{-1} p)^2,$$

where x is the vertical displacement measured from the equilibrium position of the surface of the plate, or x

$$x/a = p + n\,(1 - p^2)^{\frac{1}{2}}\,(t - n^{-1} \sin^{-1} p) - \tfrac{1}{2}\,(g/a)\,(t - n^{-1} \sin^{-1} p)^2.$$

If the particle is to fall on the plate just when it is in its equilibrium position going down, then $t = \pi/n$, or coming up $t = 2\pi/n$, while if it is to strike the plate when at its lowest point $t = 3\pi/2n$, giving respectively as equations for p,

$$0 = p + (1 - p^2)^{\frac{1}{2}}\,(\pi - \sin^{-1} p) - \tfrac{1}{2} p\,(\pi - \sin^{-1} p)^2 \qquad \text{.....(2)},$$
$$0 = p + (1 - p^2)^{\frac{1}{2}}\,(2\pi - \sin^{-1} p) - \tfrac{1}{2} p\,(2\pi - \sin^{-1} p)^2 \qquad \text{.....(3)},$$

and
$$-1 = p + (1 - p^2)^{\frac{1}{2}}\,(3\pi/2 - \sin^{-1} p) - \tfrac{1}{2} p\,(3\pi/2 - \sin^{-1} p)^2 \qquad \text{.....(4)}.$$

These transcendental equations can be easily solved by trial and error. For (2), (3) and (4) p is respectively 0·874, 0·340 and 0·586. The corresponding velocities of upward projection are, for (3) and (4), 0·940na and 0·810na. For the velocity v of the particle at a time ψ/n, we have v ψ

$$v = na\,\{(1 - p^2)^{\frac{1}{2}} - p\,(\psi - \sin^{-1} p)\},$$

which gives in case (3)
$$v = -1{\cdot}078na,$$

and in case (4)
$$v = -1{\cdot}584na.$$

Hence at a spot where $n^2a = g/0{\cdot}586 = 1{\cdot}706g$ the particle is thrown outwards at some angle ϕ with a velocity 0·810na, and strikes the plate when it is at its lowest point with a greater velocity, 1·584na, and at a greater angle ϕ/p or 1·706ϕ, if we suppose as a rough approximation that the particle is falling vertically and that the angle of the plate is proportional to the displacement. At such a spot then, the particle should be thrown inwards towards the node, and Winkelmann's general argument is quite misleading. ϕ

If the plate is vibrating vigorously the particle may drop on to it at its lowest point after the plate has executed $m + \frac{3}{4}$ complete vibrations, where m has any integer value. The solution for $m = 1$ is $p = 0{\cdot}202$. When p is small we may write, neglecting p^3 and higher powers,

$$-1 = p + (1-p)\tfrac{1}{2}\left(\frac{4m+3}{2}\pi - \sin^{-1}p\right) - \tfrac{1}{2}p\left(\frac{4m+3}{2}\pi - \sin^{-1}p\right)^2$$

as
$$-1 = p + (1-\tfrac{1}{2}p)\left(\frac{4m+3}{2}\pi - p\right) - \tfrac{1}{2}p\left(\frac{4m+3}{2}\pi - p\right)^2,$$

or
$$4p^2\{(4m+3)\pi + 1\} - p(4m+3)\pi\{(4m+3)\pi + 2\} + 4(4m+3)\pi + 8 = 0,$$

of which the solution is, to the required approximation,

$$p = \frac{4}{(4m+3)\pi}\left\{1 + \frac{16}{(4m+3)^2\pi^2}\right\}.$$

This gives for $m = 2$, $p = 0{\cdot}1173$,

and for $m = 3$, $p = 0{\cdot}0856$,

and therefore if $n^2a = 1{\cdot}706g$ or $4{\cdot}95g$ or $8{\cdot}53g$ or $11{\cdot}7g$, to take the first four values, the particle should be thrown towards the centre, in ascending order of vigour. Any plate which has a spot where n^2a has one of the higher values will, of course, have places where values of n^2a lower in the series occur. It is a remarkable fact which has not, apparently, hitherto been noticed, that when the plate is vibrating vigorously the clearance is not, as might be supposed, more complete than at feebler intensity, but on the other hand a large number of particles remain dancing in the neighbourhood of the places of most vigorous vibration as can, for example, be seen in figure 5. It is clear that, after a particle at a spot where n^2a has one of the values specified has been thrown towards a place of vigorous motion, it will soon reach a spot where it is thrown the other way, and since the particles are not in reality spherically symmetrical points, as has been tacitly assumed, it will probably eventually fall so as to bounce at a sharp angle, and be carried through the critical place and ultimately reach a place of rest where $n^2a < g$. While a particle at a place of vigorous vibration is thrown about until eventually it is carried through any difficult spots, a particle once arrived at a place of rest is withdrawn from circulation, so that the process pictured is plausible. When, however, there is a large number of regions, like contour lines on the plate, at which the particle is thrown inwards to the antinodes, it will clearly be more difficult for a trapped particle to escape to the nodal lines. This is why, when the plate is in very vigorous vibration, particles can be seen in lively motion near the antinodes.

To confirm the general correctness of this picture search was made for a place at which particles should be thrown outwards, towards the antinodes. For this purpose the bar already mentioned was set into vibration with an amplitude at the antinode somewhat greater than that corresponding to an acceleration of $1{\cdot}7g$, and

crushed glass was scattered over the surface. It was observed that there was a critical region, some 10·5 cm. from the antinode, such that the particles on the nodal side of it moved only towards the node. Particles in the critical region itself moved some towards the node, some towards the antinode. The amplitude at this critical region was measured microscopically. Five independent observations gave 113, 117, 115, 112, 112 as the amplitude in arbitrary scale divisions, the mean of which gave as the critical acceleration $1{\cdot}74g$. This agrees quite well with the calculated value $1{\cdot}706g$.

It was not possible to identify quantitatively the critical region where the acceleration has the value $4{\cdot}95g$ or one of the higher values, because the phenomenon is complicated by particles thrown outwards by the lower critical region so that the appearance is very confused.

Observations were made on a square Chladni plate also. The plate, strewn with sand, was thrown into vibration with a fairly high amplitude, corresponding roughly to figure 5. Sand particles at the outer edge of the sand figure were observed to be in motion, and some of them moved out towards the antinodal region and were finally thrown off the plate towards the middle of the edge. A small glass ball placed as nearly as possible on the nodal line showed no tendency to leave the nodal region, but if placed outside, and near the edge of, the figure it was set into motion, carried out towards the antinodal region, and finally thrown off at the edge of the plate. This is in accordance with the theory, when the vibration is so vigorous that there are many critical regions.

One of us (D. H. S.) is in receipt of a grant from the Department of Scientific and Industrial Research, and has much pleasure in acknowledging the help so rendered.

Note. After the measurements described were substantially completed, Dr R. E. Gibbs drew our attention to a brief note by Sir William Bragg on a chattering device for the measurement of small amplitudes*. After describing the instrument Sir William adds that chattering on a vibrating surface takes place if an^2 is greater than g, and says, "Illustrations can be found in the behaviour of objects on a Chladni plate," but no more.

* *J. Sci. Inst.* **6**, 196 (1929). See also A. B. Wood, *Sound*, 455 (1930).

STANDARDS OF MEASUREMENT, THEIR HISTORY AND DEVELOPMENT

By Sir RICHARD T. GLAZEBROOK, K.C.B., M.A., Sc.D., F.R.S.

The sixteenth Guthrie Lecture, delivered May 15, 1931.

§ 1. INTRODUCTION

When, some few months since, your secretary, Dr Griffiths, invited me in the name of the Council to deliver the Guthrie Lecture for 1931 and suggested I should speak on standards of measurement, dealing with their history and development, I accepted with a light heart. I welcomed the opportunity of helping to commemorate in this way the name of Guthrie. I remember him—few here can say that—when more than fifty years ago I began to take some share in the doings of the Society, then about five years old. When I was elected, Prof. W. G. Adams was president, to be followed in the next year by Lord Kelvin. Guthrie occupied the Chair five years after I became a member, and for the first ten years of its life the Society owed much to his fostering care, his zeal for its interests and his wise guidance. As I have said, I accepted your invitation lightheartedly; the subject proposed had always interested me. I thought I knew something about it and that the labour of preparation would not be overserious. And so I began to get ready, soon to be undeceived. My knowledge was but a small fraction of what there is to know. When does the history of weights and measures begin? May we start with the arm of King Henry I, A.D. 1120, the three barley corns which make an inch, or the dry grains of wheat taken from the middle of the ear by which the weight of the original silver penny was determined? or are we to go back to Roman and Greek times, or further still to the time of Solomon and the Temple or the days of Abraham, or even earlier to the days of Cheops the builder of the Great Pyramid? For there is no doubt that even before that, standards of measurement were used; the architect of the Pyramid laid out his plan with strict attention to his cubits and palms and digits.

And all this is very interesting—possibly more interesting, dare I say it, than the details of Sears's latest comparison of the yard and the metre, or Benoît, Fabry and Perot's determination of the number of wave-lengths of red cadmium light in a metre?

But what a task to describe all that has happened in the long story of weights and measures during the past 5000 years—for the date of the Great Pyramid is given by some authorities as about 2800 B.C.

§ 2. ANCIENT STANDARDS OF LENGTH

When man became a measuring animal he naturally took as his standards the parts of his own body; the cubit, the length of the forearm from the elbow joint to the tip of the middle finger, was his unit of measurement. The Egyptian hieroglyph for a cubit is a figure of the arm, and this unit was divided into spans, palms, digits, etc. There were various cubits, differing somewhat in length, and our knowledge of them has come to us in the main from Egyptian monuments. It appears probable, however, that the Egyptians obtained their cubit originally from Chaldaea; and one of the oldest standards we know of is to be found in the Louvre, where there is a bust of Gudea, a Sumerian King of about 2300 B.C. On it there is engraved a scale showing sixteen digits of the Sumerian cubit, which was divided into thirty digits. Now this scale is 264·4 mm. long, so that the cubit of thirty digits is 496 mm. or 19·6 in. in length, giving as the length of the digit 0·653 in.*.

The cubit, from which it appears probable that our foot and inch have descended, is generally known as the Olympic cubit, because it passed from Egypt into Greece.

It was, as we shall see shortly, about 18·24 in. long and was divided into two spans, the stretch from the thumb to the little finger, of about 9 in. A span was equal to three palms, the breadth of four fingers, each being about 3 in. in length, and the palm was divided into four digits, each $\frac{3}{4}$ in. approximately, or, more accurately, 0·76 in.

There was also the fathom of 4 cubits 72·96 in. or 6·08 ft., the outstretch of the two arms.

At a somewhat later date another unit, the foot, equal to two-thirds of a cubit, was introduced, and this was divided into twelve thumb-nail breadths—Greek *ουγξ* —whence came the terms uncia, ounce, inch, used for the twelfth part of the Troy pound and of the foot.

I have not been able to hear of an existing specimen of an Olympic cubit, but we can determine its length in the following manner. Herodotus (*Euterpe*, 168) tells us that "the Egyptian cubit is equal to that of Samos," i.e. the cubit used in Greece. We can obtain the length of this from the measurement of various Greek temples.

The Hecatompedon of the Parthenon is a square of 100 ft. in side. This was measured in 1846 by F. C. Penrose†, who gave as the length of the Greek foot 12·1610 in., whence the cubit being 3/2 foot is 18·2415 in.; this was corrected later to 18·2405, or (say) 18·24 in.‡.

* I owe this information and indeed much of what I have gathered about the Egyptian measures and their relations to our measures to the extreme kindness of Col. N. T. Belaiew, C.B., the distinguished metallurgist who has made a long study of these matters and most generously put his knowledge at my disposal. Mr Sidney Smith, of the British Museum, has most kindly replied to various queries.

† F. C. Penrose, afterwards the distinguished architect and Fellow of the Royal Society, was sent out to Greece by the Society of Dilettanti to test certain theories on Greek architecture by measuring the buildings. As an undergraduate he rowed in the Cambridge boat in 1841, 1842 and 1843.

‡ Notes by Col. Sir H. James, Director General Ordnance Survey, 1869.

There is another link to be obtained thus. The side of the Great Pyramid is stated to be 500 cubits in length. This has been frequently measured. Sir Henry Lyons informs me that the result obtained a few years since by Mr J. H. Cole, of the Egyptian Survey, was 230·364 metres or 755·76 ft. Earlier measures give much the same result. Sir Flinders Petrie* found the average length to be 9068·8 in. or 755·7 ft., giving for the length of the cubit, if we accept the statement that a side of the base of the Pyramid measured 500 cubits, the value 18·14 in.

This is nearly the same as the 18·24 in. found from the Greek temple, and we may perhaps accept this value as close to the truth.

This foot with its division into twelve thumb breadths (ὄνυξ) became the Greek foot.

In due course the foot passed to Rome, becoming somewhat altered in value; the Roman mile of 5000 Roman feet is generally said to be equal to 8 Greek stadia each of 600 ft. or 400 cubits, so that the Roman foot was equal to 24/25 of the Greek foot or 11·67 English inches.

This makes the Roman foot equal to 0·973 English feet. Direct measurement by Martin Folkes† and others give as the value 0·967 English feet; or, putting it the other way round, 1 English foot = 1·03 Roman feet.

In time Roman measures were established in Britain, but as to when and how the change from the Roman to the British foot took place we have no evidence. Various surmises are possible. I refer to one more fully on p. 421. For another it is known that the Gothic nations in the 4th and 5th centuries, established in the neighbourhood of the Crimea, used a large cubit. This became the basis of the Russian measures prior to the reign of Peter the Great, and may have travelled westward with the Goths and been brought to England by the Anglo-Saxons‡.

But more likely it was a chance alteration. Maybe King Henry I, A.D. 1100, did fix the yard by the outstretch of his arm and this happened to give a foot some 3 per cent. in excess of the Roman foot. There was a standard foot fixed in old St Paul's, and all measures were referred to the standard, "qui insculpitur super basem columpnae in Ecclesia Sancti Pauli."

King David of Scotland§ (*c.* 1150) is said to have ordained that the Scotch inch should be the mean measure of the thumbs of three men, "an merkle man an man of measurable stature and an lytell man," the thumbs being measured at the root of the nail.

According to Mr Nicolson, the earliest table of English linear measures is probably that in Arnold's *Customs of London* (*c.* 1500).

* Flinders Petrie, *Pyramids of Gizeh*, p. 39. For a further discussion of the relation between the cubit and the dimensions of the Pyramid, see p. 419.

† An account of standard measures in the Capitol at Rome by Martin Folkes, V.P., *Phil. Trans.* (1736). The ratio had also been determined by John Greaves, who in a visit to Italy in 1637 measured various standards, giving as the results the values 0·967, 0·972 and 0·986. For fuller details see p. 417.

‡ This suggestion has been made by Col. Belaiew in his writings on Russian measures (*The History and Development of Russian Weights and Measures*).

§ For this and much other information relating to early measures I am indebted to *Men and Measures* by Edward Nicolson.

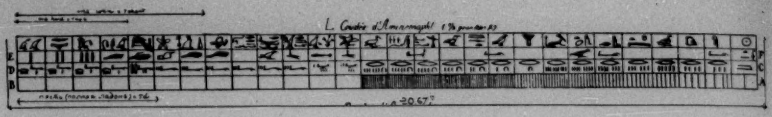

Fig. 2. The Royal Egyptian cubit = 2 half-cubits = 4 spans (of 7 digits) = 7 hands (of 4 digits).

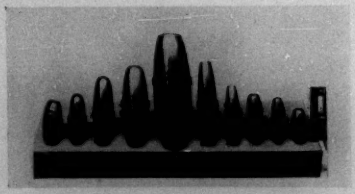

Fig. 1. Haematite weights from Warka (Erech). British Museum.

Reproduced with the kind permission of the Director and Trustees of the British Museum.

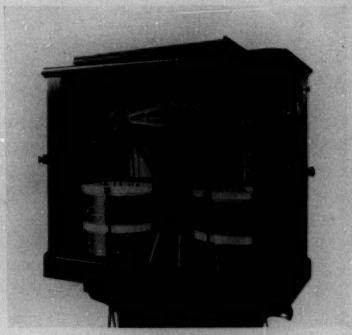

Fig. 6.

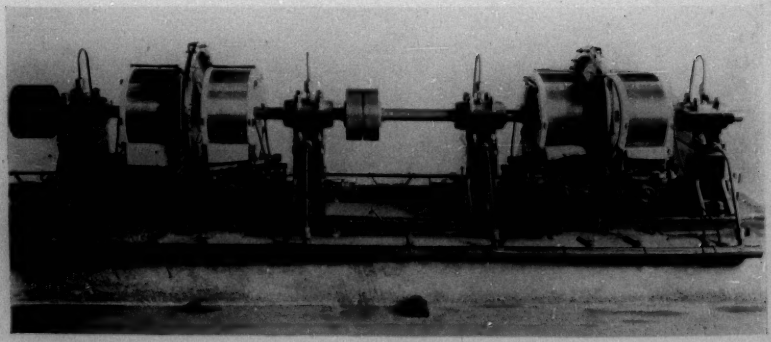

Fig. 5.

"The length of a barly corne iij tymes make an ynche
and xij ynches make a fote
and iij fote make a yerde
and v quarters of the yarde make an elle
v fote make a pace
cxxv pace make a furlong
and viij furlong make an English myle."

Thus in A.D. 1500 the furlong was 625 ft. or 208⅓ yds. and the mile 5000 ft. or 1666·6 yds. Half a century later, in the time of Queen Elizabeth, steps were taken to fix definite standards of measurement, but before I tell of these it is desirable to refer to the history of weights.

§ 3. ANCIENT STANDARDS OF WEIGHT

The standard of weight in Sumerian times was the mina, the history of which has been very fully examined by Col. Belaiew*.

Figure 1 is an illustration of a series of these mina weights made of homatite now in the British Museum. According to his investigations there were three series of standards, one of which became specially associated with the weights of coins. Another may have relation to the weight of water in a cubic cubit. From the mina in course of time came the talent, and the Alexandrian talent formed the basis of the Greek, Roman, and Western European weights until the advent of the Metric System.

The Alexandrian talent was divided by the Romans into 125 librae, each containing 12 unciae, so that there were 1500 oz. to the talent. The weight of the talent has been found to be 93·65 lb. avoirdupois or 655,550 grains. Thus the weight of the Roman ounce was 437 grains, while the pound contained 5244 grains. This weight became current in Britain, but the pound, instead of remaining at 12 oz., was raised to 16 oz. each of 437 grains and possibly became the basis of our avoirdupois weight, a pound of 6992 grains. Another suggestion is that the pound avoirdupois came from the Attic mina of 6945 grains.

The Saxon kings, however, had brought to England as a standard weight the marc of Cologne. Two marcs weighed 7216 grains and were divided into 16 oz. each of 451 grains. Twelve of these ounces, reckoned as weighing 450 grains each, constituted the Tower or Mint pound by which the coinage was weighed. In the time of Edward III (about 1350) a Tower pound of silver was coined into 240 silver pennies each weighing 22·5 grains, giving 5400 grains† for the pound and 450 for the ounce.

But there was another pound in ordinary use, the Troy pound. Its origin is quite uncertain. In France we find a *marc de Troyes* of 472 grains to the ounce, and it may have come from that. Another suggestion is that the word Troy has no reference to the French town but rather to Troy Novant, a monkish name of London

* "On the Sumerian mina, its origin and probable value," *Trans. Newcomen Soc.* 8– (1927–8).

† In 1842 a Tower pound from the Pyx Chamber was found to weigh 5404 grains.

founded on the legend of Brutus. Troy weight was mentioned in the reign of Henry IV, and in the time of Henry V goldsmiths were ordered to use "la libre de Troy" and coin 360 pennies from a pound of 5400 grains. Thus a penny weighed 22·5 grains, and an ounce 450 grains.

The Troy pound did not become the standard for the coinage until 1527, when Henry VIII enacted that "the pounde Towre shall be no more used, but all manner of golde and sylver shall be wayed by the pounde Troye which maketh XII oz Troye and which excedith the pounde Towre by III quarters of the ounce." This would make the pound Troy weigh 5400 + ¾ (450) grains or 5737·5 grains. It was afterwards defined as 5760 grains and the ounce Troy (1/12 part) as 480 grains.

But throughout this time avoirdupois weights, though not defined by statute until 1485, had been used, and from quite early days a *libra mercatoria* one-fourth greater than the Tower pound (containing therefore 6750 grains) was employed.

In the time of Edward III (1352) we read of weighings "cum ponderibus de haberdepase" and weights of this denomination existed in the time of Elizabeth. Specimens are on the table. "Haberty poie" pound is mentioned in the time of Henry VII. Henry VIII (1532) ordered that meat shall be sold by weight called haver-du-pois*. The avoirdupois pound contained 16 Roman ounces of 437 grains each—6992 to the pound.

Elizabeth, by her action in 1584, introduced what has become our modern system of weights and measures.

She ordered a jury in 1574 to examine the standard weights and measures in use, to report on them, and to construct standards "as well of Troy weight as of avoirdupois." The standards were found to be very erroneous and a second jury was appointed in 1584. The result of their work was the production of 57 sets of weights, both Troy and avoirdupois, which were distributed to the Exchequer and various local authorities. Some of these weights are still extant and do not differ by more than a grain from the Imperial standard.

Either at this period or previously the weight of the avoirdupois pound was raised by 8 grains, being brought from 6992 to 7000 grains at which it has remained ever since. Thus the avoirdupois ounce is 437·5 grains.

The Proclamation for Weights of December 16, 1587, established avoirdupois weight and ordered that no person shall use any Troy weight but only for weighing of bread, gold, silver and electuaries and for no other thing†.

In addition standards of length and capacity were constructed; the standard yard of Elizabeth, now on the table, is continually referred to later and was an important factor in determining the length of our present standard.

The work was well done, and supplied in a satisfactory manner the needs of the country for more than 150 years.

* According to the *New English Dictionary* quoted by Nicolson, to whose book I am indebted for much of this account of English weights, "averdepois" is the best spelling of the name. "Aver" is an old established English word for "goods."

† Nicolson, *Men and Measures*, p. 102.

§4. THE PYRAMIDS AND THE CUBITS. EGYPTIAN STANDARDS

But before continuing the history of these standards it is of interest to go back to Egyptian times and to the Pyramids and learn more about those standards from which ours have undoubtedly come. I have based our foot on the common or Olympic cubit of 18·24 in., but there were many other cubits. One in particular, the royal cubit of Egypt, is closely connected with the building of the Great Pyramid.

John Greaves, Savillian Professor of Astronomy at Oxford, A.D. 1648–9, became Professor of Geometry in Gresham College, London, in 1631, and after some visits to the Continent started on a journey to the East in 1637. After visiting Rome and Constantinople he arrived at Alexandria in 1638 and thence "went twice to Grand Cairo to measure the Pyramids, carrying with him a Radius of ten foot most accurately divided into 10,000 parts besides some other instruments for the fuller discovery of the truth*."

Greaves was an Arabic scholar as well as a mathematician and astronomer and was the author of works on many subjects. One deals with the Grand Seignior's Seraglio or the Turkish Emperor's Court.

In the *Dissertation on Cubits* Newton sets down the dimensions given by Greaves for various parts of the Pyramid. He then argues that the builder probably used some unit† in the design of his work and, by comparing the figures, endeavours to fix on that unit; the result is shown in table 1, in which are included the dimensions of two of the principal passages and of the great central chamber as well as the length of one side of the Great Pyramid.

Table 1. Greaves's measurements of the Pyramids.

Part measured	Dimensions (ft.)	Value of cubit deduced
Breadth of entrance gallery	3·463	2 × 1·732
Breadth of main gallery	6·870	4 × 1·718
Height of side benches in main gallery	1·717	1·717
Breadth of side benches in main gallery	1·717	1·717
Length of central chamber	34·380	20 × 1·719
Breadth of central chamber	17·190	10 × 1·719
Length of north side of base	693	400 × 1·732

* "The life of Mr John Greaves," by Thos. Birch, Sec. R.S., prefixed to the *Miscellaneous Works of Mr John Greaves, Professor of Astronomy in the University of Oxford*. Published by Thos. Birch, F.R.S., 1737.

The works include among other writings: "I. Pyramidographia—a Description of the Pyramids of Egypt." "II. A Discourse of the Roman Foot and Denarius," and to them is added: "A Dissertation upon the Sacred Cubit of the Jews and the Cubits of the Several Nations; in which, from the Dimensions of the greatest Egyptian Pyramid, as taken by Mr Greaves, the antient Cubit of Memphis is determined." Translated from the Latin of Sir Isaac Newton. Not yet published.

† The method had been used previously by Greaves himself who, to check the value of the Roman foot as deduced from one of the monuments examined, measured the stones in the foundations of various buildings, especially those of the Pantheon, and found "that most of the white marble stones in the pavement contained exactly three of those Roman feet on St Cossutius' monument, and the lesser stones in porphyry contained one and a half." Greaves's *Dissertation of the Roman Foot*, p. 211.

The mean value for the cubit, excluding the first and last, Newton takes as 1·719 ft. The last line of the table, as will appear shortly, is clearly erroneous; as to the difference between the first line and the others Newton remarks that it comes to about 1/7 in., which is "an error of no importance if we consider the much greater irregularities observed by Mr Greaves in the best buildings of the Romans."

As to the base of the Pyramid, Greaves writes: "For, measuring the North side of it at the basis by an exquisite radius of ten feet in length, taking two several stations as Mathematicians use to do when any obstacle hinders their approach, I found it to be six hundred and ninety three feet according to the English Standard."

This length we know to be wrong. Until recently there was much débris about the base of the Pyramid and it was impossible to approach the actual corners. Newton, however, finding that the length of the cubit as determined by the other measurements was about 1/400th of the base, and knowing also that an Arabic writer, Ibn Abd Alhokm, quoted by Greaves, had stated that there were 100 royal cubits in the base, assumed that by "royal cubits" were meant the *orgyia* of the Greeks—four cubits in length—and that the cubit used was therefore 1/400 of the length of the base of 1·732 ft.

If we take Newton's value 1·719 ft. as the outcome of Greaves's measurements in 1638–9, we obtain, as the length of the royal cubit used in the interior of the Great Pyramid, 20·628 in.

There is ample evidence that a cubit of about this size was in general use. There is one in the British Museum—a model of this is on the table—another in the Liverpool Museum and a third in Paris*. Mr Nicolson mentions others.

The Paris cubit, figure 2, contains many interesting features. Its length is given as 20·67 in. It is divided into 28 digits†, each of 0·736 in. in length. Of these 15, measured from the right, are subdivided respectively into 2, 3...16 parts. The number of the divisions is indicated above line *CD* by lines thus: |||, ||||, etc., ∩, ∩|, the ∩ stands for ten. On the top line are a series of hieroglyphics presumably indicating the number of digits. Reckoning from the left the digits are shown separately. Four of them constitute a palm shown as such—the royal cubit had seven palms—while five digits make a hand.

The royal cubit is found also on the outside of the Great Pyramid; at each corner were vertical walls, used by the builder to set out the slope of the faces. Two of these are extant and are marked in cubits of this size. From this it would appear that the royal cubit was used throughout the building.

Moreover, there still exist a number of Nilometers used in ancient times to measure the height of the Nile flood along the valley. These are marked in cubits. Several of them were measured by Sir Henry Lyons and Dr Borchardt when in Egypt, with the following results:

* Through the kindness of Col. Belaiew I am able to exhibit a copy of this cubit, while Prof. Wilberforce has very kindly measured the Liverpool cubit for me, with the result given in Appendix 2.

† The Olympic cubit of 18·24 in. was divided into six palms, each of 3·08 in. in length, so that the length of its digit was 0·77 in.

Nilometer	Number of cubit lengths	Average cubit (metres)
Philae I	12	0·520
Philae II	12	0·532
Philae III	16	0·535
Elephantine	12	0·523
Edfu	6	0·528
Esna	9	0·532
Luxor	6	0·529

The mean of these is 0·528 m. or 20·8 in. Other measurements have given a rather less result.

The mean value found by Mr Cole for the length of the side of the base was 755·8 ft. If this is to contain an even number of cubits of about the length found otherwise for the royal cubit, that number will be 440. This being assumed as the number of cubits in a side, the length of the cubit is 1·718 ft. or 20·62 in.

Table 2 gives the lengths of the four sides of the base as measured by Sir Flinders Petrie and Mr J. H. Cole.

Table 2. Base of the Great Pyramid.

	Sir Flinders Petrie	Mr J. H. Cole
North	755′ 9″	755′ 5″
East	755′ 8″	756′ 1″
South	755′ 9″	755′ 11″
West	755′ 9″	755′ 10″
Mean	755′ 8″·7	755′ 10″

The agreement is practically complete.

Thus we have found good cause to suppose that there were two cubits in general use—there were others besides—the one 18·24 in. in length, the other 20·62 in. It is perhaps natural to enquire what was the reason for this, and Col. Belaiew pointed out to me a fact of great interest in this connexion.

If we call l and L the lengths of the two, we have

$$l/L = 18{\cdot}24/20{\cdot}62 = 0{\cdot}885 = \tfrac{1}{2}\sqrt{\pi} \quad \text{approximately.}$$

Thus $$l^2 = \tfrac{1}{4}\pi L^2.$$

Thus the area of a square having the smaller cubit for a side is equal to that of a circle with the large cubit for its diameter*. This may, of course, be chance or it may be that some ancient Egyptian, wishing to construct a circle of the same area as the square, found the large cubit. We know that the value 256/81 was used for π, so that $\sqrt{\pi} = 16/9$.

But there are other curious facts connected with the dimensions of the Great Pyramid. Its perimeter, according to Mr Cole, is $4 \times 755{\cdot}8$ or 3023·2 ft. Earlier

* The same result follows of course approximately from the statement that the base of the Pyramid contains either 440 royal cubits and 500 Olympic cubits.

measurements made the base 760 ft. in length and the perimeter 3040 ft. Now the mean length of a nautical—meridian—mile, the 60th part of a degree, is 6080 ft. Thus the perimeter of the Pyramid is very approximately half a meridian mile. Did the wise men of old know this, and were they aware that the earth is spherical and about 8000 miles in diameter?

If the perimeter of the Pyramid was 2000 Olympic cubits then the Olympic cubit is 1/4000 of a meridian mile, and the Olympic fathom—the Greek *orgyia* of 4 cubits—is 1/60,000 of a degree.

It is all very curious; but can we credit such knowledge to the men who designed the Pyramids? They did place them very accurately with regard to the meridian; the faces of the Great Pyramid are only 3′ 43″ out, while the entrance gallery was directed* towards α Draconis, which was the Pole Star some 5000 years ago. Thales of Miletus, born B.C. 640, taught that the earth was spherical and occupied the centre of the Universe. Pythagoras, born B.C. 580, said that the sun was the centre of the planetary world and the earth one of the planets†. We know also that Muhammed Ibn Mesoud in his book (intituled in the Persian *Gehandanish*) relates that in the time of Ma' Mun‡, the learned Calif of Bagdad, by the elevation of the pole of the equator they measured the quantity of a degree upon the globe of the earth and found it to be 66 miles and two-thirds of a mile; every mile containing 4000 cubits and each cubit 24 digits and every digit 6 barley corns.

§ 5. THE ALEXANDRIAN TALENT

In connexion with the weights, equally interesting questions arise when one begins to ask how men in ancient days settled on their system of weighing. Since their standards of length represented without doubt the lengths of parts of their own bodies, their weights may well have been chosen to represent a definite quantity of some commodity of daily use, wheat or barley, the weight and length of whose grain has been in all countries intimately connected with standards of weight and length. The kilogramme was intended to be the weight of a unit of volume of water. How far back does this idea go? We have seen that the Alexandrian talent weighed 93·65 lb. or 655,550 grains. Now, taking the royal cubit as 20·62 in., the foot is 13·74 in. Thus the volume of a royal cubic foot is 2594 in.^{3}. Again, 1 lb. of water at 15° C. has a volume of 27·71 in.^{3}. Thus the weight of a royal cubic foot of water is 2594/27·71 lb. or 93·61 lb.

Is it a coincidence that the weight of the talent of Alexandria is so closely that of a royal cubic foot of water, or is the suggestion made by Mr Nicolson—that the agreement was designed—correct?

How much did the Wise Men of the East know?

* Another, possibly a more probable, explanation of the inclination of the gallery, about 20°, is that it is the angle of slope for brickwork.

† Greaves, *Discourse on the Roman Foot*, p. 193.

‡ Ma' Mun, the son of Haroun al Raschid, reigned from 808 A.D. to 833 A.D., and Muhammed Ibn Mesoud wrote about 1246 A.D.

The foot and the ounce. And here it is perhaps relevant to refer to another suggestion made by Mr Nicolson in an endeavour to account for the difference between the Olympic and the British foot. It is that the foot was chosen so as to make a cubic foot of water weigh 1000 oz. of (at the time) 437 grains each. For 1000 oz. contain 437,000 grains and a cubic inch weighs 252·62 grains. Thus the volume of 1000 oz. is 1729·9 in.^{3} and the cube root of this is 12,004 in., so that a cubic foot is very nearly the volume of 1000 Roman oz. of water at 15° C.

But I am digressing into curiosities of metrology. The weight of a cubic foot of water was investigated by the Oxford Philosophical Society in 1685. When Wren and the founders of the Royal Society moved to London, those of the members who were left behind at Oxford continued their meetings*, and at one of these two members, Messrs Caswell and Walker, described how they had made a hollow cube of very hard dry oak, each edge of which was very exactly 1 ft., with which they proposed to determine the weight of a cubic foot of water.

At a later meeting they communicated their result to the Society, finding it to be 1000 oz., and their result, along with the weights found for the same volume of a number of other substances, was sent to the Royal Society and is printed in the *Philosophical Transactions*† along with their table of specific gravities, which I have extracted.

Table 3. "A list of the Specific Gravitys of bodys, being in proportion as the following numbers."

Pump water	1000
Fir dry	546
Elm dry	600
Cedar dry	613
Wallnut tree dry	631
Crab tree meanly dry	765
Ash meanly dry, and of the outside lax part of the tree	734
Ash more dry, but about the heart	845
Maple dry	755
Yew of a Knot or root 16 years old	760
Beech meanly dry	854
Oak very dry, almost worm eaten	753
Oak of the outside sappy part, felld a year since	870
Oak dry, but of a very sound close texture	929
The same tryed another time	932
Logwood	913
Claret	993
Moil cider not clear	1017
Sea-water setled clear	1028
College plain Ale the same	1028
Urine	1030
Milk	1031
Box the same	1031
Redwood the same	1031
Sack	1033
Beer Vinegar	1034
Pitch	1150
Pit-Coal of Staffordsh,	1240
Speckled wood of Virginia	1313

* Their minute book is now at the Royal Society and from it these notes are taken.

† *Phil. Trans.* **169**, 926 (1685).

This has brought us very close to what I perhaps may call the more scientific period of our subject, and we may leave the speculations, interesting though they are, that centre round the Pyramids and their architects.

§6. THE ROYAL SOCIETY AND WEIGHTS AND MEASURES

We come now to more modern times, some two hundred years ago, and from that date onwards can trace with more certainty the history of our weights and measures.

In 1742 George II was on the throne; the Royal Society had been founded for nearly eighty years, and its Fellows were actively engaged in the pursuit of natural knowledge; the importance of exact measurement was becoming more fully recognized. Volume 42 of the *Transactions* for 1742–3 contains the account of various comparisons of weights and measures. There is also an account of the laying down of 3 ft. on a brass bar belonging to the Royal Society from a scale at the Tower.

Owing mainly to the action of Graham (November 11, 1742) sundry comparisons with French and other measures were made, and as a result he had a yard measure and a pound weight carefully made and examined. These were sent to Paris by the Society and, at the request of the French men of science, were compared with the French standards by M. du Fay and the Abbé Nollet. As the result it was reported that the half-toise of Paris contained 38·355 in. and the Paris foot 12·785 in., while the Paris two-marc weight of 16 oz. weighed 7560 Troy grains; thus the Paris ounce was 472·5 Troy grains, while an English avoirdupois pound weight belonging to the Royal Society contained 7004 grains, so that the English ounce was 437·52 grains. This seems to have led to the suggestion that a comparison of the existing standards of length and weight was desirable, and a committee consisting of Lord Macclesfield, Lord Clarendon, Mr George Graham and others, with Mr Cromwell Mortimer as secretary, undertook this in April, 1743.

Among the standards compared were a brass standard yard of Henry VII, marked with a crowned H., the Exchequer yard of Queen Elizabeth (now on the table—this is a square rod, an end standard), a yard from the Guildhall, London, another from the Tower, a yard belonging to the Clockworkers' Company and a sixth, then in the possession of the Royal Society.

The Committee reported (June 16, 1743) the differences shown in table 4, the Elizabethan yard being taken as standard.

Table 4. Comparisons of yards with the Exchequer standard of Elizabeth.

Royal Society yard	+ 0·0075 in
Henry VII yard	− 0·0071 in.
Guildhall yard	+ 0·0434 to 0·0396 in.
Clockworkers' yard	− 0·021 in.
Tower yard	+ 0·011 in.

Some standard ells also were measured and found to exceed 45 in. by from 0·04 to 0·05 in. The comparisons were made by the aid of a beam compass. The committee also examined a series of weights from the Exchequer and elsewhere.

As we have seen, from the time of Edward I the Troy pound is defined as containing 12 Troy oz. each of 20 pennyweights; no Troy standard pound was found among the Exchequer weights, but 12 oz. taken from a box containing 256 oz. weights were selected as representing the pound Troy. There was, however, a series of avoirdupois weights from 1 lb. to 14 lb. marked "Elizabeth Regina 1582," and a 7 lb. weight "VII El. 1588." Various comparisons led to the following results for the pound and ounce avoirdupois in Troy grains:

Pound	6998	7000·7	7000·2
Ounce	437·4	437·54	437·51

Of the weights in the possession of the Royal Society the avoirdupois pound was lighter than the standard by one grain, and the Troy pound lighter by half a grain.

This constitutes the earliest systematic comparison of our weights and measures that I have found. It was realized that the length of the standards varied with temperature, but it was held that since all were of much the same material the differences would not be affected.

Two standard bars were made by Bird in 1758 and 1760 respectively, for a committee appointed by the House of Commons under the chairmanship of Lord Carysfort. These were to be copies of the Royal Society standard as laid down by Graham 1742–3. One, marked "Standard Yard 1758" was presented to the House as the legal standard of length. Both were compared in 1768 with the R.S. standard, while in 1785 a scale made for General Roy's measurement of an arc of the meridian on Hounslow Heath was graduated in terms of the same standard.

Bills were presented to the House in 1765 to give effect to the recommendations of this and a subsequent committee, but from some accident they did not become law, and the country remained without legal standards for the next sixty years.

In 1792 Troughton* made a very full comparison of a standard bar constructed for Sir George Shuckburgh. In this comparison he made use of the R.S. Tower standard, the length marked on the R.S. bar from the Exchequer comparison by Graham (1743), two of Bird's bars, 1758 and 1760, Roy's bar, and the standards of Henry VII and Elizabeth. The modern method, employing micrometer microscopes, was used for the first time by Troughton in his comparison of the line standards; a beam compass was used for the end standards.

In 1814 a report was presented to the House of Commons by a new committee which asked Dr Hyde Wollaston and Prof. Playfair for advice. They stated that the length of the pendulum vibrating seconds had been found to be 39·13047 in. and that the metre of platina measured, at the temperature of 55°, 39·3828 English in., representing at 32° the ten-millionth part of the quadrant of the meridian. They remarked with great truth that although in theory the original standard of weight is best derived from the measure of capacity, yet in common practice it will generally be found more convenient to reverse the order, and they recommended, upon the

* *Phil. Trans.* **99**, 105–145 (1809).

suggestion of Dr Wollaston, "that a gallon containing ten pounds of pure water should be adopted as a substitute for the ale and corn gallons*."

In 1816 there still remained considerable doubt both as to the length of the seconds pendulum and the relation of the metre to the inch; the Astronomer Royal, Dr Pond, was instructed by Parliament to make the measurements necessary to remove the doubt. He asked for the assistance of the Royal Society and a committee† was appointed for the purpose.

A number of experiments were made by various methods‡, and in 1818 a commission§ was appointed under writ of the Privy Seal "to discuss the matter more minutely than could be done with convenience before a Committee of either House of Parliament."

The report‖ of this commission, drawn up by Dr Young, is of great interest; and as a result a bill giving effect to their conclusions was brought into the House of Commons by Sir George Clerk in 1822, and with a few amendments in 1823. This was nearly identical with an act which was passed on June 17, 1824, and came into force on January 1, 1826.

The commissioners at first recommended "for the legal determination of the standard yard that which was employed by General Roy in the measurement of a base at Hounslow Heath." This was afterwards altered to Bird's standard of 1760 in consequence of errors found by the committee in the Roy standard. The act enacted "that the straight line or distance between the centre of the two points in the gold studs in the straight brass rod now in the custody of the Clerk to the House of Commons whereon the words and figures Standard Yard 1760 are engraved shall be and the same is hereby declared to be the original and genuine standard of that measure of length or linear dimension called a yard." This distance, when the rod was at a temperature of 62° F., was denominated the "Imperial standard yard" and all other measures of extension were to be multiples or sub-multiples of this standard.

Thus the standard finally chosen was Bird's standard of 1760; it was found to agree closely with Sir George Shuckburgh's scale, with which the length of the pendulum and the metre had been compared, and of which a facsimile was known to exist in Geneva¶.

For the standard of weight the commissioners recommended "that the standard brass weight of two pounds Troy weight now in the custody of the Clerk of the House of Commons shall be considered authentic," and it was proposed "that a brass weight equal to one half of the said brass weight of two pounds gravitating in

* This quotation is from the article "Weights and Measures," by Dr Thomas Young in the seventh edition of the *Encyclopædia Britannica* (1842), to which I am indebted for much information.

† The committee consisted of the president and secretaries, Sir Chas. Blagden, Mr Gilbert, Dr Wollaston, Dr Young, Capt. Kater, General Mudge, Mr Brown, Mr Rennie and Mr Troughton.

‡ Capt. Kater's pendulum work is described in the *Phil. Trans.* for 1826, 1830 and 1831.

§ The commissioners were: Sir Joseph Banks, Sir George Clerk, Mr Davies Gilbert, Dr Hyde Wollaston, Dr Thomas Young and Capt. Henry Kater. Dr Young "with the assistance of a clerk who had studied the law" undertook the duties of secretary.

‖ The more important parts are printed in Appendix 1, p. 454.

¶ Sir Geo. Airy, *Phil. Trans.* (1857).

air (the barometer being at thirty inches and the thermometer being at 62° by Fahrenheit's scale (1822))* shall be, and the same is hereby declared to be the original and genuine standard of weight; and that such brass weight shall be and is hereby denominated the Imperial Standard Troy Pound." This Troy pound was declared to contain 5760 grains "and 7000 such grains shall be and are hereby declared to be a pound avoirdupois."

For the recovery of the standards if lost, destroyed, defaced or otherwise injured, suitable provision was made in the act; the yard was to be recovered from the length of the seconds pendulum which, when vibrating in a vacuum at sea level in the latitude of London, was stated to be 39·1393 in., while it was held that the pound could be reconstructed from the fact that a cubic inch of distilled water weighed by brass weights in a vacuum at a temperature of 62° had been found to weigh 252·724 grains. It will be noticed that, for the purpose of recovery, recourse is to be had to the length of the pendulum and the weight of a cubic inch of water, not to the old measures from which the new Imperial standards had been derived or to any copies of those standards which might be in existence.

§7. THE MODERN STANDARDS

The need for reconstruction came all too soon. In 1834 the Houses of Parliament were destroyed by fire; the pound standard was lost entirely and the yard standard was damaged, one of the gold plugs having been melted so that its centre point could no longer be determined with accuracy. In 1838 the Chancellor of the Exchequer, the Rt. Hon. T. Spring Rice, set up a commission† to advise as to the steps necessary to replace the standards.

Meanwhile, since 1824 a number of criticisms of the means suggested for recovering the standards had been made. Certain of the corrections employed by Kater in determining the length of the seconds pendulum were shown to be in error; while the agreement among the various determinations of the weight of a cubic inch of water left the true value in doubt, and the commission‡ of 1838, accepting these criticisms, threw over the methods provided in the act of 1824 and advised, with regard to the pound, that it should be recovered by comparison with copies which were known to exist and that the pound avoirdupois should be the standard. As to the yard, it was stated that "Several measures now exist which were most accurately compared with the standard yard... and by the use of these the values of the original

* In 1823 this became "That the Standard brass weight of one pound Troy weight made in the year 1758 now in the custody of the Clerk of the House of Commons."

† Full accounts of the work of this commission and of the scientific commission which followed it in 1843 will be found in the following two papers in the *Philosophical Transactions of the Royal Society*: (1) Prof. W. H. Miller, "On the construction of the new Imperial Standard pound and a comparison with the Kilogramme des Archives," *Phil. Trans.* **146**, 753–946 (1856). (2) G. B. Airy, Esq., Astronomer Royal, "Account of the construction of the new national standard of length and of its principal copies," *Phil. Trans.* **147**, 621–702 (1857).

‡ This commission consisted of the Astronomer Royal (Mr G. B. Airy), Mr F. Baily, Mr Bethune, Mr Davies Gilbert, Sir J. F. W. Herschel, Mr Lefebvre, Mr J. W. Lubbock, Rev. Geo. Peacock, and Rev. R. Sheepshanks.

standards can be respectively restored without sensible error." The commission therefore recommended a material standard, "the distance between two points or lines engraved on metal," "but that the standard be in no way defined by reference to any natural basis such as the length of a degree of a meridian on the earth's surface in an assigned latitude or the length of a pendulum vibrating seconds in a given place."

The Government approved this recommendation and appointed a scientific committee in 1843 to reconstruct new parliamentary standards of length and weight.

The pound. Prof. Miller was entrusted with the construction of the pound, Mr Baily with that of the yard. Prof. Miller has described his work very fully in the Royal Society paper already referred to. He gives a list of the Troy pounds, fourteen in number, some of platinum, others of brass or gun metal, available for comparison, and describes in detail the balance employed and the method of weighing (Gauss's method by double weighing was used). A thermometer, K. 43, was specially calibrated at Kew Observatory for the work, and great precautions were taken to secure accuracy. The standard weight of 1824 was known as U. It had been compared in 1824–5 by Capt. Kater with five gun-metal standards deposited at the Exchequer, the Royal Mint, and with the civic authorities in London, Edinburgh and Dublin.

In 1829, U had been compared by Capt. V. Nehus with a platinum pound (R.S.) belonging to the Royal Society, and also with two brass pounds and a platinum pound (Sp.) in the custody of Prof. Schumacher*. These weights were available for the construction of the new pound. The comparison showed considerable discrepancies in the values of the gun-metal and brass weights and in the end it was resolved "with the consent of the Astronomer Royal to rest for the evidence of the weight of the lost standard entirely on the comparisons of the two platinum standards Sp. and R.S."

The density of the lost standard was not known with accuracy, and this led to uncertainty. A value deduced from observations on weights of the same date as the standard (1758) was employed in the reduction of the weight to that in a vacuum.

In the course of the work quartz was examined as a material for the standard but the conclusion was reached that it was insufficiently dense, and the corrections required to the weight in air were in consequence unduly uncertain.

As the result of this work and the corresponding work on the yard our present standards came into existence, and the Weights and Measures Act of 1855 prescribes that: "Whereas by Act of the Fifth Year of King George IV a standard brass weight of one pound Troy made in the year 1758 then in the custody of the Clerk of the House of Commons should be...the genuine standard of weight...and whereas there exist weights which had been accurately compared with the said standard pound Troy which afforded sufficient means for restoring such original standard, the said weight of platinum marked 'P.S. 1844 1 lb.,' deposited in the Office of the

* Schumacher was Professor at Copenhagen. He became a Foreign Member of the Royal Society in 1821 and in 1836 communicated to the Society a paper, "A comparison of the late Imperial standard Troy pound weight with a platina copy of the same and with other standards," *Phil. Trans.* **126**, 487–495 (1836). His portrait hangs in our Library.

Exchequer as aforesaid, shall be and be denominated the Imperial standard pound avoirdupois. The one 7000th part of this shall be a grain and 5760 such grains shall be a pound Troy."

Provision is made in the act for recovery of the standard, if lost, by comparison with copies made and distributed among various authorities.

Thus the pound avoirdupois became our standard of weight and the pound Troy followed in the path of the King's Tower pound.

With regard to the comparison between the pound and the kilogramme, Prof. Miller writes, "The comparison left me in entire darkness as to the real value of the kilogramme." The standard kilo of the French Bureau denoted by $\sigma\tau$ was, by the kindness of Arago, compared with the pound with the result that

$$\sigma\tau = 15432{\cdot}3489 \text{ grains}^* = 2{\cdot}2046 \text{ lb.}$$

A platinum kilo ϵ obtained for England was compared with $\sigma\tau$ with the result that

$$\epsilon = \sigma\tau - 0{\cdot}02435 \text{ grains.}$$

But at the time the density of $\sigma\tau$ had not been determined and to this extent the result was uncertain.

The yard. Mr Baily, at the request of the committee, undertook the reconstruction of the yard. The account of the work is given by Sir George Airy in the paper of 1857 already referred to.

In addition to the damaged standard, the following were available for comparison: a scale, No. 46, belonging to the Royal Society, a scale belonging to the Royal Astronomical Society, two three-foot bars of the Ordnance Society. There was also the Shuckburgh scale of 1792 but, on account of certain imperfections, little importance was attached to it.

The standard was to be of bell metal or steel, a rectangular bar about 1 in. square in section, with the ends notched away to half the thickness. The marks defining the length were to be on plugs inserted flush with the surface and the standard was to be correct at 61° or 62° F. The ends were cut away in order that the marks might be in the median plane of the bar, and the effect of flexure when supported at two points minimized.

Baily began by making and testing the suitability for his purpose of various alloys; he finally settled on one, since known as Baily's metal, consisting of copper 16 parts, tin $2\frac{1}{2}$ and zinc 1 part. Of this the standard was ultimately constructed.

Faraday was consulted somewhat later as to the stability of such an alloy, and wrote, "I do not see why a pure metal should be particularly free from internal changes... I suppose the labour would be too great to lay down the standard on different metals and substances, and yet the comparison of these might be very important hereafter, for twenty years do seem to do or tell a great deal in relation to standard measures."

Baily also introduced the method of drilling a hole in the bar down to the median

* The equivalent legalized by the Weights and Measures Act of 1897 is 15432·2564 grains or 2·2046223 lb.

section, and fixing the plug with the mark at the bottom of this instead of notching the end as Kater had done. His work was cut short by his death in August, 1844.

At this stage Sheepshanks offered to take Baily's place, and the question immediately before the committee was whether to attempt to restore the damaged legal standard or to construct a new standard from the available material based on Shuckburgh's yard which Kater had considered satisfactory. It was plain that in the strictly legal or scientific sense the restoration of the standard must be indefinite, and the latter plan was approved. Sheepshanks suggested that the necessary apparatus for his comparisons should be set up in the cellar of Somerset House and made a report on the questions involved—the stability of the material, the thermometric difficulties, etc., as well as on the method he proposed to adopt.

A meeting of the committee was held on June 4, 1847, to inspect these arrangements, and the minutes record that "The Committee approve entirely of the course followed by Mr Sheepshanks and request Mr Airy to take measures for the exam^{n} and discharge of the instrument makers accounts."

Airy's paper already referred to gives very full details of the construction of the standards; it also contains his investigation of the effect of flexure and the determination of best points of support.

Ultimately six standards of Baily's metal were constructed which were found to have the length of 1 yd. at the temperatures indicated below:

No. 1.	Correct at	62°	F.
2.	,,	61·94°	F.
3.	,,	62·10°	F.
4.	,,	61·98°	F.
5.	,,	62·16°	F.
6.	,,	62°	F.

Of these it was arranged that No. 1 should become the standard yard, while 2, 3, 4 and 5 should be known as parliamentary copies and No. 6 should be retained by some officer of the Government for the comparison of other standard bars or for other scientific purposes.

Sheepshanks was taken ill while actually engaged in his comparisons; he died the day before royal assent was given in 1855 to the act establishing the new measures. Thus, as a result of the death of the two men who had done most of the work, the Royal Society paper in which an account of it is to be found was written by Airy. The act of July 30, 1855, provides that "The straight Line or Distance between the Centres of the two Gold Plugs in the Bronze Bar deposited in the Office of the Exchequer as aforesaid shall be the genuine standard of that Measure of Length called a Yard and the said straight Line or Distance between the Centres of the said Gold Plugs or Pins in the said Bronze Bar (the Bronze being at a temperature of Sixty two Degrees by Fahrenheit's Thermometer) shall and be deemed to be the Imperial Standard Yard." Thirty-seven copies were made and distributed. Such were the conclusions enacted in the Weights and Measures Act of 1855, and they have remained unaltered up to the present time.

Since that date there have been various supplementary or consolidating acts. In 1864 The Metric Weights and Measures Act was passed. It provided that "No contract... shall be deemed to be invalid or open to objection... on the ground that the Weights and Measures expressed... are Weights or Measures of the Metric System," and it gives a schedule of tables of metric measures expressed by means of legalized denominations of weights and measures in Great Britain, thus legalizing the metric system but not providing any metric standards for purposes of comparison and verification.

In 1866 the custody of the imperial standards of length and weight and of all secondary standards was transferred by act of Parliament to the Board of Trade. Parliamentary copies of the yard and pound were placed at the Royal Mint, with the Royal Society and at the Royal Observatory, Greenwich, and it became the duty of the Board to cause these to be compared every ten years with the standards in their charge. For this and other purposes connected with standards the Board were to constitute a department to be called the Standards Office and "shall appoint as Head of that Department an Officer to be called the Warden of the Standards" with Assistants, Clerks, etc. The act continues, "It shall be the duty of the Warden of the Standards to conduct all such comparisons... or other operations with reference to the Standards of Length Weight or Capacity in aid of Scientific Researches or otherwise as the Board of Trade from time to time authorize or direct." It was also the duty of the Warden to make an annual report to the Board to be laid before both Houses of Parliament.

The procedure was modified by an act of 1878 relating to the standards deposited in the Standards Department of the Board of Trade in the custody of the Warden of the Standards which provides (para. 33) that "The Board of Trade shall have all such powers and perform all such duties relating to Standards of Weight and Measure as are by any Act or otherwise vested in or imposed upon" various official persons "or the Warden of the Standards," and it was made the duty of the Board of Trade to conduct all such comparisons as the Board of Trade from time to time thinks convenient and to report to Parliament.

Other sections of the act definitely transfer to the Board all the various duties assigned by the act of 1866 to the Warden of the Standards.

The act also gives a schedule of metric equivalents and a list of metric standards in the custody of the Board and provides (para. 38) that "Whereas the Board of Trade have obtained accurate copies of the Metric Standards and it is expedient to make provision for the verification of Metric Weights the Board of Trade may, if they think fit, cause to be compared with the Metric Standards in their custody all metric weights and measures submitted for the purpose."

A Metric Convention (Convention du Mètre) had been agreed to among a large number of nations in 1875. To this Great Britain adhered in 1884, and comparisons between the metric and British measures were set on foot. The work was carried out by Mr Chaney of the Standards Department of the Board of Trade and M. Benoît, Director of the Bureau International des Poids et Mesures, and in 1889 certified copies of the metre and kilogramme were deposited at the Board of Trade.

In 1894 the International Committee of Weights and Measures suggested the desirability of a comparison between the standard yard and the metre. This was agreed to. Mr Chaney made a careful comparison between the standard and the parliamentary copy No. 6 which was then sent to Sèvres and compared with the metre, by M. Benoît.

A yard is about 914 mm. in length and the outstanding difficulty in any such comparison is the subdivision of the metre to make sure of the length marked as 914 mm. Special means were taken to determine the difference between the two standards accurately in terms of the inch, with the result that the value 1 metre = 39·370113 inches was finally accepted by both investigators and became the legal equivalent under the act of 1897.

As the result of these comparisons a further act was passed in 1897 giving to the Queen in Council power to make a table of metric equivalents in substitution for the table in the schedule to the act of 1878, and providing that the Board of Trade standards which may be made under para. (8) of the act of 1878 shall include standards from the metre No. 16 and the kilogramme No. 18 deposited with the Board of Trade.

Under the provisions of this act a table* of metric equivalents was issued in 1898 according to which

$$1 \text{ metre} = 39{\cdot}370113 \text{ in.}$$

$$1 \text{ kilogramme} = 2{\cdot}2046223 \text{ lb.}$$

The most recent comparison† of the yard and metre is that made at the National Physical Laboratory in 1927 which leads to the result that

$$1 \text{ metre} = 39{\cdot}370147 \text{ in.}$$

$$1 \text{ inch} = 25{\cdot}399956 \text{ mm.}$$

Here perhaps a reference to the Metric Convention‡ is desirable. An International Metric Commission was convened at Paris in 1872 to consider the question of the construction and issue of metric standards of weight and measure. A committee was appointed to make the necessary preliminary investigations, and as the result a convention was agreed to on May 20, 1875, by a large number of States. Great Britain adhered to the convention in 1884. The convention provided that the contracting parties should set up at Sèvres an International Bureau of Weights and Measures at which the international prototypes of the kilogramme and metre were to be kept. For the control of the bureau an International Committee of Weights and Measures, consisting of fourteen members belonging to different countries, was established. The committee was placed under the authority of a General

* *Statutory Rules and Orders*, 1898, No. 411.

† J. E. Sears, Jr., C.B.E., M.A., W. H. Johnson, B.Sc., H. L. P. Jolly, M.A., "A new determination of the ratio of the imperial standard yard to the international prototype metre," *Phil. Trans.* A, **227**, 281–315 (1928).

‡ International Convention of 1921 modifying the International Convention of May 20, 1875, Stationery Office, Treaty Series No. 24 (1923), Cmd. 1982.

Conference of Weights and Measures formed of delegates from all countries adhering to the convention. The conference is to meet whenever convoked by the international committee, and in any case once in every six years*. The president of the French Academy is the president of the international committee. The first duty of the committee was to make and distribute to the adhering countries copies of the prototype standards; while this work was continuing† meetings were to be held annually. At the conclusion of the work the meetings became biennial.

The prototype standards at the bureau are under the charge of the committee‡ and a special resolution of the committee is required to permit of their examination. The first Director of the Bureau was M. Benoît. The present holder is M. C. E. Guillaume, who has done so much for accurate metrology and who delivered the Guthrie Lecture in 1920.

Inter-dominion standards. Before we leave this part of our subject—the present position of our national standards—a brief reference should be made to the Conference§ on Standardization held at the Board of Trade in October, 1930, under the Chairmanship of Mr W. R. Smith, M.P., Parliamentary Secretary to the Board. The conference was attended by representatives of the United Kingdom, the Dominions of Canada, Australia and New Zealand, the Union of South Africa, the Irish Free State and India. A representation from the Office of the Crown Agents attended as an observer on behalf of the non-self-governing parts of the Empire. The work of the conference was divided into two parts relating respectively to fundamental standards and to industrial standardization. Sir Joseph Petavel acted as chairman of the section dealing with fundamental standards.

The Imperial Conference accepted the report of the Conference on Standardization and adopted the resolutions submitted therewith.

These resolutions, so far as they deal with fundamental standards, were:

(1) "That it is desirable that there should be uniformity between the standards employed for all units of measurement which are in common use among the British Commonwealth of Nations. (2) In order to secure uniformity arrangements should be made (*a*) to provide in each Dominion and in India suitable reference standards for each unit of measurement required for use in that country where not already available; and (*b*) to introduce suitable procedure whereby all such standards shall be periodically compared with the corresponding standards at the Board of Trade or at the National Physical Laboratory. (3) At least one member of the Commonwealth should undertake research work with the object of enabling the fundamental standards to be referred ultimately to natural standards such as the wave-length of light. It would be a great advantage if it were possible for research work of this character to be carried out independently by more than one member."

* The last meeting was in 1930. Mr Sears was the British delegate.

† This task was completed in 1889.

‡ A brief notice of the recent activity of the international committee in connexion with electrical standards will be found on p. 447.

§ Report of the Conference on Standardization (including resolutions adopted by the Imperial Conference). Presented to Parliament by command of His Majesty, November, 1930. Cmd. 3716–1930.

The report of the Fundamental Standards Committee defines the fundamental standards, both on the British system and on the metric system, which constitute the fundamental units of measurement legal for use throughout the British Commonwealth of Nations. It is noted that in India the railway system of weights in common use is defined in terms of the pound. It is explained that the term "reference standard" is used to denote a standard of suitable form and construction preserved under suitable conditions to serve as the basis of reference for the determination of any unit of measurement within one of the Dominions or of India. In accordance with the recommendation of the report the errors, if any, of such reference standards in relation to the fundamental standards or to the theoretical definitions of the units they represent are to be ascertained from time to time by comparison with the corresponding standards in use in the United Kingdom, and are to be allowed for in the use of the reference standards. The report concludes with a list of the principal units for which reference standards are likely to be required, with suggested procedure in regard to each.

Thus a simple and definite system has been set up by the Imperial Conference of 1930 for securing uniformity of standards throughout the Home Country, the Dominions and India.

§ 8. TIME

I have dealt at some length with the standards of length and mass. Time is for the physicist the third fundamental unit, and a lecture on standards of measurement would be incomplete without some reference to it. But the reference must be very brief; the subject is astronomical, and to attempt any full account of the measurement of time from the shadow clock or water clock of the Egyptians to the free pendulum of Mr Shortt or the vibrating crystal developed at the Bell Laboratories* in New York by Mr Marrison is a task far beyond my powers.

The apparent diurnal motion of the stars about the pole of the equator, the monthly motion of the moon, the yearly motion of the sun among the stars are the phenomena on which the measurement of time has always been based. Early observation showed that the moon returned to the same position among the stars in about thirty days—a month—while in twelve of these months—360 days—the annual cycle of the sun was, it appeared, complete. In each day the sun moved through 1/360th of its path, and this may have led to the division of the circle into 360 degrees. Division by six, or six multiplied by some power of ten, was very common in ancient measurements, while the sixths were sometimes halved, leading to division by twelves, e.g. the ounce and the inch. Thus the day—a day and a night—was divided into two equal periods, each containing twelve hours, while the hour was divided into sixty minutes and each minute into sixty seconds.

In quite early times, however, it was realized that the year contained about 365 of these days of twenty-four hours and "by attentively observing the heliacal rising of the star Sirius to which they gave the name of Thaat or Thoth (the Watch

* W. A. Marrison, "High Precision Standard of Frequency," Bell Telephone Laboratories, August, 1929, B 407.

Dog), because its appearance shortly preceded the overflow of the Nile, the Egyptians had discovered that the year consists of 365¼ days*." This was known to the Chinese, who in the time of the Emperor Yao, about 2300 B.C., divided the circle also into 365¼ degrees. The Chaldaeans in the far past discovered the cycle of 223 lunations, eighteen solar years, which bring the sun, earth and moon into approximately the same relative positions and so give rise to a repetition of the series of eclipses.

Shadow clocks were used at a very early date. The earliest known reference to a sundial† is to that of Berossus, a Chaldaean astronomer of about 300 B.C. One of these was erected in Rome about 290 B.C., and another found at Pompeii was constructed for the latitude of Memphis.

Water clocks were used in Egypt, the time being measured by the change of level in the water contained in a suitable reservoir from which it escaped by a small aperture. It is said‡ that wheel-work was employed and the hours shown on a graduated scale.

Ibn Janis and other Arabian astronomers about the year 1250 A.D. were in the habit of measuring small intervals of time by the oscillations of a pendulum which they kept in motion by a light touch of the finger when the vibrations had become too small to be easily counted. They did not connect the pendulum with a train of wheels.

A bar weighted at each end and kept in oscillation about its centre by the action on two pallets of the teeth of a wheel, maintained in continuous rotation by a spring or weight, was used to secure uniformity of motion of the wheel and thus to mark time. An early clock made about 1364 by Henry de Wyck for Charles V of France was regulated thus. *The Times* of May 9 last contains an account of a clock in Salisbury Cathedral dated about 1386 which has recently been cleaned and re-erected in the north transept.

Later the balance bar became a wheel with a heavy rim, while later still, in 1638, Hooke added a spiral spring to regulate the motion of the wheel. This, as it coiled and uncoiled, communicated to the wheel an oscillatory motion of a definite period. The motion of the balance wheel was maintained by a weight or a spring acting through a train of wheels, the last wheel of the train being connected to the balance by the escapement, a device which allowed one tooth of the wheel to pass at each oscillation of the balance and thus secured uniformity of motion in the train of wheels.

In 1600 Galileo called attention to the isochronous character of the motion of the pendulum. In 1633 he suggested its application to a clock; the problem was more fully investigated by Huyghens in 1638. Huyghens studied the relation between the period and the arc of vibration, and proved that for isochronism the

* *Encyclopædia Britannica* (*Seventh Edition*): Art. "Astronomy."

† The word translated "sundial" in the reference (Isaiah xxxviii. 8) to the "sundial" of Ahaz, B.C. 700, should be "steps"; the translation from the Hebrew is incorrect. *Encycl. Biblica.*, Art. "Dial."

‡ Dr Thomas Young, *Lectures on Natural Philosophy*, Lecture 17.

arc must remain constant unless the motion was compelled to be cycloidal. To maintain the arc constant a small impulse must be given to the pendulum at stated intervals—theoretically at each swing—and it was early recognized that this should occur as the pendulum is passing through its lowest—zero—position. Endeavours to secure this condition led to modifications of the escapement, the most important of which were due respectively to Harrison and to George Graham, who about 1700 introduced his dead-beat escapement which was almost universally used on astronomical clocks until, about 1890, Riefler of Munich produced a still more accurate mechanism.

Graham and Harrison also each introduced means to compensate for the change of length of the pendulum due to changes of temperature. One or other of these, usually Graham, was used until Guillaume's discovery of invar.

Electric clocks. The idea that the necessary energy could be communicated to the pendulum by electromagnetic forces seems first to have occurred to Alexander Bain, a mechanic in the employ of Sir Charles Wheatstone, who in 1840 took out a patent for the purpose. Since that date some eight hundred such patents have been issued. Prof. Sitter of Leyden, writing in *Nature* in 1928* of the Shortt clock, patented by Mr W. H. Shortt in association with the Synchronome Company, stated: "One of these clocks has been left entirely to itself, being however kept under rigorous observation at Greenwich during the greater part of a year, and its rate has been absolutely invariable....It looks as if this clock could be depended to keep time within a few hundredths of a second for a period measured in years instead of days." The pendulum of a clock has ordinarily two duties to perform. (1) It must execute its oscillations in equal periods, receiving in some way, from a suitable source, just enough energy to maintain the constancy of the arc of oscillation, without in any way else affecting its motion. (2) It must secure uniformity of motion in the train of wheels which constitute the works of the clock and indicate time by the motion of the hands.

Now the good time-keeping of a pendulum depends mainly on its being left alone to swing freely under gravity, receiving just enough energy to maintain its motion, but not being called upon to control directly the motion of a complex mechanism. In the Shortt clock this is secured by entire separation of the two functions of the pendulum described under (1) and (2) above. There are two pendulums; the one, the master, is free, the second known as the slave, which by suitable means is made to have exactly the same period as the free pendulum, controls the mechanism of the clock. The free pendulum swings in an air-tight case kept at greatly reduced pressure and at constant temperature. The swings of the slave are identical both in period and in phase with that of the master pendulum. Each half-minute—not each beat—as the two pendulums, moving from right to left (figure 3), approach the zero of their swing, a contact made by the slave allows a current to pass round an electromagnet in the case of the master clock, thus causing an arm, carrying a jewel at its free end, to fall on a wheel attached to the free pendulum, which is just then at the centre of its swing. The weight of this arm pressing on the

* *Nature* **121**, 99–106 (1928).

right-hand side of the wheel gives the pendulum the energy it has lost during the previous half-minute. As the arm falls off the wheel it makes a contact, actuating one electromagnet, which replaces the fallen arm in position ready for the next half-minute impulse and, at the same time, a second electromagnet controlling a spring which can act on the slave pendulum. If the slave clock is more than 1/250th of a second slow this spring comes into action and quickens the clock by this amount. The slave pendulum controls the clockwork and does all the work. Figure 3 is a diagram, taken from Mr Hope Jones's* book, which indicates the connexions. Such is a very brief description of the most remarkable clock of ancient or modern times.

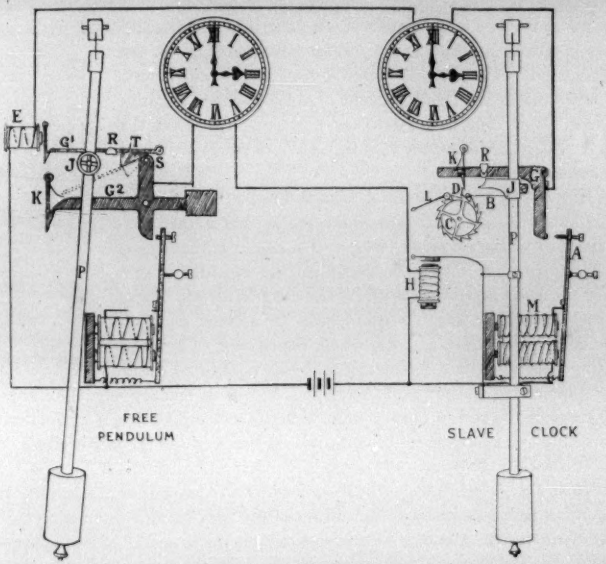

Fig. 3. The Shortt clock.

By permission of Mr Hope Jones.

The vibration clock. But the various methods of measuring time, which have been referred to, all depend on the uniform rotation of the earth for the calibration of their record over one or more days, and the uniformity of gravity or of the elastic forces brought into play by the deformation of a spring for its divisions into smaller units, hours, minutes and seconds. Now a body such as a bar of steel or a block of quartz, when set into vibration, has a number of frequencies depending on its shape, the nature of the motion, and the molecular properties of the material. The fre-

* I am indebted to the Astronomer Royal and to Mr Hope Jones for descriptions of the clock and for permission to include the figure.

quency of these is invariable so long as the body remains in the same physical state. It is possible under certain conditions by the action of an oscillating electric force to set such a body into vibration and to arrange that the body when in vibration reacts on the electric force producing the motion, in such a way as to reinforce its own vibration and thus maintain itself in an unvarying condition of motion of constant period. Such a system is not affected by the earth's rotation; the frequency remains constant so long as the physical conditions affecting the system are constant, and if the number of vibrations in an interval of time, measured by a clock, can be counted we have a means of checking the accuracy of the clock's record.

Recently a number of quartz oscillators have been set up at the Bell Telephone Laboratories in New York by Mr W. A. Marrison and the following brief account of these is taken from his paper*. "Plates of quartz cut in one direction from a crystal have a positive temperature coefficient, while other plates cut in a different direction have a negative coefficient. It is thus possible to cut a plate or ring from the crystal in which the frequency is independent of small variations of temperature.

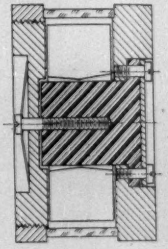

Fig. 4. Section of crystal mounting showing point support.

"A ring of the crystal, figure 4, is supported on a horizontal cylinder between two electrodes which are kept at a fixed distance apart by a ring of pyrex glass, and the hole in the crystal is so shaped that the line of contact with the cylinder is at a node for the two vibrations set up by the electric forces. There are arrangements for keeping the crystal in a fixed position relative to the electrodes and the whole forms part of a tuned oscillating electric circuit, the electrodes being connected to the earthed pole of the battery and the grid of the lamp respectively. The apparatus is very carefully lagged to reduce variations due to temperature to a minimum."

An account of a long series of comparisons between the records of these oscillators and three Shortt clocks in the laboratory of Mr A. L. Loomis at Tuxedo, U.S.A., was given to the Royal Astronomical Society by Mr Loomis on March 13, 1931†. Mr Loomis's paper contains a description of the very beautiful chronograph used to make the records. A comparison of the records made at Tuxedo was carried out by Prof. E. W. Brown and Mr Brouwer and the results were given at the same meeting of the Royal Astronomical Society. The examination showed that in any period of 30 seconds variations of as much as 0·001 second were rarely present. The records also show a very small variation in the period of the clock which was traced to a change in the value of g due to the influence of the moon‡, agreeing almost exactly with the amount expected by calculation.

During the past few years Dr Dye has carried out at the National Physical

* *Bell Telephone Laboratories Journal*, **8**, 493 (1929).

† *Monthly Notices R.A.S.* **91**, 569–575 (1931).

‡ This brings back to my remembrance and possibly to that of one or two of my hearers the attempt made in the very early days of the Cavendish Laboratory, now more than 50 years since, by George and Horace Darwin to investigate this effect, an attempt which ended in failure.

Laboratory a large amount of work on the quartz oscillator as a standard of frequency, investigating the conditions for maintenance of the oscillations as well as the influence on the frequency and power output of variations in the circuits associated with the quartz. Reference to this work will be found in the recent Reports of the Laboratory.

The same problem has been attacked in the Metrology Department of the Laboratory. Mr Sears utilizes for his vibration clock the longitudinal vibrations of a bar a metre in length held at its nodal point. This provides a very pure mode of vibration, and with a close control of the temperature it is hoped to realize a very high degree of isochronism. He has kindly furnished me with the following brief account of the arrangement as worked out by Mr Tomlinson:

"The bar is of elinvar and is self-maintained by means of electrostatic forces. Each end is highly finished, flat and square to the axis, and is associated with a small parallel fixed plate at a distance of about 0·003". These form small condensers, in one of which a feeble alternating potential is induced by the motion of the bar. This is amplified and applied as a driving potential to the other end. The electrostatic method of maintaining the vibration is expected to reduce the reaction on the frequency due to variations of the circuit to a minimum much less than would be attainable with a magnetic method of driving.

"To count the total number of vibrations, a phonic motor of special design runs in synchronism with the bar, driving a registering clock train. The first spindle of this train is arranged to rotate approximately once per second and is fitted with a contact device by means of which seconds signals, very precisely defined, are recorded by one marker of a special chronograph. A second marker records the duration of any time interval to be measured by comparison with the vibration clock.

"The temperature-control of the vibrating bar will be obtained by enclosing the bar completely in a steel vessel which is immersed in a large water bath, with a vigorous circulation kept up by a propeller. A toluene thermostat submerged in the bath regulates the temperature by controlling the current supplied to a group of electric heaters. The latter details are all that remain to be completed before the clock is ready for trial."

A chronograph for use with the vibration clock has been made and tested. An account of this appears in the *Journal of Scientific Instruments**. In a trial of the chronograph the small irregularities shown in a series of 10-second intervals obtained from the vibration clock did not exceed 0·0002 sec. When the chronograph was being used a period of 3 min. could be measured with an accuracy of about one part in a million. This very brief and incomplete account of the history of time-measurements is perhaps sufficient to illustrate human progress from the stage when the year was thought to contain 360 days to the present time, when a thousandth of a second can be measured with accuracy and the pull the moon exerts on a penny can be recorded on a clock.

* J. E. Sears, Jr., C.B.E., M.A., and O. A. Tomlinson, B.Sc., "A high precision chronograph," *J. Sci. Inst.* **8**, 21–28 (1931).

§ 9. ELECTRICAL STANDARDS

To turn now to electrical standards. There is neither time nor space for a long discussion. Besides, eighteen years ago, I delivered the fourth Kelvin Lecture* before the Institution of Electrical Engineers; it occupies, I find, thirty-three pages of their *Journal*. In that I tried to say all that I knew about electrical standards and to it I must refer you for many details. All I need do now is to give a brief summary of those first fifty years with some reference to the work of the eighteen which have followed.

Coulomb in 1785 had shown how, by the use of the proof plane and the torsion balance, a quantity of electricity could be measured, and had verified the inverse-square law from which we now derive the electrostatic unit of electricity. Faraday in 1834 discussed the "definite chemical action of electricity†." He had shown in December, 1832, "when experimenting with a battery of Leyden jars and a galvanometer that the deflecting force of an electric current is directly proportional to the absolute quantity of electricity passed," and further that "the chemical power, like the magnetic force, is in direct proportion to the absolute quantity of electricity which passes," two experimental results on which depend the electromagnetic and the electrochemical measures of a current.

But it was not until 1851 that W. Weber made his first distinct proposal of a definite system of electrical measurements according to which resistance would be measured in terms of an absolute velocity‡."

Various suggestions for a standard of resistance had been made previously to 1861 when the British Association, at the suggestion of Sir William Thomson, appointed a committee for improving the construction of practical standards of electrical measurements. The appointment of the committee was the outcome of a paper by Sir Charles Bright and Latimer Clark proposing names for the standards of resistance, current, electromotive force and quantity.

In the *Electrician*, in which this paper is printed, the editor writes: "We fear there is some danger that a system may be devised which will be followed exclusively by the eminent gentlemen at whose suggestion it is put forward." While Latimer Clark, in a letter published shortly afterwards, expressed the "fear that while bringing the highest electrical knowledge to the subject and acting with the best motives they (the committee) may be induced simply to recommend the adoption of Weber's absolute units or some other units ill adapted to the peculiar and various requirements of the electric telegraph." His paper had proposed the e.m.f. of a Daniell cell as a unit of electromotive force, and the charge on an air condenser of two parallel plates 1 m.^{2} in area and 1 mm. apart, when charged by a Daniell cell, as a unit of quantity. However, in spite of this, the committee, led by Thomson,

* "The ohm, the ampère and the volt. A memory of fifty years, 1862–1912," *J. Inst. E. E.* **50**, 560–92 (1913).

† M. Faraday, *Collected Researches*, **1**, para. 807 *et seq.*

‡ Fleming Jenkin, "Report on the new unit of electrical resistance," *Proc. R.S.* **14**, 154–164 (1865).

decided that Weber's proposal was far preferable to the use of any unit of the kind previously described. Can anyone measure the debt of gratitude due to them?

A table, given in the third report of the committee, contains a list of units of resistance which had been discussed, some of which were in actual use. The committee had commenced their work by consulting scientific men on the Continent and in America as to the system of units to be adopted, and, as a result of the discussion which followed the receipt of the replies, arrived at a provisional conclusion as to what would be the best system of units to adopt. Their first report (Cambridge, 1862) gives their reasons for the adoption of the absolute system based on the centimetre, the gramme and the second, and in their second report, 1863, will be found the celebrated appendix C* by J. Clerk Maxwell and Fleming Jenkin which explained, once for all and in a manner in which the experience of seventy years has found no flaw, the elementary relations between electrical measurements.

No doubt all electricians to-day know these relations almost by instinct, but is it too much to hope that this lecture may induce some who have never done so to go back to this appendix† and study the words Clerk Maxwell used to commend the c.g.s. system?

But the establishment of a system of units theoretically sound did not constitute the whole work of the committee. There was a second task, that of improving the construction of practical standards for electrical measurements. The ohm, the new unit of resistance, could be reproduced in material form as the resistance of a coil of wire or a column of mercury; such standards had been constructed by Prof. Weber, and in their second report (Newcastle-on-Tyne, 1863) the committee described experiments by Thomson's method of the revolving coil‡, carried out at King's College by Maxwell, Balfour Stewart and Fleming Jenkin, for the measurement of electrical resistance. The result was expressed as the resistance of a coil of wire known as "June 4" which the committee found to have a resistance of 107,620,116 m./sec. The revolving coil apparatus is among the exhibits on the table.

Meanwhile Matthiessen had been at work endeavouring to find the alloy best suited for the construction of permanent standard of resistance, while during 1863 a further set of measurements were made with the revolving coil. These are described in the report for 1864. Platinum silver in the proportion of 66 per cent.

* This appendix is called B in the index to the report, and C in the main report; there is no appendix B.

† This appendix and the whole of the reports of the committee will be found in the collected volume of British Association reports on electrical standards edited in 1913 by F. E. Smith as a record of the history of absolute units and of Lord Kelvin's work in connexion with these—a memorial volume made possible by the generosity of Mr R. K. Gray and the Council of the Association.

The following were the members of the committee from its commencement in 1861 to 1870, when it ceased its work temporarily: Lord Kelvin, Prof. A. Williamson, Sir Charles Wheatstone, Prof. W. H. Miller, Dr A. Matthiessen, Prof. Fleming Jenkin, Mr C. F. Varley, Prof. Balfour Stewart, Mr C. W. Siemens, Prof. J. Clerk Maxwell, Dr Joule, Sir Chas. Bright, Dr Esselbach, Prof. G. C. Foster, Mr Latimer Clark, Mr D. Forbes and Mr Ch. Hockin.

‡ The method had been indicated a few months previously by Weber in his paper "Zur Galvanometrie," but the committee appear not to have been aware of this.

Table 5. Resistance standards in use in 1863.

Description	Name	Absolute $\frac{\text{foot}}{\text{second}} \times 10^7$	Observations
Absolute $\frac{\text{foot}}{\text{second}} \times 10^7$ electromagnetic units (new determination)	Absolute $\frac{\text{foot}}{\text{second}} \times 10^7$	1·000	Calculated from the B.A. unit
Absolute $\frac{\text{foot}}{\text{second}} \times 10^7$ electromagnetic units (old determination)	Thomson's unit	1·0505	From an old determination by Weber
Twenty-five feet of a certain copper wire, weighing 345 grains	Jacobi	1·988	No measurement made; ratio between Siemens (Berlin) and Jacobi taken from Weber's *Galvanometrie*
Absolute $\frac{\text{metre}}{\text{second}} \times 10^7$ electromagnetic units determined by Weber (1862)	Weber's absolute $\frac{\text{metre}}{\text{second}} \times 10^7$	3·015	Measurement taken from a determination in 1862 of a standard sent by Prof. Thomson; does not agree with Weber's own measurement of Siemens's units; by Weber 1 Siemens's unit = $1{\cdot}025 \times 10^7$ metres-second
One metre of pure mercury, one square millimetre section, at 0° C.	Siemens 1864 issue	3·138	Measurement taken from three coils issued by Messrs Siemens
One metre of pure mercury, one square millimetre section, at 0° C.	Siemens (Berlin)	3·156	Measurement taken from coils exhibited in 1862 by Messrs Siemens, Halske & Co. (well adjusted)
One metre of pure mercury, one square millimetre section, at 0° C.	Siemens (London)	3·194	Measurement taken from coils exhibited in 1862 by Messrs Siemens, Halske & Co. (well adjusted)
British-Association unit	B.A. unit, or Ohmad	3·821	Equal to 10,000,000 $\frac{\text{metres}}{\text{second}}$, according to experiments of Standard Committee
One kilometre of iron wire, four millimetres in diameter (temperature not known)	Digney	30·40	From coils exhibited in 1862 (pretty well adjusted)
One kilometre of iron wire, four millimetres in diameter (temperature not known)	Bréquet	32·03	From coils exhibited in 1862 (indifferently adjusted)
One kilometre of iron wire, four millimetres in diameter (temperature not known)	Swiss	34·21	From coils exhibited in 1862 (badly adjusted)
One English standard mile of pure annealed copper wire $\frac{1}{16}$ in. diameter at 15·5° C.	Matthiessen	44·57	From a coil lent by Dr Matthiessen (of German-silver wire)
One English standard mile of one special copper wire $\frac{1}{16}$ in. in diameter	Varley	84·01	From coils lent by Mr Varley (well adjusted)
One German mile = 8238 yards of iron wire $\frac{1}{8}$ in. in diameter (temperature not known)	German mile	188·4	From coils exhibited in 1862 by Messrs Siemens, Halske & Co.

silver to 33 per cent. platinum was chosen as the metal for standard coils, but resistances of 1 B.A. unit were constructed in platinum and a number of other alloys. Some of these are exhibited. The first series of comparisons of B.A. units to be deposited at Kew Observatory was made by Hockin and reported on in the fourth report (Birmingham, 1865). This is shown in table 6.

Table 6. Comparison of B.A. units to be deposited at Kew Observatory, by C. Hockin.

Material of coil	Number of coil	Date of observation	Temperatures at which coil has a resistance of 107 m./sec. (° C.)	Observer
Platinum-iridium alloy	2	January 4, 1865	15·5	C. H.
		June 6, 1865	16·0	A. M.
		February 10, 1867	16·0	C. H.
Platinum-iridium alloy	3	January 4, 1865	15·3	C. H.
		June 6, 1865	15·8	A. M.
		February 10, 1867	15·8	C. H.
Gold-silver alloy	10	January 5, 1865	15·6	A. M.
		February 10, 1867	15·6	C. H.
Gold-silver alloy	58	April 10, 1865	15·3	A. M.
		June 6, 1865	15·3	A. M.
		February 10, 1867	15·3	C. H.
Platinum	35	January 7, 1865	15·7	C. H.
		August 18, 1866	15·7	A. M.
		February 10, 1867	15·7	C. H.
Platinum	36	January 7, 1865	15·5	C. H.
		August 18, 1866	15·5	A. M.
		February 10, 1867	15·7	C. H.
Platinum-silver alloy	43	February 15, 1865	15·2	C. H.
		March 9, 1865	15·2	A. M.
		February 10, 1867	15·2	C. H.
Mercury	I	February 2, 1865	16·0	A. M.
		July 18, 1866	16·0	A. M.
		February 11, 1867	16·7	C. H.
Mercury	II	February 3, 1865	14·8	A. M.
		August 18, 1866	14·8	A. M.
		February 11, 1867	14·8	C. H.
Mercury	III	February 11, 1867	17·9	C. H.

The fifth report (Dundee, 1867) contains Thomson's report on electrometers and electrostatic measurements, in which among others the absolute electrometer (one is exhibited) was described. I began work at the Cavendish Laboratory in 1876, and this was the apparatus which I was set to use in order to check the e.m.f. of a series of tray Daniell cells, employed by Chrystal in his experiments on Ohm's law. The report also contains Joule's determination of the dynamical equivalent of heat from the thermal effects of electric currents, in which he described for the first time the use of a balance for measurement of current.

In 1870 the committee expressed the view that the work could be better advanced if they were not reappointed, but three new committees of smaller numbers were chosen instead to determine and issue (1) a condenser representing the unit of capacity, (2) a gauge for showing the unit difference of potential, (3) an electrodynamometer adapted to measure the intensity of currents in a decimal multiple of the absolute measure. The committee was not at once reappointed, and the small committees were not set up.

Meanwhile, doubts had been thrown on the accuracy of the result found by the committee for the value of the ohm, both by the experiments of Joule already referred to and by investigations in Germany by Kohlrausch and H. Weber and in America by Rowland.

The B.A. standard resistances had been transferred by Maxwell from Kew to the Cavendish Laboratory, and compared by Chrystal and Saunder in 1876. Between 1878 and 1881 a further comparison was instituted by Fleming. The apparatus he used is on the table. It was described* in a paper read to this Society in 1880. His results were expressed in a series of curves giving the value of the coils at various temperatures in terms of the mean B.A. unit, defined as the mean of the values of the seven coils tested by Hockin at the temperatures at which each was stated to be correct. Fleming's chart was for many years the standard to which all resistances were referred.

Lord Rayleigh, appointed Cavendish Professor in 1879, soon became interested in electrical measurements; the need for standards was becoming felt by engineers; and Ayrton in 1880 at the Southampton meeting of the B.A. moved for the reconstitution of the committee. The new committee made its first report in 1881 at the jubilee meeting of the Association at York. At this meeting I became a member of the Association; and shortly afterwards Lord Rayleigh asked me to help in the issue of certificates of resistance coils from the Cavendish Laboratory. In the following year I became Secretary of the Committee and closely connected with its work, being in charge of its standards which in 1900 were removed to the National Physical Laboratory.

In 1881 there was an international congress in Paris at which Thomson supported the views of the B.A. committee, and after much discussion the c.g.s. system was accepted as the basis of an international system, while the practical units were to be the ohm, 10^9 c.g.s. units, and the volt, 10^8 c.g.s. units. During the early meetings of the congress Wiedemann and v. Helmholtz had spoken strongly in favour of Siemens's mercury unit as the standard. It was further agreed (1) that the unit of resistance, 1 ohm, should be represented by a column of mercury 1 mm.^{2} in section at the freezing point of water, and (2) that an international commission should be charged with the duty of determining by new absolute measurements the length of this column.

The commission met in 1884. Mascart presented a table giving all the results then known for the length of the mercury column; the mean was 106·02 cm., and the first resolution of the conference stated that "The legal ohm is the resistance

* *Proc. Phys. Soc.* **3** (1880). *Phil. Mag.* **9**, 109 (1880).

of a column of mercury a square millimetre in section and 106 centimetres in length at the temperature of melting ice." The values on which this resolution was based differed among themselves by some 2 per cent. Thomson and the other English representatives pressed for a result more nearly in agreement with the English determinations, with the result that the "legal ohm" never became legal in England.

An interesting relic of this period is on the table in the form of a mercury standard representing the legal ohm constructed by Dr Benoît of the Bureau des Poids et Mesures, and compared with the B.A. units in 1885*.

In 1890 the Board of Trade appointed a committee to report on a standard for use in England, and the upper part of table 7 gives the values then available to guide their decision. As a result the committee adopted as the resistance of the B.A. unit the value 0·9866 ohm and as the length of the mercury column 106·3 cm.

Table 7. Value of the ohm expressed as the resistance of a column of mercury.

Lord Rayleigh	1882	Rotating coil	106·31
Lord Rayleigh	1883	Lorenz	106·27
Mascart	1884	Induced current	106·33
Rowland	1887	Mean of several methods	106·32
Kohlrausch	1887	Damping of magnets	106·32
Glazebrook	1882	Induced currents	106·29
Glazebrook	1888		
Wuilleumier	1888	—	106·31
Jones	1891	Lorenz	106·31
Jones	1892	Lorenz	106·32
Ayrton and Jones	1897	Lorenz	106·27
Guillet	1899	Induced currents	106·21
Campbell	1912	Alternating currents	106·27
		Mean	106·29

The committee also recommended as standards for current and e.m.f. respectively the silver voltameter and Clark's cell, and prepared specifications for their use, basing their recommendation in the main on the work which had been in progress at Cambridge during the previous ten years.

Some of the results on which the decisions were based are shown in tables 8 and 9.

Table 8. Electrochemical equivalent of silver.

		mg./sec.
1884	Rayleigh and Sidgwick	1·1179
1884	Kohlrausch	1·1183
1884	Mascart	1·1156
1890	Pellat and Potier	1·1192
1898	Kahle	1·1183
1898	Patterson and Guthe	1·1192
1903	Pellat and Leduc	1·1195
1904	Van Dijk and Kunst	1·1182
1906	Guthe	1·1182
1907	Smith, Mather and Lowry	$1·1182_7$
1908	Laporte and de la Gorce	$1·1182_9$
1912	Rosa, Dorsey and Miller	$1·1180_4$

* For a detailed account see *Proc. Phys. Soc.* and *Phil. Mag.* **20**, 343 (1885).

Table 9. E.m.f. of Clark cell at 15° C.

		Volts
1872	Clark	1·4378
1884	Rayleigh and Sidgwick	1·4345
1896	Kahle	1·4322
1899	Carhart and Guthe	1·4333
1905	Guthe	1·4330
1907	Ayrton, Mather and Smith	1·4323
	Indirect determination of e.m.f. of Clark cell	
1884	Von Ettinghausen	$1{\cdot}433_5$
1885	Rayleigh	1·435
1892	Glazebrook and Skinner	1·434
1904	Trotter	$1{\cdot}432_9$

The findings of this Board of Trade committee were confirmed at the Edinburgh meeting of the Association in 1891, at which Dr v. Helmholtz was present with Dr Lindeck and Dr Kahler of the Reichsanstalt, M. Guillaume of the Bureau des Poids et Mesures, and Prof. Carhart of the United States. Dr v. Helmholtz expressed his full concurrence in the decisions which were, as he informed the committee, in accord with the recommendations which had already been laid by the Curatorium of the Reichsanstalt, as well as by himself, before the German government. Dr Lindeck laid before the committee information as to the property of the manganese alloy (manganin) used by the Reichsanstalt for resistance coils. Some of the material which was before the committee has been given in tables 7, 8 and 9. An international congress took place at Chicago in August, 1893, at which practically the same words were adopted to define the international standards. In the Chicago resolutions a distinction is drawn for the first time between the c.g.s. units or their multiples and the international standards. Thus the international ohm is said to be based on the ohm of 10^9 c.g.s. units of resistance and to be represented by the resistance of a certain mercury column.

Meanwhile experiments aimed at attaining a higher value of accuracy in all the standards were continued. In 1897 Profs. Ayrton and Viriamu Jones described a new form of Lorenz apparatus, constructed for McGill University, by the aid of which they obtained the value 106·28 for the length of the mercury column having a resistance of one ohm, and next year the same authors described an ampere balance of high accuracy. The National Physical Laboratory was opened at Teddington in 1902, and the resistance coils and other standards came under the charge of F. E. Smith, now Sir Frank Smith. Arrangements were made next year to set up an ampere balance in accordance with the Ayrton Jones designs, and a series of papers appeared in 1907 giving an account of work by Ayrton, Makin, Smith and Lowry on the current balance, the silver voltameter and the Weston cell. This cell had been recommended at a conference held at the Reichsanstalt in 1905 in place of the Clark cell. As a result of discussions at St Louis in 1904 and the Berlin Conference, it was decided to hold an International conference in London in 1908,

and this duly took place. Lord Rayleigh was chairman and Mr Duddell acted as secretary.

After much discussion agreement was reached as to the definitions of the ohm, the ampere and the volt, multiples of the c.g.s. units of resistance, current and electromotive force, and the international standards by which these units were to be represented. I will refer later to the reasons for this distinction.

The conference was attended by representatives of twenty-five countries, who all signed its report. It also set up an international committee to formulate a plan for, and to direct, such work as may be necessary in connexion with the maintenance of standards, etc. As a result of this, representatives of the N.P.L., the Reichsanstalt and the Laboratoire Centrale d'Électricité, by the kindness of Dr Stratton, met at [illegible] and conducted a valuable series of comparisons. Since that

ERRATUM

Page 444, *for* "Makin" *read* "Mather."

"					
"	volt	...	...	...	$(1{\cdot}00045 \pm 0{\cdot}00008)\ 10^8$
"	watt	...	...	...	$(1{\cdot}00039 \pm 0{\cdot}00014)\ 10^7$
E.m.f. of Weston normal cell at 20° C.					$1{\cdot}01876 \times 10^8$ c.g.s. units

Intercomparisons of the original B.A. coils have been continued from time to time, and a very complete discussion by F. E. Smith of the changes in their values will be found in the report for 1908. The coils were compared with the resistance of mercury by myself in 1888 and by Mr Smith twenty years later in 1908. On the assumption that the resistance of 1 m. of mercury is 0·95352 B.A.U., the value given by my experiments, we obtain the results given in table 11. These results being accepted, it would appear that the two coils of pure platinum D and E did not change during that period.

Thus, the differences between the various coils being known, it becomes possible to form a table giving the values of the coils in terms of the original B.A.U. at each of the periods at which complete intercomparisons were made. This is done in table 12.

* F. E. Smith, *Phil. Trans. R.S.* **214** A, 27 (1914).

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Meanwhile the work of realizing the c.g.s. units to the highest accuracy has continued. In 1902, by the generosity of the Drapers' Company and of Sir Andrew Noble, funds had been found for the construction at the N.P.L. of a greatly improved form of Lorenz apparatus, and the results of Smith's measurements with this apparatus were published in 1914*. He also set up a number of mercury resistance tubes, and as a result found for the length of the mercury column having a resistance of one ohm the value 106·27 cm.

Figures 5 and 6 show the Lorenz apparatus at the N.P.L. and the Ayrton Jones current balance, while in table 10 will be found the results of the more recent determinations of the international ohm, ampere, volt and watt.

Table 10. Values of international units (1831) in terms of c.g.s. units.

International ohm	$(1{\cdot}00051 \pm 0{\cdot}00002)\ 10^9$
,, ampere	$(1 - 0{\cdot}00006 \pm 0{\cdot}00006)\ 10^{-1}$
,, volt	$(1{\cdot}00045 \pm 0{\cdot}00008)\ 10^8$
,, watt	$(1{\cdot}00039 \pm 0{\cdot}00014)\ 10^7$
E.m.f. of Weston normal cell at 20° C.	$1{\cdot}01876 \times 10^8$ c.g.s. units

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* F. E. Smith, *Phil. Trans. R.S.* **214** A, 27 (1914).

Table 11. Values of coils at 16·0° C. in 1888 and 1908 obtained from comparison with mercury tubes, the resistance of 1 m. of mercury being assumed to be 0·95352 B.A.U.

Coil	Value in 1888 at time of determination of specific resistance of mercury	Value in 1908
A	1·00068	1·00042
B	1·00025	1·00018
C	1·00067	1·00093
D	1·00013	1·00012
E	1·00073	1·00072
F	0·99970	1·00080
G	0·99936	1·00095
H	0·99963	0·99964
Flat	1·00023	1·00045

Table 12. Resistances at 16·0° C. in terms of the original B.A.U. (1867). Values obtained through the two platinum coils D, E.

Coil	Material	1867	1876	1879–81	1888	1908
A	PtIr	1·00000	1·00077	1·00056	1·00147	1·00122
B	PtIr	1·00029	1·00121	1·00080	1·00104	1·00098
C	AuAg	1·00050	1·00141	1·00101	1·00146	1·00173
D	Pt	1·00092	1·00092	1·00092	1·00092	1·00092
E	Pt	1·00152	1·00152	1·00152	1·00152	1·00152
F	PtAg	—	—	1·00016	1·00072	1·00160
G	PtAg	1·00022	1·00030	0·99982	1·00025	1·00175
H	PtAg	1·00020	—	—	1·00042	1·00044
Flat	PtAg	—	—	1·00079	1·00120	1·00125

§ 9. FUNDAMENTAL INTERNATIONAL ELECTRICAL STANDARDS

And here it is desirable to include a brief notice of recent action taken by the International Committee on Weights and Measures with regard to electrical standards and approved by the International Conference of Weights and Measures in 1930.

Some forty years after England had accepted the metric convention there was an important change. In pre-war days there had been discussions as to an extension of the functions of the International Committee of Weights and Measures. Sir David Gill and Dr Stratton, then Director of the Bureau of Standards, had taken a prominent part in these, but it was not until 1927 that action was taken. The following resolution, ratified later by the Seventh International Conference on Standards, was approved on October 4 of that year: "The International Committee of Weights and Measures approves the organisation of a Consultative Committee for Electricity (Comité consultatif d'Électricité) to advise the International Committee of Weights and Measures on questions relating to systems of Measurement and Electrical

Standards." The first meeting of this* committee was held in November 1928, and the information which follows is taken from the proceedings† of that meeting.

After considering the question of the unit‡ to be employed, the committee proceeded to discuss the advice they should give to the International Committee of Weights and Measures as to electrical units. In this connexion it is necessary to bear carefully in mind the distinction between electrical standards and those of length and mass already in the care of the Bureau. For length and mass there are material standards assumed to be invariable in form and dimensions. These have to be maintained in safety and used from under carefully defined conditions for intercomparison with the national standards of the countries adhering to the convention. With electrical standards no such simple course is possible. No permanent standard of resistance or electromotive force has been found. Changes, some of them no doubt very slight, have occurred in all known resistance coils§, while no voltaic cell preserves its electromotive force unaltered. It is not possible therefore to equip the Bureau with invariable standards against which national standards could be compared. Recourse must of necessity be had to experimental determinations of the standards in terms of the absolute units of length, mass and time. This requires expensive apparatus and much skilled experimenting. Again, all such work is liable to error; it is desirable therefore that it should be carried out in a number of laboratories under various conditions.

Now the national and other laboratories of several countries are already equipped for such work, and the staff have experience in the measurements. It was therefore thought right by the committee to utilize the opportunities and experience thus afforded, and not to plan at the Bureau the extensive and elaborate equipment necessary if the work of determining *ab initio* international standards for electricity. Instead of this the national laboratories are invited to continue their work and to send the result of their investigations to the Bureau in such form as the committee may desire. These results will be co-ordinated by the committee, and international standards will be formulated in accordance with the combined results.

And thus the Comité consultatif d'Électricité agreed to advise the International

* The Members were: President: M. le Sénateur Volterra, President of the International Committee. Deputy President: M. Paul Janet, Director of the Laboratoire Central d'Électricité. Members: for the Reichsanstalt, Berlin, Prof. Steinwehr; for the Bureau of Standards, Dr Geo. R. Burgess, Director of the Bureau; for the National Physical Laboratory, Dr D. N. Dye, F.R.S.; for the Laboratoire Central d'Électricité, M. R. Jouast, Deputy Director; for the Electrotechnical Laboratory of the Royal College of Engineers, Rome, Prof. L. Lombardi, Director; for the Electrotechnical Laboratory, Tokyo, M. Seikighi Jimbo, Assistant; for the Central Chamber of Weights and Measures, Leningrad, Prof. D. Konorator, President; for the Bureau International des Poids et Mesures, Sèvres, M. Ch. Éd. Guillaume, Director.

† Comité consultatif d'électricité, *Rapport Procès-verbaux des Séances de* 1928.

‡ Some results of this discussion are dealt with later. See pp. 452 *et seqq.*

§ Among the most permanent of these are the two platinum B.A. coils D and E, constructed in 1863, for which the following values have been found by comparison with mercury tubes:

	1888	1908
D	1·00013	1·00012
E	1·00073	1·00072

Committee that the functions entrusted to the International Bureau of Weights and Measures in regard to electrical units should be to establish: (1) A central secretariat to organize the systematic exchange of standards (étalons) and secure the co-ordination (assurer la synthèse) of the results of comparisons made by the national laboratories. (2) A laboratory to which material standards representing the results obtained in different countries can be set for exact comparisons. (3) A collection of reference standards and working standards, including standards of inductance and capacity, together with the equipment necessary for the comparison of other standards with those of the bureau. It is pointed out that such a scheme places very definite responsibilities on the national laboratories, and the report continues with regard to the

Functions of the International Committee. "If the general plan be approved* the International Committee under the authority given to it by the General Conference will have the responsibility of fixing and promulgating the values to be employed for practical standards and of fixing the date of any fresh revision. For this purpose the Consultative Committee can continue to advise the International Committee in accordance with the functions which the General Conference has assigned to it.

The Report continues: "It is to be noted that the plan does not envisage the establishment in any one laboratory of standards to be regarded as representative of the electrical units or as having an authority superior to that of the standards of other laboratories. The essential point is that the electrical units are considered as secondary, in the sense that they are based directly on the fundamental units of length, mass and time. It is common knowledge that material electrical standards change more or less in course of time. The determination of their exact value in terms of the fundamental units, the maintenance of the standards themselves in the highest degree of precision possible, and the development of means certain to secure their constancy are problems which necessitate the combined resources of the best laboratories of the world. While a part of this task could be undertaken under the auspices of the International Committee in the laboratory of the Bureau International, the Committee and the Bureau can do a more useful work by combining the results of research made throughout the world.

"As the result of the authority given to it to co-ordinate electrical measurements the International Committee has ample power to carry into effect this plan."

The above extract makes clear the conditions and method under which the International Committee of Weights and Measures has undertaken electrical standardization.

§ 10. NATURAL AND ARBITRARY STANDARDS

If we look back over the history of the past two hundred years or so we may note some interesting changes in the views of scientific men as to the bases on which to rest their standards of measurement.

If we go to a much earlier period it is obvious that weights and measures began with some natural objects selected for their convenience, and sufficiently constant

* The plan was approved at the meeting in 1931.

in quantity for the simple needs of the times. Such were the length of a man's forearm or the weight of corn—possibly of water—contained in a vessel approximately constant in volume.

In time, uniformity became more important and so we pass from the length of the king's arm to the scale engraved on the wall or floor of the cathedral, and from the weight of so many grains of wheat taken from the middle of the ear to the mass of a lump of metal.

The jury of Queen Elizabeth's time, to which we owe our modern system, paid little attention, it is probable, to the origin of the yard or the pound; they fixed on the length of a metal bar and the weight of a lump of iron and prescribed that these were to be the standards, trusting to the permanence of these material objects rather than to any natural laws which might serve for their definition. They were no doubt severely practical; a change came in England, in the early days of the Royal Society, when we find Caswell and Walker endeavouring to connect the weight and volume of a quantity of water and thus establish a natural standard of weight in terms of the standard of length, trying possibly to repeat what had been done in Egypt long ago.

Then some fifty years later Graham, realizing that there was a definite relation between the period of a body in oscillation, its dimensions, and the distribution of its mass, endeavoured to connect the period of a seconds pendulum with its length. He set about the comparison of existing material standards, to find one in terms of which to express the length of the pendulum, thus leading to the report of Lord Macclesfield's committee in 1743.

Intercomparisons between existing standards continued to be made, and the idea of basing that of length on the seconds pendulum increased in favour until in 1814 the House of Commons asked for a formal statement of the length, and two years later the Astronomer Royal was instructed to determine it. Kater's work followed, with the result that, while in 1824 certain material standards—Bird's bar of 1760 and a brass weight of 1 lb. Troy dated 1758—were made the legal standards, they were to be recovered if lost by reference to natural standards, the length of the seconds pendulum and the weight of a cubic inch of water.

But this recourse to natural standards did not last long. Scientific men discovered that it was simpler to compare a standard with its copies, and to recover its value if lost by the aid of those copies, than to determine the length of the seconds pendulum or the mass of a cubic inch of water, and so in 1843 the recommendations of 1824 were thrown over; the Scientific Committee which had been appointed in 1838 to advise, recommended that the standards which had been lost in the fire of 1834 should be recovered from their copies "and that the standard be in no way defined by reference to any natural basis." More trust could be placed in the permanence of a material standard and on the accuracy with which it could be recovered by intercomparison of its copies, than on the result of any attempt to reconstruct it based on some natural law, and so it has remained up to the present.

In France the history has been the same. The Constituent Assembly entrusted

the Academy of Science with the duty of introducing a new system of measures, and in 1788 a report* was issued on the choice of a unit of measure drawn up by Borda, Lagrange, Laplace, Mouge and Condorcet. The report discussed the advantages of employing either the length of the seconds pendulum or that of a meridian of the earth as the basis of length-measurement and decided in favour of the latter which, in the words of Laplace, "appears to be of very high antiquity, it is so natural to man to refer measures of distance to the dimensions of the globe which he inhabits, in order that in transporting himself from place to place he may know, by the denomination of the space passed through alone, the relation of this space to the entire circumference of the earth. This method has also the advantage of making nautical measures correspond at once with celestial ones." And so the metric system came into being. Delambre† and Mechain measured the length of an arc from Dunkirk to Barcelona and hence calculated the length of the quadrant as 5,130,740 of a certain iron toise when at a temperature of $61\frac{1}{2}°$ F. The metre was fixed at one ten-millionth of this length, and a standard metre constructed to represent it, while the kilogramme was to be the mass of a cubic decimetre of water at the temperature of its maximum density and a platinum weight was constructed with great care to have this mass when weighed *in vacuo*.

But, while the material standards continue to exist, research has shown that they no longer represent the definitions. The metre is still the distance between the marks on a certain bar of platinum-iridium but the earth's quadrant is not 10^7 of these metres, while different meridians differ in length; the earth is not a spheroid. The best mean figure obtainable seems to be 1 meridional earth quadrant = 10,002,090 metres.

The platinum mass, known as the Kilogramme des Archives, is still the standard kilogramme, but the mass of a cubic decimetre of water at 4° C. approximately has been found to be 999·972 gm.; or alternatively 1000·028 cm.^{3} is the volume of a kilogramme of water at the temperature of its maximum density and under normal atmospheric pressure.

Neither the seconds pendulum nor a quadrant of the earth has proved to be more satisfactory than the material standard. This was the case when Michelson‡ in 1889, realizing that the wave-length of light of a given frequency was invariable, suggested its use as an ultimate standard of length. There was the difficulty of stepping from so small a unit as a wave-length up to the metre, but he pointed out that one can bridge over the distance by using a number of standards each approximately, but not quite, double the distance of the last, employing interference methods to determine the difference between the length of the larger standard and twice that of the smaller. Light arranged to traverse the smaller distance twice is made to interfere with light which has only once traversed the longer distance. Michelson§ had an opportunity of carrying this idea into practice somewhat later

* *Mém. de l'Académie* (Paris, 1788 H 7–17).
† Delambre, *Base du Système Métrique* (3 vols. 4to. Paris).
‡ Michelson and Morley, *Am. J. of Sci.* **38** (1889).
§ *Travaux et Mémoires du Bureau International des Poids et Mesures*, **9** (1895).

at the Bureau des Poids et Mesures at Sèvres, and found from observations on the red line of cadmium that its wave-length λ_R is given by

$$\lambda_R = 0{\cdot}64384722 \times 10^{-6} \text{ m.}$$

Somewhat later, in 1913, Messrs Benoît, Fabry and Perot* at the Bureau International repeated the measurement, using a modification of the method, and found

$$\lambda_R = 0{\cdot}6438470 \times 10^{-6} \text{ m.}$$

Since that date measurements have been made in Japan by Messrs Watanambe and Imaizumi leading to the result

$$\lambda_R = 0{\cdot}64384685 \times 10^{-6} \text{ m.},$$

while Dr Tutton communicated the results of his experiments to the Royal Society on April 3 last: according to these he finds

$$\lambda_R = 0{\cdot}64384698 \times 10^{-6} \text{ m.}$$

Mr Sears gives as a provisional result of his N.P.L. experiment

$$\lambda_R = 0{\cdot}64384714 \times 10^{-6} \text{ m.}$$

At their meeting in 1925 the International Committee of Weights and Measures agreed in principle to define the unit of length by means of the wave-length of light, subject to the determination by the national laboratories of the most satisfactory method of realization. As a result, work has been in progress at the National Physical Laboratory for some time and now is approaching conclusion, while work is in progress at the Reichsanstalt and at the Bureau of Standards. I am indebted to Mr Sears for the following description of the investigation which is being carried out at Teddington by Mr Barrell.

"The apparatus which has been constructed for this purpose is based on the method of Fabry and Perot in which the separation of two parallel semi-silvered glass plates, constituting the first étalon, is determined directly in monochromatic light by the method of coincidences, and then compared by the superposition of fringes in white light with that of similar longer étalons which are very closely but not quite exactly multiples of the first. The étalons are set up in line, and the light passes through them in succession. In the N.P.L. apparatus the difference from the exact multiple is determined by the tilting of one of the étalons through a measured angle on either side of the normal until the optical path in the longer étalon is an exact multiple of that in the shorter. The étalons take the form of invar tubes whose ends are chromium-plated and polished optically flat and parallel, the glass plates being wrung on to these surfaces. It is thus possible to evacuate the tubes and work with wave-lengths *in vacuo*, or alternatively to work in air in any desired controlled condition. The longest étalon is slightly greater than 1 m. in length, and contains a steel end-gauge of X-shaped cross-section, 1 m. in length, whose ends also are polished flat and square to its axis. The small distances between the ends of this gauge and the glass terminal plates of the étalon are determined directly by interference in monochromatic light, while the length of the étalon is determined by

* *Travaux et Mémoires*, 15 (1913).

means of light passing along the four channels formed between the arms of the X and the wall of the tube. By subtraction the length of the metre bar is finally ascertained in terms of the chosen wave-length of monochromatic light—usually the cadmium red.

"The apparatus is set up in a carefully designed case which is thermally controlled by electrical heaters governed by a thermostat. Air is circulated over these heaters and through the enclosure by means of a fan and ducts leading to rotating cowls, which vary the direction of flow and so effect complete mixing and stirring. The temperature of the whole enclosure can be maintained constant throughout to within 0·01° C. over long periods of time. On the outside of the tube of the longest étalon is wound a platinum resistance thermometer by means of which its temperature can be ascertained at any time to 0·001° C. A very high accuracy of temperature-measurement is necessary, as it is hoped that the apparatus will be capable of repeating measurements to an accuracy of the order of 1 part in 40,000,000, which is about ten times as great as can be obtained with certainty in the microscopic comparison of line standards."

Thus it will be seen that the International Committee are prepared to refer the standard metre to a natural standard against which it can be compared. This natural standard will be represented by the distance between two marks on a bar of platinum-iridium, and the constancy of this distance will be checked from time to time by comparison with the wave-length of light of definite period. Hence there is a prospect that a natural standard of length will in time be generally accepted.

As to a standard of mass, up to the present it is not probable that the kilogramme, the mass of a lump of platinum, will be disturbed until we reach the condition when the mass of (say) the atom of hydrogen or of an electron can be determined with sufficient accuracy.

For time we already have a natural standard in the sidereal day, and, while the frequency of a quartz or other oscillator will certainly be used for comparatively short intervals, it will not replace the day and hour for longer periods.

In electrical standards also a transference to natural standards is taking place.

One of the first duties of the Comité Consultatif d'Électricité set up in 1927 was, as has been stated, to determine the units in terms of which electromotive force, current and resistance are to be measured. At present, while the units are defined in terms of the c.g.s. units of length, mass and time, the international standards used in daily work are, for the ohm 10^9 c.g.s. units, the resistance of a column of mercury; for the volt 10^8 c.g.s. units, the e.m.f. of a Weston cell; and for the ampere 10^{-1} c.g.s. units, the current which deposits a certain weight of silver per second in a voltameter of specified design. This weight of silver, it should be noted, has been determined by the weighing in an ampere balance of the electromagnetic force set up by a current of one ampere. These definitions, as has already been explained, were agreed to at the international conference in London in 1908.

The appliances needed for an absolute determination of the electrical units are elaborate and expensive. At the time of the London conference it was realized that the necessary measurements could be made only in a few well-equipped standardizing

laboratories, while it was felt that the standards for daily use should be such as could be set up without serious difficulty in any laboratory provided with ordinary measuring and weighing apparatus of adequate precision. The arrangements proposed at Sèvres by the Comité Consultatif d'Électricité have altered this. When these are complete any country can have its electrical standards compared against international standards maintained at the Bureau, while by the collaboration of countries possessing national laboratories, equipped for absolute measurements, the international standards of the Bureau can be realized and maintained in much closer agreement with the c.g.s. units than is now the case with the international ohm, volt, and ampere. Dr Dye tells me that at present:

One international ohm (as defined) = $(1{\cdot}00051 \pm 0{\cdot}00002)\ 10^9$ c.g.s. units.

For an actual resistance coil the uncertainty must be increased to $0{\cdot}00004 \times 10^9$. As to the ampere the figures cannot be quite so definite, for no absolute determination of the ampere has been made for a number of years, and the comparisons which have been made all assume the "constancy of standard cells of which there is definite disproof in the international comparisons of late years." He gives, however, as the best result available the relation

10^{-1} c.g.s. units of current = $(1{\cdot}00006 \pm 0{\cdot}00006)$ international ampere.

And so the Comité Consultatif at its meeting in 1928 adopted the following resolutions. "(1) Le Comité consultatif d'Électricité institué auprès du Comité international des Poids et Mesures, considérant la grande importance qu'il y a à unifier les systèmes de mesures électriques sur une base dépourvue de tout caractère arbitraire, reconnaît dès sa première réunion que le système absolu, dérivé du système c.g.s., pourra être avec avantage substitué au système des unités internationales pour toutes les déterminations scientifiques et industrielles, et décide d'en proposer l'adoption au Comité international des Poids et Mesures. (2) Le Comité consultatif d'Électricité, tout en reconnaissant les grands progrès déjà accomplis dans le domaine des mesures électriques de haute précision, ne croit cependant pas qu'il soit possible dès maintenant de fixer avec toute l'exactitude nécessaire et dont ils sont susceptibles les rapports qui existent entre les unités absolues dérivées du système c.g.s. et les unités internationales de courant de force électromotrice et de résistance telles qu'elles ont été définies par le Congrès international de Chicago en 1893 et la Conférence de Londres en 1908, et émet le vœu que des recherches soient poursuivies dans ce but dans les laboratoires convenablement outillés suivant un programme préalablement étudié en accord avec le Comité consultatif d'Électricité."

The method by which the results obtained in the various laboratories are to be utilized by the Bureau International has already been referred to.

Since 1928, and indeed previously to that date, work has been in progress at the National Physical Laboratory in connexion with all the units. The ampère balance has been reconstituted, work has been in progress on a novel method of measuring a resistance suggested some years since by Albert Campbell, preparations are going

forward for further measurements with the Viriamu Jones Memorial Lorenz apparatus and for a redetermination of the electromotive force of a Weston cell.

At the second meeting of the Comité Consultatif d'Électricité in 1930, reports as to progress were received from the N.P.L., the Bureau of Standards, Washington, the Central Chamber of Weights and Measures of Russia, and the Electrotechnical Laboratory of Tokyo.

While, of course, it will be necessary to have at Sèvres material standards or apparatus embodying the results of these various absolute measurements, these standards and apparatus will represent, with all the accuracy possible, multiples of the c.g.s. units themselves. They will not be material standards adopted because at the time they were constructed they were believed to represent the theoretical units to the highest accuracy then possible. Thus in electricity we go back through the c.g.s. units to the natural standards of length and time, the wave-length of the red line of cadmium and the sidereal second.

§ 12. ACKNOWLEDGMENTS

I cannot conclude without expressing my indebtedness to many friends, and first and foremost to Sir Henry Lyons for the use of the Lecture room and most of the historic apparatus shown on the table, to the Deputy Warden of the Standards for apparatus in his charge, then to the Director of the N.P.L. and Messrs Sears and Dye for their valued help and to Mr Christelow for his care in preparing the slides, also to Mr Hope Jones for the slide of the Shortt Clock. For the part relating to ancient standards thanks are due to Col. N. Belaiew, C.B., to Mr Sidney Smith of the British Museum and to the Library Staff of the Royal Society who helped in looking up the references.

APPENDIX I

Report of the Royal Commission of 1818

Secretary, Dr Thos. Young, F.R.S.

First Report.

I. Upon a deliberate consideration of the whole of the system at present existing, we are impressed with a sense of the great difficulty of effecting any radical changes, to so considerable an extent as might in some respects be desirable; and we therefore wish to proceed with great caution in the suggestions which we shall venture to propose.

II. With respect to the actual magnitude of the standards of length, it does not appear to us that there can be any sufficient reason for altering those which are at present generally employed. There is no practical advantage in having a quantity commensurable to any original quantity existing or which may be imagined to exist in nature, except as affording some little encouragement to its common adoption by neighbouring nations. But it is scarcely possible that the departure from a standard, once universally established in a

great country, should not produce much more labour and inconvenience in its internal relations than it could ever be expected to save in the operations of foreign commerce and correspondence, which always are and always must be conducted by persons to whom the difficulty of calculation is comparatively inconsiderable, and who are also remunerated for their trouble, either by the profits of their commercial concerns, or by the credit of their scientific acquirements.

III. The subdivisions of weights and measures at present employed in this country appear to be far more convenient for practical purposes than the decimal scale, which might perhaps be preferred by some persons for making calculations with quantities already determined. But the power of expressing a third, a fourth, and a sixth of a foot in inches without a fraction, is a peculiar advantage in the duodecimal scale: and, for the operation of weighing and measuring capacities, the continual division by two renders it practicable to make up any given quantity with the smallest possible number of standard weights or measures, and is far preferable in this respect to any decimal scale. We would therefore recommend that all the multiples and subdivisions of the standard to be adopted should retain the same relative proportions to each other as are at present in general use.

IV. The most authentic standards of length which are now in existence being found, upon a minute examination, to vary in a very slight degree from each other, although either of them might be preferred, without any difference that would become sensible in common cases, we beg leave to recommend for the legal determination of the standard yard, that* which was employed by General Roy in the measurement of a base on Hounslow Heath, as a foundation for the trigonometrical operations that have been carried on by the Ordnance throughout the country, and a duplicate of which will probably be laid down on a standard scale by the Committee of the Royal Society appointed for assisting the Astronomer Royal in the determination of the length of the pendulum; the temperature being supposed to be 62° of Fahrenheit when the scale is employed.

V. We propose also, upon the authority of the experiments made by the Committee of the Royal Society, that it should be declared for the purpose of identifying or recovering the length of this standard, in case that it should ever be lost or impaired, that the length of a pendulum vibrating seconds of mean solar time in London, in the level of the sea, and in a vacuum, is 39·1372 inches of this scale; and that the length of the metre employed in France, as the 10,000,000th part of the quadrantal arc of the meridian, has been found equal to 39·3694 inches.

VI. The definitions of measures of capacity are obviously capable of being immediately deduced from their relations to measures of length; but since the readiest practical method of ascertaining the magnitude of any measure of capacity is to weigh the quantity of water which it is capable of containing, it would, in our opinion, be advisable in this instance to invert the more natural order of proceeding, and to define the measures of capacity rather from the weight of the water they are capable of containing, than from their solid content in space. It will therefore be convenient to begin with the definition of the standard of weight; by declaring that nineteen cubic inches of distilled water, at the temperature of 50°, must weigh exactly ten ounces Troy or 4800 grains, and that 7000 such grains make a pound avoirdupois; supposing, however, the cubic inches to relate to the measure of a portion of brass, adjusted by a standard scale of brass. This definition is deduced from some very accurate experiments of the late Sir George Shuckburgh on the weights and measures of Great Britain; but we propose at a future period to repeat such of them as appear to be the most important.

* But see later second report.

VII. The definitions thus established are not calculated to introduce any variation from the existing standards of length and of weight, which may be considered as already well ascertained. But with respect to the measure of capacity, it appears from the Report contained in the Appendix (A), that the legal standards of the highest authority are considerably at variance with each other; the standard gallon, quart, and pint of Queen Elizabeth, which are kept in the Exchequer, having been also apparently employed, almost indiscriminately, for adjusting the measures both of corn and of beer; between which however a difference has gradually, and, as it may be supposed, unintentionally crept into the practice of the excise; the ale gallon being understood to contain about four and half per cent. more than the corn gallon, though we do not find any particular Act of Parliament in which this excess is expressly recognized. We think it right to propose that these measures should again be reduced to their original equality; and at the same time, on account of the great convenience which would be derived from the facility of determining a gallon and its parts, by the operation of weighing a certain quantity of water, amounting to an entire number of pounds and ounces without fractions, we venture strongly to recommend that the standard ale and corn gallon should contain exactly ten pounds avoirdupois of distilled water of 62° Fahrenheit, being nearly equal to 277·2 cubic inches, and agreeing with the standard pint in the Exchequer, which is found to contain exactly 20 ounces of water.

VIII. We presume that very little inconvenience would be felt by the public from the introduction of this gallon in the place of the customary ale gallon of 282 cubic inches, and of the Winchester corn gallon, directed by a Statute of King William to contain 269, and by some later Statutes estimated at $272\frac{1}{4}$ cubic inches; especially when it is considered that the Standards by which the quart and pint beer measures used in London are habitually adjusted, do not at present differ in a sensible degree from the Standard proposed to be rendered general. We apprehend also that the slight excess of the new bushel above the common corn measure would be of the less importance, as the customary measures employed in different parts of Great Britain are almost universally larger than the legal Winchester bushel.

Appendix (*A*). The standards kept at the Exchequer, for the adjustment both of corn and beer measures, are a bushel, a gallon, and a quart, dated 1601, and a pint, dated 1602, all marked with an E. and a crown. They were examined by Sir George Clerk and Dr Wollaston, and the weight of Thames water which they held, at the temperature of 52°, was found as in the subjoined table. Now, since, according to Sir George Shuckburgh's experiments, a cubic inch of distilled water at 60° weighs $252\frac{1}{2}$ grains, the specific gravity of the water being to that of distilled water as 1·00060 to 1 and the apparent specific gravity of distilled water, in a vessel of brass at 52° being to that of water at 62° as 1·00046 to 1, it follows that the apparent specific gravity of the water employed was 1·00106, and that an ounce Avoirdupois corresponded to 1·731 cubic inches. Hence we obtain the contents of the measures in cubic inches, which are compared in the table with the more direct measurement of Mr Bird and Mr Harris reported to the House of Commons in 1758.

	Oz. Avoir.	Cub. in.	Gallon	Rep. 1758
Pint	20·00	34·6 (× 8 =)	276·9	34·8
Quart	40·35	69·8 (× 4 =)	279·3	70·0
Gallon	156·25	270·4 (=)	270·4	271·0
Bushel	1229·85	2128·9 (× $\frac{1}{8}$ =)	266·1	2124·0

The Exchequer standard wine gallon is dated 1707, and was found to contain 133·4 ounces, answering 230·9 cubic inches. An experiment of Dr Wollaston and Mr Carr, 1814, gave 230·8, the mean being 230·85; while the measurement of 1758 made it 231·2. A duplicate of this measure, and of the same date, is kept at Guildhall.

Dr Wollaston and Mr Carr examined also the three other wine gallons at Guildhall. The oldest of these seems to be the same that was measured by Halley and Flamsteed in 1688, and was said to contain 224 cubic inches: its actual capacity is 224·4. The wine gallon of 1773 which is in daily use for adjusting other measures, was probably in the first instance a correct copy of the Exchequer gallon, but has been reduced by a bruise and by the wear of the brim to 230·0 cub. inches, having lost $-\frac{4}{5}$ of a cub. inch or $\frac{1}{300}$ of its whole capacity. The wine gallon of 1798 contains 230·8 cub. inches.

The Excise wine gallon was found by a similar experiment to contain 230·1 cub. inches, having partaken of the progressive deficiency of the Guildhall gallon, from which it was derived."

Second Report.

We have examined, since our last Report, the relation of the best authenticated standards of length at present in existence, to the instruments employed for measuring the base on Hounslow Heath, and in the late trigonometrical operations; but we have very unexpectedly discovered that an error has been committed in the construction of some of these instruments. We are therefore obliged to recur to the originals which they were intended to represent, and we have found reason to prefer the Parliamentary Standard executed by Bird in 1760, which we had not before received, both as being laid down in the most accurate manner, and as the best agreeing with the most extensive comparisons which have been hitherto executed by various observers, and circulated throughout Europe; and, in particular, with the scale employed by the late Sir George Shuckburgh.

We have therefore now to propose that this Standard be considered as the foundation of all legal weights and measures, and that it be declared that the length of a pendulum vibrating seconds in a vacuum on the level of the sea, in London, is 39·13929 inches, and that of the French metre 39·37079 inches, the English standard being employed at 62° of Fahrenheit.

APPENDIX 2

GEORGE HOLT PHYSICS LABORATORY
THE UNIVERSITY OF LIVERPOOL

2nd April, 1931

THE bar of wood sent to me and described as an Egyptian cubit is of approximately rectangular section with one corner slightly bevelled.

The ends are rough cut and are not perpendicular to the length nor parallel to each other. Their greatest distance apart (measured along the length) is about 52·7 centimetres, and their least distance about 52·3 centimetres. The bar is subdivided into 7 parts by grooves cut through the bevel, and its *two* adjacent faces, and the middle part is divided into approximately equal segments by a similar groove.

The grooves are not accurately cut, and the distance between adjacent grooves therefore varies appreciably along their length. The last 4 of the 7 parts are each subdivided into 4 segments by grooves cut through the bevel and *one* of its adjacent faces.

The first of the 7 parts is considerably too long, varying from about 7·8 cm. to 7·6 cm. The next five are approximately equal, each varying from about 7·5 cm. to 7·4 or 7·3. The last part is considerably too short, measuring about 7·0 cm.

The 16 segments into which the last 4 parts of the bar are subdivided are very irregular in size, varying from about 2·0 cm. to 1·6 cm., except the last segment of all, which is much too short, varying from 1·5 cm. to 1·3 cm.

(*Signed*) L. R. WILBERFORCE

REVIEWS OF BOOKS

Theoretical Physics, Vol. 1, *Mechanics and Heat*, by Prof. W. Wilson, F.R.S. Pp. x + 332. (London: Methuen and Co.) 21*s*.

This is the first of three volumes which together will constitute a systematic treatise on theoretical physics, from Newtonian to the new quantum mechanics. The author's aim is to "present an account of the theoretical side of physics which, without being too elaborate and voluminous, will nevertheless be sufficiently comprehensive to be useful to teachers and students."

So far as can be judged from the single volume now before us, it may be said at once that Prof. Wilson is assured of conspicuous success in this aim, and that he is producing a book which will fill a want which has been long felt by university teachers of physics. Most teachers who have had to organize advanced degree courses have found it difficult to give adequate representation to both the "classical" and the "modern" sides of the subject (to use a distinction which may in itself be objectionable, but is in general pretty well understood). From the whole mass of physical theory and experiment, some form of selection and exclusion principle is imposed by limitations of time and human capacity, and a great deal of the classical physics is unavoidably relegated to the background, in spite of its intrinsic importance and its beauty. If this process is pushed too far, it is apt to have undesirable repercussions—more particularly on the young physicist who is destined for a career in industrial research, where he will learn by daily experience that the older physics has a value quite outside its mere antiquarian interest.

The purpose of this digression is simply to emphasize the service which a work like the one under review can render to students and teachers alike, by providing a single text in which the whole framework of the older and the newer physics is built up in a consecutive and homogeneous form. The treatment is not exhaustive—subjects like capillarity, diffusion and thermal conduction are wholly or largely omitted—but the book as a whole promises to provide just that basis which is required for the building up of a systematic knowledge of the fundamental principles and their more important applications.

There was most emphatically a need for a book of this type, and it is difficult to see how, within its prescribed limits, the present volume could have been improved upon, or how those limits could have been extended without detracting from a main purpose of the work. The author's experience as a teacher is in evidence throughout—not least in his accurate appreciation of the assistance required in purely mathematical difficulties by the average physics student. Many students, by the way, will probably find it instructive to compare his systematic derivations of formulae with the simpler, if less elegant, *ad hoc* proofs which abound in text-books of a more experimental character.

It is a pleasure to come across a book so well written, and one which can be so cordially recommended. The second and third volumes will be awaited with interest—even with some impatience.

H. R. R.

Conférences d'Actualités Scientifiques et Industrielles. (Paris: Hermann et Cie.) (x) "Température des Flammes," by G. RIBAUD. Pp. 43, 5 fr. (xi) "Anistropie des Molécules: Effet Raman," by JEAN CABANNES. Pp. 66, 8 fr. (xv) "Les Statistiques Quantiques et leur Application aux Électrons Libres dans les Métaux," by L. BRILLOUIN. Pp. 44, 5 fr. (xvii) "La Structure et les Mouvements de l'Univers Stellaire," by G. DAMOIS. Pp. 16, 3 fr.

Reports of four sets of interesting lectures, Nos. x, xi, xv, and xvii of the series *Conférences d'Actualités Scientifiques et Industrielles*, have recently come to hand. The first of these (x) is a set of two lectures on the temperature of flames and the radiation of flames and incandescent gases by Prof. G. Ribaud. The most interesting feature of these lectures is the verification that the temperature of the flame can be calculated very closely thermodynamically on the assumption that the gases supplied come (after combustion) adiabatically to thermodynamic equilibrium with the products of reaction. The second (xi) also is a set of two lectures by Prof. J. Cabannes on the anisotropy of molecules and the Raman effect. The author gives a very instructive account of the light that is thrown on the theory of molecular structure by a study of the depolarizing effect in the coherent scattered radiation and by a study of Raman lines. The third (xv) is a lecture by Prof. L. Brillouin on quantum theory statistics and its application to the electronic theory of metals. This gives a clear account of the origin and simple properties of the statistics of Bose-Einstein and Fermi-Dirac, and in the latter statistics, that obeyed by electrons, of the effect of a magnetic field in producing a weak temperature-independent paramagnetism, and of the way in which the properties of a degenerate electron gas suit the requirements of the theory of metals. Finally the last (xvii) is a single lecture by Prof. G. Damois on the structure and movements of the stellar universe. This gives an account of the modern work on the globular clusters and the speculations on the rotation of our own galaxy, a most interesting chapter of physics on the largest conceivable scale.

R. H. F.

Acoustics, a Text on Theory and Applications, by G. W. STEWART, Ph.D., and R. B. LINDSAY, Ph.D. Pp. ix + 358. (London: Chapman and Hall.) 25*s*.

In a recent presidential address to the Physical Society, Dr W. H. Eccles dealt with the subject of "The New Acoustics," and incidentally referred to the gap in the literature of the subject subsequent to the publication of the classical works of Rayleigh and Lamb. This gap is being rapidly filled. There are now on the market several good text-books dealing with recent progress in theory, practical measurements, and technical applications. In addition to these text-books there is also a number of books dealing with developments in more specialized branches of the subject. One is therefore surprised to read in the preface of the book under review that "the present work is *unique* in its stress on the important researches of the past decade and in the useful combination of material both from the theoretical and practical viewpoints."

The book is none the less welcome, as a large part of it deals with the original work of the authors. Chapters v, vi, vii and ix, which include sound-transmission through conduits, impedance theory of tubes and horns, filtration of sound, and physiological acoustics, are in this respect of particular interest and importance. In these branches of the subject Dr Stewart has contributed many important original papers which are now collected and summarized in an interesting manner. They will be read carefully by all students. Chapter ix, dealing with physiological acoustics, is unfortunately rather brief. One would have welcomed here more details of Dr Stewart's researches on binaural audition.

On the theoretical side the book fulfils the claims made in the preface, but the same can hardly be said with regard to the practical side. Chapters x (subaqueous sound-ranging and signalling) and xi (architectural acoustics) are examples of this. One cannot fail to notice the omission of references to important work recently carried out in this country. For example, the work of Drs A. H. Davis and E. T. Paris in architectural acoustics is almost completely ignored. In dealing with the methods of measuring absorption-coefficients of materials, the authors refer to Sabine's and Knudsen's full-scale methods only, no consideration being given to the testing of specimens of small area. The section dealing with the echo method of sounding at sea is particularly inadequate. Only a bare reference is made to Langevin's quartz-oscillator method of depth-recording, and no reference at all is made to the echo sounding system developed by the British Admiralty and in regular use on hundreds of ships. This weakness of the book on the practical side is revealed not only in the text but also in the illustrations, which are in almost every case simple line-diagrams to illustrate the theory rather than the method of practical application.

Apart from the description of Dr Stewart's method of measurement of acoustic impedance, Chapter viii, on acoustic instruments and measurements, is disproportionately short (14 pages). Only a few pages are devoted to intensity-measurement, some of the most important methods being reduced to mere references, and no reference is made to methods of sound-analysis and sound-recording or to measurements of frequency.

In spite of these criticisms, however, the book can be strongly recommended to students as an example of modern tendencies in the theoretical treatment of acoustic problems, the first seven chapters being particularly good in this respect.

The book concludes with an interesting chapter on atmospheric acoustics. This includes a section dealing with the general theory of the propagation of sound through the atmosphere and its application to sound-signalling in air. Tables of data and detailed mathematical processes are given in appendices. The index omits all reference to authors.

A. B. W.

THE PROCEEDINGS OF THE PHYSICAL SOCIETY

VOL. 43, PART 5 *September* 1, 1931 NO. 240

COHESION

BY J. E. LENNARD-JONES, The University, Bristol

A Lecture delivered before the Society on May 1, 1931.

§ 1. INTRODUCTION: NEW IDEAS IN PHYSICS

THERE are in nature, as in politics, two opposing forces. One of these aims at a peaceful consolidation and the other at a more active and probably more spectacular disruptive process. In nature it is cohesion between atoms which tends to produce condensation and solidification, and temperature which tends to produce dissociation, first of solids into liquids and then into isolated molecules, then of molecules into atoms and finally into electrons and protons (as in the hotter stars).

Temperature is a manifestation of kinetic energy and cohesion of potential energy, and the interplay of these two forms of energy is responsible for many of the observed physical properties of matter. A knowledge of the nature of cohesion is thus a necessary step towards an understanding of these physical properties. In this lecture I propose to review some of the progress which the new wave mechanics has made possible in this direction and, by way of introduction, to refer to some of the new conceptions which have been introduced during the last few years.

First, there is the main idea of wave mechanics that it is impossible to follow the electron in all its ways. Instead of supposing that the electron of an hydrogen atom steadily pursues for all time a definite track or orbit, we are now content to know the probability that it will do this or do that. We represent the electron by continuous distributions—probability patterns. To each such pattern there corresponds a definite average energy and normally the electron stays in the pattern of lowest energy. The density of the pattern at any point of space is a measure of the probability that the electron shall be found there. The probabilities become infinitesimally small outside regions measuring one hundred-millionth of a centimetre, so that the electrons of an atom may be regarded as almost always within this small

region. This is the interpretation to be given to the "size" of an atom from the new point of view*.

The mathematical theory does not give these density patterns directly. The wave equation gives the possible energy levels and certain mathematical functions, usually called wave functions and denoted by ψ, associated with them. The quantity here denoted by ψ is a function of the three coordinates used to specify the position of the electron in space. When ψ is a real quantity, its square is interpreted as a measure of the density of the probability pattern, just as the square of the amplitude (and not the amplitude itself) is a measure of the intensity of a sound or light wave. When ψ contains the square root of -1, the square of the amplitude of ψ, viz. $|\psi|^2$, or the product of ψ with its conjugate complex function, viz. $\psi\bar{\psi}$, is taken to be the appropriate measure of the probability of a given configuration.

§ 2. PAULI EXCLUSION PRINCIPLE

The hydrogen is particularly simple to work out because there is only one electron. It is equally easy to work out the patterns for one electron alone in the presence of a nucleus of charge Ze. The patterns are similar to those of hydrogen, but are contracted in the ratio 1 to Z everywhere.

If electrons are added one at a time to a nucleus of charge Ze until the whole structure is neutral and thus complete, we must inquire what happens to the electrons after the first. There is nothing in the wave mechanics, as formulated at present, to prevent all the electrons taking up the same pattern. The density of the electrons might be regarded as identical and superimposed, except for the electrostatic repulsion between them which would tend to swell out the pattern again.

We know from experience, however, that all the electrons of an atom do not assume the same pattern, and we have had to invoke an entirely new principle, first enunciated by Pauli, and now generally referred to as the *exclusion principle*. This principle asserts that there are never more than two electrons in the same pattern; it goes further by asserting that it is not sufficient to describe patterns in terms of the three spatial coordinates of an electron, but that a fourth coordinate must be introduced. This fourth coordinate can take only two discrete values, so that the patterns we have described up to now are to have a further property added to them which will double their number. It is as though the patterns had colour as well as shape. Actually it is usual to attribute a spin to the electron, and to suppose that the axis of the spin can take up either of two quantized directions. The exclusion principle then asserts that there is only one electron, described by its four coordinates, in every pattern. In building up atoms by the addition of one electron at a time, the electrons arrange themselves in those patterns which respect

* At this stage of the lecture, a number of diagrams of patterns of the excited states of the hydrogen atom were shown. It was intended to reproduce these, but since the lecture, a number of similar pictures have been published by H. E. White, *Phys. Rev.* **37**, 1416 (1931), to which reference may be made.

the exclusion principle, and at the same time take up the configuration of least energy. The application of this principle accounts in a very beautiful way for the main features of atoms and molecules.

§ 3. PRINCIPLE OF IDENTITY OF ELECTRONS

In this addition of electrons to an atom, we have tacitly assumed that each electron persists in one pattern. But so far as is known, electrons are identical, and a system in which two electrons are interchanged is indistinguishable from its original state. This principle of the *identity of electrons* has proved to be of great importance. A method must be found of introducing this fact into the mathematics, and this was first done about the same time by Dirac and Heisenberg.

If we suppose that there are N electrons in an atom and that the wave functions associated with the patterns which they occupy are $u_1, u_2, \ldots u_N$, these functions including spin as well as space coordinates, then the wave function of the whole system must be taken to be* N u_N

$$\Psi = \begin{vmatrix} u_1(1), & u_2(1), & \ldots & u_N(1) \\ u_1(2), & u_2(2), & \ldots & u_N(2) \\ \vdots & \vdots & \ldots & \vdots \\ u_1(N), & u_2(N), & \ldots & u_N(N) \end{vmatrix} \qquad \Psi$$

In this determinant $u_N(1)$ indicates that the four coordinates of electron 1 are to be introduced into the Nth function, and so on. Now if two electrons 1 and 2 are interchanged, the effect on the determinant is to interchange rows 1 and 2, and this has the effect of changing only the sign of Ψ. Furthermore, if we suppose two wave functions u_1 and u_2 to be the same, two columns of the determinant become identical and the determinant vanishes. The mathematical expression of the Pauli exclusion principle is thus that no two of the wave functions $u_1, \ldots u_N$ may be the same. The determinantal form of Ψ thus takes into account the identity of the electrons and the exclusion principle at the same time. $u_N(1)$

Actually when the electrostatic repulsions between electrons in an atom are taken into account, we cannot regard the wave function as made up of N different functions $u_1 \ldots u_N$, each containing only the coordinates of one electron. All that we can be certain of is that the complete wave function, when found, must have the same properties as the determinant as regards the interchange of electrons and the exclusion principle. It has not been possible, however, as yet to deal mathematically with functions more complicated than the determinant.

The probability distribution of the N electrons is now given by $\Psi\overline{\Psi}$, which

* This is subject to the condition that there be not several functions of this type with the same energy. It is possible to write the wave function in the form given above for most atoms in their state of lowest energy.

is a generalization of the probability distribution for one electron. When the two determinants Ψ and $\overline{\Psi}$ are multiplied together, we find

$$\Psi\overline{\Psi} = \begin{vmatrix} \rho(1,1), & \rho(1,2), & \dots & \rho(1,N) \\ \rho(2,1), & \rho(2,2), & \dots & \rho(2,N) \\ \vdots & \vdots & \dots & \vdots \\ \rho(N,1), & \rho(N,2), & \dots & \rho(N,N) \end{vmatrix}$$

$\rho(1,2)$ where* $$\rho(1,2) = \sum_j \bar{u}_j(1)\, u_j(2).$$

We note that $\rho(1,1)$ is the sum of terms like $\bar{u}_j(1)\, u_j(1)$ which are density functions of electron 1 in the different patterns; in fact, we have

$$\rho(1,1) = \sum_j \rho_j(1,1),$$

and we should be justified in superimposing the density patterns if it were not for the terms like $\rho(1,2)$. These indicate to us that we must no longer expect to express the charge density in three dimensions (or four, including spin), but we must use a six-dimensional function (or an eight-dimensional one if we include spin). This is not surprising. In the older theories, when electrons were assumed to be pursuing orbits, we needed a knowledge not only of space coordinates but also of the components of velocity of the electrons to specify their configuration completely. Now that we have given up the idea of attributing definite velocities and space coordinates to electrons simultaneously, we must not be surprised to find the six-dimensional character of the probability configuration appearing in another form.

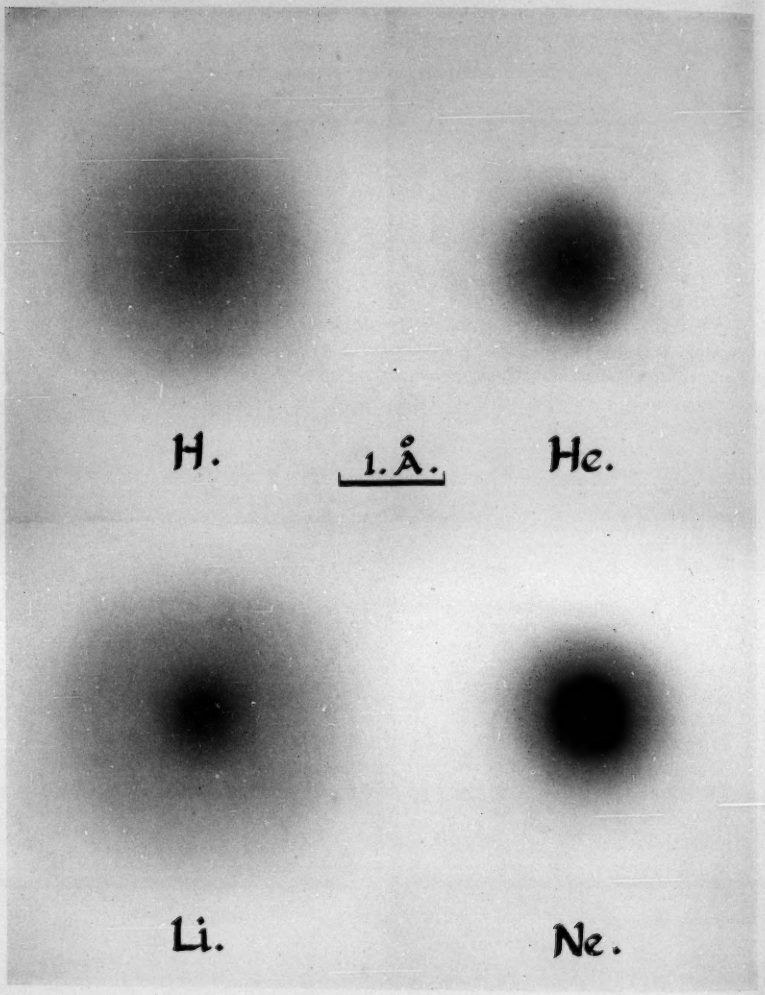
§4. ELECTRON-DISTRIBUTIONS IN ATOMS

The expression given above for $\Psi\overline{\Psi}$, multiplied by elements of volume $d\tau$ $d\tau_1, d\tau_2, \dots d\tau_N$, in the configuration space of each electron, may be interpreted as the probability that an electron (we do not specify which one) shall be found in $d\tau_1$, another in $d\tau_2$, and so on simultaneously†. The probability that an electron shall be found in $d\tau_1$, irrespective of the position of the other electrons, is obtained by integrating $\Psi\overline{\Psi}$ over the configuration space of electrons 2 to N. In the integration each electron configuration is to be counted only once, so that if $d\tau'$ indicates one specified element of volume and $d\tau''$ another, $d\tau_1'\, d\tau_2''$ is to be taken to be the same as $d\tau_1''\, d\tau_2'$. The result of the integration is $\rho(1,1)\, d\tau_1$ or $\sum_j \rho_j(1,1)\, d\tau_1$, and is thus the same as the superposition of the separate patterns ρ_1 to ρ_N‡. In this

* This form of the function $\Psi\overline{\Psi}$ has been given recently by Dirac, *Proc. Camb. Phil. Soc.* **27**, 240 (1931); cf. also Lennard-Jones, *Proc. Camb. Phil. Soc.* **27**, 469 (1931).

† P. A. M. Dirac, *loc. cit.*

‡ This result holds when the functions u_1 to u_N are normalized and orthogonal, that is, $\int \bar{u}_k u_l d\tau = \delta_{kl}$. For most atoms in their normal state, the wave functions can be adjusted to satisfy this condition.



To face p. 465]

sense we may construct pictures of many-electron atoms, and these pictures represent the probability of finding *an* electron in any element of volume in ordinary space.

When the process is carried out, it appears that many atoms—more than were previously suspected—may be considered to be spherically symmetrical. Of the first ten atoms of the periodic table, six may be so regarded. They are hydrogen, helium, lithium, beryllium, nitrogen* and neon. Diagrammatic pictures of some of these atoms are given on the accompanying plate†. They have been calculated from some approximate wave functions given by Slater‡. The pictures cannot be regarded as accurate but they give a fair idea of the relative "spread" of the various atoms. The inert gas atoms, occurring at the end of a group of the periodic table, are usually the most compact atoms of that group.

The spread of these atoms may be gauged also from figure 1. The contours give the probability of finding an electron *outside* the contour. The first is 0·2—that is, the probability of finding an electron outside it is 1 in 5. The second, third... are 0·4, 0·6.... Outside the contour 1·0, there is a *certainty* of finding one electron; outside the contour 1·2 there is a probability of finding two electrons simultaneously. The spread of lithium, which combines with others in the solid state to form a metal, is in striking contrast to that of neon, although the latter contains many more electrons. Boron in its normal state has one electron in a $2p$-state, or one with quantum numbers $n = 2$, $l = 1$, $m = 1$, 0 or -1. The corresponding probability patterns are asymmetrical. The one with quantum numbers 2, 1, 0 consists of two blobs of electron charge situated symmetrically with respect to the nucleus. The plane of symmetry contains no charge, while a line through the nucleus perpendicular to this plane of symmetry passes through the centres of the blobs. Along this line the electron charge increases from zero to a maximum and then falls away to zero§. This line may be referred to as the axis of the pattern. Those with quantum numbers (2, 1, 1) and (2, 1, -1) are like anchor rings. The concentration of charge in each is zero along the axis, and has a maximum along the core, from which it decreases to zero in each direction. The axis of the ring coincides with the axis of the (2, 1, 0) pattern so that the maximum concentration of the (2, 1, 1) pattern lies in the plane of zero concentration of the (2, 1, 0) pattern.

Carbon in its normal state may be regarded as having one of its two outer electrons in the (2, 1, 0) pattern and the other in either the (2, 1, 1) or the (2, 1, -1) pattern, while the two electron spins are the same‖. The electron distribution thus consists of an anchor ring and two "balls" of charge on its axis above and below the nucleus, all of which is superimposed on a background due to the inner electrons, which is spherically symmetrical.

* In its normal state, that is, in the ^4S state.

† I am greatly indebted to Mr H. H. M. Pike of the H. H. Wills Physical Laboratory, Bristol, for these pictures, as also for those in figure 1, and for the calculations involved in making them.

‡ J. C. Slater, *Phys. Rev.* **36**, 57 (1930).

§ Cf. H. E. White, *loc. cit.*

‖ The normal state of the carbon atom is a ^3P state.

Nitrogen has one outer electron in a (2, 1, 0) pattern, another in a (2, 1, 1) pattern and a third in a (2, 1, − 1) pattern. These three patterns when superimposed give a distribution which is spherically symmetrical. The spins of these three outer electrons in the normal state of the nitrogen atom are the same, for this configuration gives the lowest energy (the ^4S state).

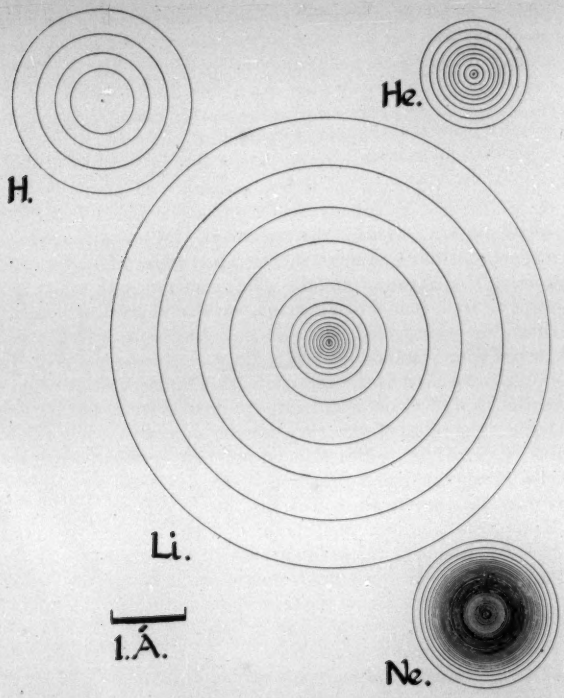


Fig. 1. The "spread" of the atoms hydrogen, helium, lithium and neon.

Neon, which has six electrons in the 2*p*-state, has two electrons, one of each spin, in each of the patterns (2, 1, 1), (2, 1, 0) and (2, 1, − 1). It also is spherically symmetrical. Flourine has one less electron than neon, and so we may regard it as an atom which requires one electron to complete its spherical symmetry*. It is an atom with a "hole" in it, and the "hole" is probably a (2, 1, 0) pattern†.

* This property is connected with the affinity which fluorine has for an electron.

† Cf. J. E. Lennard-Jones, *Trans. Faraday Soc.* **25**, 668 (1929).

Oxygen has two "holes," one of which is probably a (2, 1, 0) and the other a (2, 11), or a (2, 1, − 1) pattern. Like divalent carbon, its normal state is a ^3P state, though now this state may be attributed to two *holes*, whereas in carbon it is due to the two *electrons*. In this sense oxygen may be regarded as the counterpart of divalent carbon.

These pictures of atoms throw considerable light on their chemical properties and elucidate the problem of molecular structure, to which we shall refer below.

§ 5. PRINCIPLE OF MINIMUM ENERGY

To the new principles already discussed, we must add an old one, viz., the principle of minimum energy; an atomic or molecular system tends to take up the state of lowest energy. This is of fundamental importance in the subject of cohesion.

§ 6. TYPES OF COHESIVE FORCES

The physical and chemical properties of matter show that cohesive forces fall into certain definite categories. Helium must be cooled down to − 269° C. before it begins to aggregate, whereas hydrogen exists in the diatomic form at ordinary temperatures. All the rare gases are similar to helium in exhibiting this weak cohesion. Neon liquifies at about − 240° C. and argon at − 186° C. The weak attractive fields of these gases is responsible for their small departure from the ideal gas laws, and as they were first investigated by van der Waals, they may conveniently and fittingly be referred to as *van der Waals attractive fields*. This type of attraction probably exists between all atoms and molecules, as will appear in the next paragraph, but it is usually masked by other large attractive fields of a different type.

The cohesive forces which hold together the atoms of a hydrogen or a nitrogen molecule are about a thousand times as great as those which two helium atoms exert on each other, even when they are neighbours in the solid form. These forces are the familiar *homopolar attractive fields* of the chemist. At the same time it should be pointed out that the *molecules* of hydrogen and nitrogen behave like the inert gases as regards liquefaction and solidification and it is almost certain that such molecules are held together in the condensed form by van der Waals forces. It requires only about 500 calories to evaporate one gram molecule of solid hydrogen, whereas 100,000 calories per gram molecule are required to dissociate 1 gram molecule of molecular hydrogen.

There is distinct evidence that in some solids, such as NaCl, there is a migration of an electron from one atom (such as an alkali) to another atom (such as a halogen). The atom which loses an electron becomes positively charged and the other negatively charged. There is thus a net electrostatic attraction between neighbouring atoms and this produces a very firm structure, which is difficult to disrupt. When

such solids are vaporized (usually at high temperatures) the vapour is diatomic. Such molecules may be said to be held by *ionic attractive fields*.

There are other solids which are also difficult to melt and to vaporize, but which stand in a different class from the ionic solids or salts. They have other properties not possessed by the salts in that they are good conductors of heat and electricity and, so far as is known, are monatomic in the vapour state. These solids, the metals, stand in a class by themselves and the forces holding them together may be called *metallic attractive fields*.

Intimately connected with the subject of cohesion is the question why atoms ever repel each other. The very existence of matter leads us to postulate that when atoms approach very near to one another they begin to repel. The resistance which solids offer to compression is evidence and indeed a measure of the repulsive forces between atoms. A theory which explains cohesion may be expected also to explain *intrinsic repulsive fields* and this proves to be the case.

§7. THE NATURE OF VAN DER WAALS FIELDS

Although it is sufficient for many purposes to represent electrons in atoms as continuous distributions, such representations are not, of course, accurate. It is only the average which is continuous, and there must be rapid fluctuations about this average, corresponding to the motion in a classical sense of the electrons within the atom. Without it, van der Waals fields, as we know them, would not exist. Two hydrogen atoms in their normal state, for example, are each represented by a spherically symmetrical distribution, and as each is neutral, the electrostatic potential of the one distribution on the other at a large distance apart is vanishingly small. Nor can van der Waals fields be interpreted in terms of a static disturbance of the continuous distribution. These fields have often been attributed vaguely to polarization. But this polarization cannot be a static one, for if it were, each atom so polarized could be represented by a small permanent dipole at its centre. As the interaction in the case of equal atoms is symmetrical, the dipoles produced must be symmetrical about a plane midway between them. They must therefore point either towards or away from each other. In either case they cause repulsion. Van der Waals fields, as we shall show, are due to a *dynamic* polarization. The motion of the electrons in one atom modifies that of the electrons in the other in such a way that *they tend on the average to move in phase*. An important consequence is that van der Waals fields to a close approximation are additive, even when one is surrounded symmetrically by several others. Although there was evidence from many sources that these fields were additive, this property could not be deduced from a theory of static polarization.

The nature of van der Waals fields may probably be brought out most clearly by means of a simple molecular model. Suppose that an atom is represented by a linear oscillator, that is, by an electron vibrating linearly about the nucleus. Such a

system requires only one coordinate z to specify it. The appropriate Schrödinger equation is then z

$$\frac{\partial^2\psi}{\partial z^2} + \frac{8\pi^2 m}{h^2}(E - \tfrac{1}{2}kz^2)\psi = 0, \qquad E$$

where m is the mass of the electron, and $\frac{1}{2}kz^2$ is the potential energy of the electron. m, k
On the classical theory, an oscillator of this type, subject as it is to a restoring force of kz, would have a frequency given by

$$2\pi\nu_0 = \sqrt{k/m}. \qquad \nu_0$$

The solution of the wave equation is well known. It consists of a set of discrete values for E, viz., $E = (n + \frac{1}{2})h\nu_0$, where n is any integer. In the state of lowest n
energy, the appropriate value of ψ is given by

$$\psi = (2\kappa\nu_0/\pi)^{\frac{1}{4}} e^{-\kappa\nu_0 z^2}$$

where $$\kappa^2 = 2\pi m/h. \qquad \kappa$$

The equation is written in such a form as to bring out the dependence of ψ on the frequency ν_0 for a purpose which will be seen later.

The "smeared-out" pattern for the electron in this case is thus a Gaussian error curve, symmetrical about the origin.

We suppose now the oscillator to be subject to a uniform electric field F, so F
that the potential energy becomes $\frac{1}{2}kz^2 - eFz$ or $\frac{1}{2}k(z - eF/k)^2 - e^2F^2/2k$. The appropriate wave equation can be written

$$\frac{\partial^2\psi}{\partial\xi^2} + \frac{8\pi^2 m}{h^2}\left(E + \frac{e^2F^2}{2k} - \tfrac{1}{2}k\xi^2\right)\psi = 0,$$

which has the same form as the original equation, except that ξ, which $= z - eF/k$, ξ
now takes the place of z.

The energy values are $E = (n + \frac{1}{2})h\nu_0 - e^2F^2/2k$. The last term gives the change of energy due to the field—the so-called polarization energy.

$$\textit{Polarization energy} = -e^2F^2/2k.$$

In terms of the coefficient of polarizability α, the polarization energy is $-\frac{1}{2}\alpha F^2$ α
by definition, so that $\alpha = e^2/k$.

The corresponding density pattern is proportional to $e^{-2\kappa\nu_0(z-eF/k)^2}$, which is the same as the original one, except for a displacement of the maximum through an amount eF/k in the direction of the field. This represents a static polarization.

We next suppose the oscillator to be subject to the field of a distant parallel rigid dipole of strength μ at a distance R. The potential of the electron is then μ, R
$-2\mu ez/R^3$ and is like a parallel field except that the constant of proportionality depends on R. The theory of the preceding paragraph applies if F is replaced by $2\mu/R^3$, and so the energy of the oscillator, being proportional to the square of F, is now proportional to μ^2R^{-6}.

$$\textit{Polarization energy} = -2e^2\mu^2/kR^6.$$

Moreover, the energy being negative indicates attraction. The polarization of the oscillator is again a static one.

If now we suppose the dipole μ to fluctuate in value and to change in sign independently of the original oscillator (though this is an artificial assumption), the polarization of the oscillator is always such as to give an attraction proportional to R^{-6}; in fact, we have

$$\textit{Average polarization energy} = -2e^2\overline{\mu^2}/kR^6.$$

The polarization is now a fluctuating one and may be described as a *dynamic* one.

Actually the second dipole cannot be regarded as fluctuating independently of the first. It also will be subject to a dynamic polarization, and the two systems must be regarded as one complete coupled system. The method, however, prepares the way for a more accurate treatment* and suggests that two such oscillators will tend to move in phase.

The complete wave equation for two equal oscillators, each vibrating along the axis of z under each other's influence, is

$$\frac{\partial^2\psi}{\partial z_1^2}+\frac{\partial^2\psi}{\partial z_2^2}+\frac{8\pi^2 m}{h^2}\left(E-\tfrac{1}{2}kz_1^2-\tfrac{1}{2}kz_2^2+\frac{2e^2z_1z_2}{R^3}\right)\psi=0,$$

and this can easily be transformed to new variables

ξ_1, ξ_2

$$\xi_1=(z_1+z_2)/\sqrt{2},\quad \xi_2=(z_1-z_2)/\sqrt{2},$$

$$\frac{\partial^2\psi}{\partial\xi_1^2}+\frac{\partial^2\psi}{\partial\xi_2^2}+\frac{8\pi^2 m}{h^2}\left(E-\tfrac{1}{2}k_1\xi_1^2-\tfrac{1}{2}k_2\xi_2^2\right)\psi=0,$$

k_1, k_2 where $$k_1=k-2e^2/R^3,\quad k_2=k+2e^2/R^3.$$

The possible energy values of the equation are

n_1, n_2 $$E=(n_1+\tfrac{1}{2})\,h\nu_1+(n_2+\tfrac{1}{2})\,h\nu_2,$$

where

ν_0, ν_1, ν_2 $$\nu_1=\frac{1}{2\pi}\sqrt{(k_1/m)}=\nu_0\sqrt{(1-2e^2/kR^3)},\quad \nu_2=\nu_0\sqrt{(1+2e^2/kR^3)}.$$

This transformation is accurate and valid as long as ν_1 is real, that is as long as $2e^2/kR^3<1$. This implies that R must be greater than $(2\alpha)^{\frac{1}{3}}$. For the state of lowest energy, we thus get

$$E=\tfrac{1}{2}h(\nu_1+\nu_2)$$
$$=h\nu_0(1-\epsilon^4/2k^2R^6),$$

if the square roots be expanded, as they can for large R. This indicates that the energy of interaction of the dipoles, or the

$$\textit{van der Waals polarization energy} = -h\nu_0\epsilon^4/2k^2R^6,$$

and that there is an attractive *force* proportional to the inverse seventh power of the distance.

* Cf. F. London, *Zeit. f. phys. Chem.* **11**, 222 (1930).

The wave function for the whole system for this lowest state is equal to the product of the appropriate functions in ξ_1 and ξ_2, viz.,

$$\begin{aligned}\psi &= (4\kappa^2\,\nu_1\nu_2/\pi^2)^{\frac{1}{4}}\, e^{-\kappa(\nu_1\xi_1^2+\nu_2\xi_2^2)} \\ &= (2\kappa/\pi)^{\frac{1}{2}}\,(\nu_1\nu_2)^{\frac{1}{4}}\, e^{-\kappa\nu_0(z_1^2+z_2^2)}\, e^{(\kappa\nu_0 e^2/k)(2z_1z_2/R^3)} \\ &= (\nu_1\nu_2/\nu_0^2)^{\frac{1}{4}}\,\psi_0\, e^{-(\kappa\nu_0/k)v},\end{aligned}$$

where ψ_0 is the corresponding wave function before interaction, and v is the potential of interaction $(-2e^2 z_1 z_2/R^3)$. ψ_0, v

The probability of a specified configuration, that is, a specified z_1 and a specified z_2, is given by the square of ψ, and is thus proportional to $\psi_0^2\, e^{-\beta v}$. When v is negative (indicating attraction between the dipoles), the probability is greater than ψ_0^2 and when v is positive (indicating repulsion between the dipoles), the probability is less than ψ_0^2.

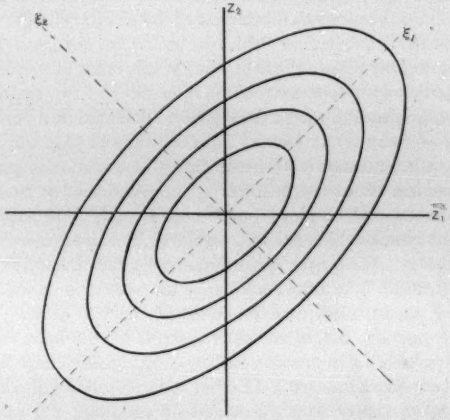


Fig. 2. The probability function for two interacting linear oscillators.

This result can be represented diagrammatically in a two-dimensional space, as shown in figure 2. The distribution function, being proportional to $e^{-2\kappa(\nu_1\xi_1^2+\nu_2\xi_2^2)}$, is like a ridge with its maximum at the origin, and with elliptical contours. The major axes of the ellipses are $\xi_2 = 0$ or $z_1 = +z_2$, and the minor axes $z_1 = -z_2$. The corresponding distribution without interaction is a round-topped mountain with circular contours. The probability of finding z_1 and z_2 with the same sign has increased, while that of finding them with the opposite sign has decreased. In other words, the interacting dipoles tend on the average to move in phase.

A similar treatment can be given for two three-dimensional oscillators*. Let the line joining the centres of the oscillators be in each case the axis of z as before,

* F. London, *loc. cit.*

x, y and let any convenient parallel sets of axes x and y be chosen in planes perpendicular to this axis through the respective centres. The mutual potential energy of the oscillators is then

$$V = e^2 (x_1 x_2 + y_1 y_2 - 2z_1 z_2)/R^3.$$

The probability-distribution function can now be represented only in six-dimensional space. It appears that z_1 and z_2 tend to have the same sign (as before), while x_1 tends to be opposite to x_2, and y_1 opposite to y_2. These are just the configurations for which the oscillators attract. Hence we may say that two systems tend to interact in such a way that the attraction is a maximum. It is in the generalized sense that we use the expression "tend to move in phase," for actually in this example the x's and y's tend to be out of phase in the usual sense.

These simple examples, which can be worked out accurately, serve to bring out the essential nature of van der Waals fields, and we may presume that in other more complicated cases, where the mathematics is more difficult to interpret, the attraction is due to a *sympathetic* movement of the electric charge.

The mathematical calculations for actual atoms are not easy. Wang* was the first to attempt the calculation of the van der Waals fields of two hydrogen atoms. He showed that the asymptotic form of the interaction of two hydrogen atoms was like that of two dipoles, and used this additional potential as a perturbation of the wave equation of two normal atoms. He thus showed that the attractive force between them at large distances varied inversely as the seventh power of the distance. The numerical value has not, however, been confirmed by later investigations. Recently Eisenschitz and London† have given a method of calculating fields of this type for any atoms, which is based on the second approximation of the usual perturbation theory. (The first approximation of this theory vanishes at large interatomic distances). The wave function of the system is expressed in terms of certain selected unperturbed wave functions of the two atoms. The method is rather unwieldy and the authors themselves expressed the hope that some simpler method would be found. The problem has also been considered by Lennard-Jones‡, Hassé§, and Slater and Kirkwood‖. The first of these authors uses a modified form of the perturbation theory which considerably simplifies the calculation of the van der Waals fields of two hydrogen atoms, while the second two use specially adapted variation methods. In each case the *potential* of the attractive field is of the form $-\lambda R^{-6}$, and the values obtained for λ are as follows:

Eisenschitz and London ...	$6{\cdot}04.10^{-60}$
Lennard-Jones	$6{\cdot}04.10^{-60}$
Hassé	$6{\cdot}05.10^{-60}$
Slater and Kirkwood	$6{\cdot}05.10^{-60}$

These values of λ give the attraction in ergs when R is measured in centimetres.

* S. C. Wang, *Phys. Zeit.* **28**, 663 (1927).
† R. Eisenschitz and F. London, *Zeit. f. Phys.* **60**, 491 (1930).
‡ J. E. Lennard-Jones, *Proc. R.S.* A, **129**, 598 (1930).
§ H. R. Hassé, *Proc. Camb. Phil. Soc.* **27**, 66 (1931).
‖ J. C. Slater and Kirkwood, *Phys. Rev.* **37**, 682 (1931).

The calculations for hydrogen atoms are valid, of course, only at large interatomic distances. At smaller distances the exchange phenomenon, referred to below, comes into prominence, and leads eventually to the formation of the diatomic molecule. The exchange forces, however, fall off very rapidly (as $e^{-\alpha r}/r$) and become small at distances at which the van der Waals fields are still important. It is unfortunate that in this case, where the theoretical calculations can be carried out with some accuracy, comparison with experiment is not possible.

An attempt has been made to extend the theory to more complicated atoms, but the main difficulty here is the lack of knowledge of the wave functions of these atoms. Even in the case of helium, the wave functions are only known approximately. London* has attempted to correlate the van der Waals fields of atoms with their coefficient of polarizability, but the correlation is only an approximate one, and only between certain rather wide limits can the magnitude of the fields be fixed by this method.

Hassé† has attempted the case of two helium atoms and has based his work on wave functions deduced by Hylleras‡ from a variation method. These wave functions are adjusted to make the energy of the helium electronic system a minimum and it does not follow that wave functions so deduced are valid over their whole range; in fact, it appears that they are probably fairly accurate near the nucleus (where the energy contributions are large) but less accurate in the outer parts of the atom. But it is just the outer parts of the atom which are most affected by outside fields, and so it is important that the wave function in those regions should be known accurately. Hassé finds that the Hylleras functions, which approximate best to the correct ionization potential of helium, do not give the best results for the polarization energy in a uniform electric field, which also is known experimentally. Similar calculations have been carried out for the van der Waals fields, but again there is a certain arbitrariness owing to the uncertainty of the Hylleras functions. The results obtained by Hassé are given below.

Slater and Kirkwood§ have given a further development of Hassé's method, also depending on the variation method. They work out the polarizability of helium and the van der Waals fields of two helium atoms by means of an approximate wave function, previously given by Slater‖. The results obtained so far for the attractive-force-constant of helium (λ in λR^{-7}) are as follows:

	λ
London	$7{\cdot}44.10^{-60}$¶
Hassé	$(7{\cdot}91–8{\cdot}21)\ 10^{-60}$
Slater and Kirkwood	$8{\cdot}94.10^{-60}$

Slater and Kirkwood give as well an expression for the field of two helium atoms at *close* distances. The potential curve obtained from their work is plotted in figure 3.

* F. London, *Zeit. f. phys. Chem.* **11**, 222 (1930).
† H. R. Hassé, *Proc. Camb. Phil. Soc.* **27**, 66 (1931).
‡ E. A. Hylleras, *Zeit. f. Phys.* **54**, 347 (1929).
§ J. C. Slater and Kirkwood, *Phys. Rev.* **37**, 682 (1931).
‖ J. C. Slater, *Phys. Rev.* **32**, 349 (1928).
¶ This figure is given as an upper limit, and is deduced from the formula given in the next paragraph, the observed value of the coefficient of polarizability being used.

Other inert gas atoms have been considered but the results can only be estimated roughly because the wave functions are not known sufficiently well. Both London and Slater-Kirkwood, unable to calculate the force-constants of these gases directly, attempt to correlate the force-constant with the coefficient of polarizability. Slater and Kirkwood give

$$\lambda = (\text{const.})\, N^{\frac{1}{2}} \alpha^{\frac{3}{2}},$$

while London gives

$$\lambda = (\text{const.})\, I\, \alpha^2.$$

λ, α In each case λ is the constant of van der Waals field, and α is the coefficient of
N polarizability; in the first formula N is the number of electrons in the outer shell,
I and in the second I is the ionization potential. It is not known yet how far either formula is correct.

The attractive force constants of gases can also be determined from a study of the equation of state, as has been shown by the author*. A collected account of the methods used and the results obtained has already been published†, but some new results, which have been worked out recently on the assumption of an attractive force of the type λR^{-7}, may appropriately be given here‡. The repulsive field, which
$\lambda_{(\text{rep.})}$, n comes into play at short distances, is represented by a force of the type $\lambda_{(\text{rep.})} R^{-n}$. Theoretical calculations of the type discussed in the next paragraph show that the repulsive field is more complicated than this and contains terms of the form e^{-aR}, but it falls off very rapidly with distance and can (in the case of helium at any rate) be represented, over the range which is most effective in atomic collisions, by a term of the type $\lambda_{(\text{rep.})} R^{-n}$.

The equation of state alone does not determine the value of the index n in this repulsive field uniquely, but, for a given n, it determines the value of the
$\lambda_{(\text{att.})}$ constant $\lambda_{(\text{rep.})}$, and the attractive constant $\lambda_{(\text{att.})}$. Other methods have to be employed to single out the right value of n from the array of possible ones thus determined. The crystal spacing of the solidified gas or its heat of sublimation may be used for this purpose. In the accompanying table the results are given for $n = 10$, 11 and 13. The experimental material from which these results are derived is cited elsewhere§, and later experimental results of Nijhoff‖, and of Gibby, Tanner and Masson¶ have been used as well. In table 1 are given calculated values of the closest distance of approach which these gases would have at absolute zero, if they set in the form of face-centred cubes. This is the form which would be expected theoretically for a force of the type $\lambda_{\text{rep.}} R^{-n} - \lambda_{\text{att.}} R^{-7}$,

* J. E. Lennard-Jones, *Proc. R.S.* A, **106**, 463 (1924), **107**, 157 (1925), **109**, 481 (1925), **112**, 214 (1926); J. E. Lennard-Jones and W. R. Cook, *Proc. R.S.* A, **115**, 334 (1927).

† J. E. Lennard-Jones, chap. x of *Statistical Mechanics*, by R. H. Fowler (Camb. Univ. Press, 1929).

‡ The author gratefully acknowledges the help of Miss M. J. Littleton (H. H. Wills, Physical Laboratory, Bristol) in the numerical calculations.

§ J. E. Lennard-Jones, chap. x of *Statistical Mechanics* by R. H. Fowler (Camb. Univ. Press, 1929).

‖ G. P. Nijhoff, *Dissert.* (Leiden, 1928).

¶ C. W. Gibby, C. C. Tanner and I. Masson, *Proc. R.S.* A, **122**, 283 (1929).

Table 1. Calculated force constants of gases from equation of state

	$\lambda_{(rep.)}$		$\lambda_{(att.)}$	Calculated closest distance in crystal (A.U.)	Calculated heat of sublimation at abs. zero in cals/gm. mol.	Observed crystal spacing	Observed heat of sublimation*
Helium	$n = 10$	$4{\cdot}38.10^{-82}$	$13{\cdot}8.10^{-60}$	3·02	106	—	—
	$n = 11$	$8{\cdot}94.10^{-90}$	$10{\cdot}3.10^{-60}$	2·93	114	—	—
	$n = 13$	$4{\cdot}55.10^{-105}$	$7{\cdot}43.10^{-60}$	2·83	126	—	—
Neon	$n = 10$	$2{\cdot}94.10^{-81}$	$8{\cdot}22.10^{-59}$	3·14	499		
	$n = 11$	$6{\cdot}285.10^{-89}$	$6{\cdot}22.10^{-59}$	3·04	542	3·20†	590
	$n = 13$	$4{\cdot}36.10^{-104}$	$5{\cdot}07.10^{-59}$	2·99	612		
Argon	$n = 10$	$7{\cdot}19.10^{-80}$	$10{\cdot}5.10^{-58}$	3·90	1730		
	$n = 11$	$2{\cdot}10.10^{-87}$	$8{\cdot}32.10^{-58}$	3·83	1847	3·84	2030
	$n = 13$	$2{\cdot}13.10^{-102}$	$6{\cdot}50.10^{-58}$	3·74	2030		
Hydrogen	$n = 10$	$4{\cdot}59.10^{-81}$	$1{\cdot}07.10^{-58}$	3·33	450	—	—
	$n = 11$	$1{\cdot}16.10^{-88}$	$0{\cdot}85.10^{-58}$	3·28	478	—	—
	$n = 13$	$7{\cdot}79.10^{-104}$	$0{\cdot}63.10^{-58}$	3·18	529	—	—
Nitrogen	$n = 10$	$1{\cdot}19.10^{-79}$	$13{\cdot}6.10^{-58}$	4·23	1380		
	$n = 11$	$3{\cdot}93.10^{-87}$	$11{\cdot}1.10^{-58}$	4·17	1460	4·0‡	1860
	$n = 13$	$4{\cdot}44.10^{-102}$	$8{\cdot}38.10^{-58}$	4·06	1640		

as it is the cubic form of least potential energy§. Argon‖ and neon¶ have already been shown experimentally to exist in this form. The calculated values of the heat of sublimation at absolute zero from this crystal structure are also given. The force constants of neon are a little uncertain because the experimental results of Holborn and Otto and those of Cath and Kamerlingh Onnes are not very consistent. The figures given are obtained by taking both sets of experimental points and adjusting the theoretical curves to fit as well as possible. Figure 3 shows the curves of potential energy of pairs of inert gas atoms as a function of their distance apart for the model $n = 13$, $m = 7$.

It is probable that many other gases are held together in the liquid and solid state by van der Waals forces, for, as London** has pointed out, many cases are known where the heat of sublimation is of the same order of magnitude as those given in the above table. CH_4, HCl, HBr, CO may be quoted as examples. Forces of this type are probably responsible also in certain cases for the adsorption of

* For reference see F. London, *Zeit. f. phys. Chem.* **11**, 240 (1930).
† J. de Smedt, W. H. Keesom and H. H. Mooy, *Proc. Amst. Acad.* **33**, 255–257 (1930).
‡ Vegard, *Nature*, **124**, 267, 337 (1929), molecules being regarded as spheres.
§ J. E.. Lennard-Jones and A. E. Ingham, *Proc. R.S.* A, **107**, 636 (1925).
‖ F. Simon u. C. Simson, *Zeit. f. Phys.* **25**, 160 (1924).
¶ J. de Smedt, W. H. Keesom and H. H. Mooy, *loc. cit.*
** F. London, *Zeit. f. phys. Chem. loc. cit.*

gases on solid surfaces. The author has shown that the order of magnitude of certain observed heats of adsorption can be explained in this way*. Other cases have been worked out recently by London†.

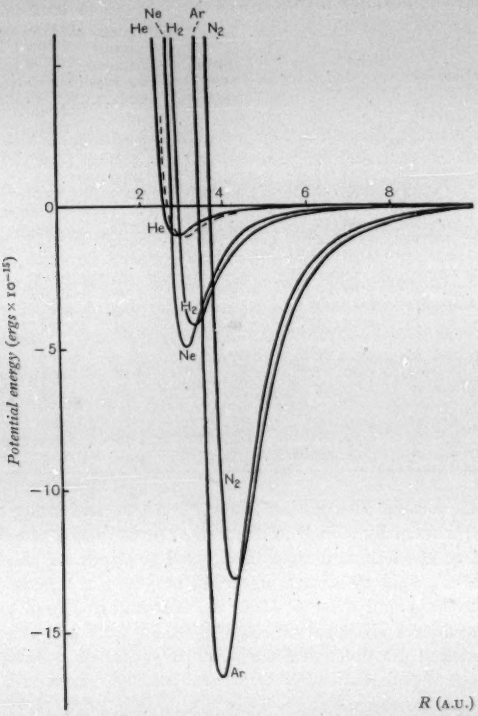


Fig. 3. The potential energy of pairs of inert gas atoms as a function of their distance apart in Å.‡

The experimental study of molecular spectra has established the existence of molecules such as HgAr, HgKr§, and lately it has been shown that a molecular form of K_2 exists quite other than the molecule formed by the usual homopolar forces‖. There is little doubt that these molecules are held together by van der Waals forces.

* J. E. Lennard-Jones and B. M. Dent, *Trans. Faraday Soc.* **24**, 92 (1928).

† F. London, *Zeit. f. phys. Chem. loc. cit.*

‡ The continuous curves are obtained from the equation of state, hitherto unpublished, assuming a law of force $\lambda_{(rep.)} R^{-13} - \lambda_{(att.)} R^{-7}$. The dotted curve for helium is obtained by Slater and Kirkwood from its electronic structure.

§ O. Oldenburg, *Zeit. f. Phys.* **55**, 1 (1929). ‖ H. Kuhn, *Naturwissen.* **18**, 332 (1930).

§ 8. HOMOPOLAR COHESION*

In an earlier paragraph we have seen that when there are several electrons in an atom, its configuration may in an approximate theory be specified by a number of wave functions $u_1, u_2 \ldots u_N$, each a function of the four coordinates of an electron, and that the probability of a specified configuration of the electrons is given by a certain determinant of N rows and columns. The importance of the new form for the probability function becomes evident when calculations are made of the *energy* of electronic systems. The average value of the electrostatic energy of the electrons can be calculated only when we know the probability that any pair of electrons will be at a specified distance apart. A detailed calculation of the energy of an atom or the interaction energy of two atoms depends then very closely on the probability function $\Psi\overline{\Psi}$.

The probability of finding two electrons in specified places $d\tau_1$ and $d\tau_2$, independently of the position of the other electrons, is obtained by integrating the probability function over the coordinates of all the electrons except two. The result proves to be

$$\{\rho(1,1)\,\rho(2,2) - \rho(1,2)\,\rho(2,1)\}\,d\tau_1\,d\tau_2,$$

with the definitions of $\rho(1,2)$, etc., given in § 3. The mutual potential of two electrons consists then of two terms; the first is the average of $(e^2/r_{12})\,\rho(1,1)\,\rho(2,2)$ integrated over the whole of space of electrons 1 and 2; the second is the average of $(e^2/r)\,\rho(1,2)\,\rho(2,1)$. Apart from certain terms of these expressions which cancel, the first represents the Coulomb interaction of the individual distributions of electric charge $u_1\bar{u}_1$, $u_2\bar{u}_2$, etc. This may be called the *Coulomb energy*. It is the mutual electrostatic potential energy of the superimposed patterns which go to make up atoms, as described in § 4.

The second term in the above expression is new. It is difficult to describe its physical nature. All that can be said of it is that it is the natural outcome of the introduction of two new physical concepts into the mathematical scheme, viz., the principle of the identity of electrons, and the exclusion principle of Pauli. It is sometimes described as the "exchange" term and the term in the energy expression arising from it as the "*exchange*" *energy*.

The "exchange" energy depends on the spin of the electrons, while the Coulomb energy does not. It is this property of the exchange term which has made it of so much importance in the theory of atoms and molecules. In a two-electron system, as in an excited helium atom, for instance, the electrons may have the same or opposite spins, and, owing to the exchange term, the energies of the states with the same spins are lower in every case than those of the corresponding states with opposite spin. The hydrogen molecule is another two-electron system, and here the energy is lowest when the electrons have opposite spin. This appears to

* The account of homopolar forces given here is necessarily brief. A fuller account has been given recently by the author in a lecture to the London Mathematical Society on "The Quantum Mechanics of Atoms and Molecules." This is to be published in the *Journal of the London Mathematical Society* shortly.

be the case in the interaction of most atoms. For certain interatomic distances the energy is lowest, and, therefore, the cohesion greatest, when the electrons of one atom are "paired" with those of the other atom. The energy of two interacting nitrogen atoms is highest when the spins of the three outer electrons of the atoms are all of them the same, and lowest when the three electrons of one atom are paired with those of the other. This latter condition corresponds to the normal nitrogen molecule, held together—as the chemist describes it—with a triple bond.

The pairing of electrons is thus brought into close connexion with the valency rules of the chemist, and chemical homopolar forces are elucidated to this extent—that they are seen to be a consequence of the same mathematical and physical principles which have been formulated for other branches of physics. This result may conceivably come to be regarded as one of the greatest achievements of the present formulation of quantum mechanics.

The nature of these forces may be described in a simpler but less accurate way by an appeal to Pauli's exclusion principle. For one electron in the field of two nuclei, there is a set of patterns with different energies just as there is for an electron in the field of one nucleus. One electron in the presence of two hydrogen nuclei will take up the pattern of lowest energy. If a second electron is added to the system, it also can take up the same pattern provided that it has a spin, which is opposite from the first one. The two electrons now exist in the same *molecular* pattern and are paired. Provided the energy of the pattern, which depends on the internuclear distance, has a minimum, a stable molecule will be formed.

If, however, two helium atoms are brought together the electrons are paired already. When the nuclei are pushed together, two of the electrons must be "promoted" to a higher energy level if the exclusion principle is to be preserved. The act of promotion requires energy and this appears as repulsion. A similar argument may be used for other closed electron groups, which are brought into contact.

The more accurate method described above also shows that closed electron groups, like those of the inert gases, repel each other. The exchange term in this case becomes predominant and leads to an increase of energy as the two systems are brought near together. *Intrinsic repulsive* fields may be attributed to this new exchange term, which quantum mechanics has produced.

The exchange term is more important than the Coulomb term in the interaction of two hydrogen atoms, but the Coulomb term becomes relatively more and more important in the interaction of larger atoms. This is due not only to the increased spread of the atoms, but also to the form of the patterns which the outer electrons have to assume. The effect is found even in the interaction of two lithium atoms*. When the Coulomb part is important, the pictures of atoms given in an earlier paragraph are useful in constructing models of molecules and helpful in understanding their various shapes. We have, for instance, seen that an atom of fluorine, or indeed any halogen gas, has a "hole" in it. It follows that the nucleus is less screened in

* M. Delbrück, *Ann. d. Phys.* 5, 36 (1930).

some directions than others. If the hole in the F atom is like the (2, 1, 0) pattern, the atom is roughly like a spherical ball of electric charge with a double conical hole scooped out of it. It is then not difficult to understand why a hydrogen atom can combine with it to give a HF molecule. The hydrogen's electron cloud sinks as far as possible into the hole of the fluorine, for in this position it gets as near as possible to the fluorine nucleus. Of course, it must be stressed that this only provides a rough picture. In an accurate treatment, the wave function of the whole system must be considered.

§ 9. IONIC COHESION

The conception of a halogen atom as an inert gas atom with a hole in it is useful in many ways. It has an affinity for another electron, and when one has been secured the electron cloud becomes spherically symmetrical. Ionic salts, like NaCl, may be pictured as arrays of spherical distributions, each on the whole charged, either positively or negatively, and so held together by electrostatic forces. This brings us to the question of determining a criterion between homopolar cohesion and ionic cohesion. When are we to suppose that atoms become ionized in the process of binding? The criterion is one of energy. Suppose that in figure 4 the curve

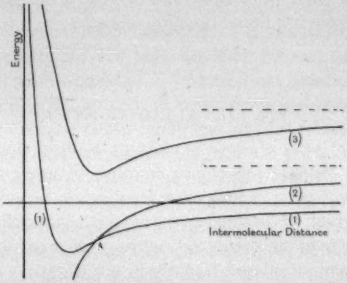


Fig. 4. Potential energy curves of atoms and ions.

(1) refers to the energy of two neutral atoms at various distances apart and that curve (2) refers to that of the corresponding ions. The difference between the energies (1) and (2) at infinite distance is equal to the difference of the ionization potential of one atom (say an alkali) and the electron affinity of the other (say a halogen). The curve (2) is then represented by $(a - b/R)$ because of the electrostatic attraction.

At the point of intersection A of the curves, the work put into the system at infinity has been recovered, and at interatomic distances less than this the ionic form has the less energy. If, however, the energy required for ionization is large, the interaction is of the type (3). Then the ions have to be brought to very close

distances before the ionic form has less energy than the atomic, but the close distances are not possible because of the intrinsic repulsion or the Pauli principle referred to in the preceding paragraph.

Table 2* shows how this condition is fulfilled in the case of some typical ionic compounds. The distance R refers to the value of the abscissa where the ionic curve cuts the axis; this is approximately the same as the place where it cuts the curve (1) when R is large.

Table 2.

	Ionization potential + electron affinity (volts)	($A.U.$)
KF	0·24	60
KCl	0·50	29
KBr	0·84	17
KI	1·23	11·8

It is likely that the terms *ionic* and *homopolar* refer to extreme cases, convenient for classification, but rarely existing in practice, actual cohesion being neither the one nor the other but partaking of both. The wave function of a perturbed system must be expressed in terms of all the wave functions of the unperturbed system and if there are two such wave functions of nearly the same energy, the wave function of the molecule is a linear function of both.

§ 10. METALLIC COHESION

From the point of view of wave mechanics, a metal must be regarded as an enormous molecule, with an enormous number of energy levels and electron patterns. The problem is to explain why a collection of atoms in the form of a metal has less energy than the isolated atoms of which it is composed. Is the cohesion due to ordinary electrostatic forces or to the exchange phenomenon or both? The problem has been considered recently in a qualitative way by J. C. Slater†, who has shown that it is likely that atoms of one spin are surrounded by others of opposite spin‡. In those metals, which are of the body-centred-cubic type of structure, each atom is at the centre of eight others of opposite electron spin.

In the bringing of the atoms together in this way the patterns of the individual atoms have been made to overlap. An electron which formerly belonged to one nucleus is now brought under the influence of several. This has the effect of increasing the electrostatic attraction, and even in the absence of any other cohesive force the metal would hold together. The metal is like a sea of electric fluid, in which the nuclei float like buoys.

* This method of illustration is due to F. London, *Zeit. f. Phys.* **46**, 475 (1928).

† J. C. Slater, *Phys. Rev.* **35**, 509 (1930).

‡ This is not true, of course, for the ferromagnetic bodies.

Now we inquire how the exchange phenomenon affects the cohesion. It is not difficult to show that the state of a crystal, in which all the electron spins are perfectly paired, combines only with the states in which two of the spins are interchanged. Thus, to use a linear example, the electron spins being denoted by α and β, the state $\alpha\beta\ \alpha\beta\ \alpha\beta \ldots$ combines only with $\alpha\beta\ \beta\alpha\ \alpha\beta \ldots$ and similar arrays. In the body-centred-cubic lattice the effect of such an interchange is to surround two electrons with seven neighbours of the same spin. The resulting structure has then considerably higher energy, because the contiguity of like spins causes a repulsion. It is thus highly improbable. The electron seems to know what will be the result of an interchange and decides that it simply is not done—or at any rate only with moderation. The net result is that the exchange effect is not as important in metals as it is in diatomic molecules, whereas the electrostatic attraction is relatively more important. An accurate quantitative treatment of the problem is, however, still required.

There are some metals, such as zinc and cadmium, in which the exchange term probably makes little or no contribution to the cohesion. The atoms of which they are composed have paired electron spins even when widely separated, and may be expected on that account to repel like inert-gas atoms when brought into contact. The fact that the metals are diamagnetic is further evidence that the metals consist of perfectly paired electron spins and are formed from normal unexcited atoms. Now the diatomic molecules Zn_2, Cd_2, are known to exist and the order of magnitude of their heats of dissociation, which is known from molecular spectra, suggests that these molecules are held together by van der Waals fields. The cohesion of the corresponding metals is several times greater, but this is probably due to the larger number of immediate neighbours and the additive property of van der Waals fields. Thus it seems likely that van der Waals fields make a greater contribution to the cohesion of some metals than has yet been realized.

Metallic cohesion cannot therefore be classified in any simple way. It is probably due partly to the electrostatic interaction of space-charge distributions, partly to the exchange phenomenon, and partly to van der Waals fields. The relative extents to which these various factors contribute to the cohesion of a metal must vary from case to case and is still a matter for investigation.

§ 11. CONCLUSION

In conclusion we may summarize the contributions of the new quantum mechanics to this branch of physics under four headings:

(1) It has provided density pictures of atoms which have permitted the evaluation of the electrostatic interaction of atoms in a way quite impossible in an orbital theory.

(2) It has led to the introduction of a new concept—an exchange force, which is responsible for the homopolar bonds of the chemist.

(3) It has provided pictures of asymmetrical atoms like the halogens and thrown light on the nature of electron affinity. This affinity is responsible for ionic structures of the NaCl type.

(4) It has provided an explanation of van der Waals fields, which formerly were not understood.

The general principles seem to be established. What is now required is a mathematical technique capable of applying them to particular cases.

THE ABSORPTION AND DISSOCIATIVE OR IONIZING EFFECT OF MONOCHROMATIC RADIATION IN AN ATMOSPHERE ON A ROTATING EARTH

PART II. GRAZING INCIDENCE

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ABSTRACT. The absorption of radiation from the sun in an atmosphere varying exponentially with the height is considered, as in a former paper; but here the earth's curvature, and that of the level layers in the atmosphere, is taken into account. The values of this absorption as previously calculated are valid so long as the sun's zenith distance does not exceed 75°, but for greater zenith distances the necessary corrections are of importance. It is shown that the absorption, and resulting ionization or dissociation of the air, should begin to increase before ground sunrise, the interval varying from about 10 minutes at the equator to about an hour at 60° latitude.

§ 1. INTRODUCTION

This paper is a sequel to one* with the same (main) title, which will here be referred to as part I. In § 4 of that paper, dealing with the absorption of radiation, it was pointed out that the equation of absorption there used was not mathematically exact (except for direct incidence) as a representation of the physical assumptions adopted at the outset; it was suggested that the approximation was probably sufficiently accurate for beams inclined to the vertical at angles up to 85°, but that for greater angles, corresponding to nearly grazing incidence, the error became appreciable. It was further stated that the values of the ion-content deduced in the paper were very nearly true, in low latitudes, except near dawn, but that in higher latitudes, *in winter*, the necessary corrections are appreciable up to noon. In the present paper the absorption of radiation at nearly grazing incidence is considered in detail, and the consequent corrections to the former results, relating to the variation of ion-content with respect to height and time, are examined.

The notation of part I will in general be used in this paper without being re-defined. The tables and diagrams in this paper are numbered in continuation of those of part I.

§ 2. THE ABSORPTION OF RADIATION

In considering the absorption of radiation we shall ignore the refraction of the beam; that is, the radiation will be regarded as travelling along a straight line.

In figure 18 let O denote the earth's centre, OS the line from O to the sun, P' any point in the earth's atmosphere, and NTM part of the boundary of the

* S. Chapman, *Proc. Phys. Soc.* **43**, 26 (1931).

section of the earth by the plane SOP'; N is thus the point on the earth immediately "below" the sun, and M the antipodal point. T is a point on the "twilight circle" of the earth, dividing the day hemisphere, from which the sun is visible, from the night hemisphere.

λ The flow of radiation is parallel to SO, and if $\angle SOP' = \lambda$, the angle of incidence
s of the beam at P' is λ. Let the beam through P' cut OT in T', and let distance s along the beam be measured from T' as origin, in the direction of travel of the
a, h', p beam. If a denotes the radius of the earth, and h' the height of P', then p, representing the distance from O of the beam through P', is given by

$$p = OT' = (a + h') \sin \lambda \qquad \text{.....(1).}$$

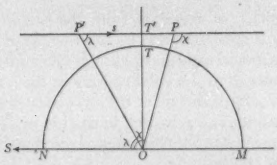

Fig. 18.

Along any particular beam p is constant, while for different points P' along it, specified by the corresponding angle λ, the height h' is given by

$$h' = p \operatorname{cosec} \lambda - a \qquad \text{.....(2).}$$

For the value of s at P' we have

$$s = -p \cot \lambda \qquad \text{.....(3);}$$

this is negative if P' is to the left of OT (as in figure 18) and positive if to the right.

Consider the absorption $-dS'$ between the point P' and a neighbouring point
ρ', S' on the beam, given by $s + ds$ or $\lambda + d\lambda$. If ρ' is the air density at P', and S' the intensity of radiation,

$$\begin{aligned} dS' &= -AS'\rho'\,ds \\ &= -AS'\rho_0 \exp(-h/H)\, p \operatorname{cosec}^2 \lambda\, d\lambda \\ &= -AS'\rho_0 \exp\{(a - p \operatorname{cosec} \lambda)/h\}\, p \operatorname{cosec}^2 \lambda\, d\lambda \qquad \text{.....(4).} \end{aligned}$$

χ Let P be any point on the beam, such that $\angle SOP = \chi$; in figure 18 this point is
S shown to the right of OT, but it may be anywhere along the beam. The intensity S of the beam, at P, is obtainable by integration along it from the extreme left, outside the atmosphere (where $s = -\infty$, $\lambda = 0$) to P. We get

$$S/S_\infty = \exp\left[-A\rho_0 \int_0^\chi \exp\{(a - p \operatorname{cosec} \lambda)/H\}\, p \operatorname{cosec}^2 \lambda\, d\lambda\right] \qquad \text{.....(5).}$$

This equation replaces equation (7) of part I, which is not strictly true for an atmosphere arranged in concentric spherical layers of equal density.

The absorption of radiation per cm.^{3} of atmosphere is $-dS/ds$; we shall consider, instead of this, the quantity $I \equiv -\beta dS/ds$, where β denotes the number of ions produced by the absorption of unit quantity of the radiation (or the number of molecules dissociated, when the effect of the radiation is dissociative rather than ionizing). By putting $\beta = 1$ in I we can obtain the actual absorption if required. I, β

From (4) the value of I at P is found, on substituting for S from (5), to be given by

$$I = \beta A S_\infty \rho_0 \exp(-h/H) \exp\left[-A\rho_0 \int_0^\chi \exp\{(a - p\operatorname{cosec}\lambda)/H\}\, p \operatorname{cosec}^2\lambda\, d\lambda\right] \quad \text{.....(6).}$$

In terms of I_0 ($\equiv \beta S_\infty/H \exp 1$) and h_0 (given by $\exp(h_0/H) = A\rho_0 H$) this may be written I_0, h_0

$$\frac{I}{I_0} = \exp\left(1 + \frac{h_0 - h}{H}\right) \exp\left[-\frac{p}{H}\int_0^\chi \exp\left(\frac{a + h_0 - p\operatorname{cosec}\lambda}{H}\right)\operatorname{cosec}^2\lambda\, d\lambda\right], \quad \text{.....(7)}$$

or, taking $z \equiv (h - h_0)/H$, as in part 1, and substituting $(a + h)\sin\chi$ or $(a + h_0 + zH)\sin\chi$ for p,

$$\frac{I}{I_0} = \exp\left[1 - z - \left(\frac{a + h_0}{H} + z\right)\sin\chi \int_0^\chi \exp\left\{\frac{a + h_0}{H} - \left(\frac{a + h_0}{H} + z\right)\frac{\sin\chi}{\sin\lambda}\right\} \operatorname{cosec}^2\lambda\, d\lambda\right] \quad \text{.....(8).}$$

This equation replaces (14) or (17) in part I, and differs from those equations in one important respect; it contains a quantity $(a + h_0)/H$ which did not occur in the former discussion. Thus the proper consideration of the earth's curvature involves the introduction of a new parameter into the formulae, and if for no other reason than this it seemed justifiable, in a first general discussion of the subject, to defer the consideration of the curvature, particularly since this made it possible to put the analysis in a very general form involving only one parameter (σ_0).

The further parameter now introduced, $(a + h_0)/H$, is equal to the distance, *expressed in terms of H as unit of length*, from the earth's centre to the level at which, at noon at the equator, the absorption of radiation is a maximum. This parameter will be denoted by R. R

It is of interest to consider the numerical value of R in the case of the ionized layers of which the heights have been measured by E. V. Appleton and his collaborators*. The value of a is 6370 km. For the lower of the observed ionized layers h_0 is about 100 km., which is small, though not negligibly so, compared with a; thus $a + h_0$ is about 6470 km. for this layer. The value of H at this height in the atmosphere is unknown; if the temperature there is 300° K. and the composition is the same as in the lower atmosphere, $H = 8{\cdot}4$ km. It is possible that H may vary with the season, and even throughout the day and night; temperatures as high as 1000° K. have been suggested as existing at great heights, and for the same com-

* E. V. Appleton, *Proc. R.S.* A, **126**, 542 (1930); *ibid.* **128**, 133 and 159 (1930), with J. A. Ratcliffe and A. L. Green respectively; and earlier papers there cited.

position as before this would correspond to $H = 28$ km. At great heights (probably much above 100 km.) the composition is likely to differ considerably from that near the ground, and I have suggested elsewhere* that in the outermost layer atomic oxygen and nitrogen, possibly ionized, may be the main constituents; if so, at these heights H would be increased, on this account, in the ratio 2 (if the atoms are not ionized) or 4 (if wholly ionized, the negative charges being electrons). Thus for very high levels values of H as large as 112 km. might require consideration. The range of R or $(a + h_0)/H$, as H ranges from 8·4 to 112 km., is from about 770 to 58, if $h_0 = 100$, or slightly more for higher values of h_0. It is convenient, in the subsequent calculations, to bear in mind this possible extreme range of the new parameter R; but for the ionized layer at about 100 km. height, H is probably about 10 km., corresponding to R about 647 or, in round figures, 650.

In terms of R, (8) may be written

$$I/I_0 = \exp\left[1 - z - \exp\left(-z\right) f(R + z, \chi)\right] \qquad \text{.....(9),}$$

where

$$f(x, \chi) = x \sin\chi \int_0^{\chi} \exp\left\{x\left(1 - \sin\chi/\sin\lambda\right)\right\} \operatorname{cosec}^2\lambda \, d\lambda \qquad \text{.....(10).}$$

Equation (9) is identical with equation (17) of part I, except that in (17) $\sec\chi$ occurs in the place of $f(R + z, \chi)$. It is of interest to show that $f(R + z, \chi)$ actually has the value $\sec\chi$ when R tends to infinity, because of course the neglect of the earth's curvature, in part 1, corresponds to taking R (or $R + z$) as infinite.

Since

$$d \operatorname{cosec}\lambda = -\cos\lambda \operatorname{cosec}^2\lambda \, d\lambda,$$

(10) may be rewritten as

$$f(x, \chi) = \int_{\lambda=0}^{\lambda=\chi} \sec\lambda \, d\left[\exp\left\{x\left(1 - \sin\chi \operatorname{cosec}\lambda\right)\right\}\right] \qquad \text{.....(11),}$$

and, by a partial integration, this gives

$$\begin{aligned} f(x, \chi) &= \left[\sec\lambda \exp\left\{x\left(1 - \sin\chi \operatorname{cosec}\lambda\right)\right\}\right]_{\lambda=0}^{\lambda=\chi} \\ &\quad - \int_0^{\chi} \exp\left\{x\left(1 - \sin\chi \operatorname{cosec}\lambda\right)\right\} \sec\lambda \tan\lambda \, d\lambda \\ &= \sec\chi - \int_0^{\chi} \exp\left\{x\left(1 - \sin\chi \operatorname{cosec}\lambda\right)\right\} \sec\lambda \tan\lambda \, d\lambda \qquad \text{.....(12).} \end{aligned}$$

It is not difficult to prove that as $x \to \infty$ the second term in the last line of (12) tends to zero provided that χ is less than 90°; such values were the only ones that came in question in the former paper. Thus when R is taken as infinite, as was done there, $f(x, \chi)$ is equal to $\sec\chi$. The revised discussion in the present paper depends essentially on the replacement of $\sec\chi$ in equation (17) of part 1 by $f(R + z, \chi)$ as here defined, and it is necessary to consider the nature of this function of the two variables x (or $R + z$) and χ.

It should be added that for any beam for which values of χ exceeding 90° have to be considered, p cannot be less than a, the radius of the solid body of the earth.

* *Phil. Mag.* **10**, 369 (1930).

This imposes an upper limit on the possible values of χ for any height h, since $p \geqslant a$ is equivalent to $(a+h)\sin\chi \geqslant a$. For example, for $h = 100$ km., the limiting value of χ is about 100°. This restriction has little actual importance, however, for the problem of upper atmospheric ionization, because the absorption of the beams for which p only slightly exceeds a is practically completed well before the limiting value of χ is attained.

§ 3. THE FUNCTION $f(x, \chi)$ WHEN $\chi = \frac{1}{2}\pi$

When $\chi = \frac{1}{2}\pi$, $\sec\chi$ is infinite, whereas $f(x, \chi)$ is finite; this infinity of $\sec\chi$ in (17) renders I zero, corresponding to complete absorption of radiation by the time the beam reaches the twilight plane (i.e. the plane through O perpendicular to SO). Actually the value of I is still positive on reaching this plane, and also for points "behind" this plane, corresponding to values of χ greater than $\frac{1}{2}\pi$.

For the special value $\chi = \frac{1}{2}\pi$, $f(x, \chi)$ may be expressed in terms of Bessel functions. For

$$f(x, \tfrac{1}{2}\pi) = x\int_0^{\frac{1}{2}\pi} \exp\{x(1-\operatorname{cosec}\lambda)\}\operatorname{cosec}^2\lambda\, d\lambda \qquad \text{.....(13).}$$

On making the substitution $\sin\lambda = \operatorname{sech} u$ .....(14),

so that
$$\left.\begin{aligned} &\cos\lambda = \tanh u, && \tan\lambda = \operatorname{cosech} u \\ &\sec\lambda = \cosh u, && \sec\lambda = \coth u \\ &\cot\lambda = \sinh u, && d\lambda = -\operatorname{sech} u\, du \end{aligned}\right\} \qquad \text{.....(15),}$$

we find that (13) becomes

$$\begin{aligned} f(x, \tfrac{1}{2}\pi) &= x\int_0^\infty e^{x(1-\cosh u)}\cosh u\, du \\ &= -xe^x\frac{d}{dx}\int_0^\infty e^{-x\cosh u}\, du \\ &= -xe^x\frac{d}{dx}K_0(x) = -xe^xK_1(x) \qquad \text{.....(16),} \end{aligned}$$

by well-known formulae in the theory of Bessel functions*. Tables of the function $e^xK_1(x)$ up to $x = 16$ are given at the end of Watson's treatise on these functions.

Since in the present application x is large (50 or more), the well-known asymptotic formula for $K_1(x)$ may be used, which gives

$$f(x, \tfrac{1}{2}\pi) = (\tfrac{1}{2}\pi x)^{\frac{1}{2}}\left\{1 + \sum_{n=1} a_n x^{-n}\right\} \qquad \text{.....(17),}$$

where $$a_n = [\{1.3\ldots(2n-3)\}^2(4n^2-1)]/(n!\,2^{3n}) \qquad \text{.....(18),}$$

so that $$a_1 = \tfrac{3}{8}, \quad a_2 = -\tfrac{15}{128}, \quad a_3 = \tfrac{105}{1024}, \ldots \qquad \text{.....(19).}$$

Thus for $x > 50$, $f(x, \frac{1}{2}\pi) = (\frac{1}{2}\pi x)^{\frac{1}{2}}$ correct to within 1 per cent. For $x = 50$ it is 8·93, while for $x = 800$ it is 35·47; these values replace infinity, the value of $\sec\chi$, when (9) replaces (17) of part I.

* Cf. Whittaker and Watson, *Modern Analysis* (2nd ed.), p. 377, ex. 40.

§ 4. THE FUNCTION $f(x, \chi)$ FOR SMALL VALUES OF χ

The smaller the value of χ, the more nearly will $f(x, \chi)$ approximate to $\sec\chi$, so that for small values of χ a formula expressing this fact should be obtainable. It may be found as follows. Let

$$y = x\sin\chi \qquad \text{.....(20),}$$

so that

$$\begin{aligned} f(x,\chi) &= ye^{x}\int_0^{\chi} e^{-y\operatorname{cosec}\lambda}\operatorname{cosec}^2\lambda\, d\lambda \\ &= -e^{x}\int_{\lambda=0}^{\lambda=\chi}\sec\lambda\, d\left(e^{-y\operatorname{cosec}\lambda}\right) \\ &= \sec\chi - e^{x}\int_0^{\chi} e^{-y\operatorname{cosec}\lambda}\sec^2\lambda\sin\lambda\, d\lambda \\ &= \sec\chi - (e^{x}/y)\int_{\lambda=0}^{\lambda=\chi}\tan^3\lambda\, d\left(e^{-y\operatorname{cosec}\lambda}\right) \qquad \text{.....(21),} \end{aligned}$$

by a partial integration similar to that which we used in obtaining (12). By further similar partial integrations we find that

$$f(x,\chi) = \sec\chi + \sum_{n=1} b_n/x^n \qquad \text{.....(22),}$$

where

$$\left.\begin{aligned} &b_1 = -\sec\chi\tan^2\chi, \quad b_2 = 3\tan^2\chi\sec^3\chi, \quad b_3 = -(15\tan^4\chi + 12\tan^2\chi)\sec^3\chi \\ &b_4 = (105\tan^4\chi + 60\tan^2\chi)\sec^5\chi, \\ &b_5 = -(945\tan^6\chi + 1260\tan^4\chi + 360\tan^2\chi)\sec^5\chi \end{aligned}\right\} \qquad \text{.....(23),}$$

and, in general,

$$b_n\sin^n\chi = -\sin^2\chi\sec\chi\,(d/d\chi)\,(b_{n-1}\sin^{n-1}\chi) \qquad \text{.....(24).}$$

This series is useful only so long as $(\tan^2\chi)/x$ is small; for $x = 800$ it is of service for values of χ up to about 80°, but for $x = 50$ its numerical convergence becomes slow at $\chi = 60°$. The following table gives values of the *ratio* $100\{1 - f(x,\chi)/\sec\chi\}$, calculated (except for $\chi = 75°$ and $R < 200$) in the above way for various values of R and χ; this indicates the *percentage* by which f falls short of $\sec\chi$ in the various cases.

Table 3. $100\{1 - f(x,\chi)/\sec\chi\}$

$\chi =$	30°	45°	60°	75°
$\sec\chi =$	1·155	1·414	2·000	3·864
$R = 50$	0·62	1·2	4·6	16·4
100	0·32	0·95	2·7	9·1
200	0·16	0·49	1·4	5·6
300	0·11	0·32	1·0	4·1
400	0·08	0·23	0·7	3·2
500	0·06	0·20	0·6	2·6
600	0·05	0·16	0·5	2·2
650	0·05	0·15	0·5	2·0
700	0·05	0·14	0·4	1·9
800	0·04	0·13	0·4	1·7

It is clear from this table that if $R > 300$ (corresponding to $H < 20$ km. approximately), the curvature of the earth reduces the factor $\sec\chi$ of part I, equations (8), (9), (12), (14), (17), by less than 5 per cent., even when χ is as large as 75°.

§ 5. GENERAL ASYMPTOTIC FORMULAE FOR $f(x, \chi)$

When χ is neither small nor equal to $\frac{1}{2}\pi$, $f(x, \chi)$ may be evaluated as follows. Using the substitutions (14) and (20), we find

$$f(x, \chi) = ye^{x-y} \int_U^\infty e^{y(1-\cosh u)} \cosh u \, du \qquad \text{.....(25),}$$

where U is the value of u corresponding to $\lambda = \chi$, so that U

$$\operatorname{sech} U = \sin \chi \qquad \text{.....(26).}$$

If $\chi < \frac{1}{2}\pi$, the positive value of U must be taken, but if $\chi > \frac{1}{2}\pi$, the negative value is the appropriate one. If we write

$$\chi = \tfrac{1}{2}\pi - \chi' \qquad \text{.....(27),} \qquad \chi'$$

U will have the same sign as χ', or, if we take χ' and U as always positive and given by

$$\operatorname{sech} U = \cos \chi' \qquad \text{.....(28),}$$

(25) may be written in the form

$$\begin{aligned} f(x, \tfrac{1}{2}\pi \mp \chi') &= ye^{x-y} \int_{\mp U}^\infty e^{y(1-\cosh u)} \cosh u \, du \\ &= ye^{x-y} \left\{ \int_0^\infty \mp \int_0^U \right\} e^{y(1-\cosh u)} \cosh u \, du \\ &= e^{x-y} f(y, \tfrac{1}{2}\pi) \mp ye^{x-y} \int_0^U e^{y(1-\cosh u)} \cosh u \, du \qquad \text{...(29).} \end{aligned}$$

Writing $v = \sinh \frac{1}{2}u, \quad \cosh u - 1 = 2v^2, \quad du = (2\,dv)/(1+v^2)^{\frac{1}{2}}$ .....(30) v

we have

$$\begin{aligned} \int e^{y(1-\cosh u)} \cosh u \, du &= 2 \int \exp(-2yv^2) \frac{1+2v^2}{(1+v^2)^{\frac{1}{2}}} dv \\ &= 2 \int \exp(-2yv^2)(1 + \Sigma c_n v^{2n}) \, dv \qquad \text{.....(31),} \end{aligned}$$

where

$$\left.\begin{aligned} c_n &= (-1)^{n-1}(2n+1)\{1.3 \ldots (2n-3)\}/n!\,2^n \\ c_1 &= \tfrac{3}{2}, \quad c_2 = -\tfrac{5}{8}, \quad c_3 = \tfrac{7}{16}, \quad c_4 = -\tfrac{45}{128}, \ldots \end{aligned}\right\} \qquad \text{.....(32).}$$

Now it is readily shown that, for all positive values of y and V,

$$\begin{aligned} \int_0^V \exp(-2yv^2) v^{2n} \, dv &= \frac{1.3 \ldots (2n-1)}{2^{2n+\frac{1}{2}} y^{n+\frac{1}{2}}} \operatorname{erf}(2yV^2)^{\frac{1}{2}} \\ &\quad - \exp(-2yV^2) \left\{ \frac{V^{2n-1}}{4y} + \frac{(2n-1)V^{2n-3}}{(4y)^2} + \frac{(2n-1)(2n-3)V^{2n-5}}{(4y)^3} + \ldots \right\} \\ &\qquad \text{.....(33),} \end{aligned}$$

where the series on the right is a terminating one, and where

$$\operatorname{erf} \eta = \int_0^\eta e^{-w^2} \, dw \qquad \text{.....(34).} \qquad \eta$$

By well-known theorems,

$$\frac{2}{\sqrt{\pi}} \operatorname{erf} \eta \to 1 \text{ as } \eta \to \infty \qquad \text{.....(35),}$$

and, in the form of an asymptotic series, useful when η is large,

$$1-\frac{2}{\sqrt{\pi}}\operatorname{erf}\eta=\frac{2}{\sqrt{\pi}}\int_{\eta}^{\infty}e^{-w^2}dw=\frac{e^{-\frac{1}{2}\eta^2}}{\sqrt{(\pi\eta)}}W_{-\frac{1}{2},\frac{1}{4}}(\eta^2)$$

$$=\frac{e^{-\eta^2}}{\eta\sqrt{\pi}}\left[1+\Sigma(-1)^n\frac{1.3\ldots(2n-1)}{2^n\eta^{2n}}\right] \qquad \ldots\ldots(36),$$

V where W denotes the confluent hypergeometric function*. Hence if V is the value of v corresponding to $u=U$, it follows that

$$ye^{x-y}\int_0^U e^{y(1-\cosh u)}\cosh u\,du$$

$$=e^{x-y}(\tfrac{1}{2}\pi y)^{\frac{1}{2}}\left\{\frac{2}{\sqrt{\pi}}\operatorname{erf}(2yV^2)^{\frac{1}{2}}\right\}\cdot\left\{1+\frac{c_1}{4y}+\frac{1.3c_2}{(4y)^2}+\ldots\right\}$$

$$-\frac{1}{2}\left[\sum_1^\infty c_nV^{2n-1}+\frac{1}{4y}\sum_2^\infty(2n-1)c_nV^{2n-3}+\frac{1}{(4y)^2}\sum_3^\infty(2n-1)(2n-3)c_nV^{2n-5}+\ldots\right]$$

$$=-ye^xK_1(y)\frac{2}{\sqrt{\pi}}\operatorname{erf}(2yV^2)^{\frac{1}{2}}-\tfrac{1}{2}\sum_{m=0}^{\infty}(4y)^{-m}\sum_{n=m+1}^{\infty}(2n-1)(2n-3)\ldots$$

$$\ldots(2n-2m+1)c_nV^{2n-2m-1} \qquad \ldots\ldots(37).$$

Hence by (29) and (16),

$$f(x,\tfrac{1}{2}\pi\mp\chi')=-ye^xK_1(y)\left\{1\mp\frac{2}{\sqrt{\pi}}\operatorname{erf}(2yV^2)^{\frac{1}{2}}\right\}$$

$$\pm\tfrac{1}{2}\sum_{m=0}^{\infty}(4y)^{-m}\sum_{n=m+1}^{\infty}(2n-1)(2n-3)\ldots(2n-2m+1)c_nV^{2n-2m-1} \quad \ldots(38).$$

In this formula

$$\left.\begin{aligned}V&=\sinh\tfrac{1}{2}U=\sqrt{\{\tfrac{1}{2}(\operatorname{cosec}\chi-1)\}}=\sqrt{\{\tfrac{1}{2}(\sec\chi'-1)\}}\\ y&=x\sin\chi=x\cos\chi'\end{aligned}\right\} \qquad \ldots\ldots(39),$$

$$(2yV^2)^{\frac{1}{2}}=\{x(1-\sin\chi)\}^{\frac{1}{2}}=\{x(1-\cos\chi')\}^{\frac{1}{2}} \qquad \ldots\ldots(40);$$

in all cases the positive square roots are to be taken.

The formula (38) is useful when χ is too great for (22) to be rapidly convergent; when χ' is positive ($\chi<90°$) and such that $(2yV^2)^{\frac{1}{2}}$ is 3 or more, the expansion (36) is helpful in evaluating (38). The binomial expansion in (31) is valid so long as $V<1$, or $\sin\chi>\frac{1}{3}$, $\chi>20°$; in calculating $f(x,\chi)$ for smaller values of χ the formulae of § 3 are available.

When χ' is positive, and x tends to infinity, the first term of $f(x,\frac{1}{2}\pi-\chi')$, in (38), may be shown, with the aid of (36), to tend to $-1/2V$. The second part of (38) tends to $\frac{1}{2}\sum_{n=1}^{\infty}c_nV^{2n-1}$, which is equal to $1/2V+\sec\chi$. Thus this expression for $f(x,\frac{1}{2}\pi-\chi')$ tends to $\sec\chi$, as it should do, when $x\to\infty$, $0<\chi'<\frac{1}{2}\pi$.

* Cf. Whittaker and Watson, *Modern Analysis* (2nd ed.), p. 335.

In calculating the first term of (38), use may be made either of tables of $e^y K_1(y)$, or of the asymptotic formula

$$-ye^x K_1(y) = (\tfrac{1}{2}\pi y)^{\frac{1}{2}} e^{x-y} \left\{1 + \sum_1 a_n y^{-n}\right\}$$

$$= (\tfrac{1}{2}\pi x \sin\chi)^{\frac{1}{2}} e^{x(1-\sin\chi)} \left\{1 + \sum_1 a_n x^{-n} \operatorname{cosec}^n \chi\right\} \quad \ldots\ldots(40).$$

Table 4 gives values of $f(R, \chi)$ and of $\ln f(R, \chi)$ for various values of R and χ; it also gives values of sec χ and ln sec χ for comparison.

When $\chi = 0$, $f(R, \chi) = \sec\chi = 1$, and $\ln \sec\chi = \ln f(R, \chi) = 0$ for all values of R.

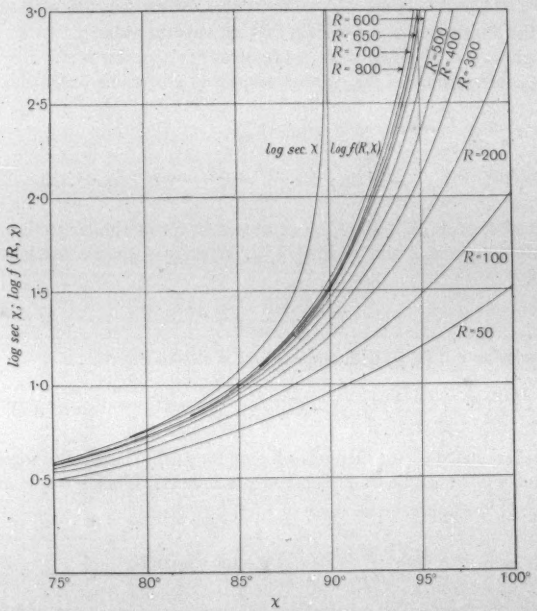

Fig. 19.

Figure 19 illustrates the functions log sec χ and log $f(R, \chi)$, for various values of R, over the range of χ from 75° to 100°. If the ordinates are multiplied by 2·303, this figure also gives the values of ln sec χ and ln $f(R, \chi)$.

§ 6. PROPERTIES OF THE FUNCTION $f(x, \chi)$

When $\chi < 90°$, or $\chi' > 0$, the principal factors in the first and most important term of $f(x, \chi)$, according to (38), are $e^{x(1-\sin\chi)}$ and $1 - (2/\sqrt{\pi})\operatorname{erf}\{2yV^2\}^{\frac{1}{2}}$; the former increases as χ decreases, while the latter decreases; these tendencies approximately neutralize one another in their product, and as χ decreases from $\frac{1}{2}\pi$, $f(x, \chi)$ steadily decreases from $\sqrt{(\frac{1}{2}\pi x)}$, approximately, and tends, from below, to $\sec\chi$ for sufficiently small values of χ.

As χ *increases* from $\frac{1}{2}\pi$, the increase of $e^{x(1-\sin\chi)}$ is no longer compensated by a rapidly decreasing factor, because the factor involving $\operatorname{erf}(2yV^2)^{\frac{1}{2}}$ is now $1 + (2/\sqrt{\pi})\operatorname{erf}(2yV^2)^{\frac{1}{2}}$, which increases from 1, when $\chi = \frac{1}{2}\pi$, and rapidly approaches the limiting value 2 when $(2yV^2)^{\frac{1}{2}}$ exceeds the value 3.

Throughout the whole range of χ, $f(x, \chi) > f(x', \chi)$, if $x > x'$.

The rate of variation of $f(x, \chi)$ with respect to χ is readily calculable:

$$\frac{\partial f(x,\chi)}{\partial\chi} = x\cos\chi\int_0^{\chi} e^{x(1-\sin\chi\operatorname{cosec}\lambda)}\operatorname{cosec}^2\lambda\,d\lambda - x^2\sin\chi\cos\chi\int_0^{\chi} e^{x(1-\sin\chi\operatorname{cosec}\lambda)}\operatorname{cosec}^3\lambda\,d\lambda \quad \ldots\ldots(41).$$

This expression could, if desired, be expressed in forms similar to those derived, for suitable ranges of x and χ, in §§ 3, 4. When $\chi = \frac{1}{2}\pi$ the result is specially simple, i.e.

$$\left\{\frac{\partial f(x,\chi)}{\partial\chi}\right\}_{\chi=\frac{1}{2}\pi} = x \qquad \ldots\ldots(42).$$

The variation of $f(x, \chi)$ with respect to x is also of interest:

$$\frac{\partial f(x,\chi)}{\partial x} = \frac{x+1}{x}f(x,\chi) - x\sin^2\chi\int_0^{\chi} e^{x(1-\sin\chi\operatorname{cosec}\lambda)}\operatorname{cosec}^3\lambda\,d\lambda \quad \ldots(43).$$

This can be evaluated by the methods adopted for $f(x, \chi)$ itself. But a general idea of the value of $\partial f/\partial x$ can be simply obtained as follows. When $\chi = \frac{1}{2}\pi$, differentiation of $(\frac{1}{2}\pi x)^{\frac{1}{2}}$, the approximate value of f (cf. § 2), gives

$$\left(\frac{\partial f}{\partial x}\right)_{\chi=\frac{1}{2}\pi} = \tfrac{1}{2}(\pi/2x)^{\frac{1}{2}} \text{ approximately} \qquad \ldots\ldots(44).$$

This is less, the greater the value of x. When $\chi = 0$, $f(x, \chi) = 1$ and $\partial f/\partial x = 0$; § 3 shows that for small values of χ the difference between $f(x, \chi)$ and $\sec\chi$ is less, the greater the value of x. This suggests that $\partial f/\partial x$ increases from 0 to approximately $\frac{1}{2}(\pi/2x)^{\frac{1}{2}}$ as χ varies from 0 to $\frac{1}{2}\pi$, being throughout smaller, the greater the value of x; this could be confirmed by a detailed discussion of (43), and is borne out by inspection of table 4. It is less easy to see how $\partial f/\partial x$ varies with x for values of χ greater than 90°, but when χ has increased sufficiently beyond $\frac{1}{2}\pi$ for the factor $1 + (2/\sqrt{\pi})\operatorname{erf}(2yV^2)^{\frac{1}{2}}$ to have become practically 2, the principal

Table 4. Values of $f(R, \chi)$ and of $\ln f(R, \chi)$

$\chi =$	30°	45°	60°	75°	80°	83°	85°	87°	90°	93°	95°	97°	100°
$\sec \chi =$	1·155	1·414	2·000	3·864	5·758	8·206	11·474	19·107	∞				
$\ln \sec \chi =$	0·1438	0·3466	0·6932	1·352	1·751	2·105	2·440	2·950	∞				
$R = 50$, $f =$	1·148	1·389	1·908	3·232	4·098	5·050	5·825	6·813	8·928	12·30	15·73	20·78	32·94
$\ln f =$	0·1377	0·3284	0·6463	1·173	1·410	1·619	1·762	1·919	2·189	2·509	2·756	3·034	3·495
$R = 100$, $f =$	1·151	1·401	1·946	3·512	4·608	5·906	7·068	8·677	12·58	20·16	29·67	49·91	106·9
$\ln f =$	0·1406	0·3370	0·6658	1·256	1·528	1·776	1·956	2·161	2·532	3·004	3·390	3·848	4·672
$R = 200$, $f =$	1·153	1·407	1·972	3·646	5·010	6·656	8·276	10·73	17·76	35·95	67·59	150·5	714·1
$\ln f =$	0·1422	0·3417	0·6788	1·294	1·611	1·895	2·113	2·373	2·877	3·582	4·214	5·014	6·571
$R = 300$, $f =$	1·153	1·410	1·981	3·706	5·281	7·020	8·918	11·96	21·74	53·57	126·9	398·2	4108
$\ln f =$	0·1427	0·3433	0·6834	1·310	1·664	1·949	2·188	2·482	3·079	3·981	4·844	5·987	8·321
$R = 400$, $f =$	1·154	1·411	1·985	3·742	5·380	—	9·327	12·82	25·09	73·94	220·1	978·4	2169×10
$\ln f =$	0·1430	0·3441	0·6858	1·320	1·683	—	2·233	2·551	3·222	4·303	5·394	6·886	9·985
$R = 500$, $f =$	1·154	1·411	1·988	3·764	5·439	—	9·620	13·48	28·05	97·92	365·6	2315	1108×10^2
$\ln f =$	0·1431	0·3446	0·6873	1·326	1·694	—	2·264	2·601	3·334	4·584	5·902	7·747	11·62
$R = 600$, $f =$	1·154	1·412	1·990	3·780	5·492	—	9·837	13·97	30·72	125·7	591·5	—	5543×10^2
$\ln f =$	0·1433	0·3449	0·6882	1·330	1·703	—	2·286	2·637	3·425	4·834	6·383	—	13·23
$R = 650$, $f =$	1·154	1·412	1·991	3·786	5·510	—	9·930	14·18	31·97	141·5	747·0	—	1233×10^3
$\ln f =$	0·1433	0·3450	0·6886	1·331	1·707	—	2·296	2·652	3·465	4·952	6·616	—	14·03
$R = 700$, $f =$	1·154	1·412	1·992	3·791	5·526	—	10·01	14·38	33·18	158·7	940·0	—	2734×10^3
$\ln f =$	0·1433	0·3452	0·6889	1·333	1·709	—	2·303	2·666	3·502	5·067	6·846	—	14·82
$R = 800$, $f =$	1·154	1·412	1·993	3·800	5·552	—	10·15	14·73	35·46	197·5	1476	—	1336×10^4
$\ln f =$	0·1434	0·3453	0·6894	1·335	1·714	—	2·317	2·690	3·569	5·286	7·297	—	16·41

factor in f, which governs the value of $\partial f/\partial x$, is $e^{x(1-\sin\chi)}$; thus $\partial f/\partial x$ is approximately equal to $(1-\sin\chi)f(x,\chi)$ for such values of χ. Since f increases with x, $\partial f/\partial x$ changes, between $\chi=\frac{1}{2}\pi$ and somewhat greater values, from a decreasing function of x to an increasing one. The change is completed, over the range of R here considered, before $\chi=93°$, as may be seen by inspection of table 4.

§ 7. THE HEIGHT-DISTRIBUTION OF THE RATE OF ION-PRODUCTION

$z(\chi)$ In part I, equation (16), it was found that the value of z, written as $z(\chi)$, corresponding to the maximum value of I at a point from which the sun's zenith distance is χ, is $\ln\sec\chi$. The corresponding height $h(\chi)$ above the ground is $h_0+Hz(\chi)$;

Z if height measured from this level as datum, in terms of H as unit, be denoted by Z,

$$Z=z-z(\chi) \qquad \text{.....(45).}$$

It is of interest to point out that in terms of Z the equation (17) of part I takes a specially simple form, namely,

$$I/I_0\cos\chi=\exp\{1-Z-\exp(-Z)\} \qquad \text{.....(46).}$$

Thus the *proportionate* height-distribution of the rate of ion-production, to the accuracy afforded by the approximate formulae of part I, is the same at all points χ, relative to the *local* height of maximum ion-production; this height increases with χ to infinity at $\chi=\frac{1}{2}\pi$; the actual rate of production at points similarly situated with respect to the local level of maximum is, however, reduced, as compared with the value at the point immediately beneath the sun, in the ratio $\cos\chi$. Thus all the curves in figure 1 of part I are identical except in their scale of ordinates, and in being bodily shifted so that their maxima occur at different points along the scale of abscissae; this point was not noted in part I. The modifications in these results due to the curvature of the earth will now be considered.

The value $z(\chi)$ at which I, as given by (9), is a maximum, is given by

$$1-e^{-z}f(R+z,\chi)+e^{-z}\partial f(R+z,\chi)/\partial z=0 \qquad \text{.....(47).}$$

For $\chi=\frac{1}{2}\pi$ it has been seen in § 5 that $\partial f(R+z,\chi)/\partial z$ is approximately equal to $f(R+z,\chi)/2(R+z)$; thus it is negligible compared with $f(R+z,\chi)$ when $R\geqslant 50$. Consequently (47) is approximately equivalent to

$$1-\exp(-z)f(R+z,\chi)=0,$$

or

$$z(\chi)=\ln f(R+z,\chi) \qquad \text{.....(48).}$$

This is valid for $\chi=\frac{1}{2}\pi$, and also for smaller values, since the ratio of $\partial f/\partial z$ to f decreases with χ when $\chi<\frac{1}{2}\pi$. As $\chi\to 0$ this equation tends to the approximate equation $z(\chi)=\ln\sec\chi$ of part I.

Reckoning height from the local level of maximum I, by the substitution (45), an approximate equation analogous to (46) is obtained, namely

$$I=\{I_0/f(R+z,\chi)\}\exp\{1-Z-\exp(-Z)\} \qquad \text{.....(49).}$$

Thus the height-distribution of I, relative to the local level of maximum I, is approximately the same as at the point immediately beneath the sun (as illustrated

by curve 7 in figure 1 of part I) except for a reduction, the same at all relative heights, in the ratio $1/f(R+z, \chi)$, or, in terms of $z(\chi)$, in the ratio $e^{-z(\chi)}$.

In part I, by neglect of the curvature of the earth, $z(\chi)$ was found to be infinity at $\chi = \frac{1}{2}\pi$, where I was given as zero at all heights. The earth's curvature being taken into account and the values of $f(x, \chi)$ given in table 4 utilized, it appears that $z(\frac{1}{2}\pi)$ varies from 2·18 for $R = 50$, to 3·45 for $R = 650$; thus the change of height of the level of maximum I, from the equator to the twilight circle, is only a small multiple of H. It should be remembered, however, that R itself depends on H (§ 2); h_0 being neglected in comparison with a, the earth's radius, $R = 50$ corresponds to a value of H 13 times as large as $R = 650$, so that $z(\frac{1}{2}\pi)$ for $R = 50$ represents $(2{\cdot}18 \times 13)/(3{\cdot}45)$ or 8·2 times as great a distance, in kilometres, as $z(\frac{1}{2}\pi)$ for $R = 650$. In the latter value, corresponding to $H = 10$ km., $z(\frac{1}{2}\pi)$ represents a distance of 34·5 km.

When χ increases beyond $\frac{1}{2}\pi$, the approximate expression for $\partial f(R+z, \chi)/\partial z$ changes fairly rapidly from $f/2\,(R+z)$ to $(1-\sin\chi)\,f(R+z, \chi)$, while $f(R+z, \chi)$ rapidly increases, approximately in proportion to the factor $e^{(R+z)(1-\sin\chi)}$; as $f(R+z, \chi)$ increases, I decreases; for the values of R that require consideration (§ 2), I is appreciable only so long as $1-\sin\chi$ is small, so that $\partial f(R+z, \chi)/\partial z$ in (47) is still negligible in comparison* with $f(R+z, \chi)$ over this range. Consequently (48) and (49) remain valid.

Thus, over the range for which I is appreciable, the function $f(x, \chi)$, in which x is to be given the value $R+z$, suffices to determine, in a very simple way, the level of maximum I, and the reduction of I at this and other levels above and below it, as compared with the corresponding magnitude at the point immediately beneath the sun. Since $\ln f = 2{\cdot}303 \log f$, the graphs of $\log f(R, \chi)$ in figure 19 thus indicate the variation in the *level* of maximum absorption of radiation, or of maximum rate of ion-production, as a function of the sun's zenith distance χ; the unit of height is taken as H.

§8. THE DENSITY OF THE AIR AT THE LEVEL OF MAXIMUM ABSORPTION

It is of interest to consider the density of the air, say $\rho(\chi)$, at the level at which, for any value of χ, the rate of absorption, or of dissociation (I), is a maximum. This occurs at $z(\chi) = \ln f(R, \chi)$ approximately, or $h(\chi) = h_0 + H \ln f(R, \chi)$: at this level $\rho(\chi)$

$$\begin{aligned}\rho(\chi) &= \rho_0 \exp\{-h(\chi)/H\} = \rho_0 \exp\{-(h_0/H) - z(\chi)\} \\ &= \rho_0/\{f(R, \chi)\exp(-h_0/H)\} \\ &= \rho_0/A\rho_0 Hf(R, \chi) = 1/AHf(R, \chi) \qquad \text{.....(50).}\end{aligned}$$

Up to $\chi = 75°$, this is approximately $(\cos\chi)/AH$.

Thus the density of the air at this level is inversely proportional to H, the height of the homogeneous atmosphere, and to A, the coefficient of absorption. The former depends on the temperature and composition of the air; also A depends on the composition.

* This may be verified also directly from table 4.

§ 9. THE SEASONAL VARIATION IN THE MAXIMUM RATE OF ABSORPTION AT NOON

The annual variation in the maximum value of I, which by (49) (putting $Z = 0$) is $I_0/f(R+z, \chi)$, over a station in latitude $l\,(=\frac{1}{2}\pi - \theta)$, may be illustrated by considering the ratio of the values at the summer and winter solstices. Reference to these two seasons will be indicated by the suffixes s and w. Since I has its maximum at noon

δ_0

$$\chi_s = l - \delta_0, \quad \chi_w = l + \delta_0,$$

where $\delta_0 = 23°{\cdot}5$. Now $I_0 = \beta S_\infty/H \exp 1$, and while β and S_∞ are unlikely to undergo any annual variation, it is possible that H may do so. Taking account of this, the required ratio is

$$I_s/I_w = \{H_w f(R+z, \chi_w)\}/\{H_s f(R+z, \chi_s)\} \qquad \text{.....(51).}$$

It may be noted that, by (50),

$$I_s/I_w = \rho_s/\rho_w \qquad \text{.....(52),}$$

where ρ_s and ρ_w are the densities $\rho(\chi)$ at the levels of maximum I at the two seasons.

For $\chi < 75°$, and therefore up to the latitude $75° - 23°{\cdot}5$ or $51°{\cdot}5$ (which happens to be the latitude of London), $f(R+z, \chi)$ is equal to $\sec\chi$ within 2 per cent. (taking R to be about 650). Thus, up to this latitude,

$$I_s/I_w = \{H_w \cos(l - \delta_0)\}/\{H_s \cos(l + \delta_0)\} \text{ approximately} \qquad \text{.....(53).}$$

The following are the values of the ratio $I_s H_s/I_w H_w$ (which is equal to I_s/I_w, if $H_s = H_w$) for various latitudes:

Latitude	0	15	30	45	50	55	60
$I_s H_s/I_w H_w$	1	1·26	1·67	2·54	3·15	4·27	7·09

§ 10. THE DAILY VARIATION IN THE RATE OF ION-PRODUCTION

The daily variation in the rate of ion-production at a given level z depends on the geographical latitude (l) of the point, and the season. If $R = 650$, then at times and places at which χ does not exceed 75°, the figures 1–5 of part I remain valid, because $f(650, \chi)$ is practically equal to $\sec\chi$, as assumed in part I, up to $\chi = 75°$.

For greater values of χ, the values of I/I_0 given in part I are in error by more than 2 per cent., if R is 650 or less. In particular, I/I_0 does not vanish when $\chi = 90°$, but is still appreciable for even greater values of χ, corresponding to points at places where at ground level the sun is below the horizon; $\chi = 90°$ corresponds to ground sunrise or sunset.

The following method may be adopted to illustrate how I/I_0 depends on height and on χ when the sun is near (above or below) the horizon.

The equation (9) may readily be transformed to

G

$$f(R+z, \chi) = e^z\{1 - z - \ln(I/I_0)\} \equiv G(z, I/I_0) \qquad \text{.....(54),}$$

or, the small difference between $f(R+z, \chi)$ and $f(R, \chi)$ being neglected,

$$f(R, \chi) = G(z, I/I_0) \qquad \text{.....(55),}$$

in which χ is involved only on the left, and z only on the right.

The graph of $G(z, I/I_0)$ as a function of z is readily plotted for any assigned value of I/I_0; G is positive from $z = -\infty$ to $z = z_0 \equiv 1 - \ln(I/I_0)$, which is positive, and beyond this value G is negative. As $z \to -\infty$, $G \to 0$ by positive values, while as $z \to \infty$, $G \to -\infty$; G has one maximum value, equal to I_0/I, for

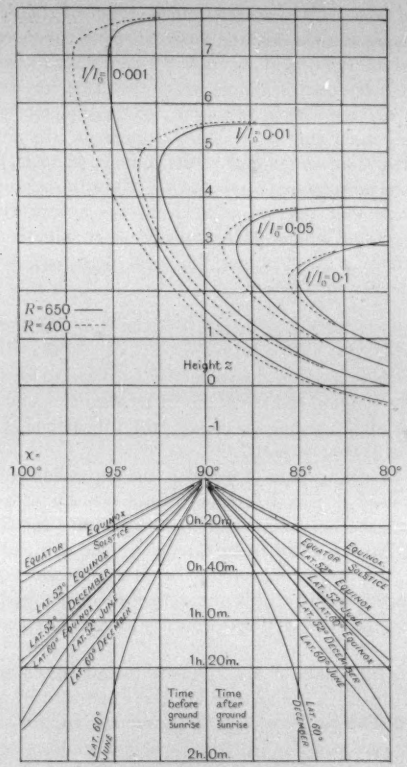

Fig. 20.

$z = z_m \equiv -\ln(I/I_0)$. Thus for any value of $G < I_0/I$, there are two corresponding values of z; moreover there is one value of χ such that $f(R, \chi)$ is equal to this value of G. The two values of z will be said to correspond to these values of χ and I/I_0; they indicate the heights at which I/I_0 has the assigned values, over any place from which the sun's zenith distance is χ. For any value of I/I_0, there is one value of χ, say χ_m, for which the two corresponding values of z coalesce; this is the χ corre-

sponding to the maximum value, I_0/I, of G, and the value of z is z_m or $-\ln(I/I_0)$. All the other values of χ corresponding to I/I_0 are less than χ_m.

Figure 20 shows the heights and times for which I/I_0 has the values 0·001, 0·01, 0·05, and 0·1, for values of χ near 90°. The graphs are drawn for two values of R, viz. $R = 650$ and $R = 400$. The lower part of the diagram indicates the relation between χ and time—now measured from the epoch of ground sunrise ($\chi = 90°$)—for various latitudes and seasons; the lower left-hand curves refer to the period before sunrise, and the right-hand curves to the period after sunrise.

It appears that if $R = 650$, I/I_0 first attains the value 0·01 at a height $z = 4·6$ approximately, at the time when $\chi = 92°·4$; at the equator this time is about 10 minutes before ground sunrise, while in latitude 52° the corresponding time before sunrise varies from 16 minutes at the equinox to 20 in December and to 22 in June; the interval increases somewhat rapidly with increasing latitude. The value $I/I_0 = 0·001$ is first attained at a height $z = 7$ approximately, at about 20 minutes before ground sunrise at the equator, or 40 minutes in latitude 52° in December. The figure well illustrates the rapid downward penetration of the radiation during the period of dawn.

§ 11. THE DISTRIBUTION AND VARIATIONS OF ION-DENSITY

The corrections here made in the value of I/I_0, allowing for the earth's curvature, modify the distribution and variations of ion-*density* found in part I, but so long as the same physical assumptions are adopted, the method of calculation is the same as indicated in §§ 9–11 of part I.

Since, for any assigned values of χ and z, the true value of I here calculated exceeds the value used in part I (except when $\chi = 0$, i.e. at noon at the point immediately beneath the sun, when the two are equal), the corresponding value of ν, or n/n_0, will be greater, at all times throughout the day and night, than was formerly deduced. The excess ion-production is due to the slightly greater absorbing area presented by the earth's atmosphere than was considered in part I; it is readily seen that the excess is proportionately least at the equator, and increases with latitude.

The changes in the values of n/n_0 formerly calculated, consequent on taking correct account of the earth's curvature, are best illustrated by graphs showing a few particular cases. This has been done for the equator and for latitude 60°, at the equinox (figures 21, 22), and for latitude 60° also in midsummer and midwinter (figures 23, 24); the parameter σ_0 has been taken as 1/25 throughout. The corrected curves are the full lines, and the original curves are shown by dotted lines, for comparison. The epoch of ground sunrise ($\chi = 0$) is indicated in each case; in the two equinoctial figures (21, 22) it is 6 a.m. It is found that the change in the predawn value of n/n_0 is too small to be shown on the diagrams.

At the equator at the equinoxes the initial rise of n/n_0 occurs about 10 minutes earlier than was inferred in part I. From one hour after ground sunrise, the corrections to the old results are small, and the curves are not reproduced for the

period after $7^h\ 30^m$. The equatorial curves at the solstices would differ only very slightly from those shown for the equinox in figure 20.

In latitude 60° at the equinoxes the initial rise of n/n_0 becomes appreciable at

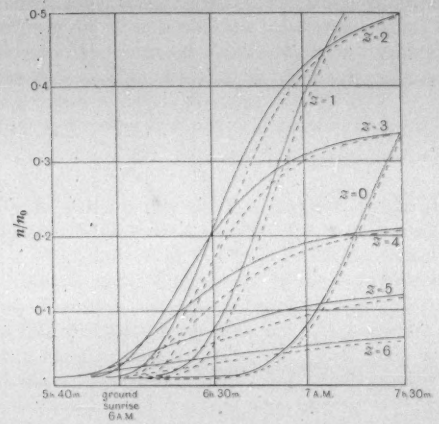

Fig. 21. Equator, equinox, $\sigma_0 = 1/25$.

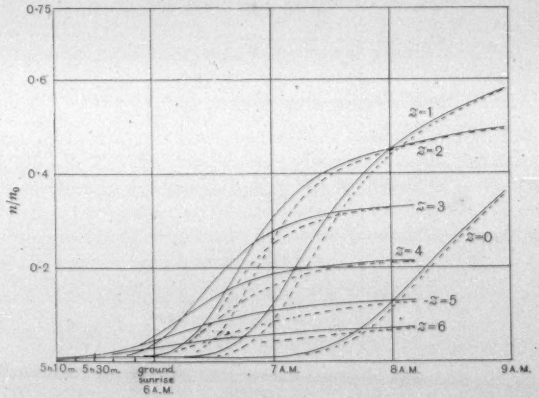

Fig. 22. Latitude 60°, equinox, $\sigma_0 = 1/25$.

20 or even 30 minutes before ground sunrise (figure 22); at ground sunrise n/n_0 is already 0·05 at $z = 4$. From about 8 a.m. the original and corrected curves are nearly identical.

In latitude 60° at midsummer (figure 23) the dawn rise of n/n_0 begins at about

1 hour before ground sunrise; at ground sunrise n/n_0 is almost 0·1 at the level $z = 4$. From 6 a.m. onwards the corrections to the original curves become small.

In the same latitude at midwinter (figure 24) the initial rise of n/n_0 begins at

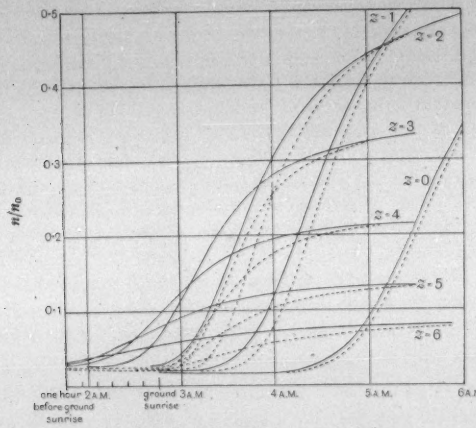

Fig. 23. Latitude 60°, summer, $\sigma_0 = 1/25$.

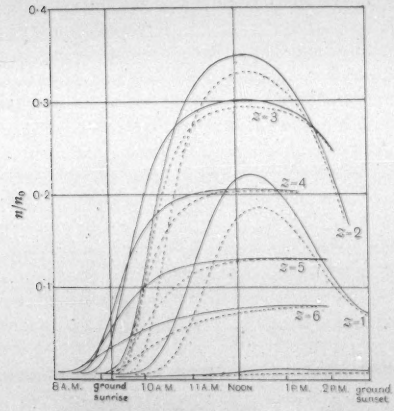

Fig. 24. Latitude 60°, winter, $\sigma_0 = 1/25$.

about 50 minutes before ground sunrise; at ground sunrise n/n_0 is about 0·07. At this season the corrections to the original curves are of importance throughout the period of daylight, though becoming small towards sunset; at noon they are quite con-

siderable. This is because χ is large throughout the day, and so there is a material difference between sec χ and $f(R, \chi)$ over this period, instead of only near dawn as in the other cases.

For the latitude of London (51°·5) the interval before ground sunrise at which n/n_0 begins to increase will be somewhat less than that calculated here for latitude 60°. It would therefore appear to agree well with the interval (rather less than an hour) observed by Prof. E. V. Appleton* in his recent measurements of the electron density in the lower ionized layer of the upper atmosphere.

It is of interest to consider how n/n_0 or ν varies near dawn. The equation of change is

$$\sigma_0 (d\nu/d\phi) = (I/I_0) - \nu^2;$$

during the night $I/I_0 = 0$, so that $d\nu/d\phi$ is negative. In the latter part of the night ν is nearly constant, and small; its value (ν', say) depends on the season, latitude, and on σ_0. In figures 21–24 ν' is about 0·01 at the equator (and, in winter, at latitude 60°), rising to 0·025 in latitude 60° in summer (these values depend on σ_0 and are purely illustrative: probably they are smaller than the true values in the ionized layers of the atmosphere). The value of ν becomes stationary when $I/I_0 = \nu'^2$, which is less than 1/1000 in the above cases; figure 20 shows that I/I_0 first attains the value 1/1000 when $\chi = 95°$. From this time ν will increase, and at first, while ν^2 is still small, and I/I_0 several times as great as ν^2, the solution of the above equation will be approximately

$$\nu = \nu' + \sigma_0 \int_{\phi_0}^{\phi} (I/I_0)\, d\phi,$$

where ϕ_0 denotes the sun's hour angle at the instant when $I/I_0 = \nu'^2$.

§ 12. ACKNOWLEDGMENTS

In conclusion I wish to acknowledge assistance received from Miss M. C. Gray, M.A., and Miss V. F. White, M.Sc., who made most of the detailed numerical calculations involved in this paper, and to Mr W. Reeve, Miss V. Hatcher and Miss R. Rossiter, who assisted in the preparation of the diagrams. I wish also to thank the Department of Scientific and Industrial Research for a grant for payment for some of the assistance received in this work.

* E. V. Appleton, *Nature*, Feb. 7, 1931.

A TIME BASE FOR THE CATHODE-RAY OSCILLOGRAPHY OF IRREGULARLY RECURRING PHENOMENA

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ABSTRACT. An account is given of a linear neon-lamp time base by means of which cathode-ray oscillograph traces due to recurring electrical or magnetic phenomena can be accurately superimposed, even though the train frequency be subject to wide and irregular variations. The underlying principle of this time base lies in the use of a glowing neon lamp which is extinguished by energy furnished either by the phenomenon under investigation or by some other phenomenon associated therewith.

§ 1. INTRODUCTION

Relatively slight coupling usually suffices in the case of either the Anson-Pearson flashing neon lamp* or the "ticking grid" (Appleton, Herd and Watson-Watt†) time base to ensure satisfactory superposition of repeated cathode-ray oscillograph traces of the relationship between time and regularly recurring, similar, electrical or magnetic phenomena such as are, for example, associated with either continuous undamped oscillations or regularly recurring trains of either damped or undamped oscillations of constant frequency and similar amplitudes. In order, however, to ensure synchronism, in the case of trains of similar phenomena where the train frequency varies, recourse must be had to tighter coupling between the circuit under examination and that of the time base, the degree of which must be increased with increasing irregularity of the train frequency, with consequent increase in the trace distortion due to such coupling. Further, the more erratic the train frequency, the poorer will be the trace synchronism, with the result that the figures become blurred and indistinct, until, finally, with even the closest permissible coupling, the individual traces can no longer be even approximately superposed.

The time base to be described below was developed with a view to the study of ignition-coil phenomena. These consist in the main of trains of either more or less damped oscillations or of non-oscillatory phenomena recurring at irregular intervals. These train-frequency irregularities are due in the main to the contact-breaker, the

* Standard Telephones and Cables, Ltd., *Bulletin*, G 577, pp. 24 and 25.
† *Proc. R.S.* A, **111**, 615 (1926).

speed of the cam drive of which is seldom constant except over short intervals. Furthermore, the cam surfaces, of which there are usually 4, 6 or 8, are not uniform (except within the limits attainable by mass production processes of manufacture), nor are the separate cam angles equally disposed about the axis of rotation of the cam. A further contributory cause of such irregularities is that due to unavoidable play of the cam spindle in its bearings.

§ 2. FLASHING HYSTERESIS IN A NEON LAMP FLASHING CIRCUIT

In order to render clear the operation of the time base to be described below, it will be necessary to discuss certain characteristic properties of a neon lamp discharge tube incorporated in a circuit such as is shown in figure 1. This is a modification of the well-known Anson-Pearson flashing circuit adapted for use as a linear, or nearly linear, time base for the purposes of cathode ray oscillography.

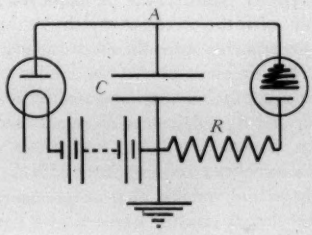

Fig. 1.

The circuit has been drawn in the conventional manner and therefore needs no further explanation, beyond pointing out that the diode must be operated under saturation conditions and that the time-scale plates of the oscillograph are connected to the point A and earth, i.e. across the condenser C.

Provided the current flowing through the diode is not excessive, C charges up to the flashing potential of the neon lamp at a linear rate and then discharges through R and the lamp until the voltage across the condenser falls to a value at which current ceases to flow through the lamp, whereupon the cycle of operations recommences, the lamp flashing once during each cycle. If, however, the current through the saturated diode be increased by brightening of the filament, a point will be reached when the lamp suddenly ceases to flash, but glows steadily. Let this critical current value be i_g. If the diode current be now lowered, flashing is not resumed until the current is reduced to a value, i_f, which may be considerably below i_g. This phenomenon may be termed "flashing hysteresis," by analogy with the "oscillation hysteresis" of an oscillating triode which has been studied by Appleton and van der Pol*. The difference, $i_g - i_f$, will serve as a measure of the flashing hysteresis.

i_g

i_f

* *Phil. Mag.* **43**, 177 (1922).

If the constants of the circuit shown in figure 1 could be so chosen that the hysteresis ($i_g - i_f$) were small and that the voltage across the lamp with i_g flowing were equal, or nearly equal, to the extinction potential, then it would appear that, under such conditions, a small momentary reduction of i_g to below i_f should result in extinction of the lamp, whereupon the ensuing charge-flash cycle should terminate in a condition of steady glow due to the renewed steady flow of i_g through the lamp. Various ways at once suggest themselves in which the necessary momentary reduction of i_g could be brought about by means of a small fraction of the energy associated with a transient electrical or magnetic phenomenon of which it is desired to obtain an oscillographic record. Further, in the case where similar phenomena are repeated, exact superpositions of the resulting traces should result, independently of uniformity or otherwise in the interval times between successive phenomena.

§ 3. EXPERIMENTAL

Whether or not the circuit, figure 1, can be employed in the manner outlined above depends primarily upon the extent to which flashing hysteresis can be reduced. The following experiments were therefore carried out with the object of elucidating this matter. The circuit employed was similar to that shown in figure 1. A three-range (0–120, 0–240, 0–360 μA) microammeter was inserted in the anode circuit between the diode and A; and, instead of a single neon lamp, two de-capped beehive-pattern "osglim" lamps with ballast resistances removed, were used in series, in order to obtain a sufficient trace-amplitude with the oscillograph employed. R was a non-inductively wound variable plug-in resistance, and C a variable air-dielectric condenser. Further apparatus consisted of a von Ardenne cathode ray oscillograph and the neon lamp time base (figure 18) which forms the subject of this communication.

§ 4. DETERMINATIONS OF FLASHING HYSTERESIS

The horizontal deflection plates of the oscillograph were connected through a suitable biassing battery to A, figure 1, and earth respectively. The vertical plates were earthed. A horizontal line traced by the cathode-ray spot on the oscillograph

Table 1.

C, μF	R, ω														
	90,000			50,000			30,000			20,000			10,000		
	i_f	i_g	$(i_g - i_f)$	i_f	i_g	$(i_g - i_f)$	i_f	i_g	$(i_g - i_f)$	i_f	i_g	$(i_g - i_f)$	i_f	i_g	$(i_g - i_f)$
0·0003	87	138	51	90·5	300	213	93	312	219	93	360	267	93	360	267
0·00055	87	96	9	90·5	252	161·5	93	255	162	96	366	270	96	390	294
0·001	90	96	6	96	105	9	100·5	114	13·5	99	270	171	100	360	258
0·0025	91·5	97·5	6	96	105	9	100·5	114	13·5	97	150	52·5	100·5	270	169·5
0·005	93	97·5	4·5	97·5	103	5·5	102	111	9	103·5	132	28·5	103·5	171	67·5

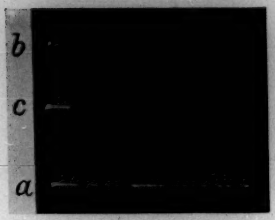

Fig. 2.

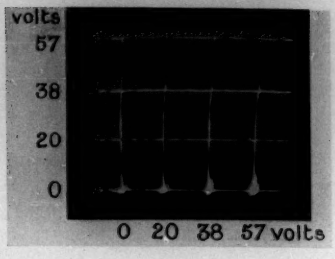

Fig. 3.

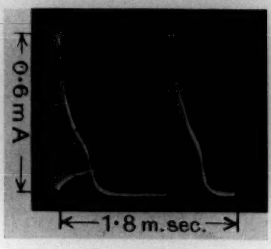

Fig. 4.

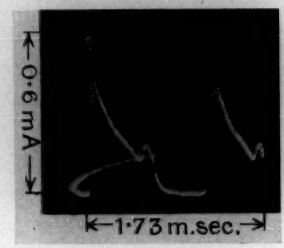

Fig. 5.

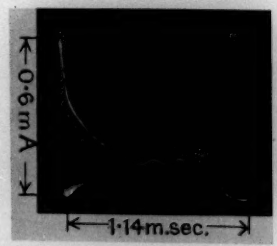

Fig. 6.

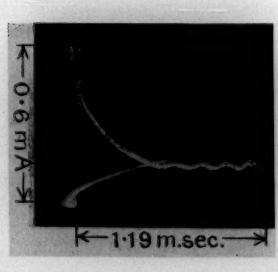

Fig. 7.

To face p. 505]

screen indicated that the neon lamps were flashing (figure 2 *a*). On increase of the diode current a point was reached when the line trace collapsed into a stationary cathode ray spot, showing that flashing had ceased (figure 2 *b*). The corresponding current i_g was read off on the microammeter. The diode current was now reduced until the spot gave way to the line trace (figure 2 *a*) on resumption of flashing conditions, the corresponding diode current being i_f. In table 1 values of i_f and i_g obtained in this manner are given in microamperes. They show the relationship between the value of C, R and flashing hysteresis for two neon lamps in series. It will be seen that low hysteresis is favoured by an increase in either C or R, and that these have critical values at which an abrupt change occurs in the magnitude of the hysteresis.

Below the heavy stepped line in the table lies a well-defined region of comparatively low hysteresis.

§ 5. FLASHING CHARACTERISTICS OF THE CIRCUIT OF FIGURE 1 UNDER LOW HYSTERESIS CONDITIONS

At first sight the results incorporated in table 1 did not appear encouraging, because the extent of the hysteresis, even under the most favourable conditions, amounted to a considerable fraction of either i_f or i_g. It was clear, however, that $(i_g - i_f)$, as determined in the manner outlined above, was virtually a measure of static hysteresis; and the fact that flashing, when once established, persisted until i_f had been increased to i_g suggested that a suitable momentary impulsive lowering of i_g to a value intermediate between i_g and i_f might suffice to extinguish a lamp glowing steadily with the current i_g. It was, indeed, occasionally observed that, with circuit conditions favourable to low hysteresis, when the diode current was gradually increased from the value i_f, the line trace on the oscillograph screen first collapsed to a much shorter line trace (figure 2 *c*) which, in its turn, collapsed to a spot when the current was further increased by an amount which could hardly be read on the microammeter; and further that, when conditions were such as to produce this short line, even such slight coupling as that due to the capacity between the oscillograph plate pairs sometimes sufficed to extinguish the glow, and thus to cause the lamp to flash once each time the vertical deflection plates were suddenly charged up to a suitable potential difference, the horizontal plates being connected to A and earth, figure 1. The short line trace was clearly due to the occurrence of oscillations of small amplitude in the lamp circuit. With a view to elucidating the rôle clearly played by the small amplitude oscillations in reducing the stability of the glowing condition of the lamps when passing the current i_g, it was decided to obtain a series of oscillograms of current time relationships and of the dynamic voltage/current characteristics of lamps flashing with various values of C and R in the circuit (figure 1).

The current time traces were obtained by connecting the vertical deflection plates of the oscillograph across R, and the horizontal plates to the neon time base circuit to be described below; and the voltage current characteristic traces by con-

necting the vertical deflection plates across R and the horizontal plates across the lamps, suitable bias being inserted in the lead from A, figure 1, to the oscillograph in order to bring the cathode-ray spot approximately into the centre of the screen when A was at a potential above earth equal to the extinction voltage of the lamps.

Of the many oscillograms recorded photographically in this manner those reproduced here (figures 4 to 17 and 19 to 24) will suffice to bring out clearly the point at issue. The exposures employed in photographing these traces were such that each oscillogram is compounded of several thousand superimposed traces. The definition of the resulting oscillograms is therefore a criterion of the degree of synchronism between the circuit phenomena under investigation and the action of the time-base employed.

§ 6. THE CURRENT TIME OSCILLOGRAMS

Ten oscillograms, showing the relationship between the current traversing the lamps under flashing conditions with various values of R and C and diode current, are reproduced in figures 4 to 11, in which the time scale is horizontal and the current scale vertical. The experimental data relating to these oscillograms are summarized in table 2.

Table 2.

Oscillogram, figure	R (ohms)	C (μF)	Diode-current (μA)	Hysteresis, $i_g - i_f$ (μA)	Remarks
4	97,000	0·0010	90		Little short of glowing
5	97,000	0·0010	93		Nearer to glowing
6	97,000	0·0010	94	6	Still nearer to glowing
7	97,000	0·0010	95		Just below glowing
8	97,000	0·0010	96		Just glowing steadily
9 *a*	32,000	0·0010	112		
9 *b*	32,000	0·0010	113	13	
9 *c*	22,000	0·0010	113		
10	33,000	0·00055	250	160	
11	53,000	0·00055	250	161	

Figure 3 is a reproduction of the voltage deflection scale which served for measuring up the oscillogram, figure 4. All other oscillograms reproduced herein, except that of figure 9, are approximately to this scale.

The oscillograms (figures 4 to 11) show that, on striking, the discharge current through the lamps rises so rapidly to a peak value that the corresponding trace is barely visible on the photographic record. The discharge current then falls much more slowly and at a non-uniform rate and, provided the diode current is sufficiently far below i_g, finally approaches zero in an exponential manner (figure 4), whereupon the steady diode current recharges C until flashing recurs. When the diode current is slightly increased a similar sequence of events occurs with the exception that the discharge current, shortly before falling to zero, executes a single or partial oscillation (figure 5) about a mean value corresponding approximately to that of the diode

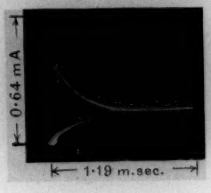

Fig. 8.

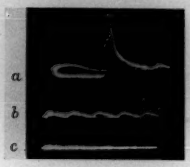

Fig. 9.

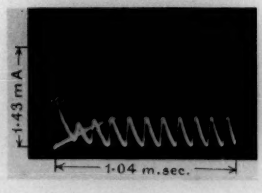

Fig. 10.

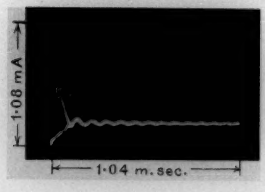

Fig. 11.

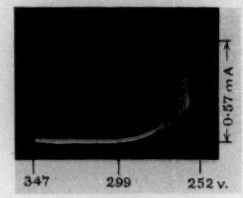

Fig. 12.

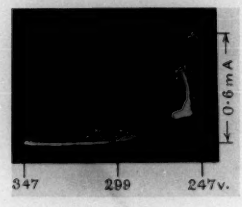

Fig. 13.

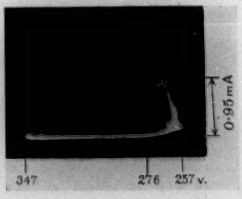

Fig. 14.

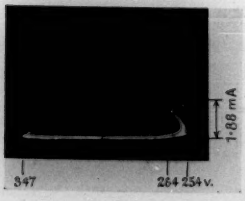

Fig. 15.

current. A further increase in the diode current results in the setting up of a train of unstable oscillations which increase in amplitude (figures 6 and 7) until, after attaining a critical amplitude value, the discharge current decays to zero (figure 6), and the lamps are extinguished. When the diode current is equal to or above i_g the discharge current falls smoothly and without executing oscillations to the diode-current value (figure 8) at which it remains constant, and the lamps in consequence glow steadily. Resistance coupling was resorted to in order to ensure synchronism in the case of the oscillograms figures 4 to 8, a variable 1-megohm resistance, in nearly every case all in, being employed for this purpose. This fact alone will serve to illustrate the ease with which is set up a train of oscillations of increasing amplitude which rapidly lead to extinction of the lamps, provided the circuit conditions are suitable.

In securing the oscillograms shown in figure 9, precautions were taken to reduce and confine the coupling between the circuit of figure 1 and that of the time base as far as possible to that due to the capacity between the oscillograph plate pairs alone. In figure 9 *a* the diode current was less than 1 per cent. below i_g. In figure 9 *b* the diode current was practically equal to i_g; the perfect superposition of the current time traces of the low-amplitude saw-toothed oscillations obtained adequately demonstrates the sensitivity of the time base circuit employed and the minute amount of coupling required to ensure synchronism. In the case of figure 9 *c* the diode current was slightly above i_g, and in order to secure a trace it was necessary to reduce the time-base diode current to below its i_f value.

The oscillograms shown in figures 10 and 11 are representative of those which could be obtained when the circuit conditions were such as to give large hysteresis values. Comparatively tight coupling had to be resorted to in order to obtain these oscillograms. According to the value of the diode current, the discharge current either (i) fell to zero, in a manner similar to that shown in figure 4 (diode current $< i_g$), or (ii) finally executed undamped saw-tooth oscillations as in figure 10 (diode current $\fallingdotseq i_g$), or (iii) gave rise to damped oscillations as in figure 11 (diode current rather higher than in (ii) but still $\fallingdotseq i_g$).

§ 7. THE DYNAMIC VOLTAGE/CURRENT CHARACTERISTIC OSCILLOGRAMS

The current/time records discussed above adequately demonstrate the metastability, under suitable circuit conditions, of the glowing state of the neon lamps passing a current equal to i_g, and the ease with which extinction may under those conditions be brought about. They further show that the initial portion of a time-base stroke produced by means of the circuit figure 1 cannot be linear; because the discharge current after falling below i_g does not fall to zero instantaneously, but does so in an exponential manner, during which time the diode current is divided between that flowing through the lamp and that which charges up C. The series of voltage/current oscillograms shown in figures 12 to 17, in which the current and voltage scales are vertical and horizontal respectively, afford a clear insight into the

effect of the values of C and R upon the lamp voltage when the current i_g or one approximately equal thereto is passing, and into the value of extinction voltage ensuing upon reduction of i_g in the manner previously outlined. The flashing potential under static conditions for the two neon lamps in series was found to be 338 volts. The experimental data relating to the oscillograms, figures 12 to 17, are given in table 3.

These oscillograms show that in every case the lamp voltage falls, on flashing, to a value considerably below that of the extinction potential under the corresponding dynamic conditions. Further, for present purposes*, it will be sufficient to draw attention to the following facts brought out by these oscillograms: The difference between the values of the dynamic extinction and flashing potentials, i.e. the length of the linear portion of the time base stroke, increases (i) as R is decreased (figures 12, 13, 14 and 15) and (ii) as C is increased (figures 14, 16 and 17). It is clear, therefore, that, in order to obtain the greatest possible length of linear stroke with the time base, it is essential that the values of C and R should be so chosen as to be as near as possible to, but below, the heavy stepped line in table 1.

Table 3.

Oscillogram figure	R (ohms)	C (μF)	Hysteresis (μA)	Remarks
12	90,000	0·001	6	Well below glowing
13	90,000	0·001	6	Nearly glowing
14	30,000	0·001	13·5	Nearly glowing
15	10,000	0·001	258	
16	30,000	0·0035	11	Nearly glowing
17	30,000	0·00055	162	

§ 8. DESCRIPTION OF THE TIME BASE

The practical circuit of the time base which has been developed on a basis of the considerations outlined above is shown in figure 18. The arrangement of filament rheostats shown therein permits of a fine control of the temperature of the tungsten filament of the R-type triode, of which grid and anode are inter-connected. The microammeter, μA, can be calibrated in terms of the duration of one time stroke for known values of C, a variable 0·0003 μF condenser, which is provided with terminals to which fixed condensers of suitable capacity can be connected. The resistance R is carried in a clip-in holder, thus permitting of easy interchange of different values of R. In the interests of linearity, the unearthed side of C is connected to the corresponding time deflection plate of the oscillograph through a bias battery, instead of the usual arrangement of a condenser and leak.

Suitable coupling between the time base and the circuit under investigation can be effected in various ways. For instance, the plate P of the oscillograph can be

* It is our intention to discuss these and other dynamic characteristics of discharge tubes in greater detail in a further communication to the Society.

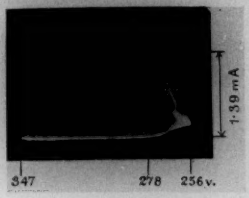

Fig. 16

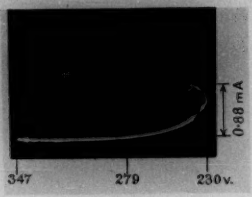

Fig. 17.

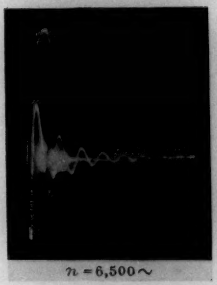

Fig. 19.

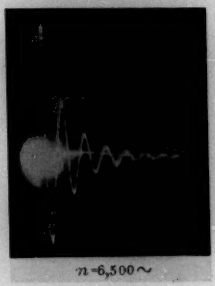

Fig. 20.

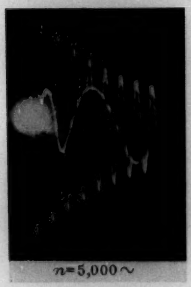

Fig. 21.

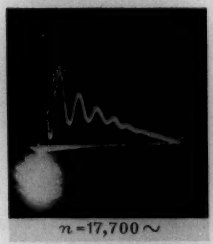

Fig. 22.

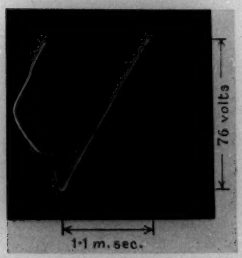

Fig. 23.

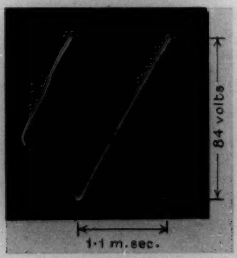

Fig. 24.

[To face p. 508

connected to the terminal B, the degree of coupling being controlled by means of the 1-megohm variable resistance shown in the diagram. Or, again, an inductance coil connected across A and B can be inductively coupled with the circuit under investigation. In certain cases we have found it of advantage to incorporate a triode adjusted to operate on bottom-bend conditions in the coupling circuit, in order to reduce interference with the phenomenon under investigation to a minimum.

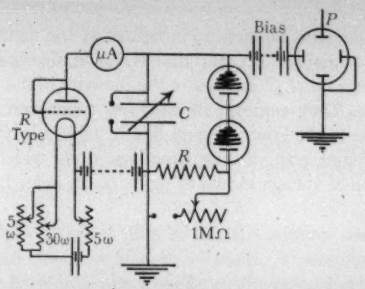

Fig. 18.

The following precautions should be taken in using the base. The high and low-tension accumulators should be first fully charged and then slowly discharged to the extent of approximately one-fifth of their capacity before use. They should be re-charged when a further two-fifths of their charge has been used up. The triode and the filament rheostats should be shielded from draughts. It is, further, essential that the insulation be good, particularly across the diode, the condenser C, and the neon lamps, and in order to ensure this being the case the pinches of both the de-capped diode and lamps should be well dried and varnished with shellac.

When the time base is to be operated the filament rheostats are adjusted until the diode current equals i_g. The voltage changes due to the phenomenon to be investigated are then applied to the plate P, and finally the coupling is adjusted until the neon lamps commence to flash in synchronism with the phenomenon. The filament and coupling adjustments will be found to be critical. Incorrect direction of coupling will result in distortion, as in figure 19, which shows the oscillations in a circuit comprising an iron-cored inductance of 3 mH and a capacity of $0{\cdot}2\,\mu$F. In this case not only was the direction incorrect, but the amount of coupling was excessive also. Similar oscillograms, but obtained with correctly adjusted coupling, are shown in figures 20 (iron-cored 3 mH $-$ $0{\cdot}2\,\mu$F) and 21 (air-cored 5 mH $-$ $0{\cdot}2\,\mu$F). It will be seen that, in the cases of the two latter oscillograms, non-linearity of the time base is confined to the initial portion of the first half-oscillation. Figure 22 shows the current oscillations in a 3000 ohms resistance shunting the secondary of an induction coil. It may be mentioned that in photographing these last four traces the camera lens was stopped down as far as possible (f : 42), and that during

exposure (about 30 sec.) the speed of the 8-cam contact breaker employed was made to vary in an erratic manner between about 100 and 800 r.p.m. A train frequency higher than 6400 per minute was not employed with the iron-cored inductance circuits because the circuit arrangements were such that, at higher speeds, the value of current flowing in the inductance at break would have varied inversely with the time of make by the contact-breaker. The definition of the oscillograms obtained is ample testimony to the degree of accuracy with which the cathode-ray traces have been superimposed.

Finally, the oscillograms reproduced in figures 23 and 24 in which the relationship between the potential across C and time is shown, demonstrate the uniformity in the rate of charging up of C, and hence the linearity of the time base, in all but a short initial stage. They confirm, further, the conclusion that the nearer the values of C and R lie to the heavy stepped line in table 1 in the region below that line, the greater is that proportion of the time-base stroke which is drawn at a linear rate. The rate of rise of voltage shown in figure 24 is, indeed, uniform practically throughout.

§ 9.

We wish to place on record our appreciation of the assistance afforded to us by Messrs Ferranti Ltd., who generously placed at our disposal such apparatus as was required in the prosecution of this investigation.

NOTES ON THE OSCILLOGRAMS, FIGURES 4 TO 11.

Flashing occurs simultaneously in the neon lamp of the time base and circuit under investigation at the origin of the coordinates, except in the case of figure 9 *a*.

Figures 4, 5 *and* 6: Time-base-diode current = i_g; the time base is "tripped," i.e. the time base lamps extinguished by flashing of the lamps in the circuit under investigation.

Figures 7, 8, 10 *and* 11: Time-base-diode current = i_g; time base "tripped" by flashing of circuit under investigation, which is, in its turn, "tripped" by flashing of time base.

Figures 9 *a*, *b*: Time-base-diode current = i_g. Time base "tripped" by low amplitude saw-toothed oscillation of circuit under investigation. In figure 9 *a* the first saw-toothed oscillation is followed (after flashing of time base) by a second one of increased amplitude which leads to extinction. In the case of figure 9 *b*, conditions are such that flashing of the time base does not increase the amplitude of the saw-toothed oscillations to an extent sufficient to bring about extinction of the lamps in the circuit under investigation.

NOTE ON THE OSCILLOGRAMS, FIGURES 12 TO 17.

Flashing occurs at 347 volts. The discharge current then rises and the voltage across the lamps falls with such rapidity that the corresponding trace is barely visible. The voltage invariably falls below the extinction potential, i.e. that voltage at which current ceases to flow (299 volts in the case of figure 12).

NOTE ON THE OSCILLOGRAMS, FIGURES 21 AND 22

The beginning of the first half-cycle trips the time base and the oscillogram is described. When the time-base lamps flash, the cathode-ray spot returns at an exponential rate to zero, the initial rate of such return being far more rapid than that of the true time stroke in the forward direction; hence, in figure 21, the expansion of oscillations which occurs on the return stroke; and in figure 22 the slope of the return stroke is seen to be much less than that of the mean line about which the current oscillates. In both cases, the spot remains at zero until the beginning of the first half-oscillation of another train trips the time base afresh.

DISCUSSION

Prof. A. O. RANKINE remarked that the traces shown in the illustrations showed very little blurring. The phenomena represented must therefore have been very similar at each recurrence.

Mr O. S. PUCKLE. I would like to ask the authors where the connexions between the tube and the wave source are made. I presume that the source is connected to the two horizontal plates of the oscillograph, that is to say to the right-hand side of the resistance R, figure 1, so that the potential across it is used to trigger the time base. In addition I would like to enquire whether it is necessary to adjust the diode current when large changes are being made in the value of the frequency under examination, say from 1000 to 1,000,000 ~.

AUTHORS' reply. The time-base-circuit condenser is connected to one pair of plates, one of which is earthed; the source leads go to the other pair of plates, one of which is earthed while the other is connected to the neon lamp through a variable 1-$M\Omega$ resistance. Energy thus tapped off through the unearthed source-lead trips the neon lamp. It is not necessary to adjust the diode current, but the coupling between the time-base-circuit and the source may require adjustment.

THE HILGER X-RAY CRYSTALLOGRAPH AND THE CUBIC-CRYSTAL ANALYSER

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ABSTRACT. A description is given of an X-ray crystallograph designed on the principle used by Seeman[1] and Bohlin[2] in which the slit, specimen to be analysed and photographic film are disposed along the circumference of a circular camera. Typical crystallograms obtainable are shown.

The cubic-crystal analyser is described and some examples demonstrating its use are given.

§ 1. INTRODUCTION

The basic principle underlying all powder methods of X-ray crystal analysis is essentially that of Bragg[3], but owing to the impracticability of obtaining single crystals of appreciable size, his original method has been modified so as to be adaptable to the various obtainable forms of the material under investigation. This has led to such methods as those used by Debye and Scherrer[4], Hull[5], Weiss[6] and others, which, although differing slightly in their experimental arrangements, owe their success in operation to identical factors. In order to arrive at a correct determination of the atomic structure of any substance by any of the powder methods, it is essential that the substance should consist of a conglomeration of small crystals offering all possible orientations to the incident monochromatic X-ray beam. This condition is ensured by powdering the substance (filings being used in the case of metals and alloys), and spreading them into flat layers or placing them in thin celluloid containers. Rotation or oscillation will further ensure the random orientation necessary, and we would then have all the
θ possible planes present at the required orientation θ to the incident beam so that reflection should result according to the Bragg relationship

$$n\lambda = 2d \sin \theta \qquad \ldots\ldots(1),$$

d d being the spacing between geometrically like planes,

λ λ the wave-length of the X-rays used, and

n n the order of reflection obtained.

Let us now consider the effect of a narrow circular beam of X-rays when incident upon a rod of powdered specimen situated at the centre of a circular camera (Debye and Scherrer method). Wherever it encounters a crystal plane of spacing d at a glancing angle θ such that the Bragg equation is satisfied, then the

ray will be reflected, the total deviation from its original path being 2θ. But owing to the haphazard distribution of the crystals, all orientations of the reflecting plane in question will be possible, and hence the reflected ray will not lie in one single plane but will form a cone of semi-vertical angle 2θ about its incident direction as axis. The intersections of these conical surfaces with the film, which is placed round the circumference of the camera, produce a series of curved lines symmetrically placed about the point of incidence of the direct beam. Measurement of the distances between these lines leads to the evaluation of the glancing angle θ and hence to the solution of the Bragg equation.

For precise measurements it is necessary to take certain precautions and make various vital corrections. Some of the principal ones are enumerated below.

(i) Since the breadth of the spectrum lines recorded is approximately equal to the breadth of the crystalline rod, inaccuracies often occur in the measurement of the inter-lineal distances owing to the difficulty of exactly determining the centre of lines not having sharp edges.

(ii) Approximate measurements only can be made of the crystal-to-film distance. This can be partially remedied by an exposure of some standard substance, such as sodium chloride, placed in a cylindrical container of dimensions identical with those used for the substance under analysis. A high order of accuracy cannot be obtained owing to the variation of the absorption properties of the substances used.

(iii) An important error is that due to absorption, which has the effect of making the calculated value of θ greater than it should be. Corrections have been suggested from theoretical considerations by O. Pauli[7] and by A. J. Bijl and N. H. Kolkmeyer[8]. Practical methods employing comparison of photographs with those of standard substances have been used by W. P. Davey[9] and R. W. G. Wyckoff[10]. If the powder used is pressed into the form of a flat layer, or if flat plates are used in the analysis of metals or alloys as in Weiss's method, the discrepancies due to absorption are modified, but further sources of error are entailed by the new experimental arrangement.

(iv) In Hull's method, where flat sheets of the powder are used either in a compressed condition or between plane glass plates, and are placed normally to the X-ray beam, the errors due to absorption are negligible. However, for accurate measurements precautions must be taken to ensure that the thickness of the sheet is constant and that the surfaces are perfectly plane.

(v) In Weiss's method a plate of the metal or alloy is oscillated in the X-ray beam through any convenient angle. Errors will occur if the surface is not coincident with axis of the cylindrical camera. Corrections as suggested by A. Müller[11] must be applied.

(vi) In the above-mentioned powder methods it is necessary, in order to obtain satisfactory photographs, that the size of the crystal grains be small, that is, about 0·1 mm. to 0·005 mm. If the crystals used are too large, the spectrum lines recorded will appear broken and disjointed, which further hinders the accurate location of the mid point during measurement of the inter-lineal distances.

(vii) Finally a great practical drawback in all of the above methods is the prolonged exposure necessary, unless a powerful and therefore more expensive plant is used.

We thus see that in order to arrive at precise determinations of crystal structures the observer must take special precautions in the preparation of his specimens and in the setting up of his instrument. He must then apply corrections to his calculated results. The following paragraphs, however, describe a method of taking powder photographs without the encumbrance of the aforementioned sources of error.

§ 2. PRINCIPLE OF METHOD

Figure 1 represents diagrammatically the camera of the crystallograph. A monochromatic X-ray beam passes through the slit S and is incident upon the scattering substance placed along BC. Consider a ray striking the powder at the point B. This will be reflected from a lattice plane of spacing d and will record a spectrum line on the photographic film at the point P. The amount of deflection from its incident path, suffered by the ray, is equal to 2θ according to the Bragg equation (1), that is

$$\angle EBP = 2\theta.$$

From the geometry of the figure

$$\angle SBP = \angle SCP,$$

whence

$$\angle FCP = 2\theta.$$

Thus if a ray is incident at C upon a lattice plane of the same spacing d as at B, it will be reflected to the point P. Similarly for all points along BC. Hence all the rays which strike lattice planes of identical spacings will be deflected and *focussed* upon the same point P, where a sharp image of the slit should be expected.

Again, from the geometry of the figure

$$\angle SOP = 4\theta.$$

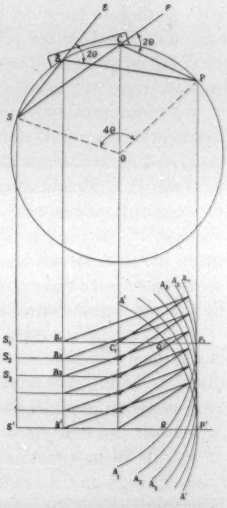

Fig. 1.

p Hence if p is the distance of spectrum line from
r slit (arc SP) and r the radius of the camera then

$$p = 4r\theta \qquad \ldots\ldots(2),$$

whence the glancing angle can be calculated. This value of θ substituted in the Bragg equation (1) will then give the spacing of the reflecting lattice plane.

§ 3. FORMATION OF THE SPECTRUM LINES

The actual formation of the spectrum lines can be seen from the vertical projection[2] as shown in the lower half of figure 1. Rays passing through the slit S_1S' are incident on the scattering substance at $B_1B_2 \ldots B'$, and will all be deflected at an angle 2θ. (We are considering only lattice planes of spacing d.) The deflected ray will in each case, as in the Debye and Scherrer method, describe a cone of semi-vertical angle 2θ about its incident direction, and will eventually intersect the film, forming curves $A_1A_2 \ldots A'$. The superimposition of these curves gives rise to the photographic image of shape $P_1P'RQ$, but with a sharply defined edge P_1P' which corresponds to the common tangent of the curves and is geometrically defined by equation (2). This image formation is very well illustrated in figure 2 which is an untouched reproduction of an enlargement ($\times$ 5 approximately) of a spectrum line that was actually recorded on the crystallograph. The scattering substance in this case was a sheet of annealed metallic aluminium having its crystal grains larger than would be practicable with other powder methods. Even in this case the edge is broken, but its definition is sharp enough for accurate measurement.

This edge does not always lie on the same side. As the glancing angle increases the breadth of the line decreases, as can be seen from the construction of figure 1, till a point is reached where $\theta = 45°$, that is, where the semi-vertical angle 2θ of the intersecting cone $= 90°$. At this position the spectral line formed is a straight-line image of the slit. If the glancing angle is still further increased the breadth of the line will also be increased, but now the sharp edge will be on the opposite side. The turning-point is given by substitution in equations (1) and (2)

$$n\lambda = 2d_0 \sin 45°, \qquad d_0$$

that is

$$d_0/n = \lambda/\sqrt{2} \qquad \ldots\ldots(3a),$$

and also

$$p_0 = \pi r \qquad \ldots\ldots(3b).$$

§ 4. COMPARISON WITH OTHER POWDER METHODS

In order to facilitate direct comparison with other powder methods, the various factors affecting the accuracy of measurement will be dealt with in approximately the same numerical order as before.

(i) The breadth of line does not affect the accuracy of measurement since readings are taken on one edge only. The breadth is, however, kept within certain limits so as to prevent diffusion of the edge. This is effected by using narrower slits and a narrower width B_1B' of the scattering material (cf. figure 1).

(ii) No crystal-to-film distance need be measured. The only dimension required is that of the radius of the instrument. This is permanently fixed and can be checked by taking an exposure of some standard substance of known lattice spacings, such as sodium chloride.

(iii) Results are unaffected by absorption, that is, by the varying depth of penetration of the X-ray beam. This is shown in figure 3 where the dotted lines represent the path of the reflected rays from the interior of the scattering medium. The geometric position of the sharp edge P is not influenced by the amount of penetration. This is demonstrated rather well in figure 2, where the images reflected from two layers of crystals can be clearly seen at P and P'.

(iv) Care must be taken to ensure that the whole surface of the reflecting material lies on the circumference of the camera. In the case of metallic sheets this condition is easily fulfilled by bending the metal into an arc of the required radius. Powders, however, may be either compressed into the requisite shape or painted on a specially provided powder holder. The latter is a glass plate which has its face ground to a concave cylindrical curve of radius equal to the focussing radius of the camera.

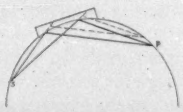

Fig. 3.

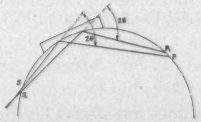

Fig. 4.

(v) There is no need to provide for the oscillation or rotation of the specimen, as the whole surface lies in the beam of the incident X-rays.

(vi) The range of crystal-grain size is shifted to a higher limit.

(vii) The exposures are greatly reduced. For example the powder photograph of copper filings was taken in 45 seconds with monochromatic CuK_α radiation generated by a Shearer tube working at peak voltage of 70,000 volts and a slit width of 0·07 mm.

(viii) A very necessary precaution is to ensure that the slit edge S, figure 4, is permanently fixed coincident with the circumference of the camera. The size of slit-width will then not affect the accuracy of readings since the sharp edge corresponds only to the position of S (cf. figure 4).

§ 5. GENERAL CONSTRUCTION OF THE CRYSTALLOGRAPH

A diagrammatic reproduction of the general assembly of the crystallograph camera is shown in figure 5. In figure 6 it is shown in the dismantled condition.

The camera itself is constructed in the form of a shallow cylinder 1 which is 4·4 cm. in height and 4·5 cm. in radius. A portion is cut away to form an angular recess for accommodating the anticathode end of a Hilger-Shearer X-ray tube. The relative position of the tube is fixed by four locating studs 4, 6, 9, 11, one of which a, 4 is adjustable. Once the correct position of the tube is found, its location is permanently secured by seeing that the anticathode end is making proper contact with all four studs. A great advantage which arises in having the crystallograph designed expressly for use with the Shearer X-ray tube is that we thus obtain a

Fig. 2.

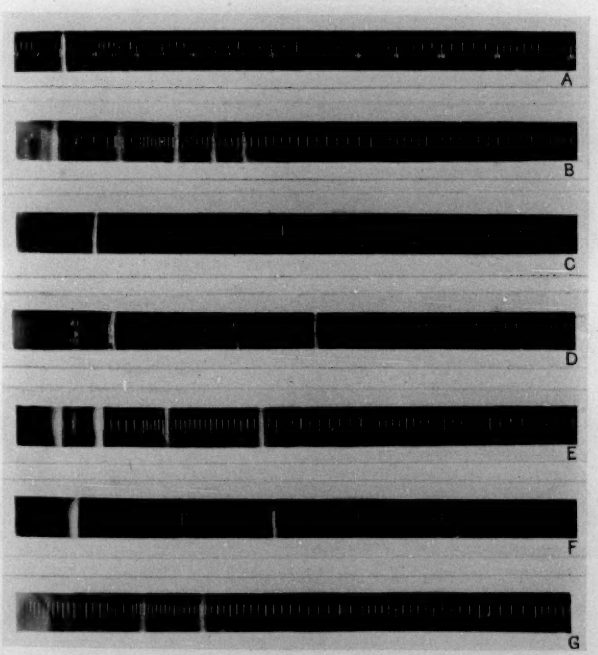

Fig. 7. Typical crystallograms. *A*, powdered sodium chloride, exposure 20 min.; *B*, powdered zinc oxide, exposure 10 min.; *C*, copper filings, exposure 5 min.; *D*, aluminium sheet (annealed), exposure 5 min.; *E*, zinc sheet, exposure 5 min.; *F*, tungsten filings, exposure 10 min.; *G*, lead filings, exposure 30 min.

[*To face p.* 516

maximum intensity of the incident X-ray beam, since the anticathode of the tube is directly behind its window which is itself almost in contact with the slits, figure 5.

The slit system consists of two brass jaws 5, 8 tipped with a bismuth-lead alloy and cut obliquely at angle of 30° to form knife-edges. The slit jaw S is permanently fixed, whilst slit jaw S_1 is adjustable by hand. A set of slit gauges is used for setting to the required aperture and the slit jaw S_1 is secured in position by two screws. A lead stop b prevents fogging of the film by direct or scattered radiation from the slit system. The glass specimen holder C, 13, 14, previously described, fits into the side of the camera and is secured by a light-tight cover d, 15 and two screws e, 7, 10.

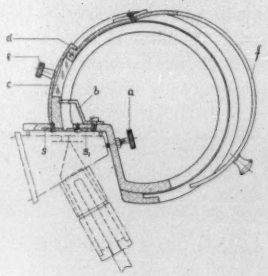

Fig. 5.

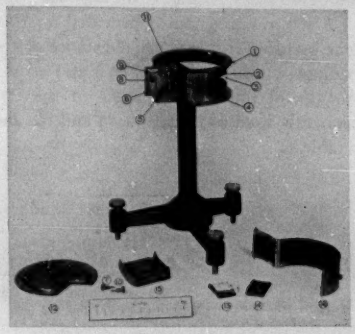

Fig. 6. Hilger crystallograph.

The photographic film is 17 cm. long and 2·5 cm. wide; it has one end punched with two accurately gauged holes and is then fitted on the two small pegs which locate it in the camera. It is then wrapped around the circumference of the camera and rests on the cylindrical surfaces 2 on either side of a wide annular slot 3 through which it is exposed to the reflected rays from the scattering medium on the powder-holder. A light-tight brass cover f, 16 clips the film into position and holds it firmly in place.

The camera is provided with a light-tight brass cover plate 12 and is mounted on a firm tripod stand provided with levelling screws, at such a height that the window of the Shearer tube is level with slit of the crystallograph.

§ 6. THE INTERPLANAR SCALE

Since the constants of the instrument are permanently fixed, it has been possible to devise a method of superimposing an interplanar scale upon the crystallogram. We get by substitution for θ from equation (2) in equation (1)

$$d/n = \lambda/\{2 \sin (\rho/4r)\},$$

that is

$$d_{hkl}/n = \lambda/\{2 \sin (\rho/4r)\} \qquad \ldots\ldots(4),$$

where d_{hkl} is the spacing between like planes whose Miller indices[12] are h, k, l. d_{hkl}

The scale was calculated from the above relationship between d_{hkl}/n and p and gives value of d_{hkl}/n at every point of the film. It was assumed that filtered Cu-radiation ($\lambda = 1{\cdot}54$ Å.U.) had been used. This condition is obtained in practice by using a Shearer X-ray tube fitted with a copper anticathode and a nickel-foil window. The interplanar scale is contained in a separate printing-frame provided with two film-locating pins so placed as to correspond approximately to a similar pair fixed in the camera. After exposure in the crystallograph the film is removed in a dark-room and placed in the printing-frame. The interplanar scale is then printed on to the negative by exposure to white light for a fraction of a second, and development brings it up together with the crystallogram.

A screw adjustment is provided for the exact setting of the scale. This can be done by using any standard substance of known spacings and then setting the scale readings, by the method of trial and error, to coincide with the theoretical values of d_{hkl}/n of the lines obtained on the crystallogram. Sodium chloride is most frequently used as the standard and the lines usually chosen for setting are given in table 1.

Table 1.

Sodium chloride ($a_0 = 5{\cdot}628$ Å.U.)	
Reflecting plane hkl (n) *	d_{hkl}/n
110 (2)	1·991
111 (2)	1·625
100 (4)	1·407
120 (2)	1·259
112 (2)	1·149
110 (4)	0·995
100 (6)	0·938

Typical crystallograms are shown in figure 7. These were taken with copper K_α radiation obtained from a Hilger-Shearer X-ray tube having a nickel-foil window and fed with about 5 mA. from a transformer working at a voltage of about 60 kV. r.m.s. A slit-width of 0·1 mm. was used, and the exposures given were in excess of that normally required so as to ensure better printed reproductions.

§ 7. THE CUBIC-CRYSTAL ANALYSER(13)

The cubic-crystal analyser has been constructed from consideration of the simple cube lattice as shown in figure 8 (1). This cube is taken as the fundamental basis of development of all other cubic structures which are encountered in crystallography, and which are obtained by the addition of lattice points in various symmetrical positions. Thus we get the body-centred cube, figure 8 (2), and the face-centred cube, figure 8 (3); but whatever the cubic structure, the measurement of the length a_0 offers a *unique specification of the substance undergoing investigation.*

a_0

* The values of (n) as given in column 1 have not been corrected to account for the internal structure of the elementary cube. See § 8.

The spacing of geometrically like planes parallel to the plane (*hkl*) is given by

$$d_{hkl} = a_0/(h^2 + k^2 + l^2)^{\frac{1}{2}} \qquad \text{.....(5).}$$

Now in practice with powder photographs it is found that the value of d_{hkl} is derived from a knowledge of the constants in the Bragg equation (1)

$$n\lambda = 2d_{hkl} \sin \theta,$$

provided of course that the order n of the spectrum considered is known. But this is not always determinable at sight, and hence solutions are found primarily for d_{hkl}/n.

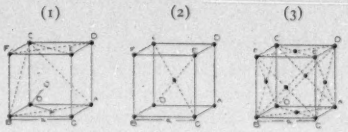

Fig. 8.

Rewriting equation (5) we get

$$d_{hkl}/n = a_0/n\,(h^2 + k^2 + l^2)^{\frac{1}{2}},$$

that is

$$a_0 = n\,(h^2 + k^2 + l^2)^{\frac{1}{2}}\,(d_{hkl}/n) \qquad \text{.....(6).}$$

Integral values being assigned to h, k, l and n, it has been possible to obtain a series of plots showing the linear relationship between a_0 and d_{hkl}/n for all the most probable crystal planes of reflections in various orders, 100 (1), 111 (3) etc. This was the simple process employed in the construction of the analyser, figure 9.

§ 8. ANALYSIS OF CUBIC CRYSTALLOGRAMS

The method used is applicable to crystal structures having cubic symmetry, and is similar to that employed by Hull and Davey[14] for determining the indices of the reflecting planes in hexagonal and tetragonal structures from graphical charts. A strip of paper is scaled off identically with that of the abscissae of the analyser. Values of d_{hkl}/n for the various powder lines recorded are read directly by means of the interplanar scale and then inscribed along one edge of the paper strip. The latter is then moved parallel to the horizontal axis with the zero mark always on the vertical (a_0) axis until a position is found where there is a full coincidence between the curves on the graphical plot and the markings on the paper strip. (Figure 9 demonstrates coincidence of lead lines.) The indices of the reflecting planes are read off directly, whilst the size of the unit elementary cube is obtained from the point of intersection of the strip edge with the vertical (a_0) axis.

The value of n given by the curves must not be confused with actual order of spectrum line obtained, since the analyser was constructed solely from considerations of the elementary unit cube without allowance for the internal distribution of the

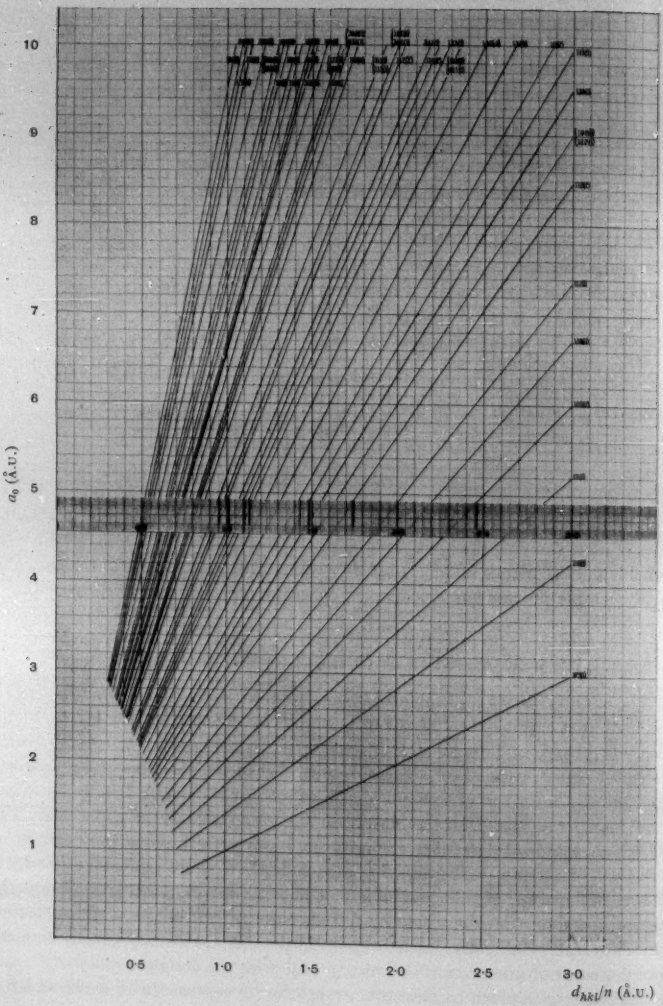

Fig. 9.

lattice points. This is by no means an objection to the use of the analyser, since it provides a method of identifying the various cubic structures by noting the different orders of reflection given by the analyser; for from a comparison of the interplanar distances of types of cubic structures indicated in figure 8, we obtain for the principal reflection planes (100), (110) and (111):

Elementary cube. $d_{100} = a_0$; $d_{110} = a_0/\sqrt{2}$; $d_{111} = a_0/\sqrt{3}$ .....(7).

Body-centred cube (*B–c*). $d_{100} = a_0/2$; $d_{110} = a_0/\sqrt{2}$; $d_{111} = a_0/2\sqrt{3}$ .....(8).

Face-centred cube (*F–c*). $d_{100} = a_0/2$; $d_{110} = a_0/2\sqrt{2}$; $d_{111} = a_0/\sqrt{3}$ .....(9).

Thus from the above expression it can be seen that:

(i) d_{100} for the *B–c* and *F–c* structures is half that for the elementary cube. Hence first-order reflection for *B–c* and *F–c* structures from (100) planes will coincide with second-order reflection for an elementary cube; the second-order for *B–c* and *F–c* structures with fourth-order for elementary cube, and so on. Thus the analyser will not show any odd order reflections from (100) planes of *B–c* and *F–c* structures.

(ii) Similarly since d_{110} for *F–c* structures is half that for the elementary cube, no odd order reflections will be shown by the analyser for (110) reflections from *F–c* structures.

(iii) And since d_{111} for *B–c* structures is half that for the elementary cube, no odd-order reflections will be shown by the analyser for (111) reflection from *B–c* structures.

It is thus possible to formulate a double rule for distinguishing between *B–c* and *F–c* structures when the orders of the various reflections are read from the analyser.

Rule 1. If for 100 (n) and 110 (n) reflections odd values of n are missing, the structure is probably *F–c*.

Rule 2. If for 100 (n) and 111 (n) reflections, odd values of n are missing, the structure is probably *B–c*.

Once the structure is known, the correct value of n is determinable if required.

§ 9. PRACTICAL ILLUSTRATIONS

The analysis of crystallograms was quickened by use of a special *T*-square which obviated the necessity for transferring readings of d_{hkl}/n on to a strip of paper. This *T*-square was fitted with a celluloid scale which was an exact replica of and coincident with the d_{hkl}/n scale of the graphical plots. Moving steel pointers were provided for setting to the readings of the powder lines, and the point of coincidence was found by moving the *T*-square along the edge of a drawing-board upon which the analyser was mounted.

Table 2. Lead filings (figure 7 G).

Spacing given by interplanar scale d_{hkl}/n (Å.U.)	Reflection plane given by analyser hkl (n)
1·74	110 (2)
1·481	113 (1)
1·422	111 (2)
1·23	100 (4)
1·133	133 (1)
1·103	120 (2)
1·015	112 (2)
0·956	111 (3)

Possible planes missing include 100 (3), 110 (3), 100 (5). Full coincidence was obtained at $a_o = 4{\cdot}92$ Å.U. Result of analysis: F–c structure, $a_o = 4{\cdot}92$ Å.U.

Table 3. Tungsten filings (figure 7 F).

Spacing given by interplanar scale d_{hkl}/n (Å.U.)	Reflection plane given by analyser hkl (n)
2·22	110 (1)
1·58	100 (2)
1·29	112 (1)
1·12	110 (2)
1·008	130 (1)
0·918	111 (2)

Possible planes missing include 100 (3), 111 (1). Full coincidence was obtained at $a_o = 3{\cdot}16$ Å.U. Result of analysis: B–c structure, $a_o = 3{\cdot}16$ Å.U.

§ 10. CONCLUSION

The instrument here described simplifies the technique of X-ray crystallography. Owing to the focussing action, the time of exposure can be decreased to such a value that the utility of X-rays for commercial analysis is enhanced. The interplanar scale eliminates hours spent in calculation, and in combination with the cubic-crystal analyser enables a complete analysis of most substances having a cubic structure to be made in less than 10 minutes. The accuracy of the method is greatly dependent upon the accuracy of the machining of the instrument. But if the instrument is calibrated by taking an exposure of some standard substance of known lattice spacings, analysis can be made quite easily with errors less than 1 in 400.

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ELECTRO-ENDOSMOSIS AND ELECTROLYTIC WATER-TRANSPORT. PART II

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ABSTRACT. A new and convenient form of apparatus is described for the measurement of liquid-transport produced by the application of an electric field across a diaphragm of parchment paper. Results obtained with solutions of copper sulphate over the concentration range 0·0005-normal to 1-normal are compared with corresponding results obtained previously, by the author, with a diaphragm of powdered glass. The liquid-transport per faraday with the parchment diaphragm decreases strongly with increasing concentration up to 0·2-normal, but maintains practically a constant value over the concentration range 0·4-normal to 1-normal. This value is compared with that deduced for the electrolytic water-transport per faraday from the values of the experimental Hittorf transport number; and evidence is adduced indicating that there is no increase in the electrolytic water-transport per faraday with decreasing concentration from 0·4-normal. The data for the solutions over the concentration range 0·0005-normal to 0·2-normal appear to be consistent with the hypothesis of a composite effect produced by a constant electrolytic water-transport per faraday (of the same magnitude as the observed transport per faraday over the concentration range 0·4-normal to 1-normal) and by a specific (electro-endosmotic) action of the parchment diaphragm. The determination of electrolytic water-transport is utilized to correct the values of the experimental Hittorf transport number obtained from the data of Hittorf and Metalka. The true transport number in the case of copper sulphate solutions decreases with decreasing concentration from 1-normal, but the values at concentrations 0·125-normal and less appear to be little different from that at infinite dilution.

§ 1. INTRODUCTION

The flow of liquid in electro-endosmosis appears to be determined by the movement of the free ions of the electrical double layer formed, according to the classical theory*, at the interface diaphragm-aqueous solution†. In the case of a diaphragm of powdered glass and aqueous solutions of copper sulphate, the author showed in a former paper‡ that the number of free ions (as indicated by their charge) decreased strongly with increasing concentration of electrolyte; at concentrations above 0·005-normal no free charge could be detected. The liquid-transport per constant applied voltage across the diaphragm also decreased strongly with increasing concentration, but there was still a considerable flow at concentration

* Helmholtz, *Wied. Ann.* **7**, 337 (1879).

† See J. O. W. Barratt and A. B. Harris, *Biochem. J.* **6**, 315 (1912); and J. W. McBain, *Trans. Faraday Soc.* **16** (Appendix), 150 (1920).

‡ H. C. Hepburn, *Proc. Phys. Soc.* **39**, 99 (1927).

0·005-normal; and over the concentration range 0·005-normal to 0·5-normal the rate of flow per constant applied voltage was not appreciably changed.

In view of the evidence adduced as to the neutralization of the free charges of the diaphragm, it was concluded that electro-endosmosis ceased to play a prominent part at concentration 0·005-normal. The liquid-transport per faraday over the concentration range 0·005-normal to 0·1-normal showed a linear relation, when plotted against dilution, similar to that given by the data of Remy* for aqueous solutions of potassium chloride with a parchment diaphragm; and the suggestion was made that the data for the copper sulphate solutions of concentrations 0·005-normal and above appeared to be characteristic of electrolytic water-transport, i.e. of the water-transference which occurs during the ordinary process of electrolysis. This suggestion was based largely on the conclusion of Remy† and others that parchment paper appears to form a suitable partition for the measurement of electrolytic water-transport.

It seemed desirable, in order to confirm the above suggestion, to obtain data for the transport in the case of a parchment diaphragm and aqueous solutions of copper sulphate. The present work is devoted to this end; and the question of the electrolytic water-transport is further investigated. A new and convenient form of apparatus of considerably greater sensitivity than that of the earlier apparatus has been devised, and a more precise regulation of working temperature has been secured by immersion of the apparatus in an electrically heated water-bath with thermostatic control.

§ 2. THE APPARATUS

The apparatus was constructed mainly from hard glass tubing, 3 cm. in diameter. The two tubes A, A', figure 1, each 14 cm. long, carried side-pieces B, B'. The extremities of the side-pieces were formed into projecting flanges, the surfaces of which were ground to fit one another. The cup-like bases of the tubes A, A' served to collect any insoluble products of electrolysis formed at the electrodes, e.g. the black colloidal substance formed at the cathode in the case of the more dilute solutions of copper sulphate. A capillary tube C', about 25 cm. long, was joined to the tube A at C; the capillary was approximately 1 mm. in diameter, calibration with a thread of mercury giving the volume per cm. as 0·01127 cm.^{3} The tube A was widened slightly at the top to carry the hollow glass stopper D, the surfaces of contact being ground to provide a good joint. The tube E was joined to the lower end of the stopper and the short tube F to the upper part, the ends of both tubes being open and permitting a length of wire to be passed through the stopper.

The electrodes, which consisted of lengths of electrolytic copper wire (1 mm. in diameter) wound into the form of flat spirals with straight projecting pieces about 11 cm. long, were practically identical with those employed in the previous work‡. The straight piece of one length of wire was passed through the tube E of the stopper until it emerged at F, leaving the spiral projecting from the end of E. The

* *Z. Elektrochem.* **29**, 365 (1923). † *Loc. cit.* ‡ H. C. Hepburn, *loc. cit.*

wire was then cemented into the tube E with Faraday cement, the position of the spiral being regulated so that its centre lay in the axis of the side tube B when the stopper was in position. The stopper was finally turned to bring the plane of the spiral normal to the axis of the tube B, and the position was recorded by means of scratches on the stopper and on the adjacent part of the tube A. A similar electrode in the tube A' was carried by the stopper G. This stopper was pierced with holes so as to leave A' open to the air.

The use of a stopper, in place of a tap, to close the tube A facilitates the filling

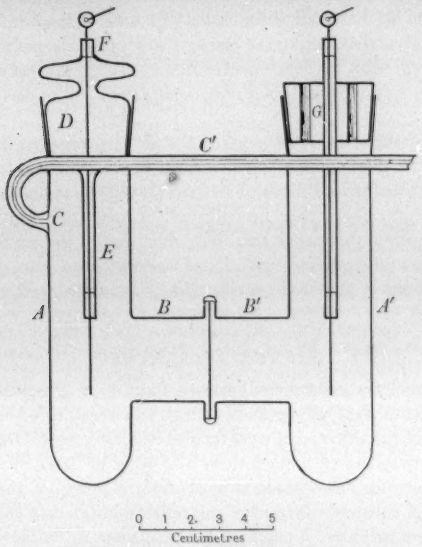

Fig. 1. The apparatus.

and emptying of the apparatus, at the same time providing a convenient means of introducing the electrode which may readily be removed for cleaning. It is possible, moreover, as explained later, to dispense with a lubricant at the joint, the risk of contaminating the solution being thus avoided.

The diaphragm consisted of a disc of parchment paper large enough to cover the projecting flange of the tube B or B'. The disc, after examination to ensure freedom from flaws and pinholes, was wetted with distilled water and placed over the flange of the tube B. The two flanges were then brought together by means of stout rubber bands which were passed round the tubes A, A'; and the projecting edge of the parchment disc (representing the expansion on wetting) was cut off with a sharp knife. Hot molten dental (sticky) wax was then applied to the joint of the two

flanges; the two parts of the apparatus thus being cemented together, the flanges holding the parchment disc tautly in position. Dental wax was adopted after trials with several cements; it was found to provide a strong and waterproof joint, at the same time eliminating the possibility of electrical leaks at the joint—a frequent source of trouble*—when the apparatus was placed in the water bath.

§ 3. EXPERIMENTAL

After the fixing of the parchment diaphragm the apparatus was at once filled with the most dilute copper sulphate solution (0·0005-normal) in the range selected (0·0005-normal to 1-normal), a long-stemmed funnel being used for this purpose to avoid the introduction of air bubbles. The process of filling and emptying the apparatus was several times repeated, a quantity of solution being allowed to flow through the diaphragm under a hydrostatic pressure head. After the final filling the stopper D was placed in position, care being taken to avoid the formation of air bubbles under the stopper. The channel between the stopper and the top of the tube A was dried with filter paper and then filled with rubber solution. Finally the level of the solution in the tube A' was adjusted to that of the capillary, and the apparatus clamped in a thermostat tank of capacity 50–60 litres. The apparatus was allowed to remain in the tank for 24 hours to ensure equilibrium between the diaphragm and solution, a coating of rubber meantime forming over the stopper and providing an airtight joint.

As a preliminary to the measurement of liquid-transport the apparatus was tilted to deliver drops from the capillary tube, until finally, when the capillary was restored to the horizontal, the liquid ran back to a point in the tube. The subsequent movement of the meniscus when an electric field was applied at the electrodes furnished a measure of the liquid-transport through the diaphragm†.

A rigorous cleansing of the capillary was found to be necessary to avoid the tendency of the meniscus to cling to the walls of the tube. The tube was treated first with benzene and then with alcohol, and finally was allowed to dip into a mixture of chromic and nitric acids for some days. Further subsequent treatment with hot chromo-sulphuric acid was needed also in the intervals between the flow measurements.

Preliminary experiments showed that the meniscus, after coming practically to rest after the delivery of drops from the capillary tube, began to move slowly towards the end of the tube. This "slow" flow, the magnitude of which was found to depend on the mechanical resistance of the diaphragm, was evidently produced by a small pressure-head developed in the apparatus. The flow through the diaphragm due to the application of an electric field at the electrodes was taken as the difference between the observed flow when the field was applied and the "slow" flow previously observed.

The experimental method adopted was to bring the meniscus initially to or

* See F. Fairbrother and H. Mastin, *J. Chem. Soc.* **125**, 2322 (1924).

† Cf. J. Perrin, *J. Chim. phys.* **2**, 617 (1904).

below the point 4·00 cm. on the engraved scale attached to the capillary tube. The meniscus, after traversing at least 1 cm. of the tube, was then timed by stop watch over the half-centimetre from 5·50 to 6·00, a lens being used in this and the following measurement to locate accurately the positions on the scale. The current, which was derived from a battery of storage cells, was switched on immediately the meniscus had passed the point 6·00, and the time taken to traverse the centimetre from 7·00 to 8·00 observed; the time for the half-centimetre 7·00 to 7·50 also was observed, but without the stop watch being stopped. The velocity of liquid-transport due to the applied electric field was taken as the difference between the rate of flow over the length 7·00 to 8·00 and that over the length 5·50 to 6·00. (N.B. The rate of "slow" flow was found by preliminary experiment to be practically uniform over the length 5·50 to 8·00 of the capillary.)

Measurements of liquid-transport were made with the solutions in order of increasing concentration, the procedure indicated in the preceding paragraphs being followed. At each concentration the rate of flow for the half-centimetre 7·00 to 7·50 was compared with that for the next half-centimetre 7·50 to 8·00, the determination being rejected if a difference greater than 2 per cent. was disclosed; in actual practice the difference rarely exceeded 1 per cent. In the event of the difference exceeding 2 per cent. the apparatus was refilled with fresh solution and the measurement repeated after the apparatus had remained in the thermostat tank for a further period of 24 hours. Successive determinations also were made, as described, to confirm that results could be reproduced.

The method described for applying a test of consistency, in preference to repetition of the measurement without change of solution, was adopted in view of the earlier work of the author which indicated that a progressive disturbance in the rate of liquid-transport, possibly as the result of concentration changes, etc. in the region of the electrodes, often arises when a measurement is several times repeated or when the current from the battery is repeatedly reversed. In view of the possibility of such disturbance the time of observation was reduced to a minimum, and the electrodes were placed at some distance from the diaphragm.

The mean current through the diaphragm was obtained from observations made with the meniscus at the beginning and end respectively of the timed centimetre; no appreciable variation in the current was, however, observed during the transit. The current value at a given concentration was approximately the same as that with the powdered-glass diaphragm, e.g. 0·3367 amp. with the parchment diaphragm and 0·3140 amp. with the powdered-glass diaphragm in the case of normal copper sulphate solution.

In order to ensure that there was no thermometric effect due to a change in temperature during the course of a flow-measurement the thermostat tank was heavily lagged with felt, and the water surface of the tank covered with a layer of B.P. paraffin to minimize heat losses through evaporation. The tank temperature was controlled within a limit of $\pm$ 0·1° C. by means of an electrical device similar to that described by Clack and Jarvis*; matters being so arranged, to avoid

* *J. Sci. Inst.* **4**, 330 (1927).

thermometric effects, that no heat was supplied to the tank immediately before or during the course of the flow-measurements. As certain of the observations were made in the summer months, it was not practicable to adopt a uniform working temperature of 18° C., and the determinations of liquid-transport at concentrations 0·0005-normal to 0·2-normal were made at temperatures in the range 18·4° C. to 21·4° C. Control experiments with solutions of these concentrations indicated, however, that the variation in the liquid-transport per faraday over the temperature range 18·0° C. to 21·4° C. is less than the limit of experimental error (between 1 and 2 per cent.) in the flow-measurements. The measurements over the concentration range 0·4-normal to 1-normal were made at temperatures within 0·1° C. of 18·0° C.

A thermometric action arises in consequence of the heating effect of the current on the solution between the electrodes. The energy developed between the electrodes, which is almost completely liberated in the form of heat, is proportional (per unit time) to the product of the current and the applied voltage. In the flow-measurements already described a constant voltage was applied over the whole range of concentrations investigated, the thermometric action due to the heating effect of the current being practically constant when expressed as a flow per faraday. The magnitude of the correction was determined by means of control experiments in which the liquid-transport was measured with the current in the reverse direction from that previously adopted, correction being made for the "slow" (gravitational) movement referred to earlier. The thermometric action, which had previously assisted, then opposed the flow, the correction being given by half the difference between the two corresponding values of the liquid-transport per faraday. Measurements with 0·01-normal and 1·0-normal solutions gave the values 41·0 and 40·4 respectively for the heating correction in cm.^{3}/faraday. These values are in satisfactory agreement, and a mean correction of 40·7 cm.^{3}/faraday has been adopted. The correction of 40·7 cm.^{3} gives a value of 30·0 cm.^{3}/faraday as the true liquid-transport with normal solution. In order to confirm this value, further measurements similar to those already described, but in which a lower current value (0·0993 amp. in place of 0·3367 amp.) was adopted, were made with normal solution. The values obtained in two sets of measurements for the true liquid-transport per faraday were 30·8 cm.^{3} and 30·4 cm.^{3} respectively, which are in satisfactory agreement with the value (30·0 cm.^{3}) relating to a current of 0·3367 amp.

The direction of liquid-transport throughout the whole range of concentrations investigated was from anode to cathode as in the case of the powdered-glass diaphragm.

§4. RESULTS AND CONCLUSIONS

The experimental results are given in table 1. $1/T_m$ (reciprocal seconds) is the reciprocal of the observed time of liquid-transport per 1 cm. of the capillary tube, less the correction $1/T_s$, T_s being the time of "slow" flow per cm. for the gravitational movement. The liquid-transport in cm.^{3}/faraday (v_f) is calculated from the expression $[(1/T_m)(1/I_m)(0{\cdot}01127)(96{,}540)] - [40{\cdot}7]$, I_m being the mean current

T_m

through the diaphragm in amperes, and 0·01127 the volume of the capillary in cm.^{3} per cm. The deduction of 40·7 cm.^{3} represents the correction for the heating effect of the current referred to in § 3. The values obtained by the author* for the liquid-transport per faraday in the case of the powdered-glass diaphragm and solutions of copper sulphate are added for purposes of comparison.

Comparison of results with diaphragms of parchment and of powdered glass. It will be seen from table 1, columns 4 and 5, that the values of the liquid-transport per

Table 1.

Concentration of $CuSO_4$ (gm.-equiv./ litre)	Mean current I_m (amp.)	Reciprocal $1/T_m$ of time of flow per cm.	Liquid-transport v_f (cm.^{3}/faraday)	Liquid-transport v_f (cm.^{3}/faraday)
	Parchment diaphragm	Parchment diaphragm	Parchment diaphragm	Powdered-glass diaphragm
0·0005	·000684	·001412	2201	324100
0·001	·001484	·002054	1464	202500
0·002	·002549	·002884	1190	72100
0·005	·00572	·004363	789	18340
0·01	·01049	·005252	504	9320
0·014	·01380	·005752	413	—
0·02	·01848	·006131	320	4754
0·03	·02524	·006397	235	—
0·05	·03705	·006666	137	2276
0·1	·0654	·007076	76·9	1238
0·2	·1070	·007823	38·8	680
0·4	·1845	·01198	29·9	—
0·5	·2113	·01356	29·1	330
0·6	·2465	·01570	28·5	—
0·8	·2860	·01837	29·1	—
1·0	·3367	·02189	30·0	†

† No perceptible flow obtained. (N.B. The apparatus employed was not designed for the measurement of slow rates of flow.)

faraday with the powdered-glass diaphragm are considerably greater than those with the parchment diaphragm, even over the concentration range 0·005-normal to 0·5-normal; and if we conclude that the parchment diaphragm exerts only a small influence on the liquid-transport‡ it seems necessary to postulate a specific action of the powdered-glass diaphragm on the flow at concentrations of 0·005-normal or more. In view, however, of the evidence adduced in the former paper as to the neutralization of the free charge of the powdered-glass diaphragm at a concentration approaching 0·005-normal, no material part of the liquid-transport obtained with further increase in concentration can apparently be attributed to electro-endosmosis in the sense of the surface phenomenon discussed by Helmholtz§; the action of the diaphragm then appears to be mainly of a mechanical rather than electrical character. The author hopes to pursue this question further in a subsequent investigation.

* H. C. Hepburn, *loc. cit.*

§ *Loc. cit.*

‡ This question is further considered below (page 534).

The relation between the liquid-transport per faraday and the dilution of the electrolyte in the case of the parchment diaphragm and the solutions of copper sulphate is shown graphically in figure 2. The apparent discontinuity between dilutions 10 and 20 (see figure 2 *A*) is considered below (page 534).

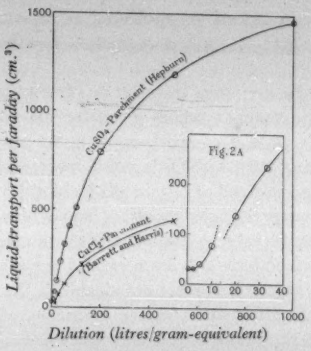

Fig. 2. Liquid transport per faraday (N.B. An enlargement of the lower part of the $CuSO_4$-parchment curve appears in Fig. 2*A*).

The values of the liquid-transport per faraday over the dilution range 20 to 1000 lie satisfactorily on a smooth curve, though the value for dilution 2000 lies above the extension of the curve; but there is no linear relation as in the case of the powdered-glass diaphragm and copper sulphate solutions of dilutions 10 to 200 or of the parchment diaphragm and potassium chloride solutions of dilutions 10 to 100 employed by Remy*. A curve of type similar to that relating to the parchment diaphragm and the copper sulphate solutions of dilutions 20 to 1000 is given, however (see figure 2), by the data of Barratt and Harris† for a parchment diaphragm and cupric chloride solutions; the values of the liquid-transport per faraday in the latter case are lower than those relating to the copper sulphate solutions. A result similar to that found with the powdered-glass diaphragm has been obtained by Boutaric and Doladilhe‡ in measurements of liquid-transport through a porous pot in the case of copper sulphate solutions of dilutions 100 or less. Boutaric and Doladilhe report that the liquid-transport is proportional to the current and to the electrolyte dilution; the data for the powdered-glass diaphragm over the dilution range 100 to 200 lead to approximately the same result.

Bearing of the data obtained with the parchment diaphragm on the electrolytic water-transport and comparison with other data. It is evident that in migration experiments where a parchment partition is interposed in the current path, as in the present work and in the work of Remy and others, the results will not give a satis-

* *Loc. cit.* † *Z. Elektrochem.* **18**, 221 (1912). ‡ *Compt. Rend.* **187**, 1142 (1928).

factory indication of the electrolytic water-transport unless the specific influence of the parchment partition on the liquid-transport can be neglected.

The work of Bein* and Hittorf† indicates that a parchment diaphragm is of little influence, especially with solutions of the alkali salts. Remy‡ and Baborovsky§ have recently concluded, however, that even with parchment the specific influence of the diaphragm cannot in general be neglected at concentrations below normal. Baborovsky‖ has attempted to apply a correction for the diaphragm effect, but without success.

It thus appears to be desirable, in order to obtain some confirmation of the estimates of electrolytic water-transport given by the results of the parchment diaphragm method, to make a comparison with the results of an independent method. This has been effected with fairly satisfactory results in the case of normal solutions of hydrochloric acid and of certain alkali chlorides by means of the Nernst method of indifferent reference-substance¶. Comparison over an extended range of concentration is not, however, possible, as determinations by the indifferent-reference-substance method have only been carried out at comparatively high concentration. Moreover, no comparison is practicable in the case of solutions of the copper salts owing to the difficulty in finding a suitable reference-substance which would not interact with the $Cu^{\cdot\cdot}$ ion. A similar difficulty arises when we attempt to find a basis of comparison in the results of an indirect method, such as that of distribution**.

In view, however, of the general conclusion of Remy and Baborovsky††, which is based on results obtained with solutions where independent confirmatory evidence is available as to the magnitude of the electrolytic water-transport, it seems probable that the specific (electro-endosmotic) effect of the parchment diaphragm may be neglected in the case of normal copper sulphate solution. It will be observed, moreover (see table 1) that the liquid-transport per faraday maintains practically a constant value over the concentration range 0·4-normal to 1-normal, but decreases strongly with increasing concentration (a characteristic of electro-endosmosis) in the case of concentrations below 0·4-normal. This latter result suggests that the specific effect of the parchment diaphragm may be neglected in the case of solutions of 0·4-normal and higher concentrations.

The mean value of the liquid-transport per faraday (29·3 $cm.^3 \equiv 1{\cdot}63$ mols) over the concentration range 0·4-normal to 1-normal is considerably less than that (4·10 mols) deduced for the electrolytic water-transport per faraday from the values of the experimental Hittorf transport number. The calculation from the values of the Hittorf transport number is based on the relation $x = dn/dc$, deduced by Riesenfeld and Reinhold‡‡ from equation (1) on page 535, x being the electrolytic water-trans-

* *Z. Phys. Chem.* **28**, 439 (1898).
† *Ibid.* **39**, 613 (1902); **43**, 237 (1903).
‡ *Trans. Faraday Soc.* **23**, 383–385 (1927).
§ *Z. Phys. Chem.* **129**, 129 (1927); *Coll. Czech. Chem. Comm.* **1**, 315 (1929).
‖ *Loc. cit.*
¶ See H. Remy, *Trans. Faraday Soc.* **23**, 382, 383 (1927).
** See J. N. Sugden, *J. Chem. Soc.* 174 (1926).
†† *Loc. cit.*
‡‡ *Z. Phys. Chem.* **66**, 672 (1909).

port in mols per faraday, n the Hittorf transport number, and c the concentration in equivalents per mol. The difference between the two results appears to be due to the fact that the true transport number is not independent of concentration as required by Riesenfeld and Reinhold's expression for x. This question is further considered on p. 535. n, c

Dependence of the electrolytic water-transport per faraday on the electrolyte concentration. The electrolytic water-transport per faraday has been shown to be dependent on the transport numbers and the difference in the hydration of the anions and cations of the electrolyte*. The ionic hydration may be considered as including also the water which is carried along by the ions in consequence of the frictional forces due to (i) the impact of the ions on the surrounding water molecules†, and (ii) the electrophoresic effect of the so-called ionic atmosphere‡.

Taylor and Sawyer§ have pointed out that notwithstanding the divergent estimates of the actual number of water molecules associated with or influenced by any specific ion, the opinion seems to prevail that this number is only slightly, if at all, influenced by concentration. In the general case of solutions of the sulphates, however, the distribution-measurements of Sugden‖, made over the concentration range 2-normal to 0·1-normal, indicate some diminution in the associated water with decreasing concentration; but this result must apparently be accepted with reserve in view of the secondary actions, such as that of depolymerizing action on the solvent, probably associated with the method of distribution.

The true transport number of the anion in the case of copper sulphate solutions of concentrations 1-normal and below appears to diminish with decreasing concentration (see page 536). This effect, corresponding to an increase in the transport number of the cation, would tend to increase the liquid-transport (which is directed towards the cathode) and to oppose the effect of diminution in the magnitude of the associated water suggested by the results of Sugden. The resultant action, viewed in relation to the observed liquid-transport per faraday over the concentration range 1-normal to 0·4-normal, appears to produce little variation in the magnitude of the electrolytic water-transport per faraday. At 0·125-normal or lower concentrations the values of the true transport number appear to be little different from that relating to infinite dilution, the general evidence then pointing to a constant or slightly diminishing value of the electrolytic water-transport per faraday with decreasing concentration.

Further evidence regarding the dependence of the electrolytic water-transport per faraday on the concentration may be adduced from a consideration of the linear relation between the Hittorf transport number and the concentration, referred to on page 536. Comparison with equation (1) shows that the linear relation, which

* See W. Nernst, *Theoretical Chemistry*, p. 447 (1923).

† Cf. F. A. Lindemann, *Z. Phys. Chem.* **110**, 394 (1924). Cf. H. Remy, *Trans. Faraday Soc.* 23, 385 (footnote) (1927).

‡ P. Debye and E. Hückel, *Phys. Z.* **24**, 305 (1923). Cf. H. Remy, *Trans. Faraday Soc.* 23, 385 (footnote) (1927).

§ *J. Chem. Soc.* 2095 (1929). ‖ *Loc. cit.*

holds over the concentration range 1-normal to 0·0212-normal*, leads to a constant value for the electrolytic water-transport per faraday, if—as seems probable (see page 536)—the true transport number also is related to concentration by a linear law. The electrolytic water-transport x per faraday is then given by the difference between the slopes $(dn/dc - dw/dc)$ of the two straight lines representing respectively the Hittorf transport number and the true transport number.

The specific effect of the parchment diaphragm. In view of the conclusion of the preceding paragraphs, the regular increase in the liquid-transport per faraday with decreasing concentration from 0·4-normal, observed in the present work, must evidently be attributed to the specific effect of the parchment diaphragm. If, as a first approximation, we assume that the electrolytic water-transport per faraday is independent of concentration and equal in magnitude to the observed transport per faraday over the concentration range 1-normal to 0·4-normal, we obtain the expression $(v_f - 29{\cdot}3)$ for the part of the transport attributable to the specific effect of the parchment diaphragm, v_f being the observed transport per faraday.

The diaphragm effect is usually measured in terms of the liquid-transport produced by the application across the diaphragm of a constant potential difference. If the specific conductivity within the diaphragm is no different from that in the bulk of the solution, the liquid-transport for a constant applied potential difference across the diaphragm is proportional to $(v_f - 29{\cdot}3)\lambda$, λ being the specific conductivity of the electrolyte solution. Some disturbance in the specific conductivity may be produced by the following possible factors: (i) diaphragm-surface current†, and (ii) change in the ionic mobilities whilst the ions are passing through the diaphragm‡; but in view of the comparatively inactive character of parchment paper when employed as diaphragm material, it is probable that the factors mentioned may be neglected for our present purposes without the introduction of any appreciable error.

Values of $v_f\lambda$ and of $(v_f - 29{\cdot}3)\lambda$ are shown graphically in figure 3; the values of λ have been obtained from the data of Kohlrausch. It will be seen that the curve relating to $v_f\lambda$ shows an apparent discontinuity between concentrations 0·05-normal and 0·1-normal (see also figure 2 *A*), but the curve relating to $(v_f - 29{\cdot}3)\lambda$ appears to be continuous. This latter curve, representing the specific effect of the parchment diaphragm, passes through a pronounced maximum and is of the same characteristic shape as that met with in certain cases where the diaphragm effect is predominant, e.g. with dilute solutions of hydrochloric acid in a powdered-glass diaphragm§.

The present data thus appear to be consistent with the hypothesis of a composite effect produced by a constant electrolytic water-transport per faraday and by a specific action of the parchment diaphragm, the latter action apparently becoming negligible at a concentration between 0·2-normal and 0·4-normal.

Determination of true transport number. It has been pointed out by Riesenfeld

* No data appear to be available for the Hittorf transport number in the case of concentrations below 0·0212-normal.

† Cf. H. C. Hepburn, *Proc. Phys. Soc.* **39**, 102 (1927).

‡ Cf. A. A. Green, A. A. Weich and L. Michaelis, *J. Gen. Physiol.* **12**, 473 (1929).

§ See H. C. Hepburn, *Proc. Phys. Soc.* **38**, 363 (1926).

and Reinhold* and others that the values obtained for the transport numbers of ions in solution by the Hittorf method† require correction in consequence of concentration-changes in the neighbourhood of the electrodes, produced by the electrolytic water-transport. The present determination of electrolytic water-transport provides a basis for this correction.

In the case of solutions of copper sulphate, where the direction of electrolytic water-transport is from anode to cathode, the following relation between the experimental Hittorf transport number and the true transport number is given by Riesenfeld and Reinhold:

$$w = n - cx \qquad (1).$$

Here w is the true transport number, n the experimental Hittorf transport number, c the electrolyte concentration in equivalents per mol, and x the electrolytic water-transport in mols per faraday. The symbols w, n refer to the anion, the corresponding numbers for the cation being $1 - w$, and $1 - n$. w, n c, x

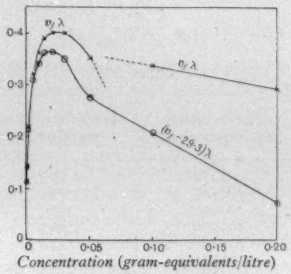

Fig. 3. Liquid transport per constant applied voltage across the diaphragm.

The data available for the experimental Hittorf transport number in the case of copper sulphate solutions over the concentration range 0·08-normal to 2-normal are conveniently shown in graphical form by Riesenfeld and Reinhold‡. The most regular results over the concentration range 0·3-normal to 2-normal appear to be those of Hittorf§ (determinations at normalities 1·96, 1·34, 0·712, and 0·327; temperature of determinations 4°–6° C.), the plot of his values occupying roughly a mean position relative to the plots of the other data. Hittorf's values for concentrations below 0·327-normal are somewhat irregular. Metalka‖, working at a temperature of 18° C., obtained the value 0·672 for the Hittorf transport number with solutions of concentrations 0·49-normal and 0·25-normal. This value is somewhat higher than that (0·659) obtained for concentration 0·5-normal by large-scale

* *Loc. cit.*
† W. Hittorf, *Pogg. Ann.* **89**, 177 (1853) *et seq.*
‡ *Loc. cit.* N.B. No later data appear in the Landolt-Börnstein *Tabellen* (1923 and 1927).
§ *Pogg. Ann.* **89**, 177 (1853).
‖ H. Jahn and collaborators, *Z. Phys. Chem.* **37**, 673 (1901).

graphical interpolation of Hittorf's data, but Metalka's values relating to 0·164-normal and lower concentrations lie satisfactorily on the continuation of the plot of Hittorf's data relating to the concentration range 1·96-normal to 0·327-normal. Metalka does not record a measurement with 1-normal solution, but interpolation of Hittorf's data gives the value 0·695 as against 0·69 obtained by Kermis* at room temperature.

The above analysis suggests that the magnitude of the transport number is not greatly dependent on the temperature within the limits 4° C. to room temperature. In this connexion it is pointed out that Hittorf† obtained the value 0·713 for the transport number with 1·34-normal copper sulphate solution at room temperature (18°–21° C.) as against 0·712 at 5·5° C., and he concluded that there was no temperature effect within the limits 4°–21° C.

The values given in table 2 for the Hittorf transport number appear to be the most probable of the available results (see the foregoing analysis) and are taken from the data of Hittorf (1-normal to 0·327-normal) and of Metalka (0·1639-normal to 0·0212-normal‡). The solution density data in the Landolt-Börnstein *Tabellen* have been used to obtain the concentration values in equivalents per mol.

Table 2.

Concentration of $CuSO_4$ (gm.-equiv./litre)	Concentration (gm.-equiv./mol)	Hittorf transport number n	True transport number w
1·0	0·01806	0·695	0·666
0·712	·01285	·675	·654
·5	·00902	·659	·644
·327	·00590	·645	(·635)
·1639	·00296	·634	(·629)
·125	·00225	·627	(·623)
·0833	·00150	·626	(·624)
·0625	·00113	·624	(·622)
·0419	·00076	·625	(·624)
·0306	·00055	·625	(·624)
·0254	·00046	·625	(·624)
·0212	·00038	·625	(·624)

The values of the Hittorf transport number n in table 2 are related, within close limits, to the concentrations in equivalents per mol by a linear law, the mean deviation being less than 0·1 per cent. of the linear value; the maximum deviation is 0·63 per cent., and the deviation at the lowest concentration 0·24 per cent. The values of the true transport number w in the case of concentrations 1-normal, 0·712-normal and 0·5-normal have been calculated by means of equation (1), the mean value of 1·63 mols for x being based on the observed water-transport over the concentration range 1-normal to 0·4-normal. These three values, when plotted against concentrations in equivalents per mol, lie also on a straight line which meets at infinite dilution the straight line representing the Hittorf transport number.

* *Wied. Ann.* **4**, 503 (1878). † *Loc. cit.*

‡ No data appear to be available for copper sulphate solutions of concentrations below 0·0212-normal.

The values of w for concentrations 0·327-normal to 0·0212-normal have been obtained on the assumption that x has a constant value of 1·63 over this concentration range and they agree fairly closely with the corresponding values read from the straight-line graph representing the true transport number. The values of the true transport number at 0·125-normal and lower concentrations appear to be little different from that (0·622) relating to infinite dilution as indicated by the point where the two straight-line graphs meet the axis.

§ 5. ACKNOWLEDGMENTS

The author desires to thank Prof. A. Griffiths for facilities provided and for the interest he has taken in this investigation.

The glass apparatus employed was made from the author's design by Messrs Gallenkamp.

DISCUSSION

Dr L. Simons. Does not the actual presence of the material of the diaphragm affect the concentration of the electrolyte in its immediate neighbourhood? This effect was dealt with by J. J. Thomson*. In view of Thomson's remarks it seems that very little meaning can be attached to the mean value of the concentration throughout the body of the solution. This would seem to have some very close relation to the apparent constancy of the liquid transport for high average concentrations as shown in table 1.

Dr V. Cofman. The author has explained that the observed flow across the diaphragm in dilute solutions is due to the combined effect of the electrolytic water-transport and the membrane effect caused by the Helmholtz double layer at the interface between parchment and solution (or within the capillaries of the membrane). How far does the curve calculated by the author for the Helmholtz effect† agree with that obtainable independently from cataphoretic measurements —i.e. from the rate of displacement, in an electric field, of small particles of parchment suspended in copper sulphate solution?

Author's reply. Adsorption of the electrolyte by the walls of the diaphragm would produce a change in concentration, but the equilibrium concentration in the diaphragm pores, after several washings with copper sulphate solution, appears to be little different from that in the body of the solution‡. It is pointed out also that the specific (electro-endosmotic) effect of the parchment diaphragm appears to be negligible in the case of copper sulphate solutions of 0·4-normal and higher concentrations.

A direct comparison between figure 3 and the results of cataphoretic measurements, as suggested by Dr Cofman, does not appear to be practicable owing to lack of experimental data relating to suspensions of parchment particles in copper sulphate solutions. The curve of figure 3 is of the same general character, however, as that met with in other cases of cataphoretic and allied electro-kinetic measurements.

* *Applications of Dynamics to Physics and Chemistry*, p. 190 (1888).

† Figure 3 $(v_f - 29{\cdot}3)\lambda$.

‡ Cf. H. C. Hepburn, *Proc. Phys. Soc.* **38**, 372 (1926).

THE SPECTRA OF TREBLY AND QUADRUPLY IONIZED ANTIMONY, Sb IV AND Sb V

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ABSTRACT. A study of the spark spectrum of antimony under varying conditions of excitation has led to an extension of the existing classification of the spectrum of Sb IV. Many new lines are classified as secondary members of the series. A second P term has been established for the Sb V spectrum.

§ 1. THE SPECTRUM OF Sb IV

An analysis of trebly-ionized antimony was given by Green and Lang* and partly independently by Gibbs and Vieweg† who also modified some conclusions of the former authors and extended their results. The investigations of Gibbs and Vieweg were, however, confined chiefly to the extreme ultra-violet region and extended only to λ 2113·14. The present communication is an extension of the investigation of the spectrum towards the region of longer wave-lengths. Many strong lines characteristic of the trebly-ionized atom have been found and have been classified as secondary members of the series system.

§ 2. EXPERIMENTAL

The spectrum of a condensed spark between poles of metallic antimony was studied under varying conditions of excitation, both in an atmosphere of air and in one of hydrogen. A large induction-coil running on 110 volts d.c. with mercury break was used. The spark in hydrogen easily brought out the lines of Sb IV. These appear relatively intense at tips with the spark in air, and fade away rapidly on diminution of the auxiliary spark gap in series and reduction of the capacity in parallel with the circuit. Plates were taken also with a one-meter vacuum-grating spectrograph, extending down to about λ 1280, a fluorite window being used, and the effect of inductance on the spark in a hydrogen atmosphere was studied. Kimura and Nakamura‡ and Soulillou§ have assigned some of the stronger lines of the spark spectrum to the various stages of ionization. Except for the line λ 2557·45, the lines used in the present investigation are in conformity with their assignments. λ 2557 has been assigned to Sb II by Soulillou and to Sb III by Kimura and Nakamura. Experiments show however that it certainly cannot belong to Sb II and may well belong to Sb IV. The lines given by the source are very diffuse and the weaker ones especially can be measured only with difficulty. For the stronger lines existing measurements by Soulillou have been utilized.

* *Proc. Nat. Acad. Sc.* **14**, 706 (1928).
† *Phys. Rev.* **34**, 400 (1929).
‡ *Jap. Journ. Phys.* **3**, 217 (1924).
§ *Comptes rendus*, **188**, 1103 (1929).

§ 3. PREDICTED TERMS

The term-scheme for the spectrum of Sb IV is presented in table 1. The term-prefix indicates the electronic configuration in a form convenient for writing. The notation used is that proposed by Russell, Shenstone and Turner*. The symbol ° denoting the odd terms is omitted as this is considered unnecessary when the configuration is given.

Table 1. Predicted terms of Sb IV.

Orbits of outer electrons												Term-prefix	Terms	
4_1	4_2	4_3	4_4	5_1	5_2	5_3	5_4	5_5	6_1	6_2	6_3	7_1		Singlets	Triplets
2	6	10		2									$5s^2$	^1S	
2	6	10		1	1								$5s5p$	^1P	^3P
2	6	10		1		1							$5s5d$	^1D	^3D
2	6	10			2								$5p^2$	$^1D\ ^1S$	^4P
2	6	10		1					1				$5s6s$	^1S	^3S
2	6	10		1						1			$5s6p$	^1P	^3P
2	6	10	1	1									$5s4f$	^1F	^3F
2	6	10		1							1		$5s6d$	^1D	^3D
2	6	10		1								1	$5s7s$	^1S	^3S

§ 4. CATALOGUE OF NEWLY CLASSIFIED LINES OF Sb IV

A catalogue of newly classified lines of Sb IV is given in table 2. The first column gives the wave-lengths and intensities, wave-lengths to λ 2000 being in air and those smaller than λ 2000 *in vacuo*. The second column gives the wave-numbers *in vacuo* followed in the third column by the classification.

§ 5. IDENTIFICATION OF TERMS

Some difficulty was experienced in assigning the $5s6p\ ^3P_0$ level. Two possible choices for the $5s5d\ ^3D_1$–$5s6p\ ^3P_0$ transition were λ 2786·00 and λ 2857·07. Kimura and Nakamura† put λ 2786·00 in the list of Sb IV lines, but Soulillou† assigns it to Sb III and it has been classified by Lang‡ as $5s^25d\ ^2D_{2\frac{1}{2}}$–$5s^25f\ ^2F_{2\frac{1}{2}}$. It may be noted that the similar transition is often very faint in spectra like Sb III. My experiments however indicate that the line more probably belongs to Sb III; no other satisfactory combination could be found with the $^3P_0$ level so obtained. λ 2857 being taken as the required transition, a fairly strong line at λ 2451·67 could be classified as $5s6p\ ^3P_0$–$5s5d\ ^3D_1$, but no combinations were obtained with the ^3S terms. Along with the two lines λ 3687·11 and λ 3425·89, classified as the $5s6s\ ^3S_1$ combinations with $5s6p\ ^3P_1$ and $^3P_2$, a remarkably similar line at λ 3735·26 led me to classify it as the $5s6s\ ^3S_1$–$5s6p\ ^3P_0$ transition, giving the $^3P_0$–$^3P_1$ difference identical with the $6s5d\ (^3D_1$–$^3D_2)$ difference, so that λ 2740·99 was classified doubly as $5s5d\ ^3D_2$–$5s6p\ ^3P_1$ and as $5s5d\ ^3D_1$–$5s6p\ ^3P_0$. This was further supported by the location

* *Phys. Rev.* **33**, 900 (1929). † *Loc. cit.* ‡ *Phys. Rev.* **35**, 445 (1930).

of $5s6p\ ^3P_0$–$5s7s\ ^3S_1$ in the calculated position. The line $5s6p\ ^3P_0$–$5s6d\ ^3D_1$ was not found and is probably masked by λ 2543·82, a strong line of Sb II which is very near the calculated position.

Table 2. Newly classified lines of Sb IV.

λ air (Int.)	ν	Classification
3922·46 (1)	25487·0	$5s4f\ ^1F_3$–$5s6d\ ^3D_3$
3735·26 (5)	26764·3	$5s6s\ ^3S_1$–$5s6p\ ^3P_0$
*3687·01 (8)	27114·6	$5s6s\ ^3S_1$–$5s6p\ ^3P_1$
3618·99 (3)	27624·2	$5s4f\ ^3F_2$–$5s6d\ ^3D_1$
3611·49 (3)	27681·5	$5s4f\ ^3F_3$–$5s6d\ ^3D_2$
3599·37 (3)	27774·8	$5s4f\ ^3F_4$–$5s6d\ ^3D_3$
3578·8 (0)	27934	$5s4f\ ^3F_3$–$5s6d\ ^3D_3$
*3425·89 (10)	29181·2	$5s6s\ ^3S_1$–$5s6p\ ^3P_2$
*3287·70 (4)	30407·7	$5s6s\ ^3S_1$–$5s6p\ ^1P_1$
2803·79 (1)	35655·5	$5s6p\ ^1P_1$–$5s6d\ ^3D_1$
2792·7 (0)	35797	$5s6P\ ^1P_1$–$5s6d\ ^3D_2$
*2740·99 (10)	36472·4	$5s5d\ ^3D_1$–$5s6p\ ^3P_0$
†2740·99 (10)	36472·4	$5s5d\ ^3D_2$–$5s6p\ ^3P_1$
*2714·96 (5)	36822·1	$5s5d\ ^3D_1$–$5s6p\ ^3P_1$
2710·7 (00)	36880	$5s6p\ ^3P_2$–$5s6d\ ^3D_1$
*2700·11 (5)	37024·6	$5s6p\ ^3P_2$–$5s6d\ ^3D_2$
*2681·77 (6)	37277·7	$5s6p\ ^3P_2$–$5s6d\ ^3D_3$
*2632·10 (10)	37981·2	$5s5d\ ^3D_3$–$5s6p\ ^3P_2$
*2594·10 (6)	38537·5	$5s5d\ ^3D_2$–$5s6p\ ^3P_2$
2570·9 (0)	38885	$5s5d\ ^3D_1$–$5s6p\ ^3P_2$
2566·92 (2)	38945·5	$5s6p\ ^3P_1$–$5s6d\ ^3D_1$
*2557·45 (4)	39089·7	$5s6p\ ^3P_1$–$5s6d\ ^3D_2$
*2514·08 (4)	39764·0	$5s5d\ ^3D_2$–$5s6p\ ^1P_1$
2501·19 (4)	39968·9	$5s6p\ ^3P_2$–$5s7s\ ^3S_1$
2492·15 (1)	40113·9	$5s5d\ ^3D_1$–$5s6p\ ^1P_1$
2378·32 (2)	42033·6	$5s6p\ ^3P_1$–$5s7s\ ^3S_1$
2358·6 (1)	42385	$5s6p\ ^3P_0$–$5s7s\ ^3S_1$
2278·03 (4)	43884·0	$5s6p\ ^1P_1$–$5s7s\ ^1S_0$
2116·1 (0)	47242	$5s5d\ ^3D_2$–$5s4f\ ^3F_2$
λ vac.		
1915·39 (2)	52209	$5p^2\ ^3P_2$–$5s6p\ ^3P_1$
1801·79 (6)	55500	$5p^2\ ^3P_2$–$5s6p\ ^1P_1$
1585·17 (2)?	63085	$5s5d\ ^1D_2$–$5s6p\ ^1P_1$
1406·59 (1)	71094	$5s5d\ ^1D_2$–$5s4f\ ^3F_2$
1358·13 (5)	73631	$5s5d\ ^1D_2$–$5s4f\ ^1F_3$

* λ by Soullillou. † Used twice.

It is interesting to compare the ratio $(^3P_1$–$^3P_2) : (^3P_0$–$^3P_1)$ for the $5s6p$ configuration in the iso-electronic spectra as is done in table 3. It is evident from the last column that with the increase of nuclear charge there is a gradual deviation from the Russell-Saunders coupling which leads to the theoretical ratio 2 : 1. If we classify λ 2857 as the $5s5d\ ^3D_1$–$5s6p\ ^3P_0$ transition the ratio in question becomes 2066 : 1832 = 1·12. Comparison of this with the ratios for other iso-electronic spectra further justifies us in rejecting λ 2857·07 and using λ 2740·99 twice.

The $5s5d\ ^1D_2$ and $5p^2\ ^1D_2$ levels as given by previous authors are interchanged, as it is thought that the $5s5d\ ^1D_2$–$5s5p\ ^3P, {}^1P$ combinations would be more likely

than the $5p^2\,{}^1D_2$–$5s5p\,{}^3P$, ^1P combinations. The conclusion that the $5s5d\,{}^1D_2$ level then becomes deeper than the $5s5d\,{}^3D$ levels is justified from comparison with the Cd I spectrum, where both $5s5d\,{}^1D_2$ and $5s6d\,{}^1D_2$ are deeper than the respective ^3D terms. In In II also the $5s5d\,{}^3D$, ^1D–$5s4f\,{}^3F$, ^1F combinations as given by McLennan and Allin* show that the $^1D_2$ is much deeper than the ^3D terms of the same $5s5d$ configuration. A similar change may be suggested for the Sn III spectrum.

Table 3. $5s6p\ ({}^3P_1$–$^3P_2) : ({}^3P_0$–$^3P_1)$.

Element	$^3P_1$–$^3P_2$	$^3P_0$–$^3P_1$	Ratio
Cd I	174	71	2·45
In II	558	182	3·07
Sn III	1223	273	4·48
Sb IV	2066	350	5·90

λ 2278·03, given as the $5s6p\ {}^1P_1$–$5s7s\ {}^1S_0$ combination of Sb IV, seems to be the most suitable line in the probable region.

A strong line, λ 1584·59, appears to be double, with a fainter component at λ 1585·17. The strong line itself does not seem to be characteristic of Sb IV, and the fainter one being near the calculated position of $5s5d\,{}^1D_2$–$5s6p\,{}^1P_1$, the transition is classified as such.

The spectrum of Sb IV is given in multiplet form† in table 4, previous results of Green and Lang and of Gibbs and Vieweg being also included for the sake of completeness. The term-values for the deepest terms, as calculated from the available series by Gibbs and Vieweg, are used, the others being calculated from the observed frequencies.

§ 6. THE SPECTRUM OF Sb V

While plates taken with an antimony spark in hydrogen with a Hilger E. 1 quartz spectrograph were being examined, two lines, $\lambda\lambda$ 3362·94 and 3036·16, which were intense and yet appeared as distinct from other lines only at the tips, were observed. No trace of these lines was found on plates taken with the spark in air.

* *Proc. R.S.* A, **129**, 208 (1930).

† In agreement with the known term-values of the Sn III spectrum (Gibbs and Vieweg, *loc. cit.*), the following $5s6p\ {}^3P$, ^1P–$5s6d\ {}^3D$, ^1D combinations in Sn III have been identified. The wavelengths are taken from Kayser's *Handbuch*, Band VI, and corrected to I.A. The lines are classed as due to Sn III by Kimura and Nakamura (*loc. cit.*).

		$5s6p\ {}^3P_0$	274	$^3P_1$	1222	$^3P_2$	$^1P_1$
$5s6d$	$^3D_1$	27974		27700			
	78	3573·7 (1*u*)		3609·1 (–)			
	$^3D_2$			27778		26556	
	134			3598·9 (2*u*)		3764·5 (1*u*)	
	$^3D_3$					26690	
						3745·6 (2*r*)	
,,	$^1D_2$						26961
							3708·0 (2*u*)

Table 4. Combinations in Sb IV.

Terms $\Delta\nu$	$5s5p\ ^3P_0$ 291721	$^3P_1$ 289456 2265	$^3P_2$ 283596 5860	$^1P_1$ 260204 23392	$5s6p\ ^3P_0$ 140770	$^3P_1$ 140420 350	$^3P_2$ 138354 2066	$^1P_1$ 137128 1226	$5s4f\ ^3F_2$ 129097	$^3F_3$ 129012 85	$^3F_4$ 128852 160	$^1F_3$ 126564 2288
$5s^2\ ^1S_0$ 356156		66700 1499·24 (25)		95950 1042·21 (75)		215763 463·47 (0)		219058 456·50 (1)				
$5s6s\ ^3S_1$ 167534	124187 805·24 (15)	121920 820·21 (25)	116063 861·60 (25)		26764·3 3735·26 (5)	27114·6 3687·01 (8)	29181·2 3425·89 (10)	30407·7 3287·70 (4)				
$5s7s\ ^3S_1$ 98385	193334 517·24 (00)	191064 523·38 (3)	185219 539·90 (6)		42385 2358·6 (1)	42033·6 2378·32 (2)	39968·9 2501·19 (4)					
5141												
$^1S_0$ 93244								43884·0 2278·03 (4)				
$5s5d\ ^3D_1$ 177242	114474 873·56 (20)	112212 891·17 (20)	106351 940·28 (6)	82975 1205·18 (15)?	*36472·4 2740·99 (10)	36822·1 2714·96 (5)	38885 2570·9 (0)	40113·9 2492·15 (1)	48146 2076·33 (5)			
350												
$^3D_2$ 176892		112562 888·40 (25)	106704 937·17 (15)	83311 1200·32 (00)		*36472·4 2740·99 (10)	38537·5 2594·10 (6)	39764·0 2514·08 (4)	47795 2091·59 (2)	47880 2087·86 (10)		
556												
$^3D_3$ 176336			107259 932·32 (25)				37981·2 2632·10 (10)		47242 2116·1 (0)	47323 2112·47 (3)	47483 2105·33	
23864												
$^1D_2$ 200200		89255 1120·38 (8)	83396 1199·10 (25)	59991 1666·93 (15)				†63085 1585·17 (2)?	71094 1406·59 (1)			73631 1358·13 (5)
$5s6d\ ^3D_1$ 101473		187956 532·04 (00)			[39297]§ 2543·96	38045·5 2566·92 (2)	36880 2710·7 (00)	35655·5 2803·79 (1)	27624·2 3618·99 (3)			
143												
$^3D_2$ 101330		188132 531·54 (1)	182255 548·68 (0)			39080·7 2557·45 (4)	37024·6 2700·11 (5)	35797 2792·7 (0)		27681·5 3611·49 (3)		
253												
$^3D_3$ 101077			182515 547·90 (4)				37277·7 2681·77 (6)			27934· 3578·8 (0)	27774·8 3599·37 (3)	25487·0 3922·46 (1)
479												
$^1D_2$ 100598				159606 626·54 (2)								
$5p^2\ ^3P_0$ 204080		85370 1171·38 (20)										
4312												
$^3P_1$ 199768	91942 1087·64 (8)	89682 1115·05 (25)	83828 1192·92 (25)	60426 1654·93 (00)								
7136												
$^3P_2$ 192632		96817 1032·88 (30)	90964 1099·33 (30)	67565 1480·05 (15)?		52209 1915·39 (2)		55500 1801·79 (6)				
19277												
$^1D_2$ 173355				86844 1151·49 (40)?								

* Used twice.

† Hardly resolved from a strong diffuse line λ 1584·6 of lower stage of ionization.

§ Probably masked by a strong line λ 2543·82 of Sb II.

They are attributed to Sb V and are classified as the first doublet of the second principal series. The irregular doublet law applied to the particular transitions in Ag-I-like isoelectronic spectra* justifies this classification beyond doubt, as the following table 5 shows.

Table 5. $6\,{}^2S_{\frac{1}{2}}$–$6\,{}^2P_{\frac{1}{2},\,1\frac{1}{2}}$ transitions in Ag-I-like spectra.

Element	$6\,{}^2S_{\frac{1}{2}}$–$6\,{}^2P_{\frac{1}{2}}$			$6\,{}^2S_{\frac{1}{2}}$–$6\,{}^2P_{1\frac{1}{2}}$		
	ν	$\Delta\nu$	$\Delta^2\nu$	ν	$\Delta\nu$	$\Delta^2\nu$
Ag I	(5741)†			(5946)†		
		5978			6447	
Cd II	11719		14	12393		208
		5992			6655	
In III	17711		11	19048		188
		6003			6843	
Sn IV	23714		10	25891		193
		6013			7036	
Sb V	29727			32927		

By application of Sommerfeld's fourth power law $\Delta\nu = K(Z-s)^4$ to the $6\,{}^2P_{\frac{1}{2}}$–$6\,{}^2P_{1\frac{1}{2}}$ separation, the values of the screening constant s calculated for Ag-I-like spectra are compared in table 6.

Table 6. Screening constants for $6\,{}^2P_{\frac{1}{2}}$–$6\,{}^2P_{1\frac{1}{2}}$ separations in Ag-I-like spectra.

Z	Element	$\Delta\nu$	$Z-s$ or $\sqrt[4]{(\Delta\nu/0{\cdot}0135)}$	s	Δs
47	Ag I	202·9	11·08	35·82	
					2·76
48	Cd II	672·8	14·94	33·06	
					1·80
49	In III	1337·4	17·74	31·26	
					1·30
50	Sn IV	2177·4	20·04	29·96	
					1·02
51	Sb V	3199·4	22·06	28·94	

Lang‡ has classified some lines in the extreme ultra-violet. Of these, λ 891·41, classified as $5\,{}^2P_{1\frac{1}{2}}$–$5\,{}^2D_{2\frac{1}{2}}$, has been shown to belong to Sb IV by Gibbs and Vieweg§ and they have suggested λ 888·97 instead. The suggestion seems to be justified by the location of the $5\,{}^2D$–$5\,{}^2P$ combination in the calculated position. The actual frequency calculated differs from that observed by an amount which may be accounted for by an error of about 0·3 Å.U. in the wave-length measure of the $5\,{}^2P$–$6\,{}^2S$ combinations. Table 7 shows the variation in the separations of $5\,D$

* The values for Ag I, Cd II, In III, and Sn IV are taken from a paper on the spectrum of Sn IV by K. R. Rao, *Proc. Phys. Soc.* **29**, 468 (1927).

† Calculated from known terms.

‡ *Proc. Nat. Acad. Sc.* **13**, 343 (1927).

§ *Loc. cit.*

terms in Ag-I-like spectra along with the screening constants. As will be seen from the table, the abnormal rise in the value of s for Sn IV is not continued for Sb V, and again for Te VI there is a normal decrease.

Table 7. Screening constants for $5\,^2D_{1\frac{1}{2}}$–$5\,^2D_{2\frac{1}{2}}$ separations for Ag-I-like spectra.

Z	Element	$\Delta\nu$	$Z-s$ or $\sqrt[4]{(\Delta\nu/0{\cdot}00776)}$	s	Δs
47	Ag I	20·3	7·15	39·85	
					3·71
48	Cd II	154	11·86	36·14	
					0·95
49	In III	283	13·81	35·19	
					−3·98
50	Sn IV	107	10·83	39·17	
					7·64
51	Sb V	1116	19·47	31·53	
					0·97
52	Te VI*	1644	21·44	30·56	

Lang classified λ 1505·70 and λ 1524·47 as the $5\,^2D_{1\frac{1}{2},\,2\frac{1}{2}}$–$5\,^2F$ transitions. These lines appear to be due to lower stages of ionization, probably Sb II or III.

As for Sb IV, the results as known to-day are shown in a multiplet form in table 8. The term-value 449300 for $5\,^2S$ given by Lang is retained.

Table 8. Combinations in Sb V.

Terms $\Delta\nu$	$5\,^2P_{\frac{1}{2}}$ 367734	$5\,^2P_{1\frac{1}{2}}$ 358747	$6\,^2P_{\frac{1}{2}}$ 194920	$6\,^2P_{1\frac{1}{2}}$ 191721
	8987		*3199*	
$5\,^2S_{\frac{1}{2}}$ 449300	†81566 1226·00 (12)	†90553 1104·32 (8)	—	—
$5^2\,D_{1\frac{1}{2}}$ 247373	120337 831·00 (6)	†111356 898·02 (1)	52453 1906·47 (7)	‡55652 1796·89 (5)
1116				
$5^2\,D_{2\frac{1}{2}}$ 246257		§112490 888·97		54536 1833·65 (8)
$6^2\,S_{\frac{1}{2}}$ 224647	†143017 699·22 (3)	†134037 746·06 (1)	29727·4 3362·94 (8)	32926·8 3036·16 (10)

§ 7. ACKNOWLEDGMENTS

In conclusion the author takes this opportunity of expressing his thanks to Prof. A. Fowler, F.R.S., for his interest and encouragement throughout the course of this investigation. His thanks are also due to the University of Bombay for the award of a Scholarship.

* The author's thanks are due to Dr K. R. Rao for supplying the data for Te VI prior to publication.
† $\lambda\lambda$ and classification by Lang. ‡ This line may also belong to Sb II or III.
§ Suggested by Gibbs and Vieweg.

THE HIGH-FREQUENCY SPECTRUM OF MERCURY AND THE FINE STRUCTURES OF λ 6123 (Hg I) AND λ 4797 (Hg II)

By S. TOLANSKY, B.Sc., Armstrong College, Newcastle-upon-Tyne, Durham University

Communicated by G. W. Todd, M.A., D.Sc., *April* 30, 1931.

ABSTRACT. It is shown that the previously reported intensity-modifications, found to take place in the high-frequency electrodeless discharge in mercury vapour at low pressures (0·001 mm.), can be completely explained in terms of recently determined electron-impact excitation curves. The modified spectrum arises from the fact that the mean free path is large, enabling the electrons to attain high velocities which result in the strengthening of lines involving upper singlet levels. The high-frequency spectrum at a pressure of 3 mm. is found to be arc-like, supporting the above conclusion.

The fine structure of the unclassified line λ 6123 (Hg I) is determined with a Fabry-Perot interferometer and found to have eight components. Four of these form a multiplet and may arise from one isotope only. The line λ 6123 probably involves an upper singlet level. The unclassified spark line λ 4797 (Hg II) shows four fine-structure components, widely separated and of similar intensities.

§ 1. INTRODUCTION

A description of the spectrum of mercury vapour excited at very low pressures by a high-frequency electrodeless discharge has already been given*. The discharge was produced in pure mercury vapour at room temperature, contained in a pyrex tube 10 cm. long and 2 cm. in bore, having a central capillary portion 1 cm. long and 0·25 cm. in bore. Strips of tinfoil were wrapped round the wider parts and these were coupled inductively to a lecher-wire system in which oscillations of a wave-length of about 7 metres were maintained by a valve (a Mullard D.O. 40 operated with a 6-volt filament battery and 480-volts high-tension†.

The triplet line λ 3341 being taken as the basis of comparison, many other lines show a marked change of intensity in the high-frequency discharge as compared with the arc‡. The intensity-modifications may be summarized in the statements that all allocated lines involving singlet levels as upper states are considerably strengthened whether they be singlet or intercombination lines, whilst intercombination lines involving upper triplet levels are somewhat weakened. In all the

* *Proc. Phys. Soc.* **42**, 556 (1930).

† S. F. Evans, *J. Sci. Inst.* **7**, 261 (1930).

‡ The line λ 3341 is the second number of the series to which λ 5461 belongs and is used because the latter is very strongly absorbed in the high-frequency discharge.

observed series, including triplets, the higher series-members are relatively stronger than in the arc, but this effect is not very marked. The unclassified lines show three types of behaviour, namely marked strengthening, marked weakening and very slight indication of change. Since strengthening in all allocated lines is invariably associated with upper singlet levels, it was suggested that the strengthened unclassified lines involve upper singlet levels. It is also to be expected that the weakened unclassified lines involve upper triplet levels and that those which are unchanged may arise from displaced triplet terms. The lines λ 5461 and λ 4047, which involve metastable levels, are very heavily absorbed. The degree of intensity-modification is found to be reduced when the current density through the tube is increased.

In the first part of this paper an attempt is made to account for these intensity-modifications, and in the second part the fine structures of two of the enhanced lines are discussed.

§ 2. EXPERIMENTAL

It was found that a very great increase in both the relative and the absolute intensities of the enhanced* lines took place if the capillary tube was replaced by a tube 20 cm. long and 3 cm. in bore. In order to obtain the maximum illumination possible, a tube 150 cm. long and 3 cm. in bore was finally employed, and when viewed end on this gave a brilliant enhanced spectrum. Part of the spectrum obtained is shown at (*a*) in the plate. For this particular photograph a 250-watt oscillator (wave-length 20 metres)† was used. The tube remained cold when running, the vapour-pressure of the mercury being of the order of 0·001 mm. The spectrum was photographed with the Hilger constant-deviation instrument on Ilford soft-gradation panchromatic plates developed in hydroquinone. In the plate (*b*) is the spectrum of an ordinary vacuum mercury arc carrying 2 amp. The enhancements are very striking.

A small capillary tube as previously employed was placed in a furnace and showed the enhanced spectrum in the cold. As the temperature was raised the singlets weakened and the triplets strengthened until at 150° C. the discharge was practically identical with that of the arc; see (*c*) in the plate.

This behaviour of the high-frequency discharge in mercury vapour at different pressures is explained by a recent investigation of Schaffernicht‡ on the excitation of the mercury spectrum by electron impact. Working at low pressures (0·001 mm.), Schaffernicht bombarded mercury vapour with electrons of varying velocity. He found that every line of the spectrum showed an optimum electron voltage, for which the line was emitted with maximum intensity. Voltage/intensity curves were plotted, and it was found that the form of each curve was entirely dependent upon the upper level involved. In general, lines of the same series had very similar

* The term "enhanced" when employed in this paper is meant to indicate strengthening in the high-frequency discharge relative to the arc, and has no reference to spark lines.

† R. L. Smith-Rose and J. S. Mc. Petrie, *Exp. Wireless*, **6**, 532 (1929).

‡ *Zeit. für Phys.* **62**, 106 (1930).

High frequency discharge in mercury vapour: (*a*) pressure 0·001 mm., (*b*) mercury arc, (*c*) pressure 3 mm.

Fabry-Perot fringes of unclassified arc line λ 6123. Plate separation = 6·5 mm. Separation of orders = 0·286 A.U.

[*To face p.* 546

excitation curves. There were two main types of curve, singlet type and triplet type, and these are shown in figure 1, which is taken from Schaffernicht. Lines with upper singlet term, as a rule, show a flat maximum at about 30 volts, whilst lines with upper triplet term show a very steep maximum at about 11 volts. Only two exceptions occur, namely lines involving $^1S_0$ and $^3D_2$ as upper levels. These have two maxima of singlet and triplet types, the curves however tending rapidly to become normal for terms of higher principal quantum number n. According to Schaffernicht, the exceptions are due to excitation of terms partly by direct impact and partly by stepwise transitions from previously excited triplet terms in the case of the $^1S_0$ levels, and singlet terms in the case of the $^3D_2$ levels. With very high voltages both singlet- and triplet-type curves become flat.

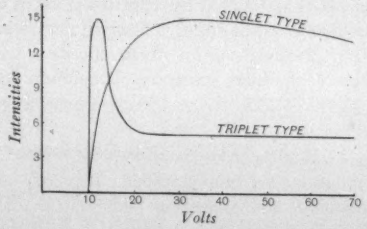

Fig. 1. Electron-impact excitation curves in mercury (after Schaffernicht)

It is at once apparent from the curves that in changing from low- to high-voltage excitation there is a very marked relative strengthening of the lines involving upper singlet terms. The spectrum produced with low-voltage electrons is identical with that of the arc (and high-frequency discharge at 150° C.), whilst that formed by high-voltage electrons is similar to that given by the high-frequency discharge at room temperature. It is reasonable to conclude that in the arc and high-temperature high-frequency discharge the effective electron voltage is small, whilst in the case of the cold discharge it is high. Consideration of the mean free paths shows that this is very probably the explanation of the enhanced spectrum. In the long tube at room temperature the mean free path of mercury atoms is about 7 cm. The electrons produced by the discharge have therefore sufficient time to acquire a high velocity before striking mercury atoms. The electrons are then effectively high-voltage electrons, the result being a spectrum in which lines with singlet-type excitation curves are markedly strengthened. When the temperature of the tube is raised to 150° C. the pressure of mercury vapour is about 3 mm. and the mean free path is of the order of only 0·003 cm. With such a short mean free path the probability of electrons acquiring a high velocity is small, so that their effective voltage is low. Since this low-voltage excitation condition also exists in the arc, the discharge reverts to arc type, wherein the triplets are much stronger than singlets; see (*c*) in the plate. There should be an optimum pressure,

depending on the power used and on its frequency, which will give a maximum enhancement of the singlets.

The increase in intensity of the singlet-type lines on changing over from the capillary tube to the wider and longer tube is further evidence that the mean free path determines the degree of enhancement. It is very probable that the restricted dimensions of the capillary tube were such as to prevent electrons from attaining their full velocity, i.e. they were equivalent to a reduction in mean free path. The influence of mean free path on the type of spectrum produced by the high-frequency discharge is shown in a different manner by the discharge in iodine vapour. A tube containing iodine at room temperature shows the arc spectrum. If a side limb is cooled with ice the pressure is reduced and the arc lines diminish rapidly in intensity, strong spark lines appearing instead. With the reduction in pressure the mean free path increases and with it the effective electron voltage. As a result of the more intensive bombardment strong ionization occurs and practically none but spark lines appear. Pressure-control, then, affords a convenient means of producing either the arc or the spark spectrum. It is found that bromine behaves in an exactly similar way*.

§ 3. FINE STRUCTURE OF λ 6123

The arc line λ 6123 has not yet been classified. It is very strongly enhanced in the high-frequency discharge and, since both temperature and pressure are low, very sharp lines result. This line was photographed along with the singlets previously examined†, but analysis was deferred since a great deal of trouble was experienced

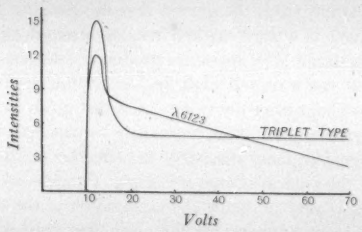

Fig. 2. Excitation curves of triplet-type lines and of λ 6123 (after Schaffernicht).

from overlapping orders. While the author was engaged upon the work a communication was received from Venkatesachar and Sibaiya‡ giving the structure of this line. Their results are not in agreement with those obtained here and will be discussed later.

The fine structure was determined with a silvered Fabry-Perot interferometer (Hilger N. 71) having a plate-separation continuously variable from 2·6 to 100 mm. The plates are of quartz and have an aperture of 6 cm. The interferometer was

* Unpublished data of S. F. Evans.

† *Proc. R.S.* A, **130**, 558 (1931).

‡ *J. Mysore Univ.* **5**, **145** (1930).

crossed with a Hilger large quartz E. 1 spectrograph, and exposures varied from one to seven hours. The spectrograms were taken on Ilford soft-gradation panchromatic plates developed in hydroquinone. The tube employed was 150 cm. long, and a 7-metre oscillator was used to produce the discharge. Since the fine structure extends over a width of 0·395 Å.U., overlapping of adjacent orders takes place even for small gaps such as 5 mm. A completely unambiguous determination of the structure was only obtained by the use of gaps increasing steadily by half-millimetres from 4 to 9 mm. The fringes reproduced in the plate were obtained with a 6·5 mm. gap and 2 hours' exposure. The weaker components do not appear in the reproduction, and the closer components are completely resolved with higher plate-separations. There are eight components and these with their estimated intensities are shown in figure 3.

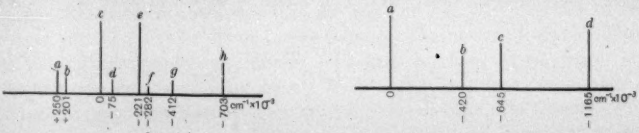

Fig. 3. Fine structure of λ 6123 of Hg I (left-hand curve) and λ 4797 of Hg II (right-hand curve).

Table 1 gives the observed structure together with that found by Venkatesachar and Sibaiya. The structures are given in thousandths of a wave-number. The extreme limits of error are indicated but the probable error is certainly less, since the values are the means of several determinations, mostly close together.

Table 1. Fine structure of λ 6123 (Hg I).

Venkatesachar and Sibaiya		Author		
		+ 250 ± 3	(3)	*a*
+ 171	(5)	+ 201 ± 2	(2)	*b*
0	(10)	0	(10)	*c*
− 117	(3)	− 75 ± 3	(2)	*d*
− 221	(8)	− 221 ± 1	(10)	*e*
− 323	(1)	− 282 ± 4	(1)	*f*
		− 412 ± 2	(2)	*g*
		− 703 ± 4	(4)	*h*

A comparison of the two structures shows that there is exact agreement in the case of the strong lines *c*, *e*, but none in the case of other lines. In considering the discrepancy a number of factors must be taken into account, since both the sources and interferometers used differed in the two cases. The Fabry-Perot interferometer used in the present work is of course free from ghost images, and with it difficulties due to overlapping may be completely resolved. The fringes are extremely narrow in the high-frequency discharge and there is no evidence of reversal since the structure is not dependent on the depth of vapour penetrated, and very sharp fringes are still obtained with big plate-separations such as 6 cm. Venkatesachar

and Sibaiya used Lummer plates and an echelon, the source being a low-current branched arc. As has been shown, the intrinsic intensity of enhanced lines is very much greater in the high-frequency discharge than in the arc, and this explains why these investigators observed fewer components. It is noteworthy that exact agreement is obtained with the strongest pair only, a result which is to be expected if one source is weak. The other lines given by Venkatesachar and Sibaiya are either ghosts or, more probably, misinterpretations due to the excessive overlapping. This would cause serious trouble with any interferometer not continuously variable. A further possibility is that the structures actually differ in the two discharges, but this is not very likely, as has previously been shown.

An attempt to fit the components of λ 6123 into a term scheme has met with partial success. The four components *e*, *h*, *b*, *f* can be arranged in square form, and give two pairs of equal differences. This result is shown in table 2. If it is not

Table 2.

− 282	(*483*)	+ 201
(*421*)		(*422*)
− 703	(*482*)	− 221

spurious, it means that the four lines are the four transitions obtainable from two upper and two lower levels, the level-separation in one case being 422 and in the other 482. This is shown in figure 4, where the bigger separation has provisionally been allotted to the lower level.

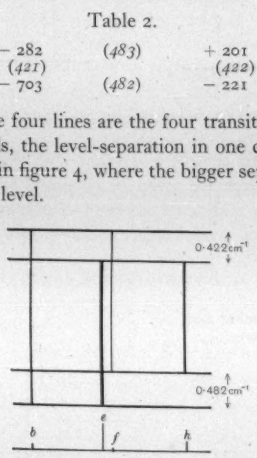

Fig. 4. Fine-structure multiplet in λ 6123.

It has previously been shown that the fine structures of singlet lines suggest that in mercury the two odd isotopes 199, 201 have different nuclear spins, each resulting in a characteristic fine structure, while the even isotopes have no nuclear spin and give single lines. If this is correct, then the above-mentioned quartet may arise from one isotope. Of the remaining four lines the strongest would belong to the even isotopes and the other three to the second odd isotope. The main difficulty of this viewpoint is the same as that experienced in considering the ringlets, namely, that since the abundance-ratios of even to odd isotopes, the even being summed, are about 5·1 : 1·2 : 1, only one very strong component would be expected, whereas two are observed.

It is well to point out that a spurious quartet can easily arise. Suppose three lines are distributed at random: if a fourth is accidentally placed so that a pair of equal differences results, then another pair of equal differences automatically exists. Hence the existence of the quartet can depend on the accidental situation of one line only. However, the differences in this case are very exact, agreeing to a thousandth of a wave-number, and they are therefore probably genuine.

§ 4. ALLOCATION OF λ 6123

Since this line is strongly enhanced in the high-frequency discharge, it is probable that it involves an upper singlet level. Schaffernicht has determined its excitation-function. This is shown in figure 2, a triplet-type curve also being plotted for comparison. Since the curve for λ 6123 shows a steep maximum at about 12 volts, Schaffernicht concludes that the line involves an upper triplet level. However, attention may be drawn to two facts: (*a*) The curve is distinctly different from the ordinary triplet-type curves. All lines involving upper triplet levels (other than the abnormal terms) have a steep maximum near 11 or 12 volts and then become practically flat and horizontal at about 20 volts. The curve for λ 6123 shows a steep maximum at 12 volts and then falls off steeply, showing no tendency to flatten even up to 70 volts. (*b*) If the line has an upper triplet level it should be stronger in the arc, but it is not.

A distinct possibility is that the mode of production of this line is different according as high-frequency excitation or electron-impact excitation is employed. This possibility may be connected with the difficulty hitherto experienced in classifying the line. Hanle* has found that the singlet and triplet excitation curves in helium are of a similar nature to those in mercury, showing the same types of maxima. However the singlet *He* line $2P - 4S$ (λ 5048) shows a curve not very different from λ 6123 in mercury, so that the curve itself does not give decisive evidence.

The fine structure of the line shows that a $^1S_0$ level cannot be involved in its production. It has been previously shown† that $7\,^1S_0 - 9\,^1P_1$ has seven components, and reasons were given why this should be the maximum number of components of a line involving a $^1S_0$ level. λ 6123 has been shown to possess eight components. A further point is that a $^1S_0$ level must always remain single for any given isotope, no matter what the value of the nuclear spin. If the quartet structure given in figure 4 is due to one isotope, then a $^1S_0$ level cannot be involved. If this quartet structure comprises the total structure for one of the isotopes, then the nuclear spin for this isotope must be $\frac{1}{2}$. However if one of the levels were triple and the other double the multiplet could have five components and there is no way of detecting whether or not one of the remaining lines is a fifth member, since j-values are unknown and no common differences involving the fifth line occur.

* *Zeit. für Phys.* **56**, 94 (1929). † *Proc. R.S. loc. cit.*

§ 5. FINE STRUCTURE OF λ 4797 (Hg II)

Very little attention has been given to the fine structures of the lines of ionized mercury. Schüler* mentions intensity-measurements in the Hg II lines λ 7944 and λ 6150 but gives no details as to structure. Venkatesachar and Sibaiya† state that λ 3984, which is $^2D_{\frac{5}{2}} - 2^2P_{\frac{3}{2}}$ according to Paschen, has six components, but they give no details. The writer has been unable to find any other reference to the fine structure of Hg II. Naudé‡ gives the line λ 4797 as by far the strongest line of the Hg II spectrum and it has not yet been classified. Paschen§ found many stronger lines in the spectrum, but he employed a discharge-tube with mercury in helium, and Naudé has shown that helium modifies the intensity-distribution. The line can be brought up in the high-frequency discharge if the discharge is sufficiently intense: (*a*) in the plate. Since two high-frequency oscillators were available, one of 250 watts with wave-length 20 metres and the other of 40 watts with wave-length 7 metres, both were coupled to the tube simultaneously. This resulted in a marked brightening of the line, and as the pressure and temperature were still low the line was not broadened. It was photographed with the Fabry-Perot interferometer on Ilford Monarch plates developed in hydroquinone, exposures up to two hours being sufficient. Since fine-structure separations increase with the degree of ionization, the structure is therefore much coarser than that appearing in the mercury arc lines. This resulted in frequent overlapping of orders, so that plate-separations of from 2·65 to 8 mm. only could be employed. The resolving power in this region being much smaller than in the red, the accuracy of the measurements is lower than for λ 6123. The separations are given in thousandths of a wave-number and are likely to be accurate to within 5 units. Four components have been observed, but there may be closer unresolved members. It has been impossible to determine this with certainty, but an exposure with a 17·5 mm. plate-separation still showed four components only. The structure is given in table 3 and figure 2. Component *a* appears diffuse and may be complex.

Table 3. Structure of λ 4797 (Hg II).

0	(5)	*a*
− 420	(2)	*b*
− 645	(3)	*c*
− 1165	(4)	*d*

The structure is so coarse, the nearest components being 225 units apart, that it could be resolved on a good grating. It is hoped to make an attempt to examine the mercury spark lines with a 21-ft. grating. Schüler has pointed out that in a number of mercury lines the weaker components B_n group themselves about a very strong component A, so that the optical centre of gravity of ΣB_n coincides with A. He has examined the intensity-distribution in the arc lines λλ 4047, 4078, 4916, 5461, 5769, 5791 and in the spark lines λλ 6150, 7944, and in most of these he

* *Naturwiss.* **43**, 895 (1930).
‡ *Ann. d. Phys.* **3**, 1 (1929).
† *Loc. cit.*
§ *Sitz. Preuss. Akad. Wiss.* **32**, 536 (1928).

finds that the ratio $\Sigma B_n/G$ is between 25 and 32 per cent. where G is the total intensity of all the components. Since the odd isotopes form about 30 per cent. he suggests that the weaker fine-structure lines come from the odd isotopes. However, the arc lines $\lambda\lambda$ 5461, 2536 and the spark line λ 6150 do not give this ratio at all. It is at once apparent that λ 4797 also constitutes an exception, there being no single strong component, the structure is in fact somewhat similar in nature to that of the resonance line λ 2536* as regards intensity-distribution.

It may be pointed out, with reference to Schüler's results, that the intensity-distribution in mercury-fine structures is partly dependent upon the mode of excitation and the degree of self-absorption.

§ 6. ACKNOWLEDGMENT

I wish to express my thanks to Prof. W. E. Curtis, who has made many helpful suggestions both during the work and in the preparation of the paper. This work was carried out during the tenure of the Armstrong College Fellowship.

* A. Schrammen, *Ann. d. Phys.* **83**, 1185 (1927).

THE SPECTRUM OF BARIUM FLUORIDE IN THE EXTREME RED AND NEAR INFRA-RED

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Imperial College of Science

Communicated by Prof. A. Fowler, F.R.S., *May* 15, 1931.

ABSTRACT. A considerable number of new heads in the known system of bands in the extreme red, due to BaF, have been observed. A new system, also due to BaF, has been discovered in the near infra-red, and assigned to the transition $^2\Pi \rightarrow {}^2\Sigma$.

§ 1. INTRODUCTION

In Kayser's *Handbuch der Spectroscopie*, volume **7**, are given the wave-lengths of ten double-headed bands in the region λλ 6909–7431 observed by George* in a barium-fluoride arc and attributed by him to barium oxide. In the original paper are given, in addition, the wave-lengths of a number of bands of unknown origin between λ 7873 and λ 8224. The present paper gives an account of a further investigation of these bands. A photograph of the spectrum of the flame of a barium-hydroxide arc has failed to show any bands in the region λλ 6909–7431 but it exhibits some weak bands in the region λλ 7900–8500; these, however, do not agree in wave-length with the bands given by George. The spectrum of the barium-fluoride arc, on the other hand, has been found to exhibit a striking set of bands in the region λλ 6710–8750, of which the bands recorded by George undoubtedly form part. It therefore appears that these bands cannot have a barium-oxide origin but must be due to barium fluoride. This assignment is confirmed by an analysis of the bands. When this work was partly completed, it was found that the ten double-headed bands measured by George had already been ascribed to barium fluoride and that vibrational quantum numbers had been assigned to the heads by Johnson†. However, as many new bands in this system have been observed, the wave-lengths are recorded in the present paper.

§ 2. EXPERIMENTAL

The source for the production of the bands was an arc between carbon poles in air, fed with a current of 10 amp. from 220-volt mains. The lower and positive pole was hollow and packed with barium fluoride.

In the preliminary work a glass spectrograph having a dispersion of about 31 A.U./mm. at λ 6685 and 67 A.U./mm. near λ 8675 was used. In the region λλ 6600–

* W. George, *Zeit. für Wiss. Phot.* **12**, 237 (1913).
† R. C. Johnson, *Proc. R.S.* A, **122**, 161 (1929).

7700 panchromatic plates and plates stained with kryptocyanin were used. The former plates were not effective beyond λ 7200. In the infra-red from λλ 7600–8800, neocyanin plates, hypersensitized by bathing in dilute ammonia, were used. Plates for measurement were obtained in the first order of the 10-ft. grating which in this region has a dispersion of about 5·4 A.U./mm. An exposure of from half to one hour was sufficient with panchromatic and kryptocyanin plates. With a slit-width slightly greater than usual, a plate of the bands in the region λλ 7900–8750 was obtained on a neocyanin plate in about seven hours.

Table 1. Extreme red system of BaF, $^2\Sigma \rightarrow {}^2\Sigma$.

λ (in air)	i	ν (in vac.)	v', v''	λ (in air)	i	ν (in vac.)	v', v''
6715·81	2	14886·1	(2, 0)	7138·79	8	14004·0	(1, 1)
18·28	2	880·6		41·96	8	13997·8	
39·63	3	833·4	(3, 1)	61·63	5	959·4	(2, 2)
41·84	3	828·6		64·92	5	952·9	
54·58	3	800·6	(12, 9)	7359·21	3	13584·6	(0, 1)
56·90	3	794·9		63·28	3	577·1	
63·41	4	780·5	(4, 2)	81·66	4	543·3	(1, 2)
65·68	4	776·3		85·67	4	535·9	
87·16	4	729·6	(5, 3)	7404·18	5	502·1	(2, 3)
89·59	4	724·0		08·23	5	13494·8	
6810·62	3	14679·2	(6, 4)	26·92	6	460·6	(3, 4)
13·33	3	673·0		30·78	6	453·7	
35·36	2	625·7	(7, 5)	7449·73	5	419·5	(4, 5)
37·63	2	620·8		53·78	5	412·2	
59·98	2	14573·2	(8, 6)	72·54	4	13378·6	(5, 6)
62·95	2	566·9		76·08	4	372·2	
6909·33	4	14469·1	(1, 0)	95·80	3	337·0	(6, 7)
12·12	4	463·3		99·80	3	329·9	
32·49	5	420·8	(2, 1)	7519·01	2	13295·9	(7, 8)
35·15	5	415·3		23·40	2	288·1	
55·92	5	14372·5	(3, 2)	7661·22	2	13049·1	(2, 4)
58·70	5	366·5		—*	—	—	
79·40	4	323·9	(4, 3)	83·50	2	011·3	(3, 5)
82·07	4	318·4		88·34	2	003·1	
7002·29	3	14277·0	(5, 4)	—†	—	—	(4, 6)
05·75	3	271·0		7711·02	1	12964·8	
25·77	2	229·3	(6, 5)	28·56	1	935·5	(5, 7)
29·65	2	221·5		33·70	1	926·8	
7115·98	8	14049·0	(0, 0)	—	—	—	—
19·18	8	042·6		—	—	—	—

* Obscured by Ba line 7664·89. † Obscured by Ba line 7706·58.

For convenience a spectrum of the iron arc in the second order was used for comparison. The error in this procedure was estimated by measuring the wave-length of first-order lines in terms of the second order, and it was found that it could be approximately corrected by subtracting 0·03 A.U. from the calculated wave-lengths. The wave-lengths of the ultra-violet and violet second-order lines were taken from Kayser's *Handbuch der Spectroscopie*, volume **7**, and the wave-lengths of the first-order infra-red lines from a paper by Meggers and Kiess‡.

‡ W. F. Meggers and C. C. Kiess, *Scientific Papers, Bureau of Stds.* No. 479 (1924).

§ 3. BANDS IN THE REGION λλ 6715–7734.

This system consists of five sequences of bands with double heads separated by about 6 cm.$^{-1}$. George has observed the (0, 0) and (1, 1) bands and four in each of the sequences $\Delta v = \pm 1$. The bands as already mentioned have been assigned to barium fluoride by Johnson. The wave-lengths, intensities, wave-numbers and vibrational quantum numbers are given in table 1. The intensities have been corrected, as far as possible, for the varying sensitivity of the photographic plates in this region by a comparison of spectra taken on panchromatic and kryptocyanin plates. The wave-numbers of the heads can be represented by the formulae:

$$\nu = 14064{\cdot}4 + 424{\cdot}46\,(v' + \tfrac{1}{2}) - 1{\cdot}805\,(v' + \tfrac{1}{2})^2 - 468{\cdot}75\,(v'' + \tfrac{1}{2}) + 1{\cdot}800\,(v'' + \tfrac{1}{2})^2 \quad \ldots\ldots(1),$$

$$\nu = 14070{\cdot}6 + 424{\cdot}33\,(v' + \tfrac{1}{2}) - 1{\cdot}945\,(v' + \tfrac{1}{2})^2 - 468{\cdot}16\,(v'' + \tfrac{1}{2}) + 1{\cdot}816\,(v'' + \tfrac{1}{2})^2 \quad \ldots\ldots(2).$$

Johnson considers that the less refrangible head of a pair is a Q head and that the bands arise from the transition $^2\Delta \rightarrow {}^2\Sigma^*$. This suggestion seems scarcely feasible, as the transition violates the selection rule for the component of the electronic angular momentum in the direction of the internuclear axis, $\Delta\Lambda = 0$ or ± 1. It seems more probable that the bands are due to a $^2\Sigma \rightarrow {}^2\Sigma$ transition. Such a band would consist of two strong R and two strong P branches, the lines $R_1(K + \frac{1}{2})$ and $R_2(K - \frac{1}{2})$ forming a "natural" doublet and likewise the lines $P_1(K + \frac{1}{2})$ and $P_2(K - \frac{1}{2})$. On this assumption the two heads are the heads of the R branches.

§ 4. BANDS IN THE REGION λλ 7860–8750

A part of this system, up to λ 8224, has been measured by George. The bands are undoubtedly due to BaF. A number of the heads in George's list have not been recorded in the present work and they appear to be merely fortuitous condensations of fine-structure lines. The analysis leads to the conclusion that the bands are due to a $^2\Pi \rightarrow {}^2\Sigma$ transition. The data for the system are given in tables 2, 3 and 4. Assuming that the $^2\Pi$ doublet is normal, the component of shorter wave-length is $^2\Pi_{\frac{3}{2}} \rightarrow {}^2\Sigma$ and its companion $^2\Pi_{\frac{1}{2}} \rightarrow {}^2\Sigma$. The sequences $\Delta v = 0, \pm 1$ have been observed for the $^2\Pi_{\frac{3}{2}} \rightarrow {}^2\Sigma$ component. The distance between the R and Q heads in the $\Delta v = 0$ sequences is about 19·5 cm.$^{-1}$, and in the $\Delta v = -1$ sequence it is about 15 cm.$^{-1}$. The R heads in the $\Delta v = +1$ sequence have not been observed. The Q heads are exhibited in the form of v' and v'' progressions* in table 3. The data for the $^2\Pi_{\frac{1}{2}} \rightarrow {}^2\Sigma$ component are not so complete. Seven Q heads in the $\Delta v = 0$ sequence have been measured, and three additional faint heads can be seen on the low dispersion plates. It is probable that the $\Delta v = +1$ sequence is present further in the infra-red but owing to the great drop in sensitivity of the plates beyond λ 8800 it cannot be measured, though on a very heavily exposed plate taken under low dispersion, bands can be very faintly observed in that region.

* The notation used in this paper is that recommended by Mulliken, *Phys. Rev.* **36**, 611 (1930).

Table 2. Wave-lengths and wave-numbers of heads $^2\Pi_{\frac{3}{2}} \rightarrow {}^2\Sigma$ component.

| | | |

Table 2. Wave-lengths and wave-numbers of heads $^2\Pi_{\frac{3}{2}} \rightarrow {}^2\Sigma$ component.

(v', v'')	Q head			R head			(v', v'')	Q head			R head			(v', v'')	Q head		
	λ (in air)	i	ν (in vac.)	λ (in air)	i	ν (in vac.)		λ (in air)	i	ν (in vac.)	λ (in air)	i	ν (in vac.)		λ (in air)	i	ν (in vac.)
(0, 0)	8151·03	8	12265·0	8136·99	7	12286·2	(1, 0)	7872·69	2	12698·6	7862·90	1	12714·4	(0, 1)	8472·10	2	11800·2
(1, 1)	8172·30	8	12233·1	8158·68	7	12253·5	(2, 1)	7895·04	1	12662·7	7885·02	1	12678·8	(1, 2)	8492·49	3	11771·9
(2, 2)	8193·64	8	12201·2	8186·44	6	12220·9	(3, 2)	7917·39	3	12626·9	7907·37	3	12642·9	(2, 3)	8513·02	4	11743·5
(3, 3)	8215·29	7	12169·1	8202·26	5	12188·4	(4, 3)	7939·14	3	12592·4	7929·77	3	12607·2	(3, 4)	8533·81	3	11714·9
(4, 4)	8237·12	6	12136·8	8224·55	3	12155·4	(5, 4)	7962·51	4	12555·4	7953·08	3	12570·2	(4, 5)	8554·60	3	11686·4
(5, 5)	8259·17	5	12104·5	8246·26	3	12123·4	(6, 5)	7985·66	4	12518·9	7975·98	3	12534·1	(5, 6)	8575·71	1	11657·7
(6, 6)	8281·15	4	12072·3	8268·62	2	12090·6	(7, 6)	8008·68	4	12483·0	7999·40	4	12497·5	(6, 7)	8597·4	1	11628·2
(7, 7)	8303·27	3	12040·1	8291·18	2	12057·7	(8, 7)	8031·28	4	12447·9	8022·56	3	12461·4	—	—	—	—
(8, 8)	8324·73	2	12009·0	8311·8	1	12027·8	(9, 8)	8054·96	3	12411·2	8045·88	3	12425·3	—	—	—	—
(9, 9)	8348·19	2	11975·3	—	—	—	(10, 9)	8080·09	4	12372·7	—	—	—	—	—	—	—
—	—	—	—	—	—	—	(11, 10)	8103·27	4	12337·3	—	—	—	—	—	—	—
—	—	—	—	—	—	—	(12, 11)	8125·51	2	12303·6	8116·82	2	12316·7	—	—	—	—

Table 3. v' and v'' progressions Q head $^2\Pi_{\frac{3}{2}} \rightarrow {}^2\Sigma$ component.

v' \ v''	0	1	2	3	4	5	6	7	8	9
0	12265·0 464·8 11800·2 433·6	432·9	—	—	—	—	—	—	—	—
1	12698·6 465·5 12233·1 461·2 11771·9	429·6	429·3	—	—	—	—	—	—	—
2	—	12662·7 461·5 12201·2 457·7 11743·5	425·7	425·6	—	—	—	—	—	—
3	—	—	12626·9 457·8 12169·1 454·2 11714·9	423·3	421·9	—	—	—	—	—
4	—	—	—	12592·4 455·6 12136·8 450·4 11686·4	418·6	418·1	—	—	—	—
5	—	—	—	—	12555·4 450·9 12104·5 446·8 11657·7	414·4	414·6	—	—	—
6	—	—	—	—	—	12518·9 446·6 12072·3 444·1 11628·2	410·7	411·9	—	—
7	—	—	—	—	—	—	12483·0 442·9 12040·1	407·8	—	—
8	—	—	—	—	—	—	—	12447·9 438·9 12009·0	402·2	—
9	—	—	—	—	—	—	—	—	12411·2 435·9 11975·3	397·4
10	—	—	—	—	—	—	—	—	—	12372·7

Some Q heads in the $\Delta v = -1$ sequence have been measured. The earlier heads in the sequence may be present in small intensity, but if so they are blended with the R heads of the $\Delta v = 0$ sequence of the $^2\Pi_{\frac{3}{2}} \rightarrow {}^2\Sigma$, the later members of which fall very near the positions in which the earlier Q heads may be expected. The measurement of the observed heads has been rendered somewhat difficult by the fact that they are obscured by fine-structure lines of the $^2\Pi_{\frac{3}{2}} \rightarrow {}^2\Sigma$ component. It is worthy of note that the R heads of this component have not been observed. The Q heads of the $\Delta v = 0$ sequence are strong and the R heads seem unlikely to be so faint as to escape observation. It seems more probable that the R branch turns back close to the origin, the resultant R head being blended with the Q head.

Table 4. Q heads of $^2\Pi_{\frac{1}{2}} \rightarrow {}^2\Sigma$ component.

λ (in air)	i	ν (in vac.)	(v', v'')	λ (in air)	i	ν (in vac.)	(v', v'')
8571·51	5	11663·4	(0, 0)	8337·21	4	11991·1	(4, 3)
8595·35	8	11631·0	(1, 1)	8359·12	3	11959·7	(5, 4)
8618·83	7	11599·3	(2, 2)	8383·80	3	11924·5	(6, 5)
8641·91	5	11568·3	(3, 3)	8408·51	3	11889·4	(7, 6)
8665·85	3	11536·4	(4, 4)	8433·83	2	11852·7	(8, 7)
8691·00	2	11503·0	(5, 5)	8459·27	2	11818·1	(9, 8)
8713·14	1	11473·8	(6, 6)	8484·53	2	11782·9	(10, 9)
8737·66	0	11441·5	(7, 7)	—	—	—	—

The wave-numbers of the Q heads of the two components can be represented by the following formulae:

$$\nu = 12281{\cdot}1 + 436{\cdot}70\,(v' + \tfrac{1}{2}) - 1{\cdot}818\,(v' + \tfrac{1}{2})^2 - 468{\cdot}71\,(v'' + \tfrac{1}{2}) + 1{\cdot}828\,(v'' + \tfrac{1}{2})^2 \quad \text{.....(3),}$$

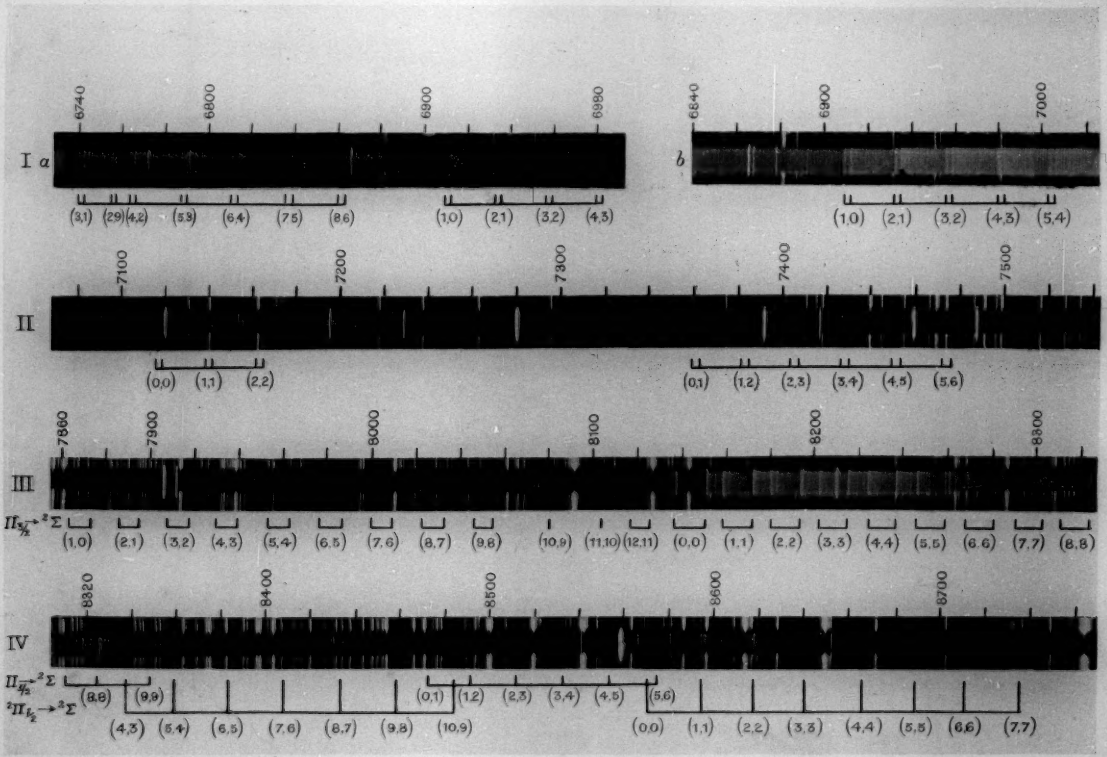
$$\nu = 11679{\cdot}0 + 438{\cdot}88\,(v' + \tfrac{1}{2}) - 1{\cdot}680\,(v' + \tfrac{1}{2})^2 - 470{\cdot}42\,(v'' + \tfrac{1}{2}) + 1{\cdot}450\,(v'' + \tfrac{1}{2})^2 \quad \text{.....(4).}$$

The accuracy of the latter formula is probably affected by the blending of the R and Q heads but the discrepancy seems outside the limit of error. Similar discrepancies are found in the analogous $^2\Pi \rightarrow {}^2\Sigma$ bands of CaF and SrF. The width of the $^2\Pi$ doublet is 602·1 cm.$^{-1}$ which is of the order of magnitude to be expected from an extrapolation from the known separation in the other alkaline-earth fluorides. Moreover, the formulae for the $\Delta v = 0$ sequences are almost exactly the same in the two components except for a constant term, and the assignment of the system to the transition $^2\Pi \rightarrow {}^2\Sigma$ seems reasonably certain.

It can be seen from a comparison of the formulae (1), (2), (3) and (4) that the two systems have the same final level. This level is also the final level of the well-known green bands of BaF.

§ 5. ACKNOWLEDGMENTS

My thanks are due to Prof. A. Fowler for his interest in this investigation, and to the Commissioners of the 1851 Exhibition for the award of a studentship which has made the work possible.



Spectrum of BaF λ 6740–λ 8750 photographed in first order 10 ft grating. I. $\Delta v = 0$ and -1 sequences of extreme red system: (*a*) panchromatic plate, exposure 2 hrs; (*b*) kryptocyanin plate, exposure 2 hrs. II. $\Delta v = 0$ and $+1$ sequences of extreme red system, exposure 45 min. III and IV. Infra-red bands, $^2\Pi \rightarrow {}^2\Sigma$ system, neocyanin plate, exposure 7 hrs.

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THE REFRACTION AND DISPERSION OF GASEOUS PENTANE AND CHLOROFORM

By H. LOWERY, Ph.D., F.Inst.P., Head of the Department of Pure and Applied Physics, and T. S. HARTLEY, M.Sc.Tech., College of Technology, Manchester

Received May 9, 1931.

ABSTRACT. The refractive index of gaseous n-pentane is found to be 1·001683 for the green mercury line, λ 5461, the result being expressed in relation to the density of the gas. The dispersion over the range λ 4358–λ 6708 is represented by the Sellmeier equation: $(\mu - 1) = 14{\cdot}605 \times 10^{27}/(8978{\cdot}4 \times 10^{27} - \nu^2)$, where ν is the frequency of the incident light. Revised values for gaseous chloroform are:

$$\mu = 1{\cdot}001448 \text{ for } \lambda\ 5461 \quad \text{and} \quad (\mu - 1) = 15{\cdot}391 \times 10^{27}/(10933 \times 10^{27} - \nu^2).$$

§ 1. PENTANE

The only previous determination of the gaseous refractivity of pentane is that of Mascart*. He found that, for sodium light, the ratio of the refractivity of pentane to that of air is 5·82 at 12° C. Dufet† deduces from this that the refractive index under n.t.p. conditions is 1·001701, assuming the index for air to be 1·0002923. No previous determinations of the dispersion of pentane have been made.

The gaseous refractive index of normal pentane was determined in the present work for the green mercury line, λ 5461, by means of a Jamin interferometer and Hilger monochromatic illuminator, a mercury-vapour lamp being employed as source of light. The general arrangement of apparatus was similar to that already described in the case of carbon disulphide‡. The number of interference fringes displaced as the vapour gradually entered the refraction tube was observed, and the amount of vapour producing retardation of the light was estimated by means of the density bulb connected to the refraction tube.

Ten determinations of the refractivity of normal pentane for the green mercury line λ 5461 gave the values:

$(\mu - 1)_d \times 10^6$	...	...	1686, 1686, 1680, 1679, 1683, 1686, 1684, 1684, 1675, 1683;
Mean	...	...	...	1683.

* *Comptes Rendus*, **86**, 1182 (1878).
† *Recueil de Données Numériques* (1898).
‡ *Proc. Phys. Soc.* **38**, 470 (1926).

Here $(\mu - 1)_d$ denotes the refractivity in relation to the density, i.e. the values show the refractivity of gaseous pentane by the same number of molecules per unit volume as hydrogen contains at n.t.p.

The data employed in obtaining the reduced values were: C = 12·000, H = 1·008 and the density of hydrogen = 0·08985 gm./litre. These lead to the value 3·21 gm./litre for the theoretical density of pentane, the corresponding experimental value being 3·29 gm./litre.

The above value of the refractive index as determined for λ 5461 being assumed, the dispersion was measured over the range λ 4358–λ 6708 with the results shown in table 1. In these experiments the number of interference fringes counted was at least 600.

Table 1. Dispersion of gaseous n-pentane.

λ in A.U.	$(\mu - 1)_d \times 10^6$		Difference
	Observed	Calculated	
Li 6708	1663·7	1663·8	+ 0·1
Cd 6438	1666·8	1667·0	+ 0·2
Li 6104	1671·5	1671·7	+ 0·2
Hg 5770	1677·3	1677·2	− 0·1
Hg 5461	1683·0	1683·2	+ 0·2
Cd 5086	1691·8	1692·2	+ 0·4
Cd 4800	1699·9	1700·7	+ 0·8
Li 4602	1706·7	1707·5	+ 0·8
Hg 4358	1716·6	1717·3	+ 0·7

The values in the second column lead to the Sellmeier equation:

$$(\mu - 1)_d = 14{\cdot}605 \times 10^{27}/(8978{\cdot}4 \times 10^{27} - \nu^2),$$

where $(\mu - 1)_d$ represents the refractivity in relation to density and ν is the frequency of the incident light.

Comparison of the present measurements with the determination of Mascart may readily be made; thus the Sellmeier equation gives the value of $(\mu - 1)_d$ for sodium light (λ 5893) as 0·001675. On altering this in the ratio

$$\text{(experimental density at n.t.p.)}/\text{(standard density)} = 3{\cdot}29/3{\cdot}21,$$

the value of $(\mu - 1)$ under n.t.p. conditions is found to be 0·001717. This corresponds to Mascart's result as reduced by Dufet, 0·001701.

The refractivity of isopentane (boiling-point = 30° C.) also was obtained but while the value differs by 2½ per cent. from that obtained for normal pentane (boiling point = 37° C.) it was felt that a differential method of experiment would be more suitable in the study of the changes of refractivity among isomeric compounds, and the work is not, therefore, being followed up with the present apparatus.

§ 2. CHLOROFORM

Some results for the refraction and dispersion of gaseous chloroform have already been given*. Further experiments with various samples of chloroform give the most probable value of the gaseous refractive index of the substance in relation to the density as 1·001448 for λ 5461. With this revised value, the dispersion has been redetermined over the range λ 4358–λ 6708 as shown in table 2. The Sellmeier equation corresponding to the results in the second column is:

$$(\mu - 1)_d = 15{\cdot}391 \times 10^{27}/(10933 \times 10^{27} - \nu^2).$$

Table 2. Dispersion of gaseous chloroform.

λ in A.U.	$(\mu - 1)_d \times 10^8$		Difference
	Observed	Calculated	
Li 6708	1433·9	1434·0	+ 0·1
Cd 6438	1436·2	1436·3	+ 0·1
Li 6104	1439·9	1439·6	− 0·3
Hg 5770	1443·5	1443·5	0·0
Hg 5461	1448·0	1447·7	− 0·3
Cd 5086	1454·7	1454·0	− 0·7
Cd 4800	1460·2	1459·9	− 0·3
Li 4602	1464·6	1464·7	+ 0·1
Hg 4358	1471·3	1471·5	+ 0·2

§ 3. ACKNOWLEDGMENT

Thanks are due to the Government Grant Committee of the Royal Society for a grant in support of the research.

* *Proc. Phys. Soc.* **40**, 26 (1927).

THE REFRACTION AND DISPERSION OF GASEOUS ETHYL BROMIDE

By H. LOWERY, Ph.D., F.Inst.P., Head of the Department of Pure and Applied Physics, and J. K. ELLIOT, M.Sc., College of Technology, Manchester

Received May 9, 1931.

ABSTRACT. The refractive index of gaseous ethyl bromide in relation to density is found to be 1·001261 for the green mercury line, λ 5461. The Sellmeier formula representing the dispersion over the range λ 4358–λ 6708 is:

$$(\mu - 1) = 12{\cdot}407 \times 10^{27}/(10138 \times 10^{27} - \nu^2),$$

ν being the frequency of the incident light.

Only one determination of the refractivity of gaseous ethyl bromide is recorded, namely that of Mascart* who used a pressure method and found the refractivity relative to that of air at 12° C. for sodium light, λ 5893, to be 4·16. This result, when reduced to n.t.p. conditions, leads to the value of the refractive index as 1·001216, the assumed value for the refractive index of air being 1·0002924. The dispersion of gaseous ethyl bromide does not appear to have been studied previously.

The gaseous refractive index of ethyl bromide has been determined in the present work for the green mercury line, λ 5461, by means of a Jamin interferometer, Hilger monochromatic illuminator and mercury-vapour lamp as source of light. The number of interference fringes displaced was observed visually while the vapour was gradually allowed to enter one of the refraction tubes, the other tube being kept permanently evacuated. The amount of vapour producing retardation of the light was estimated by means of a density bulb connected to the refraction tube.

The results of ten determinations of the refractivity for λ 5461 are as follow:

$(\mu - 1)_d \times 10^6$	...	...	1258, 1257, 1262, 1261, 1261, 1257, 1262, 1263, 1265, 1268;
Mean	...	...	1261,

where $(\mu - 1)_d$ represents the refractivity in relation to the density, i.e. the results show the refractivity of gaseous ethyl bromide by the same number of molecules as 1 cm.^{3} of hydrogen contains at n.t.p.

* *Comptes rendus*, **86**, 1182 (1878).

Dispersion measurements over the range λ 4358–λ 6708 gave the results shown in the table; these were made relative to the refractivity determination for λ 5461

Dispersion of gaseous ethyl bromide.

λ in Å.U.	$(\mu - 1)_d \times 10^8$		Difference
	Observed	Calculated	
Li 6708	1249·1	1248·4	− 0·7
Cd 6438	1250·1	1250·6	+ 0·5
Li 6104	1253·7	1253·7	0·0
Hg 5770	1257·0	1257·3	+ 0·3
Hg 5461	1261·3	1261·4	+ 0·1
Cd 5086	1267·2	1267·3	+ 0·1
Cd 4800	1273·0	1272·8	− 0·2
Hg 4358	1283·4	1283·8	+ 0·4

given above. The observed values of the second column, treated by the method of least squares, give the Sellmeier equation:

$$(\mu - 1)_d = 12{\cdot}407 \times 10^{27}/(10138 \times 10^{27} - \nu^2),$$

ν being the frequency of the light.

Thanks are due to the Government Grant Committee of the Royal Society for a grant in support of the research.

TWO PRECISION CONDENSER BRIDGES

By ALBERT CAMPBELL, M.A.

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ABSTRACT. In Heydweiller's modification of the Carey Foster bridge as ordinarily used (with variable mutual inductance) the capacitance of the condenser can be read directly, but the power-factor has to be deduced by calculation. To facilitate direct reading of both capacitance and power-factor the author has developed two bridge systems, *A* and *B*, based on that of Carey Foster. In both of these a fixed mutual inductance *M* is used and there is no added resistance in the condenser arm.

The capacitance $C \simeq M/PR$, where *P* and *R* are the resistances of the other two arms. To give direct reading for *C*, *P* can be varied in simple steps (e.g. 10, 100, 1000, 10,000), giving range multipliers, while *R* consists of a conductance box reading in millimhos. In system *A* a fourth arm *Q* of variable low resistance is added to the bridge. The power-factor is given by $Q/M\omega$ and can be read directly (at given pulsatance ω) if a slide wire is used for *Q*. In system *B* the power loss in *C* is balanced by the addition of impurity to the mutual inductance *M* by means of a closed-loop circuit variably coupled to both the primary and secondary coils of *M*. When the resistance of this loop circuit is set proportional to the frequency, the scale of the double inductor which varies the couplings can be graduated to read the power-factor directly. When a simple amplifier is used in the detector circuit, a capacitance range of 100 $\mu\mu$F up to 10 μF can be obtained and a power-factor range from 0·0001 to 0·01 with high accuracy of reading.

§ 1. INTRODUCTORY

For the measurement of the capacitance and power-factor of condensers over a wide range of values the Carey Foster method with Heydweiller's modification is, without doubt, one of the simplest and most accurate. The standards of reference, being resistance and mutual inductance, can be constructed to have high accuracy and good permanence. By using a variable mutual inductance the capacitance can be read directly, but unfortunately the power-factor requires to be calculated from observed numbers. To avoid this difficulty the author has developed from the Carey Foster bridge two new bridge systems* which are direct reading for both capacitance and power-factor. They may be designated (*A*) Slide-wire system and (*B*) Closed-loop compensator system.

§ 2. SLIDE-WIRE SYSTEM (*A*)

The *A*-system is a four-arm bridge, as shown in figure 1, in which *C* is the capacitance to be measured and *s* the internal resistance representing its power loss. It is a simple Carey Foster system with the addition of a fourth arm *Q*, and without added resistance in the *C* arm. The resistances *P*, *Q* and *R* are variable and the

C

s

P, *Q*, *R*

* British Patent Specifications, No. 317,642 and No. 350,789.

mutual inductance M is fixed. Let the P arm have self-inductance L, and let q be the power-factor of C at pulsatance ω; *M, L, q ω*

then $$q = C\omega s.$$

It can then be shown that, for balance,

$$C = M/(PR - Qs) \qquad \text{.....(1)},$$

and $$\text{power-factor } q = \frac{Q}{M\omega} + \frac{(L - M)\,\omega}{P - Qs/R} \qquad \text{.....(2)}.$$

Let Q be so small compared with R that Qs can be neglected. The equations then become

$$C \fallingdotseq M/PR \qquad \text{.....(3)},$$

and $$q \fallingdotseq Q/M\omega + (L - M)\,\omega/P \qquad \text{.....(4)}.$$

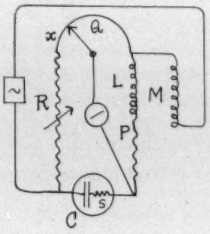

Fig. 1. System *A*.

Usually L is made equal to M, in which case

$$q \fallingdotseq Q/M\omega \qquad \text{.....(5)}.$$

In practice the approximations in equations (3) and (5) can be made very close to exactness.

In order that C may be read without calculation, the value of M is made an integral power of 10 (e.g. 1 mH) and the arm R consists of a conductance box reading in millimhos, while P can be arranged to go in steps such as 10, 100, 1000, 10,000 Ω, giving a series of ranges for C.

For a given value of ω, q is directly proportional to Q. This latter is conveniently made a slide-wire, which can be graduated to read q directly for a standard value of ω.

For other frequencies a multiplying factor is required. The total resistance of the slide-wire is made so small, relatively to R, that the part x in the R arm is negligible except in extreme cases. Power-factors outside the normal range can be determined by altering L by known amounts, so that $L - M$ is no longer zero, and calculating by equation (4).

Complete instruments have been built on this system giving, with two values of M, a range of capacitance-measurement from 1 $\mu\mu$F up to 10 μF, and reading power-factor directly at frequencies of 50 and 800 cycles per sec. from 0·0001 up to 0·01. But system B is more convenient, as it is quite direct reading over a long range of frequencies.

§ 3. CLOSED-LOOP COMPENSATOR SYSTEM (*B*)

The B-system is shown in figure 2. It consists of an ordinary Carey Foster bridge without external resistance in the C arm, but with a closed loop H coupled to the primary and secondary coils of the fixed mutual inductance M by means of mutual inductances μ and m which are both variable. The total resistance and self-inductance of the loop are ρ and λ respectively, λ being constant and ρ variable.

μ, m
ρ, λ

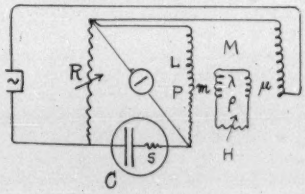

Fig. 2. System B.

A closed circuit of this kind may be called an "M-compensator," for it can be used to adjust the impurity of a mutual inductance, reducing it to zero if desired*, as m and μ can have the same or opposite signs. When the bridge is balanced we have

$$M\lambda - m\mu/CR = P\lambda + (L - M)\,\rho - M\rho s/R \qquad \text{.....(6)},$$

and

$$M/CPR = 1 - (\omega^2/P\rho)\,[(L - M - Ms/R)\,\lambda + m\,(m - \mu) + m\mu s/R] \qquad \text{.....(7)}.$$

In this particular application M is usually made equal to L, and ρ is always much greater than $\lambda\omega$. With these conditions, when the bridge is balanced, we have

$$\text{power-factor } q = Cs\omega \fallingdotseq m\mu\omega/M\rho \qquad \text{.....(8)},$$

and

$$M/C \fallingdotseq [P + m\,(m - \mu)/\rho]\,R - m\mu\omega^2 s/\rho \qquad \text{.....(9)}.$$

By suitable choice of m, μ and ρ the last term in equation (9) can be made negligible compared with PR, and thus

$$M/C \fallingdotseq [P + m\,(m - \mu)/\rho]\,R \qquad \text{.....(10)}.$$

In the practical instrument m and μ are varied together so as to be always nearly

* For other applications, see A. Campbell, *Proc. Phys. Soc.* **29**, 347 (1917) and British Patent Specification No. 294,053.

equal except when both are small. In this way we can ensure that the $m(m-\mu)$ term shall be negligible and then

$$C \fallingdotseq M/PR \qquad \text{.....(11).}$$

As in system *A*, the capacitance is read directly, a conductance box being used for the arm *R*. To make the power-factor readings direct, the frequency is observed and ρ is set proportional to it, say

$$k\rho = \omega.$$

Then
$$q = km\mu/M = m\mu \times \text{const.} \qquad \text{.....(12).}$$

The variation of μ and m is carried out by means of a little inductometer whose fixed coils are in the primary and secondary *M*-circuits, while the rotatable coil is in the loop circuit. The scale is marked proportionally to $m\mu$ and reads q directly.

The actual instrument, which is made by the Cambridge Instrument Company, has a capacitance range from 10 μF down to 100 $\mu\mu$F, with $M = 1$ mH, but much smaller values can be measured by taking differences. Within the frequency range of 50 to 2000 $\sim$ it reads power-factors directly from 0·01 down to 0·0001. The detecting instrument can be either a vibration galvanometer or a telephone, according to the frequency used. For the smaller capacitances, below 0·1 μF, it is desirable to use an amplifier in the detecting circuit. With a simple amplifier and moderate power from the source it is possible to read the capacitance over the whole range to about 1 part in 10,000, and the power-factor to about 0·0001.

§ 4. CONDUCTANCE BOX

The high accuracy of reading is attained by the use of the specially designed conductance box which forms the *R* arm. This consists of a number of variable resistances in parallel, beginning with three step-switch dials which give consecutive resistances of

(1)	∞,	100,	50,	33·33, 25	...	...	11·111, 10 Ω,
(2)	∞,	1000,	500,	333·3, 250	...	...	111·11,
(3)	∞,	10000,	5000,	3333, 2500	...	...	1111·1, 11000 Ω,

thus affording a range of 0 to 100 millimhos in steps of 0·1 millimho. It should be noticed that it is practically impossible to provide continuous adjustment down to zero, but a 1:100 range is here convenient. To continue this parallel-step system for finer adjustment would involve sets of resistance coils mounting up to megohms, which would be costly and troublesome. To avoid this difficulty, two more circuits *D* and *E* are added in shunt, each of the order of 500 Ω. *D* is a single step-switch giving

0·50, 0·51, 0·52 ... ... 0·60 mmho (2000 Ω down to 1666·7 Ω),

and E consists of two step-switches and a slide-wire, all in series, giving

0·50000, 0·50001 ... ... 0·51000 mmho (2000 Ω down to 1960·8).

D and E thus keep a minimum value (0·1 mmho) always in circuit, and this is taken account of by marking dial (2) with 1, 2, 3 ... 10 mmhos instead of 0, 1, 2, 3 ... 10 mmhos. Then dials (4), (5), (6) and (7) are marked 0·01, 0·02, 0·03 ...; 0.001, 0·002, 0·003 ... and so on.

In the E circuit the small conductance-variations made by the series switches do not add quite accurately, but the error is negligible. Even the minimum value (1 mmho) can be read with high accuracy.

MAGNETOSTRICTION AND HYSTERESIS

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ABSTRACT. The magnetostriction of unannealed wrought iron is measured, by means of an optical lever of length 0·12 mm., up to an intensity of magnetization of 985 e.m.u. The longitudinal extension and the intensity of magnetization are measured for a cycle of magnetization, both being found to show hysteresis. The extension is approximately proportional to the square of the intensity but seems to depend slightly on the previous magnetic history as well as on the intensity of magnetization. This hysteresis indicates that the extension is a consequence of the magnetization, in the same sense that the magnetization is a consequence of the applied field.

For a bibliography of papers on magnetostriction since its discovery by Joule and investigation by Shelford Bidwell reference may be made to *The International Critical Tables*.*

In the present experiments the longitudinal extension of a wrought-iron bar when magnetized was measured by means of an optical lever such as I have described previously†. The iron bar was 82·7 cm. long and 0·78 cm. in diameter. To its ends brass extension-pieces were soldered, and the bar was placed in a long vertical solenoid, having 19·3 turns per cm. The lower brass piece rested in a "geometrical constraint" in a brass disc cemented to the floor. The lever had its outer legs resting on the pole-pieces of a small strong horse-shoe magnet which was supported on a slate bracket let into the wall. The third leg of the lever rested on the upper brass extension-piece. The length of the lever, 0·12$_1$ mm., was only estimated approximately by direct measurement, but was found accurately by comparison with a less sensitive lever of length 1·41 mm. Deflections of the lever were measured by a simple lamp-and-scale method, the scale distance being 99·4 cm. The magnification of the lever was thus 16,400-fold. An extension of the specimen by one millionth of its length, i.e. $8{\cdot}27 \times 10^{-5}$ cm., gave a scale reading of 1·36 cm. A change in length by 1 in 136,000,000 would be just detectable on the scale.

To make full use of the large magnification, it would be necessary to keep the temperature of the bar of iron constant to within about 1/1000° C. In the present experiments no thermostat was used, but the readings were taken in rapid succession and a correction was applied for the gradual change of zero. The accuracy thus attained is of the order 1/20,000,000 of the length of the specimen. Since a maximum

* *Int. Crit. Tables*, **6**, 441. See also W. L. Webster, *Proc. Phys. Soc.* **42**, 435 (1930).
† *Phil. Mag.* **7**, 1166 (1929).

scale-reading of 5·85 cm. was obtained, the percentage error in the reading of this scale was little, if any, greater than the percentage error in the reading of the scale of the ammeter in series with the solenoid.

With the specimen supported by the floor, and the measuring apparatus supported from the wall, it was found necessary to carry out the experiments without anyone walking within about two metres of the apparatus.

$\delta l/l$, I The longitudinal extension $\delta l/l$ and the intensity of magnetization I were measured for applied fields of from 0 to 31·7 gauss, and also for cycles of magnetization with a maximum applied field of 31·7 gauss. The intensity of magnetization I was measured by a magnetometer method, a simple declination magnetometer being used with a lamp and scale, and a coil to compensate for the effect of the

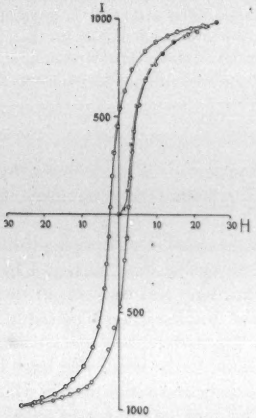

Fig. 1.

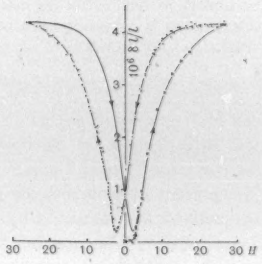

Fig. 2.

field of the solenoid on the magnetometer. The results of these experiments are given in figures 1 and 2. In both of these diagrams the results are plotted against
H the magnetizing field H, which has been deduced by deducting the demagnetizing field from the applied field. A demagnetizing coefficient of $0{\cdot}005_4$ was assumed.

In figure 3 the fractional extension $\delta l/l$ is plotted against the intensity of magnetization I. In figures 2 and 3, the measurements have for clearness all been inserted on one half of the hysteresis curve. The experiments were not accurate enough to detect any difference between the shapes of successive cyclic curves.

It was found, figure 2, that when the coercive-force field was applied the iron bar did not become quite as short as when the specimen was properly demagnetized. Figure 3 shows that for small intensities of magnetization the extension is approximately proportional to the square of the intensity. This diagram also shows

that the specimen returns to its original length only when properly demagnetized. The diagram shows other hysteresis effects, the magnetostriction depending not only on the intensity of magnetization but also on the previous magnetic history of the specimen. The vertical arrow in figure 3 indicates the remanent magnetization and magnetostriction.

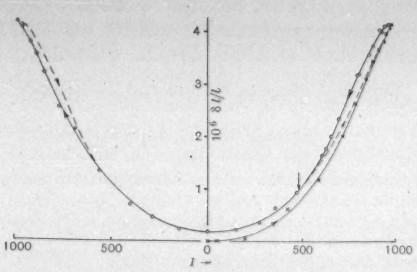

Fig. 3.

The direction in which the hysteresis loops in figure 3 are traversed indicates that the longitudinal extension, or magnetostriction, is a consequence of the intensity of magnetization, in the same sense that the intensity is a consequence of the applied magnetic field.

In conclusion I should like to thank Prof. J. A. Crowther for the interest he has taken in these experiments.

ON THE MEASUREMENT OF THE TOTAL HEAT OF A LIQUID BY THE CONTINUOUS MIXTURE METHOD

By H. R. LANG, Ph.D., F.Inst.P.,
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Received May 12, 1931.

ABSTRACT. A development of Callendar's continuous-mixture method for the measurement of the total heat or specific heat of a liquid is given. It consists of the determination of the amount of heat extracted by a stream of water from a hot stream of the liquid. A discussion of the heat losses and the method of reducing and allowing for these is included, as well as experimental tests of the theory of the method.

§ 1. INTRODUCTION

In recent years attention has repeatedly been directed to the demand for experimental data that will help in the attempt to produce a rational system of equations representing the thermodynamic properties of substances in liquid and vapour states. This is important not only for testing various theories, but also in many industrial problems. Such a system has been devised for the properties of water and steam by Callendar and others, and the use of steam tables is familiar to engineers.

With the assistance of the Institution of Petroleum Technologists an attempt is now being made to obtain such measurements for petroleum oils. Following the methods used by Callendar, the continuous electric method* was first used, and recently the continuous mixture method† has been suitably modified for use at temperatures at which the electric method is more troublesome. The object of the present paper is to describe this modified method, with particular reference to its use for oils, and the method of eliminating the heat loss.

§ 2. THE TOTAL HEAT

In practical calculations the quantity usually required is the change of total heat, and it is this that has to be calculated from the specific heat data. The advantage of the present method is that it directly measures this change over any range of temperature desired, and by the introduction of suitable throttles can be used over any range of pressure‡. Thus the energy-change per unit mass of the substance

* H. L. Callendar and H. T. Barnes, *Phil. Trans.* A, **199**, 55 (1902); H. R. Lang, *Proc. R.S.* A, **118**, 138 (1928).

† H. L. Callendar, *Phil. Trans.* A, **212**, 1 (1912).

‡ H. L. Callendar, *Howard Lectures*, R.S.A. (1926).

in passing from one set of conditions to another is directly measured and tabulated. Usually, to obtain this information an uncertain calculation has to be made, in which a knowledge of the specific heat and its change with temperature is required, as well as certain other thermal properties of the substance. An example will make this clear. A quantity frequently required is the amount of heat necessary to change a liquid at one temperature and pressure to a vapour at another temperature and pressure. This is the difference of total heat between the two cases, and is obtained by simple subtraction, in place of the more usual intermediate steps which involve uncertainty.

The variation of the specific heat of many liquids with temperature is quite complex, and unless some form of integrable equation can be fitted to the curve it is only possible to obtain the mean specific heat over a range by a graphical method, or by making an approximate equation to the curve. By the use of the total heat tables such a mean specific heat, if it is required, can be found by dividing the change of total heat between the two temperatures by the corresponding temperature difference.

§ 3. ELEMENTARY THEORY

The principle of the method is to measure the amount of heat extracted from a hot stream of the liquid by a cold stream of water. The necessary measurements are the rise in temperature of the water and the fall in temperature of the liquid, as well as the rate of flow of each stream. As in all other calorimetric experiments it is only possible to measure changes of total heat, so that, in order to find the value at the higher temperature (inflow), it is necessary to know that at the lower temperature (outflow). Let H_1 and H_2 be the total heat of the liquid in cal./gm. at the temperatures t_1 and t_2 respectively; H_1, H_2 t_1, t_2

s the mean specific heat of water in cal./gm.-deg. between the temperatures t_3 and t_4; s

t_3 the inflow temperature of the water; t_3

t_4 the outflow temperature of the water; t_4

m the rate of flow of the liquid (oil) in gm./sec.; m

M the rate of flow of the water in gm./sec.; and let $d\theta = (t_4 - t_3)$. $M, d\theta$

Then the energy equation per second is:

$$(H_1 - H_2)\, m = sMd\theta + \text{heat losses} \qquad \ldots\ldots(1).$$

§ 4. THE APPARATUS

The main features of the apparatus have been described in an earlier paper*, and only a brief outline will be needed here. The heat-exchanger, which is shown in figure 1 without the external jacket, consists of a central tube along which is passed a central rod carrying a spiral to cause good circulation of the oil. The

* H. R. Lang, *J. Inst. Petr. Techn.* **16**, 792 (1930).

annular space between the rod and the tube is about $\frac{1}{8}$ in. The water enters at the bottom, passes up the outside of the internal lagging, and then washes the outside of the central oil tube, and leaves again at the bottom. The internal lagging is of asbestos placed between two brass tubes. Special provision is made for the relative expansion of the various parts of the exchanger, and also for the escape of any air that may find its way into the apparatus. The exchanger is mounted vertically and is about three feet in length. The temperature rise of the water is measured with a differential pair of platinum resistance thermometers, reading to thousandths of a degree centigrade. The temperature of the oil is also measured with platinum resistance thermometers. Jacketed thermometer pockets of the type used by Callendar* are utilized to make the heat losses from them independent of the flow-rate.

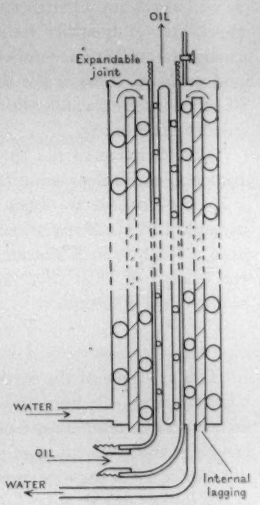

Fig. 1. The heat-exchanger.

Both flow circuits are fed from overflow devices; the rate can be varied for the oil by means of glass capillary-tube throttles, and for the water by raising or lowering the height of the overflow chamber. Experiment showed that the inflow temperature of the oil could safely be taken as that of the thermostat bath from which it passes into the exchanger. This bath is controlled by a modified form of recorder in conjunction with a resistance thermometer†, and its temperature varied by less than 0·03° C. even at the highest temperatures used (250° C.). The connecting tube between the spiral of pipe immersed in this bath and the heat-exchanger was made as short as possible in order to minimize the heat-loss from this region.

§ 5. METHOD OF ELIMINATING THE HEAT-LOSS

In considering the heat-losses from the apparatus it has to be remembered that the temperatures of the water stream and the outflow temperature of the oil stream are never very different from the surrounding temperature. In its present form it has been possible to reduce the heat-loss from the apparatus to about 2 per cent. on the fast flows, and a correspondingly larger percentage on the slower ones, since the heat-loss itself is independent of the flow. The greater part of the loss is from the water stream and is dependent only upon its rise in temperature $d\theta$, provided the temperature-distribution over the apparatus is independent of the rate of flow. By surrounding the heat-exchanger with a jacket fed from the same

* H. L. Callendar, *Phil. Trans.* A, **212**, 16 (1912).
† H. R. Lang, *Proc. Phys. Soc.* **42**, 589 (1930).

tank of water, the external temperature is always maintained the same as the inflow temperature. The loss from the water may be written,

$$\text{heat-loss} = f(d\theta, M).$$

The terms in M have been included to allow for the possibility of a change in temperature-distribution due to conduction with change in flow, and are known to be very small. As a first approximation we may therefore write,

$$\text{heat-loss} = k_1\, d\theta,$$

k_1

as far as the part lost from the water stream is concerned. In the earlier experiments there was a comparatively large and uncertain heat leak from the connecting pipe between the thermostat bath and the exchanger. This loss has been reduced and made quite definite and measurable in the following way. The pipe is lagged with asbestos, over the outside of which is fixed a heavy copper cylinder B, figure 2. This serves to distribute the heat generated by the passage of an electric current

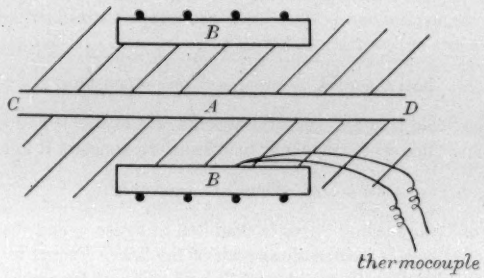

Fig. 2. Lagging of the connecting pipe.

through a coil wound on the cylinder and suitably insulated from it. The power supplied to the coil is varied so as to maintain a zero reading of a thermocouple, one junction of which is fixed to the copper cylinder while the other junction is in the thermostat bath. The outside of the lagging is therefore always at the same temperature as the inside, but a new difficulty is thereby introduced, namely that heat is able to creep along the pipe itself and so into the water, thus causing the temperature-rise of the water to be a little too large. Thus, in order to reduce the heat-loss from the oil, the radial temperature-gradient A–B, figure 2, from the centre of the pipe to the outside of the surrounding lagging must be reduced. This, however, changes the axial gradient C–D and thereby increases the conduction into the water stream. These two effects are antagonistic. It was found best to reduce to zero the loss from the oil by maintaining the radial gradient at zero, and to measure and allow for the conduction into the water stream. By making this connecting pipe as thin as possible and of a bad conductor (German silver) this negative heat-loss was reduced to the order of 2 per cent. of the total heat exchanged

in the experiments. By taking "a cold reading*" (i.e. the rise in temperature of the water stream when no oil is flowing, all other conditions being the same) and subtracting this from that found when the oil is flowing, this loss, together with other small conduction losses, is eliminated. This involves the assumption that the temperature gradients remain the same. Such an assumption was made in a similar case by Regnault†, in his classical experiments on the specific heat of gases at constant pressure. This has been criticized by Swann‡, who considers that this would give too low a value. The special feature of the present method, devised to overcome the difficulty, is that the gradients are maintained the same, in the manner described above. The radial gradient is always zero, and the axial gradient is sufficiently nearly the same when the oil is flowing as when it is not. For one end C of the pipe, figure 2, is in the thermostat bath, and the other D is washed by the cold stream of water immediately on entering the exchanger. It is this end of the pipe that may possibly change in temperature when the oil flow is turned on, but the experimental tests of the method given below show that the change may safely be neglected. It was found that the cold readings varied inversely as the rate of flow M, and the explanation is simple. If the temperature-distribution is independent of the rate of flow then we have

$$\text{heat absorbed by water} = Ms d\phi = \text{constant},$$

$d\phi, s$ where $d\phi$ is the "cold reading" and s the specific heat of water; and as the specific heat of water over this short range may be considered constant it follows that

c $$d\phi = c/M, \text{ where } c \text{ is a constant.}$$

A typical set of "cold reading" lines is exhibited in figure 3, and the temperature of the thermostat bath is written along each of the lines. Except for work of the highest precision, it suffices then to observe "a cold reading" for one rate of flow, and from the line drawn through this point and the origin to read off the appropriate value for other rates of flow.

In any experiment (set of flows) since the inflow temperature is constant and quite independent of the flow, and since the rise in temperature of the water is arranged to be the same for different flow-rates, the temperature-distribution over the apparatus will be almost the same in each case, and thus the heat-loss from the oil stream will be constant and independent of the rate of flow, and small compared to that from the water stream. Moreover, as the temperature-rise of the water ($d\theta$) is nearly enough constant throughout the experiment, this loss may conveniently be expressed in the form,

k_2 $$\text{loss from oil stream} = k_2 d\theta + f(m, d\theta).$$

The additional terms have again been included as a reminder of the possibility

* H. T. Barnes, *loc. cit.* p. 195.

† *Mémoires de l'Académie des Sciences de l'Institut Impérial de France* (1862).

‡ W. F. G. Swann, *Phil. Trans.* A, **210**, 231 (1910).

of small changes of temperature-distribution with flow. The full equation for the heat-exchange per second may now be written:

$$(H_1 - H_2)\, m = sMd\theta + k_1\, d\theta + k_2\, d\theta + f(m, M, d\theta) \quad \text{.....(2)},$$

neglecting for the present the end terms, and writing

$$z = k_1 + k_2, \qquad z$$

we have

$$H_1 = H_2 + sd\theta M/m + zd\theta/m \quad \text{.....(3)},$$

or

$$Y = (H_1 - H_2)\, m/d\theta M = s + z/M \quad \text{.....(4)}. \qquad Y$$

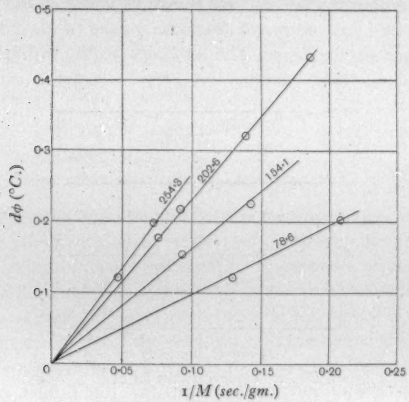

Fig. 3. Typical cold-reading lines.

If then the small terms dependent upon the flow-rates are negligibly small, and if the other assumptions made are to be experimentally justified, a plot of Y against $1/M$ should be linear. Before the present form of the apparatus was proceeded with some preliminary experiments were made with water in both flow circuits and with a fourfold change of flow-rate. The experimental points were colinear to rather better than 1 part in 1000. In this case the values of H_1 and H_2 were taken from the standard tables for water. The apparatus was therefore developed along these lines, and the following additional test was devised. Equation (3) may be written

$$H_1 = A + zd\theta/m \quad \text{.....(5)},$$

where

$$A = H_2 + sd\theta M/m \quad \text{.....(6)}. \qquad A$$

The quantity A represents the total heat uncorrected for heat-loss, and will therefore depend to some extent on the rate of flow. It should be noted that only the ratio M/m of the two flows is required accurately, and the actual value of m is needed only approximately for the calculation of the small heat-loss term. The advantage of this is that the exact time of switching over the flows into the collecting

flasks need not be known, as being mechanically coupled they are switched simultaneously. Further, as the densities of the oil and water are of the same order, the correction to the ratio of the weights on account of buoyancy becomes vanishingly small. Since H_1, the total heat at the inflow temperature of the oil, is constant in any one experiment, a test of the above equation is provided by plotting the quantity A against $d\theta/m$. As z is small the value of A does not change very much from flow to flow, so that it is possible to use a very open scale. It has been found that for all the experiments so far tried this equation holds to an order of accuracy even better than the estimated observational accuracy, provided the flows are turbulent. Some typical "A curves" are shown in figure 4, and in order to save space a constant has been subtracted from the values of A. This and the temperature t_1 are given on the figure. The intercept on the ordinate axis gives the

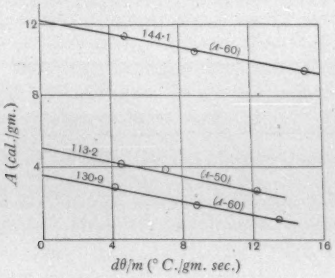

Fig. 4. Typical curves for A

value of the total heat H_1 less the appropriate constant, and the slope gives that of z. The value of H_2 has been obtained in each case from the integration of the specific heat/temperature curve found by the continuous electric-flow method*. It is usual to take the total heat as zero at 0° C., and this forms the convenient arbitrary zero. When reckoned from this point H_2 is small compared to the second term in equation (3), and it is not therefore required to any great accuracy. Further, as the temperature t_2 does not vary by more than about 15° C. from laboratory temperature, it is only necessary to know the variation of the total heat of the substance over a short range of temperature, in order to be able to calculate the value of H_1.

§ 6. METHOD OF REDUCTION OF RESULTS

The rise $d\theta$ in temperature of the water is calculated from the mean of a series of "cold" and "hot" readings, and is reduced to the gas scale in precisely the same way as for the experiments by the electric-flow method*. The value of A, equation (6), is next calculated, and when a "run" has been repeated the average is taken at this stage. In the earlier experiments the heat-loss term z was eliminated between

* H. R. Lang, *Proc. R.S.* A, **118**, 141 (1928).

two equations of the form (6) for a fast and a slow flow. When an intermediate one had been used this value of z was substituted in the third equation. The two values of H_1 found in this way agreed in general to 1 part in 600. An abridged set of observations is given in table 1, which shows the magnitude of the various quantities to be measured and the agreement between "runs."

Table 1. Abridged set of observations.

"Miri" fraction No. 4. Total heat at $t_1 = 131{\cdot}55°$ C.							
t_2	t_3	$d\theta$	m	M	H_2	A	H_1
° C.			gm./sec.		cal./gm.		
18·50	17·98	7·703	0·7064	4·858	8·092	61·067	—
18·50	17·98	7·691	0·7031	4·852	8·092	61·166	—
22·12	17·84	7·904	2·0432	13·921	9·715	63·568	64·94
22·10	17·84	7·886	2·0317	13·885	9·706	63·601	—
19·52	17·82	7·789	1·2993	9·077	8·547	62·962	—
19·54	17·82	7·747	1·2987	9·132	8·557	63·031	65·09

Mean value: $H_1 = 65{\cdot}01$ cal./gm.; $z = 0{\cdot}351$ cal./° C.

A series of determinations of the specific heat of this oil had previously been made by the electric-flow method up to 100° C. and, by integration and extrapolation in the equation that had been fitted to these results, a value of 64·97 cal./gm. was obtained for the total heat at this temperature. This close agreement of values obtained by two entirely different methods, which is typical of many cases, shows that any constant errors that may exist can safely be neglected.

From a large number of experiments that have now been performed with this apparatus, it has been found that the small variation in the value of the heat-loss per degree (z), is independent not only of the rate of flow but also of the rise in temperature of the water. A fourfold range of flows and a twofold range of temperature-rise have been tried. The value of z is calculated from the difference between two large and nearly equal quantities, and consequently an error of 1 part in 1000 in one of these would make about 5 per cent. difference in the calculated value of z. Provided sufficient data are available, it is more satisfactory therefore to use an average value of z*. Besides improving the accuracy of the results, this also halves the number of the necessary observations, as there is then only one unknown in equation (6), so that one flow suffices to obtain a result.

§ 7. MEASUREMENT OF SPECIFIC HEAT BY THE METHOD

Although primarily designed for the measurement of the total heat, the apparatus has been used to measure specific heat. The knowledge of the total heat H_2 at the outflow temperature may be obtained with the mixture apparatus itself by means of a series of preliminary determinations of the mean specific heat over comparatively short ranges of temperature, say 20°–30° C.

* Cf. H. L. Callendar and J. H. Brinkworth, *Phil. Trans.* A, 215, 391 (1915).

S If S is the mean specific heat of the oil, then

$$(H_1 - H_2)\, m = S(t_1 - t_2)\, m = sd\theta M + zd\theta \qquad \text{.....(7),}$$

B or if

$$B = sd\theta M/(t_1 - t_2)\, m,$$

$$S = B + zd\theta/(t_1 - t_2)\, m \qquad \text{.....(8).}$$

If, as before, the experiment is arranged so that the same temperature-rise is used with different rates of flow, and if the inflow temperature of the oil is kept the same, then the outflow temperature cannot be quite the same. As a result the mean temperature of the oil is changed, and since S in equation (8) is no longer constant it is impossible to eliminate the heat-loss between two such equations in the usual way. In cases where the value of z is known from previous experiments it may be considered as a constant of the apparatus, as was explained above, and in this way an immediate solution of the difficulty is provided. When this is not so, the difficulty may be overcome in two ways. First, if the difference of mean temperature is small the values of B may be corrected to the same temperature by the assumption of an approximate value for the rate of change of specific heat with temperature. In a second and more satisfactory method the inflow temperature of the oil is adjusted to keep the mean temperature the same for the different flow-rates. This is rather a laborious process since the outflow temperature is partly dependent on the inflow temperature, and partly on the temperature-rise of the water stream. After a little experience, however, it is possible to estimate the required values fairly closely. Now this change of conditions may seriously affect the assumptions regarding the heat-loss, on which the equations are based. This was tested experimentally by plotting B, equation (8), against $d\theta/(t_1 - t_2)\, m$ and resulted in the conclusion that the assumptions were still justified to at least the order of accuracy of the observations. Actually it is the mean specific heat that is measured, but over this short range of temperature it may be supposed to be equal to the true specific heat at the mean temperature, i.e. the differential $(dH/dt)_p$ of the total heat at constant pressure. By integration, the values of the total heat H_2 at the outflow temperature are then calculated, for use in the main series of experiments at higher temperatures.

THE DETERMINATION OF THERMAL CONDUCTIVITY AND ITS TEMPERATURE-VARIATION FOR MEDIUM CONDUCTORS

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ABSTRACT. A plate method for the determination of thermal conductivity and its variation with temperature between 0 and 100° C. is described. Although specially adapted for medium conductors (of the order 0·001 to 0·02 c.g.s. units) such as rocks, it can be used for substances of lower conductivity.

§ 1. INTRODUCTION

In previous determinations of the thermal conductivities of poor or medium conductors by direct steady-state methods it has been customary to use specimens in the form of either a spherical shell, a hollow cylinder, or a plate or slab. While methods involving the use of a spherical shell or a cylinder possess certain advantages, yet on account of the ease with which specimens in the form of rectangular plates can be prepared and of the simplicity of construction of the necessary apparatus, the plate method has found most favour. It is with this method that the present paper deals.

If heat be supplied to one of the large faces of a rectangular plate at a constant rate, the determination of the thermal conductivity of the material of the plate involves the accurate measurement of two quantities, namely the heat-flow per unit area across the plate and the temperature-gradient normal to the surface to which heat is supplied. The rate at which energy is supplied can be measured with great accuracy when electrical heating is adopted, the chief difficulty being the estimation of the heat which is lost by conduction and radiation from the sides of the plate; this loss should therefore be reduced to a minimum or eliminated altogether. The losses can be made very small by using very thin plates*, a procedure which is satisfactory with poor conductors but becomes impracticable with medium conductors such as rocks owing to the small temperature-gradient. A thickness of at least 1 cm. is desirable, particularly if the specimen is not homogeneous for in that case a thin plate would not be a representative specimen. Thus by using thick plates increased accuracy in the measurement of temperature-gradient is secured at the expense of a greater lateral heat-loss. Guard-ring methods† have been used to eliminate the lateral heat-loss, such methods being ideal theoretically though they involve a complication of apparatus which it is the aim of the present method to avoid. In the present investigation a method has been used by means of which the losses can be reduced to about 5 per cent. or less, even with thick specimens, the

* Eg. E. Griffiths and G. W. C. Kaye, *Proc. R.S.* A, **104**, 71 (1923).
† E. Griffiths, "Heat Insulators," *Special Report No.* 35. *S.I.R.* (1929).

heat-loss or emissivity being determined by a subsidiary experiment with the same apparatus. Determinations of conductivities over the ranges 15 to 30° C. and 105 to 120° C. have been made for several substances, the values ranging from 0·002 to 0·014 c.g.s. units.

§ 2. DESCRIPTION OF APPARATUS

The system of plates in position in the constant-temperature enclosure is illustrated in figure 1. In order to ensure symmetry in the heat-flow two similar specimens were used, one on either side of the hot plate. The latter consisted of a unit made by sandwiching two nichrome-wire-asbestos heating mats, connected in parallel, between two brass plates each 15 cm. × 14 cm. × 1 cm. which are screwed together tightly. Each mat had a resistance of 100 ohms. Two mats in parallel were used for reasons of symmetry, their similar sides facing each other. The mats were insulated from the brass plates by sheets of mica and from each other by a wad of asbestos paper. On the outer faces of the specimens are mounted two similar channelled brass castings, each 15 cm. × 14 cm. × 1·5 cm., through which a rapid stream of water can circulate so as to carry away the heat transmitted through the specimens. These will be referred to as the cold plates.

The temperatures of the cold faces of the specimens being in this way kept practically constant and equal to that of the enclosure, the mean excess temperature of the specimens is never allowed to become very high, whereas the heat-supply and consequently the temperature-gradient can be made quite large. Such a method, which can be called "calorimetric" (as distinct from an "emissivity" method, which differs in having no water-circulation through the cold plates and is similar in principle to the classical method of Lees*), has marked advantages in permitting large values of the energy supplied (say 50 to 100 watts) and of the temperature-gradient (say 5 to 20 degrees/cm.) and thus increasing the percentage accuracy with which these quantities can be measured. As the mean excess temperature of the specimens above that of the enclosure is very much less in the calorimetric method than in the emissivity method for the same heat-supply, the heat-loss (which is approximately proportional to the excess temperature) will also be very much less. The following figures, taken from tables 1 and 2, afford an excellent comparison of the two methods; it will be seen that the heat-loss from the sides of the specimens amounts to 31 per cent. of the heat generated in the heating coil in one case, but only 3 per cent. in the other.

	Emissivity method	Calorimetric method
Energy W supplied to coil	10·19	52·41 watts
Loss w_1 from sides of hot plate	1·44	1·40 ,,
Loss w_2 from sides of specimens	3·18	1·55 ,,
Mean flow $\overline{W}$ through specimens (i.e. $W - w_1 - w_2/2$)	7·16	50·23 ,,
Mean temperature θ_1 of hot face	25·70° C.	27·93° C.
Mean temperature θ_2 of cold face	23·61° C.	12·49° C.
Mean temperature drop $(\theta_1 - \theta_2)$	2·09° C.	15·44° C.
Thermal conductivity	$5{\cdot}33 \times 10^{-3}$	$5{\cdot}06 \times 10^{-3}$ c.g.s.

* *Phil. Trans.* A, **191**, 399 (1898).

An experiment performed without water-circulation through the cold plates gives the value of the emissivity which is used in calculating the heat-losses in the main (calorimetric) experiment. Since these losses are small, the emissivity is not

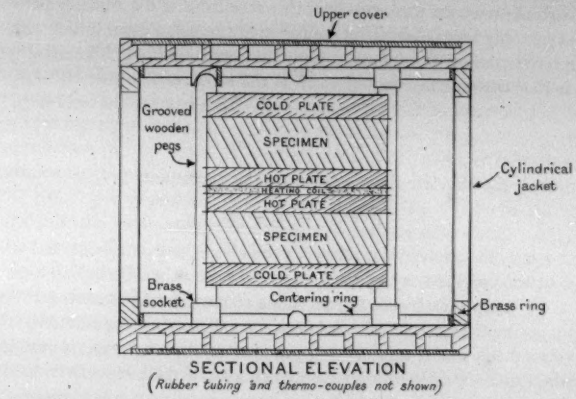

Fig. 1.

required to a high order of accuracy, an error of 10 per cent. in its value producing an error of less than 0·5 per cent. in the value of the conductivity. It is rather surprising that the value of the conductivity deduced by the emissivity method should

differ by only 5 per cent. from the more reliable value given by the calorimetric method in spite of the smallness of the temperature-drop and the uncertainty in the value of the relatively large heat-loss in the former method. Assuming for the moment that no errors are introduced on this account, the discrepancy between the two values can only be explained by the fact that the usual assumption with regard to the effective heat-flow through the specimen is not justified. The assumption in question is that the effective heat-flow $\overline{W}$ is the mean of the heat $(W - w_1)$ which enters the hot face and of that $(W - w_1 - w_2)$ which reaches the cold face, i.e.

$$\overline{W} = W - w_1 - w_2/2.$$

For example, if the effective flow is given by the expression

$$\overline{W} = W - w_1 - rw_2,$$

where $0 < r < 1$, the appropriate value of r which in the example quoted will make the values of the conductivity agree is 0·63, the values then becoming $5{\cdot}02 \times 10^{-3}$ and $5{\cdot}04 \times 10^{-3}$ respectively, the former being reduced by 6 per cent. and the latter by only 0·5 per cent. The difference between the values of the conductivity obtained by the two methods was always about 5 per cent. so that the error, if any, involved in taking the value of r as $\frac{1}{2}$ will only be 0·5 per cent. of the conductivity as deduced by the calorimetric method. In short, the experimental results lead one to suppose that the effective heat-flow should be given by

$$\overline{W} = W - w_1 - rw_2,$$

where $\frac{1}{3} < r < \frac{2}{3}$, the error in the conductivity due to the uncertainty in the correct value of r never exceeding 0·5 per cent.

The heating current is supplied by a battery of accumulators, the wattage being calculated from the resistance of the coil and the potential-difference between its terminals as measured with a calibrated voltmeter. The wattage can be measured in this way to the same order of accuracy as the other quantities, the necessity of using a more elaborate potentiometer method being thus dispensed with.

Measurement of temperature. It is important that the temperatures be measured at two points a known distance apart along the lines of heat-flow in the specimen. The practice of embedding thermocouples in the specimen itself is undesirable from the point of view of the inevitable distortion of the lines of heat-flow produced by the material used to cement the couples in place. It is also inadvisable to measure the temperatures of the hot and cold plates on the assumption that these give the temperatures of the specimen surfaces, particularly when the latter are hard or uneven, for then the thermal contact is poor and it is preferable to use thermocouples in direct contact with the surfaces, as pointed out by Griffiths*. The thermal contact can be improved by using between the surfaces a cementing material, the thickness of which should be very small in comparison with the thickness of the specimen, a condition difficult to fulfil with thin plates. With thick specimens (1 cm. or more) the accuracy of measurement of temperature-gradient is increased, and it

* *Loc. cit.*, p. 14.

is not necessary to adopt special methods of ensuring perfect plane parallelism of the faces or of measuring the thickness.

In the present work the temperatures at the centres of the actual faces were measured by means of thermocouples in good thermal contact with the faces, the couples being in the form of very thin strips sandwiched between the specimens and the hot and cold plates. With brass plates of the thickness used here the temperature is uniform over the surface to within a short distance of the edges. The thermocouples were made by hard-soldering together 28-gauge copper and constantan wires and rolling them out to form a strip about 15 cm. long and 0·03 mm. thick, with the junction in the centre. Latterly constantan-manganin couples were used in preference to copper-constantan couples owing to the fragility of thin copper strip, the e.m.f. per degree being only slightly less. There is also the additional advantage that the temperature-coefficients of resistance of both constantan and manganin are small, so that the galvanometer deflection per microvolt has practically the same value at 0° C. as at 100° C. Readings of the e.m.f. were made with a slide-wire potentiometer adjusted to give a potential drop of 1 microvolt per mm. down the wire; the galvanometer, which was of the moving coil type, had a voltage-sensitivity of 2·5 divisions per microvolt. The e.m.f. could easily be measured to 1 microvolt, corresponding to a temperature of about 0·02° C. Subsequently the slide-wire potentiometer was replaced by a thermoelectric potentiometer of the Cambridge Scientific Instrument Co. The e.m.f./temperature relations were obtained experimentally with sample couples over the range 0° to 100°, one junction being in melting ice and the other in a vacuum flask containing water, the temperature of which could be controlled by a small heating coil and measured with a platinum resistance thermometer. If E represents the e.m.f. of the couple in microvolts for a hot-junction temperature of t° C. the parabolic equation

E
t

$$E = at + bt^2,$$

a, b

fits the observed values closely over the range 0° to 100° and can safely be extrapolated to 130°, the values of the constants being

$$a = 38{\cdot}5, \quad b = 0{\cdot}0425 \text{ for copper-constantan,}$$
$$a = 35{\cdot}6, \quad b = 0{\cdot}0412 \text{ for constantan-manganin.}$$

The e.m.f. for $t = 100$ is practically the same for all couples provided the latter be made from wire taken from the same two reels; in one case, for example, five constantan-manganin couples had the following e.m.f.'s: 3962, 3963, 3963, 3965 and 3967 microvolts. In calibrating the couples used in the apparatus it is therefore sufficiently accurate to determine the e.m.f.'s at 100° and apply a proportionate correction on the appropriate parabolic formula. It has been shown by Adams* that the relation for a copper-constantan couple over an extended temperature-range (0° to 350° C.) is accurately given by the formula

$$E = 74{\cdot}672t - 13892\,(1 - e^{-0{\cdot}00261t}).$$

* *J. Amer. Chem. Soc.* **36**, 65 (1914).

Over the range of temperatures used in the present work (0°–130°) the simpler parabolic formula gives values of the e.m.f. which agree with those obtained from the above formula to within 2 or 3 microvolts.

The four thermocouples are insulated from the hot and cold plates by thin sheets of mica 0·05 mm. thick and of area slightly greater than that of either plate. These sheets must make good thermal contact with the hot and cold plates and with the surfaces of the specimens. A suitable substance for this purpose is British pitch, which is sufficiently fluid at 100° to flow over the surface and sufficiently viscous to be retained between the plates at 130°, the highest temperature reached. The four mica sheets are first attached to the surfaces of the hot and cold plates by means of the pitch, layers of pitch are then formed on the surfaces of the mica sheets; and the whole system of plates, including the two specimens with the thermo-junctions in place, is mounted in a framework to prevent slipping. Steam is now circulated through the cold plates, and under the pressure of a 28-lb. weight the surplus pitch is gradually squeezed out until very thin layers exist between the surfaces. Owing to the high viscosity of pitch even at 100° this may take a considerable time, but it is inadvisable to expedite matters owing to the danger of creating small air pockets in the layers of pitch.

The constant-temperature enclosure. In order to make the external conditions definite, and to investigate the thermal conductivity above 100° C., the apparatus must be placed in a constant-temperature enclosure. The latter consists of three portions, a hollow cylindrical jacket through which water or steam may circulate, and an upper and a lower cover channelled for water- or steam-circulation in the same way as the cold plates of the apparatus. Through the lower of the brass rings which form the ends of the cylindrical jacket are bored four small holes on either side for the passage of the thermocouple and heating-coil leads, and two $\frac{3}{8}$-in. holes, one on either side, for the passage of rubber tubing carrying the water circulating through the lower cold plate. A similar pair of holes through the brass ring at the upper end of the cylindrical jacket suffices for the rubber tubing connected to the upper cold plate. On the inner surface of the lower cover are soldered four brass sockets, the centres of which are at the corners of a rectangle of the same size, 15 cm. × 14 cm., as the faces of the system of plates. Four $\frac{3}{8}$-in. wooden rods are fixed in these sockets, the rods being grooved along their length to within an inch of their base so as to support the system of plates at the corners, thus preventing any sliding of one plate over another.

§ 3. EXPERIMENTAL PROCEDURE

In carrying out a determination at room temperature, a rapid stream of water from the tap is passed through the lower cover, lower cold plate, cylindrical jacket, upper cold plate and upper cover in the order named. The heating current is then established and after the lapse of an hour the e.m.f.'s of the four thermocouples are measured until constant values are indicated. Temperature-readings are obtained in this way for different values of the potential-difference between the heating coil

terminals. The stream of water is sufficiently rapid to give, for a supply of 100 watts to the heating coil, a rise of temperature of not more than 0·4° C. in passing through the cold plate, this rise of temperature being observed by means of mercury thermometers inserted in the circulating medium just before and after passage through one of the cold plates. A series of temperature-readings having been obtained for different values of the heating current, a series of emissivity readings is made over the same temperature-range by disconnecting the rubber tubing from the cold plates, the water-flow being through the jacket and covers only. Much smaller values of the heating current suffice in this case, but the realization of steady conditions is prolonged to about 8 to 10 hours, instead of one hour as in the calorimetric method.

The procedure was similar in making determinations of the conductivity above 100° C., except that steam was passed through the system in the reverse order, i.e. downwards instead of upwards, any tendency to superheat being thus avoided; the steam was then condensed and returned to the boiler.

§ 4. THEORY OF THE METHOD

Determination of the emissivity. Let

W be the energy supplied to heating coil in watts;	W
d_h the thickness of the hot plate in cm.;	d_h
d_c the thickness of each cold plate in cm.;	d_c
d_s the mean thickness of the specimen in cm.;	d_s
p the perimeter of the specimen in cm.;	p
A the area of face of the specimen in cm.^{2};	A
θ_1 the mean temperature of the hot faces in degrees C.;	θ_1
θ_2 the mean temperature of the cold faces in degrees C.;	θ_2
ϕ the temperature of the enclosure in degrees C.;	ϕ
h the emissivity in watts/cm.^{2}-degrees; and	h
K the thermal conductivity in c.g.s. units.	K

Then it follows that in the steady state

$$W = h\left\{pd_h(\theta_1 - \phi) + 2pd_s\left(\frac{\theta_1 + \theta_2}{2} - \phi\right) + 2(pd_c + A)(\theta_2 - \phi)\right\},$$

from which the value of h at a mean temperature of $(\theta_1 + \theta_2)/2$ can be determined. The assumption is here made that h is the same for all the surfaces and is independent of the temperature. As the difference between θ_1 and θ_2 is never more than 3 or 4 degrees in this experiment the latter assumption is justified. The first assumption is not strictly legitimate even if the different surfaces be made similar by varnishing, so that the above equation gives only a rough average value for the emissivity. As was pointed out previously, however, a comparatively large error on this account influences the value of the conductivity to only a small extent.

K The approximate value of the conductivity K as given by this method is

$$K = \overline{W} d_s / 2A\,(\theta_1 - \theta_2) \times 4{\cdot}18,$$

$\overline{W}$ where $\overline{W}$, the effective flow through the specimens, is given by

$$\overline{W} = W - hpd_h\,(\theta_1 - \phi) - hpd_s\{(\theta_1 + \theta_2)/2 - \phi\}.$$

Determination of the thermal conductivity by the calorimetric experiment. If, as before, θ_1 and θ_2 represent the mean temperatures of the hot and cold faces respectively in the calorimetric experiment, the hot plate will be at an excess temperature of $(\theta_1 - \theta_2)$ above that of the enclosure, and the specimens will be at an excess temperature of $(\theta_1 - \theta_2)/2$ approximately, since the temperatures of the cold faces and of the enclosure are practically the same.

The effective flow is therefore given by

$$\overline{W} = W - h_1 pd_h\,(\theta_1 - \theta_2) - h_2 pd_s\,(\theta_1 - \theta_2)/2,$$

h_1, h_2 where h_1 is the emissivity at a temperature θ_1, and h_2 is the emissivity at a temperature $(\theta_1 + \theta_2)/2$, these values being given by the emissivity experiment.

The thermal conductivity is, as before, given by

$$K = \overline{W} d_s / 2A\,(\theta_1 - \theta_2) \times 4{\cdot}18,$$

at a mean temperature $(\theta_1 + \theta_2)/2$.

§ 5. SPECIMEN OF OBSERVATIONS

As an example of the method the observations are given in some detail for slate

$$d_h = 2{\cdot}35 \text{ cm.},$$
$$d_c = 1{\cdot}565 \text{ cm.},$$
$$d_s = 2{\cdot}795 \text{ cm.},$$
$$p = 58{\cdot}7 \text{ cm.},$$
$$A = 215{\cdot}0 \text{ cm.}^2.$$

Table 1. Values of the emissivity.

W (watts)	θ_1 (° C.)	θ_2 (° C.)	ϕ (° C.)	$(\theta_1 + \theta_2)/2$ (° C.)	h (watts/cm.²-degree)
5·73	19·31	18·19	9·85	18·75	$6{\cdot}15 \times 10^{-4}$
6·98	21·88	20·52	10·50	21·20	$6{\cdot}23 \times 10^{-4}$
8·94	24·21	22·49	10·25	23·35	$6{\cdot}53 \times 10^{-4}$
10·19	25·70	23·61	9·55	24·65	$6{\cdot}47 \times 10^{-4}$
11·91	28·45	25·97	9·90	27·21	$6{\cdot}60 \times 10^{-4}$
14·78	32·33	29·24	10·05	30·79	$6{\cdot}84 \times 10^{-4}$

Values obtained above 100° C. ranged from $9{\cdot}34 \times 10^{-4}$ at a mean temperature of 108·62° C. to $10{\cdot}72 \times 10^{-4}$ at a mean temperature of 127·51° C. By plotting

emissivity against mean temperature graphs are obtained from which the appropriate value of h can be read off in deducing the losses in the main experiment.

Table 2. Values of the thermal conductivity.

W (watts)	θ_1 (° C.)	$(\theta_1 - \theta_2)$ (° C.)	w_1 (watts)	w_2 (watts)	$\overline{W}$ (watts)	$(\theta_1 + \theta_2)/2$ (° C.)	$K \times 10^3$ (c.g.s.)
32·87	22·76	9·71	0·85	0·96	31·54	17·90	5·052
35·63	23·20	10·49	0·92	1·04	34·19	17·96	5·067
47·95	27·53	14·02	1·27	1·41	45·97	20·52	5·100
52·41	27·93	15·44	1·40	1·55	50·23	20·21	5·060
63·16	31·52	18·62	1·74	1·92	60·46	23·21	5·049
69·76	33·22	20·47	1·94	2·12	66·76	23·23	5·072
74·54	35·32	21·92	2·10	2·28	71·30	24·36	5·057
80·46	37·13	23·64	2·31	2·48	76·91	25·31	5·059
93·89	41·00	27·53	2·76	2·93	89·66	27·23	5·064
104·0	44·33	30·51	3·14	3·28	99·22	29·07	5·058
109·2	46·00	32·12	3·43	3·50	104·0	29·94	5·035
115·7	47·99	33·98	3·60	3·74	110·2	31·00	5·044

The following values of $K \times 10^3$ were obtained for mean temperatures ranging from 105·43° C. to 113·88° C., the values of W ranging from 28·15 watts to 73·96 watts, and $(\theta_1 - \theta_2)$ from 8·76° C. to 22·82° C.:

4·694, 4·684, 4·697, 4·690, 4·694, 4·670, 4·679, 4·672, 4·681, 4·702, 4·671, 4·690.

The above values of the conductivity are plotted against the mean temperature in figure 2.

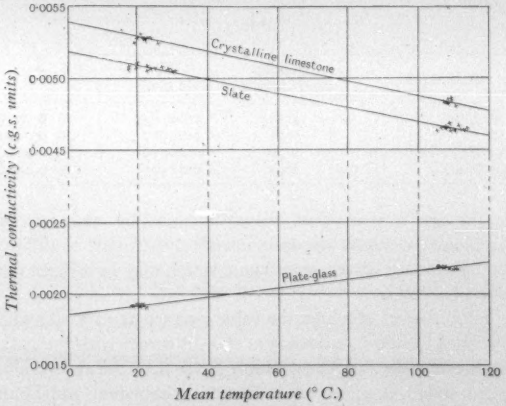

Fig. 2.

On the assumption that a linear relation

$$K_\theta = K_0 (1 + \alpha\theta),$$

K_θ, K_0, α

holds over the range 0° C. to 130° C. the following values are obtained:

$$K_0 = 0{\cdot}00517 \text{ c.g.s. units},$$

$$\alpha = -0{\cdot}00085.$$

It will be noted that for this specimen the lateral loss amounts to 3·4 per cent. of the total heat supplied at room temperature. At 100° C. the loss is nearly 5 per cent. By cutting down the plates to a thickness of $\frac{1}{4}$ in. it would be possible to reduce the lateral loss to 1 per cent., while still maintaining an accurately measurable temperature-difference. In spite of the comparatively large heat-loss the values obtained show a fair consistency, as will be seen from the table; the consistency is even greater in the case of plate glass, for which, owing to its small thickness ($\frac{1}{4}$ in.), the percentage loss is less. It may be remarked that the determinations at steam temperatures were generally more erratic than those at room temperature.

§6. SUMMARY OF RESULTS

The following substances were investigated: plate glass, slate, crystalline limestone and quartzite. The quartzite was obtained from South Africa; the sources of the limestone and the slate are not known. The specimens of slate show evidence of laminations in planes parallel to the largest faces (i.e. at right angles to the direction of heat-flow).

The values of the thermal conductivity at 0° C. and the temperature-coefficient are given in table 3.

Table 3.

Substance	Density (gm./cm.³)	K_0 (c.g.s.)	α
Plate glass	2·55	0·001845	+ 0·00168
Slate	2·78	0·00517	− 0·00085
Crystalline limestone	2·70	0·00539	− 0·00095
Quartzite	2·95	0·01493	− 0·00163

Except in the case of plate glass, values obtained by other investigators vary considerably. These variations are undoubtedly due in part to differences in the composition of substances of the same name which may have been quarried from widely separated localities.

For plate glass Meyer* obtained the value 0·00179 at 12·5° C., while the value given by Niven and Geddes† (0·001923 at 20° C.) agrees with the author's.

Previous values for slate are considerably lower than the above value; Lees and Chorlton‡ give 0·00357 at 94° C. while Herschel, Lebedour and Dunn§ obtained a value of 0·0034 perpendicular to the cleavage planes, the value in a direction at right angles being much higher. The low values could be explained if the specimens

* *Wied. Ann.* **34**, 596 (1888).
† *Proc. R.S.* A, **87**, 535 (1912).
‡ *Phil. Mag.* **41**, 495 (1896).
§ *Brit. Ass. Report*, **49**, 58 (1879).

of slate used by these observers were more markedly laminar than those used in the present work.

Limestones and marbles vary considerably, as judged by the wide range of values of conductivity; Poole* gives values ranging from 0·0046 to 0·0057 at 40° C. and 0·0039 to 0·0049 at 100° C., those at the lower temperature being in substantial agreement with the determinations of Herschel, Lebedour and Dunn†. Values as high as 0·007‡ have been obtained for marble.

The comparatively small number of determinations of temperature-coefficients shows even greater discrepancies between the values obtained by different observers. According to the present work slate, limestone and quartzite possess negative coefficients, and this result is in agreement with those obtained for the majority of rocks. In connexion with the positive value obtained for plate glass it may be noted that Lees§ gives a value of + 0·0025 for window glass.

§ 7. ACKNOWLEDGMENT

The above investigation was suggested by the late Prof. H. L. Callendar, F.R.S., from whose constant interest and advice the author derived much valuable help, which he gratefully acknowledges.

* *Phil. Mag.* **24**, 45 (1912).
† *Loc. cit.*
‡ A. Eucken, *Ann. d. Phys.* **34**, 185 (1911).
§ *Loc. cit.*

THE ATTENUATION OF ULTRA-SHORT RADIO WAVES DUE TO THE RESISTANCE OF THE EARTH

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ABSTRACT. This paper describes an investigation of the attenuation of radio waves, of wave-lengths between 5 and 10 metres, when transmitted directly along the earth's surface. A brief description is given of the transmitters employed, one being a fixed installation used with an input power supply of the order of 500 watts, the other a transportable set operated with an input supply of about 50 watts obtained from batteries.

The observations of field-intensity were obtained by measurement of the audio-frequency voltage across the telephones of a simple two-valve loop receiver, a brief description of which is given. Measurements were carried out at both Slough and Teddington of the field-intensity at various distances from the transmitter up to 700 metres. In some of the experiments a negative attenuation effect was observed in the radiated field over a distance of about 4 wave-lengths. Later measurements carried out under more favourable conditions did not show this effect, which was, therefore, attributed to the interference of waves reflected from trees and buildings in the neighbourhood of the transmitter. Some qualitative observations were made at distances up to 20 miles with a single-loop direction-finder. These observations showed that the signal intensity on such short wave-lengths depends to a great extent upon the existence of obstacles in the path of transmission. The signal-intensity at a distance of 20 miles over a direct air line was of the same order as that obtained at 4 miles for transmission along the ground.

A comparison has been made between the experimental results and those calculated from a simple wave-attenuation theory, the electrical constants of the earth being taken into account. As a result of this comparison, the value of the conductivity of the earth appears to lie between 5×10^8 and 30×10^8 e.s.u. for the frequencies of 30 to 60 megacycles per second employed. These values correspond to resistivities of from 1800 to 300 ohm/cm.[3] The most suitable value of the dielectric constant of the earth is about 10, although the experimental method does not enable this to be obtained with any great accuracy.

A brief description of some experiments carried out with a single-loop direction-finder on the wave-lengths under consideration is appended to the paper.

§ 1. INTRODUCTORY

During the past few years considerable progress has been made in the technique of radio transmission and reception on ultra-short wave-lengths (below 10 metres), although such wave-lengths have scarcely yet been applied to practical communication. While radio transmission to long distances depends upon the propagation-facilities provided by the upper regions of the atmosphere, trans-

mission over the earth's surface depends rather upon the scattering and absorption-effect of obstacles on or near the earth's surface, and upon the absorption of energy in the earth. On the first point it is easily understood that such objects as buildings, hills, trees, etc., whose dimensions are negligible compared with the longer wave-lengths, may become serious obstacles in the path of ultra-short waves whose wave-length is comparable with or smaller than the principal dimensions of the obstacles. On the matter of the attenuation of radio waves due to the finite conductivity of the earth, a certain amount of data has been made available during the past few years as a result of experiments carried out on longer wave-lengths [1, 2, 3, 4, 5, 6]. These data have been used on the one hand to compare the experimental results with the various theoretical formulae which have been suggested to account for the effect of the earth's resistance, and on the other hand to deduce values for the conductivity and dielectric constant of the earth. Even were complete agreement reached on the theoretical side it would still be desirable to ascertain to what extent these electrical constants of the earth vary with the frequency of the oscillations employed.

The objects of the investigations described in the present paper were, firstly, to extend the existing data on ground-wave attenuation into the region of wave-lengths below 10 metres (frequencies above 30 megacycles/sec.), and secondly, to endeavour to deduce values for the electrical constants of the earth at such frequencies.

§ 2. DESCRIPTION OF TRANSMITTERS

The experimental portion of the work was carried out partly at the Radio Research Station, Slough, and partly at the National Physical Laboratory, Teddington. A description of the principles underlying the design of the transmitters employed has been given in a former publication [7]: which contained illustrations

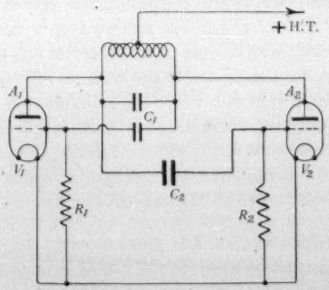


Fig. 1. Schematic circuit diagram of two-valve push-pull transmitter.

of the equipment installed at Slough. At this station the transmitter comprised a push-pull oscillator using two 250-watt valves feeding a single tuned oscillatory circuit, the arrangement being as shown in the schematic diagram in figure 1. The

valves were erected on a skeleton wooden panel, and were so arranged that the connexions from the valve electrodes to the tuned circuit were as short as possible. Suitable choke-coils were inserted in the leads to filament and anode, in order to restrict the oscillatory current as far as possible to the tuned circuit. The valve filaments were operated from a 16-volt battery and the anode current was obtained from a motor-generator giving a d.c. supply at from 2000 to 5000 volts. When operating with an input supply of a few hundred watts to the set the circulating current in the tuned circuit was several amperes at wave-lengths between 5 and 10 metres, this range of wave-length being obtained by the use of different sizes of single-turn inductance loops. In some of the experiments the radiation direct from this loop was adequate for measurement purposes, and the arrangement possessed the advantage of forming practically a point source. For other measurements at greater distances, however, the loop was coupled to a Lecher-wire system by means of which the radio-frequency energy could be supplied to a half-wave aerial erected at some 10 metres from the transmitting hut. When both the Lecher-wire system and the aerial were accurately tuned to the working wave-length, the current at the centre of the aerial was of the order of 0·5 amp. The aerial was constructed in a telescopic form so that it could easily be adjusted to resonate at the desired wave-length.

In addition to this installation a smaller transmitter was constructed. This employed an essentially similar circuit arrangement, but used two 25-watt valves of a type which could be operated from a 300-volt anode-current supply. For this purpose the set could be operated from a 12-volt battery, a small motor-generator being used for obtaining the anode current, or alternatively the set could be operated from a 6-volt filament battery and a 300-volt h.t. battery. The latter method was found to be preferable as it avoided the necessity of a smoothing circuit in the h.t. supply, and gave greater constancy of frequency of the generated oscillations. The transmitter, with the exception of the inductance loop of the oscillatory circuit, was built into a screened box which was mounted on a table, which carried the batteries and was provided with castors. The whole set was thus transportable as a unit, and could be placed on any suitable site with no difficulty arising from interference due to power-supply cables. Since the main portion of the transmitter was screened and radio-frequency currents were excluded from the battery-supply leads by the use of suitable choke coils and condensers, the inductance loop formed practically a point source of radiation, the strength of which was found to be adequate for the experiments for which this transmitter was used.

§3. DESCRIPTION OF RECEIVER AND METHOD OF MAKING MEASUREMENTS

The type of receiver employed has been described in the publication to which reference has already been made[7]. It comprised a single-valve retroactive detector stage followed by one stage of audio-frequency amplification, the schematic circuit diagram being given in figure 2. A single-turn loop was used as the receiving aerial and as the inductance of the sole oscillatory circuit employed. This loop was

mounted directly on the top of a screened box which contained the whole of the rest of the receiver including the batteries. By the use of single-turn loops of diameters between 5 and 12 in., constructed of $\frac{1}{8}$ in. copper tube, wave-lengths from 4·8 to 10·8 metres could be covered.

The output from this receiver was measured by connexion of a suitable valve-voltmeter directly across the output terminals, a measure of the audio-frequency voltage across the telephones being thus given. In some of the experiments the radio-frequency current at the transmitter was modulated at a constant audio-frequency and no difficulties arose from note variation: in the majority of the experiments, however, continuous waves were used, and the signal current was reduced to an audible frequency by the autodyne method, the detector stage of the receiver being made to oscillate. Although a certain amount of difficulty was ex-

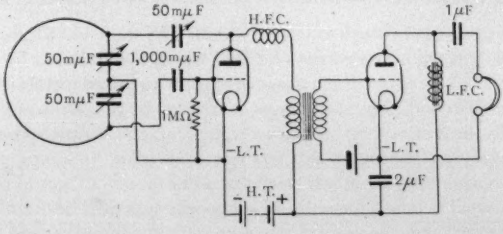


Fig. 2. Two-valve short-wave loop receiver for field-strength measurements and direction-finding (wave-length range 4 to 10 metres).

perienced due to note variation under these conditions, it was found that in favourable circumstances the beat frequency could be maintained constant long enough for a measurement to be made, and that the readings could be repeated to a reasonable degree of accuracy. This was especially the case with the battery-driven transmitter with which the difficulties due to frequency variation were relatively small.

The receiver used in this way for relative signal-intensity measurements was calibrated directly from the transmitter by variation of the oscillatory current while measurements of the signal output were made with the receiver at a fixed position. In this way it was determined that up to outputs of nearly two volts, the measured output was proportional to field-intensity. Three ranges were provided on the valve-voltmeter, and the sensitivity of the receiver could be altered by variation of the retroaction adjustment, but care was taken not to use too critical a value of this, so that small accidental variations would not produce an appreciable effect.

The procedure adopted in obtaining attenuation curves was first to mark out the line of propagation required with wooden pegs at intervals of 10 metres or 100 metres from the transmitter. The receiver was then taken in succession to each position and a measurement of the output made for each of two or three values of transmitter current. No alteration of receiver conditions was made in changing

from one position to the next, and the transmitting-aerial current was measured throughout the experiments by means of a thermo-ammeter inserted at its mid-point. During the measurements the receiver was placed directly on the ground and the observer lay alongside it to avoid any variable effects due to his presence, since in the upright position a man can produce quite a marked variation in the field-strength in his proximity on these short wave-lengths. In each experiment measurements were made in the above manner at each position over the full length of the run and were then repeated during the return journey to obtain a check. The readings were then reduced to a unit value of transmitting aerial current and also to the relative intensity at some position arbitrarily chosen for the purpose of the tests.

§ 4. EXPERIMENTAL PROCEDURE AND RESULTS OBTAINED

(*a*) *Measurements at Slough at short distances.* The short-wave transmitting hut at Slough was erected near a corner of a field which was bounded by a moat and a bank of trees. The trees were at a distance of from 50 to 100 metres from the hut, and there were two buildings at distances of from 25 to 50 metres. Over an arc of some 30° from the transmitting hut, however, there was a clear path which extended to a distance of over 600 metres and was free from trees, buildings, or other obstacles. The transmitting aerial was erected at a distance of 10 metres from the hut and was connected to the oscillator by a Lecher-wire system. The line of the Lecher wires was arranged along one side of the clear arc just referred to, and for the purpose of the measurements to be described below this line was taken as the zero direction.

In order to ascertain the effect of the various obstacles surrounding the transmitting hut, measurements were made of the field-intensity in various directions at distances varying from 7 to 100 metres, all conditions at the transmitter being maintained constant. At distances up to 18 metres the radiation-distribution was found to be fairly uniform in all directions around the transmitter, except where the hut was between transmitter and receiver. At a radius of 100 metres, however, the distribution of radiation varied greatly with the direction of transmission: the results obtained on the three wave-lengths employed are shown plotted in rectangular co-ordinate form in figure 3. The positions at which the circle of reception crossed the surrounding trees and the electric power cable supplying the transmitter are indicated in this diagram. It is clear from these graphs that while within the open sector of 0° to 30° mentioned above the field-intensity is reasonably constant for each wave-length, erratic variations in field-intensity were experienced in other directions. There is a pronounced drop in intensity, for instance, in the 120° direction where the receiving measurements were made beyond a belt of trees. In other directions the effect varies with the wave-length employed and indicates not only the absorption due to the trees but also the reflection and scattering effects which would be expected from obstacles several wave-lengths high. The power cable is seen to produce effects varying with the wave-length, the resultant intensity

being due possibly to resonant phenomena in the cable itself. Since a vertical half-wave aerial was employed at the transmitter the radiation was polarized with the electric force in a vertical plane and it is, therefore, to be expected that trees, the lower portions of which are sensibly vertical, would form an effective screen to the passage of such waves. Further, it is reasonable to suppose that waves polarized with the electric force horizontal would penetrate the trees more easily, but their propagation along the ground itself would be very difficult. The investigation was concerned with propagation along the ground free from the effects of obstacles, and the use of horizontally polarized waves has not so far been investigated experimentally.

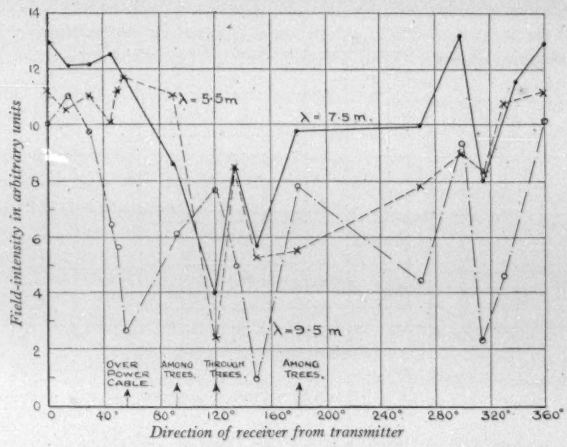
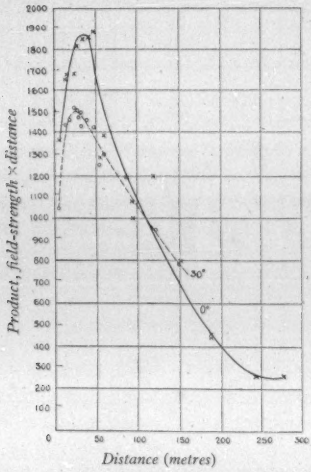


Fig. 3. Short-wave transmitter at Slough. Measurements of field-intensity in different directions at constant distance of 100 metres (wave-lengths 5·5, 7·5 and 9·5 metres).

Attention was now turned to the carrying out of field-intensity measurements at various distances from the transmitter in the clear sector. In the first series of tests the transmitting aerial was removed and the radiation direct from the loop of the oscillator used for measurements at distances of from 6 to 300 metres. Figure 4 shows the results of a series of such measurements made on a wave-length of 8·7 metres and in the two directions 30° apart which form the boundaries of the "clear" sector. The ordinates are given as the product of the field-intensity and distance for each position employed, and it may be pointed out that this quantity would be constant at all distances for the ideal case of propagation over a flat perfectly conducting earth. The graphs in figure 4 indicate that in each case, for short distances between transmitter and receiver, the product of field-intensity and distance *increases* with the distance, attaining a maximum value at a distance of about 36 metres. The suggestion has been made that this might be due to a mutual coup-

ling effect between the transmitter and receiver tuned circuits, but this was considered to be unlikely since no difficulty was experienced in maintaining constant the conditions at the transmitter. It was next considered that the peak in the curve indicated a negative attenuation effect, as has been suggested by Ratcliffe[3, 5], to explain some of his own experimental results on wave-lengths of 1600 and 30 metres. Some doubt was felt about the validity of this explanation, however, because of the possible reflection effects which might occur from surrounding obstacles; and these doubts were confirmed by the fact that within the range under consideration the readings obtained with a direction-finder on the transmitter showed that large



——×—— Direction from transmitter = 0°.
– – –○– – – Direction from transmitter = 30°.

Fig. 4. Field-strength measurements at distances of 6 to 300 metres from a small loop transmitter (wave-length 8·7 metres).

changes in the indicated bearing occurred with small changes in position. Furthermore, it may be recalled that in their experiments carried out in Bushey Park in 1905, Duddell and Taylor[8] found that the product of received current and distance from transmitter did not always decrease as the distance from the transmitter was increased, and in some cases a definite increase in the value of the product was recorded. These experiments were carried out on a wave-length of the order of 100 metres and these anomalous results were ascribed to scattering effects of the trees between transmitter and receiver. It was, therefore, decided to attempt the carrying out of more accurate measurements on a site free from the obstacles referred to above.

(*b*) *Measurements at Teddington at short distances.* For the purpose of these measurements a transportable type of transmitter was developed on the lines already described in § 2. The great advantage possessed by the use of this set was that it could be taken to a clear site and no difficulties due to the attachment of power-supply cables would arise. The next series of measurements was carried out on a field at the National Physical Laboratory approximately 250 metres square and clear of any obstacles. The transmitter was set up at a point at least 100 metres from any interfering obstacle along the boundaries of the field, so that a clear run 100 to 150 metres in length was obtainable before the receiver came under the influence of the conditions on the boundaries of the field.

As in the previous case, a measurement of the radiation in various directions from the transmitter was carried out. Since in this case the radiation was obtained entirely from the transmitting loop, this loop was turned for each measurement so

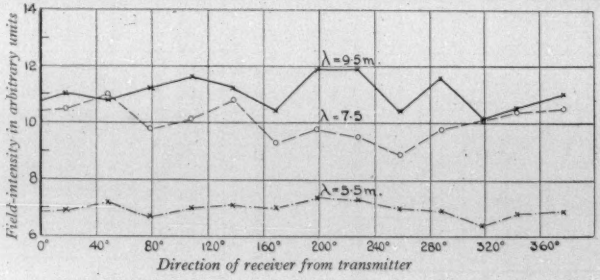


Fig. 5. Polar curves of portable transmitter at N.P.L., Teddington. Measurements of field intensity in different directions at constant distance of 30 metres (wave-lengths 5·5, 7·5 and 9·5 metres).

that its plane was in the direction of the receiver. The results of measurements made in this way at a constant radius of 30 metres from the transmitter, and on the three wave-lengths of 5·5, 7·5 and 9·5 metres are shown by the graphs plotted in figure 5. Similar results were obtained at a radius of 50 metres in a check experiment made on a wave-length of 7·5 metres. From these graphs it is seen that within extreme limits of about ± 10 per cent. the radiation from the transmitter in the plane of the loop is uniform in all directions on each of the three wave-lengths employed. It was concluded, therefore, that the site in the neighbourhood of the transmitter was satisfactory, and that attenuation measurements might be made without risk of interference from the effects of external objects.

The results of a series of measurements carried out during a period of four days in July, 1930, are shown by the graphs plotted in figure 6 for the three wave-lengths employed. Measurements made on consecutive days under the same conditions on the same wave-length showed good agreement to within the experimental accuracy for observations on such short wave-lengths. This, as was previously stated, is of the order of 10 per cent. Further measurements made during a two-day period in

August, 1930, on the same site are shown in figure 7. These measurements were made with the direction of transmission reversed from that of the July experiments, by placing the transmitter at the opposite end of the site. A direct comparison test was made for the two directions of transmission on 27–28 August, 1930, and the results are shown by the two graphs plotted for a wave-length of 7·5 metres in figure 7. Within the limits of experimental error the two curves are in good agreement. This implies that the observed attenuation is independent of the direction of transmission and the results are, therefore, likely to be free from external interference conditions.

Viewing the curves in figures 6 and 7 as a whole, it is seen that there is no sign

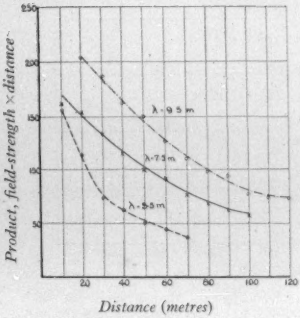


Fig. 6. Field-intensity measurements at distances up to 120 metres from a loop transmitter at the N.P.L., July 1930.

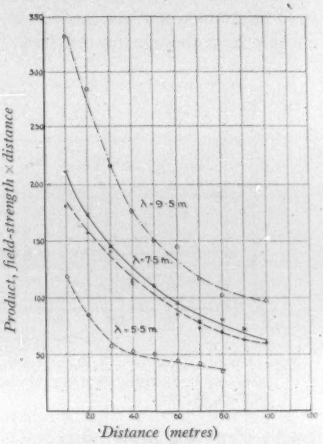


Fig. 7. Field-intensity measurements at distances up to 100 metres from a loop transmitter at the N.P.L., August 1930.

of the "negative-attenuation" effect previously observed in figure 4 at a distance of 30 to 40 metres from the transmitter, and it is likely that the peak in figure 4 is attributable to the interference of two sets of waves, one of which has reached the receiver after reflection from some object such as a building or a group of trees. In some of the measurements carried out under very dry conditions a slight rise in the curve was noticed at a distance of about 50 metres. On inspection of the site it was found that two strips of dried-up grass crossed the line of transmission at this point. These strips were subsequently found to mark out the directions of two disused tunnels at a depth of from 1 to 2 metres below the ground. As a result of the increased drainage conditions prevailing here, these strips of ground became very dry and produced a change in the ground conductivity, giving rise to interference effects which show up on the attenuation curves. After a period of wet weather both the

discoloration of the grass and the inflection of the curves disappeared. This is a result to be expected, since under normal conditions the surface layer of about 1 metre of soil is a sufficiently good conductor to screen the lower materials, unless these have a markedly superior conductivity.

(*c*) *Measurements from Slough at greater distances.* Some further experiments were carried out in an endeavour to obtain quantitative attenuation curves covering distances greater than the 120 metres to which the measurements given above were restricted. The site at Slough was chosen for this purpose since, as already stated, a clear path was available up to 600 or 700 metres in the most favourable direction. In these tests the transmitter was coupled to the vertical half-wave aerial, in which currents of from 0·1 to 0·5 amp. were employed. In order to avoid interference due to radiation from the transmitter itself the whole of the inside of the transmitting hut was screened with iron wire netting of half-inch mesh. This was found to be effective in reducing the radiation from the transmitter to an amount too small to

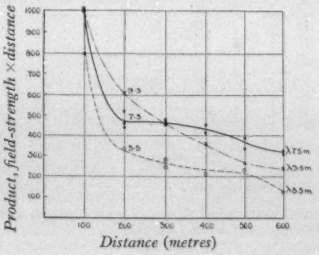


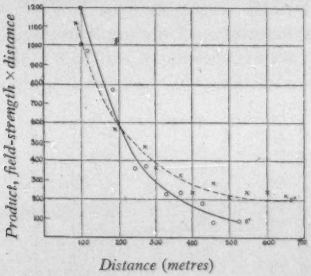
Fig. 8. Field-strength measurements at distances of 100 to 600 metres from a half-wave transmitting aerial (wave-lengths 5·5, 7·5 and 9·5 metres).

be measured except at very short distances from the hut. The receiver was used in the same manner as previously, measurements of the signal output being made at each position for two or three values of transmitting aerial current.

The results of a series of measurements made at distances of from 100 to 600 metres on the three wave-lengths of 5·5, 7·5 and 9·5 metres are shown in the graphs in figure 8. These measurements were taken for a direction of transmission 7° from the arbitrary zero direction defined above. The graphs show that while at first the attenuation increases as the wave-length is reduced, there is a marked change in the effect produced on the wave-length of 7·5 metres as compared with the other two wave-lengths. The fact that the curve for a wave-length of 7·5 metres crosses that for $\lambda = 9{\cdot}5$ metres at a distance of 300 metres from the transmitter indicates that the receiver is still under the influence of irregular propagation due to interference and reflection effects. Although at these positions the receiver was several hundred metres from the trees surrounding the site, it is evident that the effects were such as to increase the field intensity on the 7·5-metre wave-length as compared with that on 9·5 metres. Further tests showed that the field-intensity was not only dependent upon the wave-length, but also, to a fairly critical extent,

upon the direction of transmission. For example, the results of two tests made upon the same wave-length of 7·9 metres for two directions 5° apart are shown in figure 9. All the measurements recorded in these graphs were taken with the receiver placed at a satisfactory distance from trees, huts, etc., and it is noteworthy that on the whole, the measured values for each set of conditions lie on a smooth curve.

Other measurements were made to ascertain the effect of an individual tree upon the field-intensity in its neighbourhood. As an example of the results obtained, it may be stated that at a distance of nearly 200 metres from the transmitter a move-



——○—— Direction from transmitter =0°. – – –×– – – Direction from transmitter =5°.

Fig. 9. Field-strength measurements at distances of 100 to 700 metres from a half-wave transmitting aerial (wave-length 7·9 metres).

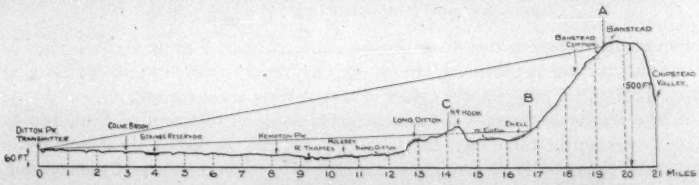


Fig. 10. Vertical section through path from R.R.S. Slough to Banstead.

ment of the receiver to 5 or 10 metres either side of the tree produced changes in field-strength of ± 50 per cent., so that in one position the field had three times the intensity obtaining in another position only 10 or 15 metres away.

For distances in excess of 700 metres it has been possible so far to make only qualitative experiments, the results of which are, however, very useful. At distances up to about 1 mile from the transmitter the received signal-intensity was adequate for taking readings on a direction-finder, although the accuracy of the bearings obtained varied in an erratic manner. At greater distances of from 4 to 7 miles over flat open country the signals were much weaker, but the d.f. bearings were more

consistent and were accurate to 2° or 3°. At 8 miles the signals were only just definitely audible, and although many tests were made at this distance in various directions over the flat country, no position was found at which satisfactory reception was possible with the simple two-valve receiver employed. When, however, the receiver was taken to a point nearly 20 miles distant from the transmitter and at an elevation of over 500 ft., the signal-intensity was of the same order as that obtained at only 4 miles from the transmitter. This point is marked as *A* in the sectional contour of the path of transmission shown in figure 10, from which it can be seen that there is a clear optical path from *A* to the transmitter. As the receiver was brought towards the transmitter the signals became inaudible at the point *B* 17 miles distant, and at point *C* 14 miles from the transmitter the signals were audible but much weaker than at the more distant point *A*.

It is thus evident from such tests that even at distances of several miles the field intensity at a receiving point is to a great extent dependent upon the topography of the ground along the line of transmission.

§ 5. DISCUSSION OF RESULTS

The first deduction to be drawn from the experience gained in this investigation is that in order to make reliable measurements of the attenuation of ultra-short waves at short and medium distances it is necessary to have available a large expanse of flat open ground clear of all obstacles, both natural and artificial, particularly in the neighbourhood of the transmitter and receiver. It is not easy to obtain such a site more than a few hundred metres in extent. Although in the experiments described above the quantitative measurements have been limited to ranges of 600 or 700 metres, it is to be noted that this distance is of the order of 100 times the wave-length employed. The most satisfactory measurements were those made at Teddington and illustrated by the graphs in figures 6 and 7, and it was thought that these were sufficiently reliable to be compared with the theory of attenuation of electromagnetic waves transmitted along the ground.

In the second place figures 6 and 7 show no sign of the negative attenuation effect which is deducible from the theoretical work of B. Rolf[9], and which appears to have been confirmed by the experimental work of Ratcliffe[3, 5], already referred to.

The results of the present experiments would indicate, however, that it is possible that in Ratcliffe's experiments the phenomena under investigation were subject to disturbance by some spurious effect which could produce a result of the type illustrated in figure 4.

Next, any comparison of experiment with theory must be made on the basis of the substitution in the theoretical formula of various values of the conductivity and dielectric constant of the earth, in order to ascertain the theoretical conditions which best fit the practical results. A simple theory of the attenuation of electromagnetic waves has been worked out by J. S. McPetrie and will be described in detail in a

later paper. From this theory it is shown that the reflection-coefficient at the earth is complex and is equal to

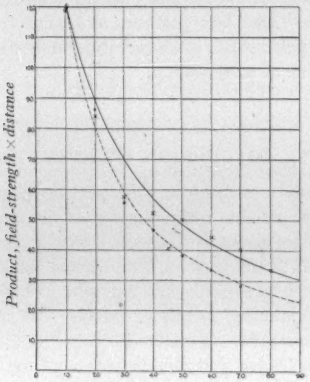
$$\frac{(k^2 + 4\sigma^2/f^2)\cos^2\theta - (c^2 + d^2) - 2j\cos\theta\,(2c\sigma/f + kd)}{(k^2 + 4\sigma^2/f^2)\cos^2\theta + (c^2 + d^2) + 2\cos\theta\,(kc - 2d\sigma/f)},$$

θ, k, σ, f, c, d where θ is the angle of incidence on the earth, k and σ are respectively the dielectric constant and conductivity of the ground, f is the frequency of the incident wave, and c and d are determined by the relations

$$k - \sin^2\theta = c^2 - d^2$$

and

$$\sigma/f = -cd.$$



Calculated curves: ——— $K = 10\,\sigma/n = 40$, ------ $K = 10\,\sigma/n = 10$.
Experimental results: • July 1930, Fig. 6. × Aug. 1930, Fig. 7.

Fig. 11. Theoretical and experimental results obtained at N.P.L. (wave-length 5·5 metres).

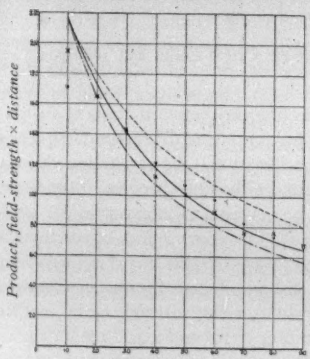
K, K' If the reflection-coefficient be expressed in the convenient form $K - jK'$, the product of the field-intensity and distance from a simple vertical Hertzian dipole for the case in which both transmitter and receiver are near the earth's surface is proportional to

$$\sqrt{(1 + K^2 + K'^2 + 2K\cos\lambda - 2K'\sin\lambda)},$$

λ in which λ represents the optical path difference between the ray direct from transmitter to receiver and that reflected by the earth's surface, and K and K' are the components of the complex reflection-coefficient for the angle of incidence which this indirect ray makes with the earth's surface.

In figure 11 a comparison is made between the experimental results for a wave-length of 5·5 metres, as given in figures 6 and 7, and theoretical curves calculated

from this formula for the following values in electrostatic units: $\sigma/f = 10$ and 40, $K = 10$. Similar graphs are given in figures 12 and 13 for the results obtained on wave-lengths of 7·5 and 9·5 metres. From these it will be seen that by a suitable choice of values of σ and K, satisfactory agreement can be obtained between theory and experiment. The nature of the variation of the theoretical curves for different values of σ/f is particularly shown in figure 13. Similar calculations have been made for different values of K between 5 and 80, but it was found that between these limits the value of K made little difference to the position of the theoretical



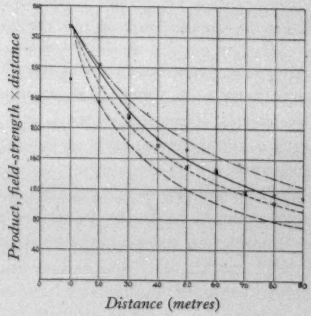
Calculated curves:

– – – – – – – $K=10\ \sigma/n=200$.
——————— $K=10\ \sigma/n=80$.
— • — • — • $K=10\ \sigma/n=50$.

Experimental results:

• July 1930, Fig. 6.
× August 1930, Fig. 7.

Fig. 12. Theoretical curves and experimental results at N.P.L. (wave-length 7·5 metres).



Calculated curves:

— • — • — • $K=10\ \sigma/n=200$.
——————— $K=10\ \sigma/n=80$.
– – – – – – – $K=10\ \sigma/n=60$.
— — — — $K=10\ \sigma/n=25$.

Experimental results:

• July 1930, Fig. 6.
× August 1930, Fig. 7.

Fig. 13. Theoretical and experimental results obtained at N.P.L. (wave-length 9·5 metres).

curves. This result is partly to be expected from the fact that over the range required K is still smaller in value than the quantity $2\sigma/f$, with which it is associated in the theoretical formulae. In addition, if K is increased much beyond its assumed value of 10 the theoretical curve becomes too convex to the axes to fit the experimental results.

A graphical comparison between theory and experiment carried out in this manner to values of σ and K for the observations made at the National Physical Laboratory in July and August, 1930 (figures 6 and 7). The results obtained from figures 11–13 are given in the following table.

Table showing the values of the conductivity (σ) of the earth as deduced from the experimental measurements of attenuation made at the N.P.L., Teddington, during July and August, 1930 (see figures 6 and 7).

The dielectric constant $K = 10$ throughout.

Wave-length (metres)	Frequency (mega-cycles/sec.)	Graphs in figure	For July		For August	
			σ/f	σ (e.s.u.)*	σ/f	σ (e.s.u.)*
				$\times 10^8$		$\times 10^8$
5·5	55	11	10	5·5	40	22
7·5	40	12	80	32	80	32
9·5	32	13	80	24	60	19

* A conductivity value of 10^8 e.s.u. is equal to $1{\cdot}1 \times 10^{-13}$ e.m.u. and corresponds to a resistivity of 9000 ohm/cm.^{3}.

It will be seen from this table that the values of σ vary between limits of $5{\cdot}5 \times 10^8$ and 32×10^8 e.s.u. for the three wave-lengths of 5·5, 7·5 and 9·5 metres (frequencies of 55, 40 and 32 megacycles/sec.). It is not considered that the variation with wave-length is significant in view of the experimental difficulties involved in measurements at such high frequencies; also there is no consistent difference in the results obtained in the two months of July and August. The above values of conductivity may be compared with those obtained on longer wave-lengths by previous workers. For example, on wave-lengths between 350 and 750 metres (frequencies between 860 and 400 kc./sec.) Smith-Rose and Barfield [1], using a tilting aerial method, obtained values of conductivity lying between $0{\cdot}14 \times 10^8$ and $4{\cdot}2 \times 10^8$ for the soil in different parts of the south of England; while in later investigations on attenuation Barfield [4] obtained values of from $0{\cdot}2 \times 10^8$ to $1{\cdot}0 \times 10^8$, the lower values of conductivity in this case being ascribable to the effects of trees. In their experiments on attenuation Ratcliffe and his co-workers [3, 5] have obtained values for σ of $0{\cdot}5 \times 10^8$ for a wave-length of 1600 metres (frequency 190 kc./sec.); of $1{\cdot}5 \times 10^8$ for a wave-length of 360 metres (830 kc./sec.); and $0{\cdot}18 \times 10^8$ for a wave-length of 30 metres (10,000 kc./sec.). Thus the results of these investigators indicate that on wave-lengths between 30 and 1600 metres (frequencies from 10,000 to 190 kc./sec.) the conductivity of the earth lies between values of 0·14 and $4{\cdot}2 \times 10^8$. It is difficult to assess the relative accuracy of the various methods of measurement employed, and it is therefore uncertain whether the variation in the values obtained is outside the overall limits of accuracy of the various experiments.

More recently M. J. O. Strutt [10] has experimented on the very short wave-length of 1·42 metres (210 megacycles/sec.) and has obtained a value of conductivity in excess of 10×10^8 e.s.u. for soil which at lower frequencies possessed a normal value of about 1×10^8. This result, taken in conjunction with those reported in this paper, appears to indicate that there is a definite tendency for the apparent conductivity of the earth to rise appreciably at radio-frequencies in excess of 30 megacycles/sec.

It is hoped that with the aid of a more sensitive receiver the present experiments

may be extended so that further quantitative data will be obtained for greater distances. Tests of various receivers suitable for use on ultra-short wave-lengths have been carried out recently, and the results suggest that the supersonic heterodyne type will be the most suitable for this purpose.

Also in order to deduce values of K for the earth with more accuracy, it will be necessary to extend the experimental measurements to still shorter wave-lengths. In this case it will probably be desirable to employ different methods of attacking the problem, for example, by raising the transmitter by a few wave-lengths above the ground and carrying out observations at short distances.

For the experimental results at longer distances already obtained, as illustrated in figures 8 and 9, it can be shown that the field-strength tends to decrease inversely as the square of the distance for distances in excess of 200 or 300 metres. This is a result which follows from the theory for transmission over uniform ground at distances of many wave-lengths from the transmitter, and it is suggested that this result is of direct importance in the application of very short waves to signalling purposes. When, however, either the transmitter or the receiver is elevated so that there is a direct optical-ray path between the two, and the propagation is independent of the effect of the earth, then the field-strength will decrease inversely as the distance. It is this law which would undoubtedly apply to the qualitative observations reported on page 602 (figure 10), in which the field-strength at 14 to 20 miles by the direct optical path was comparable with that at 4 miles, when the transmission was along the ground and the inverse-square law would apply.

§ 6. ACKNOWLEDGMENTS

The work described in this paper was carried out as part of the programme of the Radio Research Board, and acknowledgment is due to the Department of Scientific and Industrial Research for granting permission for publication. The authors desire to thank Messrs B. J. Byrne and A. C. Haxton for much assistance both in the experimental measurements and in connexion with the various theoretical calculations involved.

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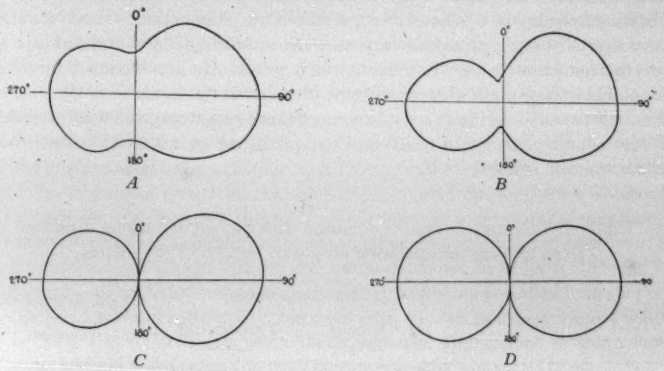
APPENDIX: AN EXPERIMENTAL DIRECTION-FINDER FOR USE ON ULTRA-SHORT WAVES

Mention has been made in the paper of the fact that observations were made with a direction-finder to ascertain approximately the nature of the field at both short and long distances from the transmitter. It is thought that a brief description of this instrument would be useful as an appendix to the paper.

(i) *Arrangement of d.f. set.* In the field strength measurements recorded above a simple type of closed loop receiver was employed, the circuit of which was shown in figure 2. This whole receiver is placed on a turntable mounted on a tripod and a means is thus provided for the taking of bearings on the transmitter. Since the whole of the receiver, other than the loop, is contained in a screened box, the set is free from the direct pick-up of signals on the valve circuits. Further, it will be seen from figure 2 that the receiving loop is tuned by two condensers in series, the lead between them being connected to the valve filaments and the screening box; also it was found that the capacities between the ends of the loop and the box were of the same order. It might, therefore, be expected that the loop is approximately electrically symmetrical about its midpoint, and would give a satisfactory figure-of-eight cosine diagram for direction-finding purposes. Simple aural tests, however, showed that this was not the case, and that the quality of the signal minima given by the set was much improved by the addition of an electric screen around the loop, in the manner commonly adopted in direction-finders on longer wave-lengths. Accordingly some measurements of the reception characteristics of the set were carried out when the loop was oriented in different directions relative to that of the transmitter.

(ii) *Experiments with screens round loop.* The measurements were made in the manner adopted above. The d.f. set was installed at a distance of 50 metres from the transmitter on the site which had proved most satisfactory for the attenuation measurements described in § 4 (*b*) of the paper. Readings of the audio-frequency output were made for a series of orientations of the set while conditions at the transmitter remained constant on a wave-length of 8 metres. Care was taken that the observer should remain in a fixed position below the set, and a repetition of the measurements over a portion of the 360° showed that the measurements could be made to a reasonable accuracy. A polar reception diagram obtained in this manner for the loop receiver with no screen around the loop is shown in figure 14 (*A*). The two lobes of this diagram are dissimilar and the positions of signal minima are not well defined. When a rectangular framework 15 × 4 in., comprising 50 wires 14 in. long, was placed around the coil, the wires being bonded together but insulated from the metal box, the diagram shown in figure 14 (*B*) was obtained. It is seen that the minima have become a little sharper but the two lobes are still asymmetrical, the strength of signal received on one side of the coil being about 30 per cent. less than on the other side. When the screen was connected to the metal screening box containing the receiver the signal minima became practically zero, as shown in figure 14 (*C*), although the polar diagram was still asymmetrical. By trial of various

arrangements of spacing and bonding of the screen wires, it was found that this asymmetry could be compensated for by earthing to the box the wires on one side of the loop while the remainder were left insulated. The polar diagram of the receiver with this arrangement is shown in figure 14 (*D*) from which it will be seen that the two signal zeros are sharp and well defined at positions 180° apart, and that the diagram is approximately symmetrical about the line through the minima. These experiments indicate that the fault with the unscreened loop is that the two sides are slightly out of balance in respect of their capacities to the box, to which the midpoint of the loop is virtually connected, and that this lack of symmetry can be compensated for by a suitable arrangement of the screen wires. It is possible that if the effect of the screen were applied in the reverse sense the asymmetry of the



A. Loop unscreened.
B. Loop surrounded with symmetrical screen, insulated.
C. Loop surrounded with symmetrical screen connected to box.
D. Loop surrounded by screen adjusted to compensate for asymmetry of loop.

Fig. 14. Polar reception diagrams of ultra-short wave d.f. set on wave-length of 8 metres.

polar diagram of *A* or *B* could be accentuated to produce a unidirectional short-wave receiver, but the experiments have not been continued in this direction.

(iii) *Performance of d.f. set.* The diagram given in figure 14 (*D*) above shows that when both transmitter and receiver were erected on a satisfactory site the d.f. bearing is very accurate for the distance of 50 metres employed. This was the condition in which the measurements of attenuation illustrated in figures 6 and 7 were measured. When the experiments were repeated on a less satisfactory site it was found that the observed bearings were subjected to erratic variations of the order of $\pm$ 15°, which were attributable to the effect of reflected waves from objects local to the transmitter.

The effect of extraneous conditions in the neighbourhood of the transmitter was also demonstrated in some experiments made with the direction-finder when

bearings were taken upon the main transmitter at Slough, a half-wave aerial with currents of from 0·1 to 0·5 amp. at the centre being used. The results of these measurements are given in table 2.

As the distance from the transmitter was increased the signal-intensity became rapidly less and at 8 miles the signals were just strong enough for bearings to be taken. At distances of more than 8 miles the signals were inaudible until the receiver was taken to the top of a hill, 500 feet higher than the transmitter, when, as already recorded in § 4 (*c*) of the paper, the signal-intensity at a range of 20 miles was comparable with that at 4 miles along the flat ground, and accurate d.f. bearings were again possible. While some of the errors recorded in the above table were probably due to conditions local to the direction-finder (e.g. at Poyle where the receiver was near telegraph wires) it is evident that there is a tendency for the errors in bearing to be less at a distance of a few miles than close to the transmitter. It was also observed that the signal minima were more clearly defined at a distance, despite the reduction in signal strength which necessarily accompanied the observation. These facts are consistent with the idea that at the transmitter the trees and other objects surrounding it are acting as efficient reflectors, and thus provide an effective source of radiation which is distributed over an appreciable area around the transmitting antenna itself.

Table 2. Summary of results obtained with d.f. set at various distances from transmitter operating on a wave-length of 7·9 metres.

Position of d.f. set	From transmitter		Error in observed d.f. bearing (degrees)
	Distance (miles)	True bearing (degrees)	
R.R.S. (A)	0·12	0	− 17
(B)	0·12	20	− 1
(C)	0·12	300	+ 15
Datchet	0·6	75	− 6
Sunnymeads	1·0	16	− 7
Wraysbury	2·0	342	− 5
Poyle	2·5	307	− 9
Stanwell	4·0	297	− 1
Ashford (A)	7·0	299	0
(B)	7·0	303	− 2
Feltham	8·0	293	− 2

These simple experiments serve to illustrate both the application and limitations of short-wave transmission and direction-finding. Unless the site around the transmitter is clear over a considerable area, reliably accurate bearing observations will not be possible except at distances for which the angle subtended at the receiver by the obstacles surrounding the transmitter is very small. Apart from this, direction-finding on ultra-short wave-lengths would appear to offer possibilities under receiving conditions which have been found to give good results on longer wave-lengths, so long as there is a direct-ray path from transmitter to receiver.

DISCUSSION

PROF. L. S. PALMER. I am, personally, needing to know the nature of the elliptical polarization of the electric vector of the wave near the surface of the earth which is consequent upon the earth's conductivity. A rough calculation from the authors' data, if $\sigma = 10^9$ e.s.u. and $f = 4 \cdot 10^7$ for a 7·5-metre wave, leads to an approximate value of 12·5 per cent. for the ratio of the horizontal component of the electric vector to the vertical component. From some measurements at this frequency which I have made recently with a frame aerial I deduce this ratio to be about 14 per cent. over the low wet soil immediately north of Hull. As this figure serves to check other results, I should be very grateful to know if the value of 12·5 per cent. calculated from the authors' data has been directly measured by them. I recollect that Dr Smith-Rose, with Mr Barfield, has done measurements of this kind with a tilting aerial but on longer wave-lengths. Secondly, I would be interested to know if the authors, in designing their radiating frame, gave any special consideration to its dimensions. I think it is quite possible that the strong signals received at Banstead may have been the result of a concentration of the radiation in that direction due to the particular frame-dimensions, which can very appreciably alter the angle at which the radiation leaves the aerial. I think this question would have to be considered as well as the reflected radiation from local trees which the authors suggest. As regards a third and minor point of interest in the question of the transmitter design, I should like to ask whether the screening box referred to in the appendix was of copper. If so, did the authors not have great difficulty in maintaining the oscillations when so much energy was being absorbed in the copper screen? I feel that the authors have obtained data, the real practical value of which will be fully appreciated only when the commercial applications of these short waves are matters of everyday life, and that time may not be very far off.

Mr L. L. K. HONEYBALL. I should like to ask the authors whether, in the design of their transmitting and receiving aerials for these ultra-short waves, they have explored the possibilities of making aerials of large surface area per unit length of conductor. I notice in the photographs shown that the aerials are of very small dimensions. My own experience in the design of gear for these wave-lengths suggests that there is an all-round gain to be obtained from the use of conductors of this large surface-area in the radiating portion of the oscillatory circuit. Apart from the increase in mechanical rigidity (necessitating the employment of less dielectric material to support the structure) and the obvious decrease in resistance, it is possible by this means to obtain actually a smaller value of inductance with a larger aerial. It seems to me that in the quest for frequencies as high as 5.10^7 or $10^8 \sim$ it is preferable to concentrate on reducing as far as possible the inductance rather than the capacity of the circuit, as any reduction in capacity limits the power and renders the oscillation less stable. Apart from any question as to the optimum dimensions of the aerial in relation to the wave-length, it is clearly better to reduce the inductance by increasing the area of the radiating surface rather than by a general all-round

reduction in the size of the aerial. I think it is of importance too that the aerial should be, if possible, of larger dimensions than the rest of the apparatus associated with it, even if the latter is screened or the former is fed by a high-frequency line.

AUTHORS' reply. In reply to the first of Professor Palmer's questions: no measurements have yet been made by us of the relative values of the vertical and horizontal components of the electric vector near the surface of the earth on the short wavelengths in question. While the tilting-aerial method as employed on longer wavelengths might be used for such measurements, it is doubtful if the method would be accurate enough to distinguish between the values of 12·5 per cent. and 14 per cent. mentioned by Prof. Palmer. It must be remembered that for a complete specification of the electric field at the earth's surface the value of the dielectric constant is required in addition to that of the conductivity. Our own experiments are being continued on these and higher frequencies with a view to evaluating these constants with greater accuracy.

Although we are aware of and appreciate very much the work carried out by Prof. Palmer and Mr Honeyball on the use of low-inductance loops for short-wave transmitters, we have not yet had an opportunity of applying them to our own investigations. In the case of the reception of signals at Banstead, mentioned in the paper, the transmitting antenna was a half-wave vertical dipole fed by a transmission line from a loop inside a screened hut. The question of the non-uniformity of radiation around a loop did not, therefore, arise, and the angle of elevation of the radiation above the ground was less than one degree. For the short-distance measurements we particularly desired a radiating loop of small area so as to simulate a point source. The use of specially designed loops of large surface area and low inductance, mentioned by Mr Honeyball, is worthy of consideration, and it is probable that such loops may have important applications to directional receivers as well as to transmitters.

We have usually employed tinned iron sheet for our screening boxes and have not experienced any great difficulty, due to absorption of energy in the screens, in maintaining oscillations. A more serious difficulty is that of keeping the dimensions of the screens and boxes small compared with the wave-lengths in use.

PRESENTATION OF THE DUDDELL MEDAL
to SIR JOHN AMBROSE FLEMING, M.A., D.Sc., F.R.S.

At the Annual General Meeting of the Physical Society, held at the Imperial College of Science, South Kensington, on Friday, March 20, the President of the Society, SIR ARTHUR EDDINGTON, F.R.S., presented the Duddell Medal of the Society to SIR JOHN AMBROSE FLEMING.

Sir Arthur Eddington, in making the presentation, spoke as follows: A careful review of the contributions to science and technology which have led to this award has been drawn up by those best qualified to judge, and I shall base what I have to say on their memorandum. We cannot sum up in a sentence researches extending over an active career of 50 years, but in general we may say that the work of Sir Ambrose Fleming has lain in the applications of electrical science, and perhaps particularly in the invention of instruments for exact measurement.

Educated at University College, London, the Royal College of Chemistry, and St John's College, Cambridge, he held for some time a lectureship at Cambridge in applied mechanics. From 1877 to 1879 he was working in the Cavendish Laboratory on a comparison of the British Association resistance coils and the standard British Association units—a work undertaken at Clerk Maxwell's suggestion. For this purpose he designed in 1879 a special form of resistance balance, which was in use there for many years for comparing standard coils. He also designed a special form of standard coil capable of taking up quickly the temperature of the water in which it was placed.

In 1882, when practical incandescent electric lighting began, his attention was turned to the need for quick and accurate workshop methods of electrical measurement. He made in 1883 the first direct-reading potentiometer, set to read current and potential directly in amperes and volts. He described this method in a journal which is now defunct. The method was almost immediately copied in various forms by all electrical instrument makers.

In connexion with incandescent lamp manufacture by the Edison and Swan Electric Lighting Company, Sir Ambrose Fleming designed in 1895 a form of large bulb incandescent lamp as a standard of light more constant and easy to use than the flame. This has been much used since. When alternating-current working began in 1883, and the alternating-current transformer began to be used, the necessity arose for careful determination of efficiencies. He designed a form of wattmeter for this purpose, with which he made extensive researches on transformer-efficiencies; these were reported to the Institution of Electrical Engineers in 1892. In this paper he first suggested the use of the term "power factor' which at once came into everyday use.

When practical wireless telegraphy first began under Marconi in 1898 no appliances were obtainable for measuring wave-lengths and frequencies. In 1904

Sir Ambrose Fleming invented a simple instrument for this purpose, which was capable also of measuring small capacities and inductances. In conjunction with Prof. Clinton he also devised a rotating commutator for measuring the capacities used in wireless. In 1904 he invented his now famous 2-electrode thermionic valve, which provided at once a simple detector of wireless waves, far more easy to manage than the coherer then in use. It prepared the way for a subsequent improvement, the 3-electrode valve, which is now the most important weapon of the radio engineer, and without which wireless as we now have it would not exist.

In 1905 he described to the Physical Society an instrument called the campograph, which enables curves such as the hysteresis curve of iron to be delineated optically without the tedious procedure involved in other methods.

Between 1892 and 1895 he co-operated with Sir James Dewar in a remarkable series of researches on the electric and magnetic properties of matter at low temperatures, including the great increase of electric conductivity which accrues. He devised special forms of resistance-coil and instruments for the measurement of inductance, capacity and thermo-electromotive force. I may remind you of the immense advance in modern theoretical physics to which the study of low temperatures has led.

He was Professor of Electrical Engineering at University College, London, for 42 years, and he established there the first laboratory for high-frequency and radio measurements. Besides his papers in scientific journals, he has published many books and treatises on applied electricity. It requires no great imagination to appreciate the importance of the numerous instruments of measurement designed by Sir Ambrose Fleming for use in the electrical industry generally and for scientific research, but I think there is one invention which comes uppermost in all our minds. I understand that the psychologists or psycho-analysts for their mysterious purposes sometimes fire at their victim a word or name which he has to respond to instantly by naming the first associated idea which comes into his head. If in this way I pronounce the word "Fleming," the instantaneous response is "valve," or we may vary the procedure and put it the other way round, and say when the word "valve" is mentioned, "Oh, Fleming, of course." The discovery of the 2-electrode valve in 1904 has paved the way for the great development of wireless to-day and so has led to one of the greatest influences of science on our civilization. This gift of science we can contemplate without those mixed feelings with which, for example, I should regard the telephone or the motor car, which sometimes occasion our gratitude and sometimes occasion our profanity. I do not suppose that Sir Ambrose foresaw all the strange results that would ultimately come from these experiments carried out in University College 25 years ago. As we sit here unheard voices are all around us; from all parts of the earth they speak. As in the magic Isle of Caliban,

> ... the isle is full of noises,
> Sounds and sweet airs that give delight and hurt not,
> Sometimes a thousand twangling instruments
> Will hum about mine ears: and sometimes voices ....

I think it might be safest to end the quotation there, because Shakespeare is

sometimes a little too apt. But if those in this audience who from time to time give talks before the microphone will shut their ears, I will complete the quotation:

> ... that, if I then had waked after long sleep,
> Will make me sleep again.

It is a proud distinction to have been one of those responsible for an essential part of this device, to have overcome one of the major obstacles to the fruition of those ideas which started with Maxwell and Hertz. There are names which we honour in connexion with the pure theory. We honour in a different way, though no less sincerely, those who by breadth of vision developed a scientific toy into something of world-wide service. Intermediately we honour those who by their skill in turning the results of scientific discovery into practical invention bridged the gulf, and among those the name of our medallist stands out conspicuously.

Now I turn to a feature of the award to which I have up to now deliberately suppressed all reference. I have, so to speak, placed our medallist on the dissecting table, and tried to show you from his writings and inventions what manner of man he is. But Sir Ambrose Fleming is not altogether a stranger to the Physical Society. He is one of ourselves. In fact, if I may so put it, he is one of ourselves more than any of us.

To go back to a period when the age of the present speaker belonged to the domain of negative numbers, Fleming was one of the original members of this society, and in March, 1874, he read the first paper at the first meeting of the society, his subject being, "The new contact theory of the galvanic cell." That was a good start, and he kept it up. During the ensuing 40 years he contributed regularly to our *Proceedings*—35 papers in all, amounting, I suppose, to about one-third of the total of his published papers. Once a year the occasion of the award of the Duddell Medal gives the society the pleasure of recognizing and honouring distinguished merit. It is a double pleasure when the honour falls to one of our intimate colleagues in the society. It is a triple pleasure, a manifold pleasure, when it falls, as it does on this occasion, to a founder of the society, whose papers started our scientific proceedings, and who has been loyal to the society during the 57 years of its existence.

Turning to Sir Ambrose Fleming, Sir Arthur Eddington said: Sir Ambrose, your life-work survives and bears fruit in the great electrical industries of the world. Your monument is in the circumambient ether. But you will not despise this mark of recognition and appreciation from the society of which you have so long been a member. In handing you the Duddell Medal I would express to you on behalf of the Physical Society its esteem for your achievements as a man of science, its regard and affection for you as a colleague, and its good wishes for your continued health and enjoyment.

Sir Ambrose Fleming, after expressing his thanks, added: It has been my good fortune for now 50 or more years, since this society was founded, to have been closely connected with the introduction into Great Britain of four scientific inventions which have subsequently founded great industries: the telephone, the incandescent electric lamp, the alternating-current transformer for high-tension

working, and wireless telegraphy. When I went up to Cambridge in 1877, with the chief object of working under Maxwell in the then recently completed Cavendish Laboratory, the telephone had just been invented by Graham Bell, who, by the way, was not an American, but an Englishman, or rather a Scotsman, although naturalized in America. Edison had added to that invention by inventing the carbon button transmitter. The Bell telephone was a good receiver, but a poor transmitter; Edison, therefore, availed himself of the discovery of the alteration of the resistance by means of a button of carbon made of lamp black, but he had no receiver, and had to invent a curious receiver, a chalk cylinder, moistened with various chemicals, which one had to keep on turning whilst one listened to it.

In 1878 David Hughes invented the microphone, and described it in a paper to the Royal Society. He did not patent it but placed it at the disposal of the public generally, and so the Bell Company availed themselves of this. Edison considered that that was an infringement of his rights, so the way was prepared for a nice quarrel, and the question was whether the microphone acted on the principle of the Edison arrangement. As a matter of fact, what the microphone did was to make small jerks and vibrations audible through a Bell telephone. An experiment which was very popular at that time was to put a domestic fly into an empty matchbox and to put on the outside a microphone connected with a battery, when one was supposed to hear the fly marching about with a tramp like that of an elephant. As a matter of fact, it was sometimes found when the matchbox was opened that the fly had been dead for some time. The noises heard with the microphone were due to agitations of that instrument by little movements in the room and so on. Some people even wondered whether it might be used to detect an incipient earthquake. I was brought into the arguments as to whether the Edison button was different in its mode of action from the microphone. As I say, the stage was set for a great quarrel, but the two telephone companies had soon to turn their faces towards a common enemy, namely, the Post Office, which claimed that the instrument could not be used without the payment of royalties to itself. That necessitated a very important action, and nearly all the eminent people of that day were engaged as witnesses either for or against the Crown. It was a question of the interpretation of certain words in an Act of Parliament.

At about that time Swan and Edison had patented the invention of the incandescent lamp. Edison sent to England two of the earliest samples of the carbon filament lamp, and his agent instructed me to show the action of these lamps in the offices of the Edison Telephone Company. The only thing that is worth remembering in that connexion is to show how one's memory sometimes fails one. Years after that, when we were engaged in fighting patent actions, it became necessary to know when the first lamps came over from Edison to London, and I was asked whether I knew anything about it. I had completely forgotten this early occurrence, and I said, "No." Then the secretary of the company which brought forward the Edison lamps said in reply to a question that the only man who knew anything about this first introduction was Fleming, and with that it all came back to me in a flash that in 1879 these specimens of the earliest lamps had been sent over to me.

Electric lighting began on a small scale. The first station established by the Edison people was in the basement of a house at No. 57, Holborn Viaduct. In those days we thought it a great thing if a machine ran through the night without a breakdown. The late Dr John Hopkinson, who was appointed scientific adviser with me, directed his attention to improving the design of the Edison dynamo. I devoted my attention to the lamp, and I will show you one slide which is interesting because it indicates the starting point for a number of other observations. My attention was directed to the fact that these lamps in some cases were blackened uniformly, while in other cases there was a curious white line which was in the plane of the filament, and when the filament came to be examined it was generally found to have been broken on the opposite side. In other words, there had been a projection of carbon in straight lines. In the majority of cases there was a volatilization of carbon from the filament generally. The next thing I found out was that on putting a metal plate into the bulb of a lamp and sealing it into the bulb, an expulsion of negative electricity took place from the filament. In those days we knew nothing about the electron; it was 10 or 12 years before the experimental discovery of the electron by Sir J. J. Thomson. I was astonished to find what an enormous quantity of electricity came off the filament. If I connected up with wire to the positive terminal of a large condenser the charge disappeared. The question was, where did that electricity come from? The founder of this society made the discovery that an iron ball heated red hot would hold a charge of negative electricity, but not of positive. I thought the phenomenon might be connected with the carbon, that the carbon particles thrown off carried a negative charge. I found that the greater part of the negative charge came off from the negative end of the filament. Here I show you one lamp which was covered with a deposit. Edison fastened the carbon to the platinum wires, and you will see that in one place there is a line in which there is no deposit, and that line is in the plane of the filament.

The next experiment I made was to construct a lamp with two filaments in it, and I found out that if one of those filaments—the positive—was rendered incandescent and not the other, then many thousands of volts were necessary to send the smallest current; but if the negative filament was rendered incandescent even one volt would send a current. It occurred to me that a bulb of this kind might act as a valve; if one of the carbon filaments was made incandescent, electricity could be sent in one direction and not the other.

All these observations were made between 1895 and 1896, and in a long paper which I presented to this society in 1896 I described these experiments. I also gave a discourse on the subject at the Royal Institution. But there the matter ended until wireless telegraphy came into the field. In 1897 Marconi came over with a new plan for conducting wireless by means of Hertzian waves. He had put together a number of old elements, which he had improved, and out of them he had made a practical system of telegraphy. His first achievement was to get across the English Channel in 1899. He then covered the longer distance between the Isle of Wight and the Lizard. But he was anxious to conquer the Atlantic, and that, he knew, would require greater power. Therefore, as I had been connected with the old London

Electric Supply Corporation, he asked me to design for him the arrangements for making powerful oscillations. I show a photograph of the first plant I arranged for him in the early stages of the Poldhu experiment.

On December 12, 1901, Marconi got his first signals across the Atlantic. Before that time we had often discussed whether it was possible to do that. We knew nothing in those days about the Heaviside layer, and my own view was that the chief objection would be the curvature of the earth. I repeatedly impressed on Marconi that he must lengthen the wave as much as possible. About that time I designed the cymometer by means of which we were able to measure the length of the waves actually used.

The question of detectors next came up. The early work was done with the coherer and telephone, but the coherer was troublesome to use and was very easily upset by atmospherics and by oscillations near it. I wanted to get hold of a receiver of some kind which would not be affected by atmospheric conditions or by powerful discharges in its neighbourhood. It was obvious that one would have to rectify the high frequency oscillations, and after trying a great many experiments with chemical rectifiers which did not work, my own experiments with these lamps came back into my mind, and I constructed a lamp in which the filament was surrounded by a cylinder. The space between cylinder and filament was unilaterally conductive. Under the influence of a battery it became useful for rectifying oscillations. I show you one of the first of these which was made for the Marconi Company. My attention was taken up at that time with other matters, and unfortunately I did not go on to put in the grid as I ought to have done. In America De Forest put in the first grid. Curiously enough, I had put a grid into a bulb, and you will see here a photograph of it, also a plate in another bulb, but what De Forest did was to put the two together into one bulb. Here is one of his receivers, and that at once made an amplifier. Soon after I constructed a 3-electrode valve with a spiral as the grid, which was the forerunner of the modern valves. It did not precede what De Forest did. That was the 3-electrode amplifying valve from which everything else has evolved.

My chief point is that the whole of these practical discoveries resulted from this single observation of the white line on the lamp which was otherwise covered with carbon.

I apologize for keeping you so long with these memories of early work, and I beg you to accept once more my sincere thanks for the great distinction you have conferred upon me by the gift of this Duddell Medal.

Prof. H. E. Armstrong, who was invited by the President to add a few words, said: I have known my friend Fleming since he was a student in this college with Guthrie. He set an example from the very beginning, and I think all of us who have known him have loved him sincerely. Among scientific men he has always ranked high on account of his extraordinary devotion to his work and his honesty in putting it forward.

Sir Ambrose Fleming and I can overlook practically everything that has taken place in the development of electricity. As he has told you, the dynamo at the

beginning was a very miserable machine, until Hopkinson took it in hand, and it had not been turned round and made into the efficient motor machine that we know to-day.

Not only have we witnessed in the period of which Sir Ambrose Fleming has been speaking the whole development of applied electricity, but we have seen scientific development in many other fields. For example, we have witnessed the age of the ironclad. The first ironclad, the *Warrior*, was built in 1860; I saw her in 1865 at Gibraltar, alongside the *Marlborough*, which was the last of the wooden three-deckers. I saw the *Great Eastern* when she declined to be launched on the stocks at Greenwich, and, as you know, she laid the first efficient cable. Then there has been the development of the internal combustion engine. I have an acute recollection of our troubles at the City and Guilds of London Technical College at Finsbury in the early days; how, for example, the piston had a habit of falling down backwards. I recollect, too, a most terrible quarrel between Ayrton and someone at Birmingham—Smith or Brown, I think, was the name—over this machine. It was a truly awful machine.

One may say that one has seen a whole vast development of scientific knowledge and its application to machinery in general, all in the space of about 60 years, and I think that is a matter that we ought to reflect upon a little more than we do. We do not sufficiently realize the astounding rate at which man has gone forward since he has learned to use the experimental method—the method in which your medallist has shown himself to be such a master. The world is in no way alive to the fact that we have discovered a new method, because it is practised only by the few, and nearly all of them engineers. We have been so much concerned with improvements in machinery, that we have forgotten the most important machine of all, that is to say, ourselves. We have not touched the politician, we have not touched government in any shape or form. I should like to see carried out a suggestion which was made by a bishop a few years ago, and put these interfering scientific inventors to sleep for a time, while we have leisure to think of other things. It seems to me that unless we can do that we shall be overcome by our own creations. Although we have to congratulate ourselves on this astounding development of the experimental method as applied particularly in physics, we have to bear in mind that there is this very much more important work awaiting our direction.

OBITUARY NOTICES

PROFESSOR H. L. CALLENDAR, C.B.E., M.A., LL.D., F.R.S.

HUGH LONGBOURNE CALLENDAR, eldest son of the Rev. Hugh Callendar of Magdalene College, Cambridge, was born at Hatherop, Gloucestershire, on April 18, 1863, and died at Ealing on January 21, 1930. He entered Trinity College, Cambridge, from Marlborough in 1882 and was placed in the First Class in the Classical Tripos, in 1884 and in the First Class in the Mathematical Tripos in 1885. He was appointed Professor of Physics at Montreal in 1893, at University College, London, in 1898 and at the Imperial College of Science in 1902.

Elected a Fellow of Trinity in 1886, he became a Fellow of the Royal Society in 1894 and Rumford Medallist in 1906. He was Treasurer of the Physical Society from 1900 to 1909, President during the period 1910–12 and first Duddell Medallist in 1923.

There is good reason to believe that Callendar regarded platinum thermometry and his investigation of the thermal properties of water and steam as his greatest achievements. It therefore seems appropriate that a brief tribute to his memory should confine itself to these aspects of his work.

Callendar's development of the platinum thermometer, which made it a standard instrument, is familiar to every physicist. Not the least of the virtues of such thermometers is the ease and certainty with which, by the use of two of them, the difference between one temperature and another can be determined from a single reading.

Callendar quickly saw the advantages of continuous flow methods in calorimetry and developed these in practice with extraordinary skill and ingenuity. In such methods there are no discontinuities due to the stopping or starting of the flow. The water equivalent of the apparatus is not required. The heat-loss can be reduced to a minimum and its magnitude can usually be obtained, as accurately as desired, by varying the speed and other conditions of flow. The Joule-Thomson porous-plug or throttling experiment is an early example of a continuous flow device to which, incidentally, Callendar applied differential platinum thermometry with conspicuous success.

Callendar applied a similar principle when he invented the continuous flow method of measuring the specific heat of water, used by Barnes and himself. At a later date, when the accuracy of the results obtained in these experiments was questioned, he showed his unfailing ingenuity by devising and using what was, in effect, an adaptation of continuous flow to the time-honoured "method of mixtures" familiar to every student of physics.

Another application of the same principle led to the determination of the specific heat of steam. This determination, in conjunction with that of the cooling effect in throttling experiments, enabled him to measure the variation with pressure of the total heat of steam. At a still later date he devised a general method of measuring the total heat at any pressure and temperature. Briefly, this method consisted in throttling to any desired pressure at constant total heat and then measuring, in a continuous flow calorimeter, the heat removed when the steam was reduced to water at measured temperature, of which the total heat was known. The same method was used to find the total heat of water, at any temperature and pressure, with reference to that at some temperature and pressure taken as standard. It was also applicable to wet steam.

With high courage Callendar set himself the task of measuring total heats in this way up to and beyond the critical region. The results are of the greatest interest. Before this work was done it had been commonly supposed that, at the critical point, the differences between saturated steam and water disappear. The specific volumes, it was thought, were then identical and the latent heat of vaporization zero. As is well known, the simplest method of determining the critical temperature of a liquid is to heat an appropriate amount of it in a sealed glass tube and to note the temperature at which the meniscus separating liquid from vapour disappears. This method presents no difficulties with substances like CO_2 and SO_2; but until fused-quartz tubes became available it was impracticable to apply it to water owing to the disintegrating effect of the latter upon ordinary glass.

In agreement with other observers Callendar, using quartz-glass tubes, found the critical temperature of water to be 374° C. Using different quantities of water in separate tubes of similar volume, he also obtained values for the densities of the liquid and the saturated vapour at temperatures below the critical point by observing the temperatures at which the tubes became filled with the liquid or the vapour as the case might be. In order to get consistent results in such experiments, it was necessary to use very pure water free from air.

Similar experiments can be performed very easily with liquid SO_2 instead of water. In this case ordinary glass suffices and the critical temperature and pressure are so much lower than those of water that there is neither difficulty nor danger in observing what happens in a series of tubes like those used by Callendar. Indeed, some of my old students of thirty years ago will remember this as a laboratory exercise devised for their instruction. Anyone who has performed such experiments and compared the phenomena with those exhibited by liquids which do not mix at ordinary temperatures, but become miscible in all proportions at higher temperatures, will agree with the view that the temperature at which the meniscus vanishes, in the case of a single substance in the tube, is not necessarily that at which the liquid and vapour phases become identical. It required the determination and the experimental skill of Callendar, however, to produce quantitative evidence. He found, for example, that the latent heat of vaporization of water at the critical temperature is not zero but 72·4 cal./gm. Not until a temperature of 380·5° C. is reached does the latent heat vanish. In obtaining these results, based upon

measurements of total heat, special methods had to be devised in the case of water to avoid contamination by air, traces of which were enough to make accurate determinations impossible.

Any attempt to indicate the character of Callendar's work on steam must contain some reference to the analytical skill which he displayed on the theoretical side. Almost every equation he used to correlate his results had a reasonable, if not an entirely "rational," basis. His equations, unlike those of his predecessors, were all consistent with the laws of thermodynamics.

One of the earliest applications which he made of platinum thermometry was to the study, at Montreal in conjunction with Nicolson, of the law of pressure/temperature variation in the adiabatic expansion and contraction of dry steam. This led him to the formulation of his well-known characteristic equation for an imperfect gas, which, in its earliest form,

v, p, θ, n

$$v - b = R\theta/p - a/R\theta^n,$$

was a modification of the Rankine-Thomson equation. Following previous workers, he replaced v by $(v - b)$. The novelty lay in the substitution of θ^n for θ^2 in the last term.

Assuming, as the simplest hypothesis suggested by the kinetic theory, that the variable part of the internal energy u of a gas is proportional to $p\,(v - b)$ and applying the second law of thermodynamics, Callendar deduced from this equation the expression

$$u = np\,(v - b) + \text{const.}$$

and expressions for n in terms of definable quantities, of which the simplest are

$$n = (c_v)_0/R, \quad \text{and} \quad (n + 1) = (c_p)_0/R,$$

$(c_v)_0, (c_p)_0$

$(c_v)_0$ and $(c_p)_0$ being the values to which the specific heats at constant volume and constant pressure, respectively, approach when the pressure is reduced without limit.

It also followed that the pressure/temperature variations for adiabatic changes should be given by the equation

$$p/\theta^{n+1} = \text{const.}$$

This was in accord with the experimental results for dry steam and the experimental value for n did not differ greatly from that calculable from the kinetic theory by a "reasonable" method.

H, ϕ

Correspondingly simple expressions for the total heat H and the entropy ϕ were:

$$H = (n + 1)\,p\,(v - b) + bp + \text{const.} \qquad \ldots\ldots(1),$$

$$\phi = (n + 1)\,R \log \theta - R \log p - ncp/\theta + \text{const.},$$

c

in which c represents the value of $a/R\theta^n$.

It was characteristic of the man that he should try to find a simple physical interpretation of c.

Writing his equation in the form

$$v - b = R\theta/p - c,$$

he pointed out that $R\theta/p$ represents the volume which the vapour would occupy if the effective volume were correctly represented by $(v - b)$ and if the vapour behaved like an ideal gas, consisting of molecules of the simplest possible type, R being the appropriate gas constant. The presence of the term c shows that the actual volume is less than the ideal volume. He suggested that the discrepancy is due to the fact that the vapour does not consist wholly of simple molecules but contains also a certain proportion of molecular aggregates. Increase in the number of such "molecules" of greater mass would cause $R\theta/p$ to become increasingly greater than the ideal volume. In this interpretation, inter-molecular forces do not appear explicitly. They are replaced by molecular complexes of which, however, they may be the cause.

Another striking example of Callendar's ingenuity is his equation for the total heat of water, which he used with great success in his correlation of the facts. To obtain it he developed a particular form of an argument previously used by Poynting, and assumed that water in equilibrium with its saturated vapour contains the same number of vapour molecules per unit volume as the vapour itself. He then supposed that the formation of such vapour molecules, within the liquid, consumed the same amount of heat as would be required to produce the corresponding volume of steam. Hence his equation

$$h = st + wL/(v - w) \qquad \text{.....(2),} \qquad h,\ t$$

expressing the value of the total heat of water at $t°$ C. reckoned from that at the freezing point as zero. In this, w and v are the specific volumes of the water and the saturated vapour respectively, and L is the latent heat of vaporization, at the temperature t. The constant s was fixed by taking h to be 100 calories at 100° C. $w,\ v$ L t

The equation leads to an expression for the entropy of saturated steam which, with the expressions for H and ϕ already mentioned, gives an equation connecting the saturation pressure of steam with the temperature. This well-known equation contains nothing, in addition to b and c, that is not directly measurable, and is in close accord with experiment up to 200° C. Beyond this, the experimental results began to deviate appreciably from those calculated and it became necessary to regard c as a function involving the pressure as well as the temperature.

In order to maintain thermodynamic consistency between the equation of state and the expression for the internal energy, it is necessary that the relation between c and p should satisfy certain conditions. From the many possible forms which the relation might take, Callendar selected the simplest for his purpose. It is contained in the equation

$$(v - b) = R\theta/p - c/(1 - Z^2) \qquad \text{.....(3),}$$

in which $c = a/R\theta^n$ as before and $Z = kcp/\theta$, where k is an additional constant. $Z,\ k$

He found that the experimental curve for the density of the saturated vapour could be represented by this equation right up to the critical temperature without

alteration of the value of c, provided that the value of the constant k were chosen to fit the density at that temperature. Further, the temperature/pressure curve for saturated vapour, the easiest of all the curves to determine experimentally, gave values which were in close agreement not only with those which could be deduced from his theoretical equation, but also with those obtained by other observers, by other methods, up to 374° C. The theoretical equation continued to give values of p agreeing with those observed by himself, until the temperature at which saturated steam and water become indistinguishable was attained.

Finally, the equation

$$(H - st)/(h - st) = v/w,$$

in which $H = h + L$, following at once from equation (2), gave a relation connecting total heats and specific volumes, which was used with success to test the accuracy of the measurements made in the critical region.

Callendar's last words on the practical aspects of this great work, which he pursued untiringly, will be found in his Hawksley Lecture delivered to the Institution of Mechanical Engineers shortly before his death. Referring to his three fundamental equations—(1), (2) and (3) above—he said: "They constitute the only rational and consistent standard which has yet been proposed for steam, and would suffice for all practical requirements if they were generally adopted."

Beyond doubt it was conviction and not dogmatism which impelled him to use these words, just as it was resignation and not impatience which led him to add: "But it has taken thirty years to reach international agreement with regard to the temperature scale, which is a much less exacting problem, and we may have to wait a long time before other nations are prepared to accept such a simple and unpretentious scheme."

Great designs require great consideration.

S. W. J. S.

PROFESSOR A. A. MICHELSON, Sc.D., LL.D., Ph.D.

When Michelson delivered the sixth Guthrie Lecture before the Physical Society in March 1921, more than one man of middle age who heard him felt it strange to see a man apparently no older than themselves who yet had seemed almost an historical personage in their own college days. It was indeed difficult to believe that this alert, vigorous, black-haired man had with Edward W. Morley published the famous paper "On the Relative Motion of the Earth and the Luminiferous Æther"[1] in the year of Queen Victoria's Jubilee, 1887. Yet he had been working at his first attempt as early as 1881[1].

Of the milestones in Michelson's life journey Sir Oliver Lodge[2] has left little to be said. I repeat the passage here, with some omissions and a few additions from various sources[3].

Albert Abraham Michelson was born in Strelno, Poland, on December 19th, 1852. In 1854 his parents brought him with them when they migrated to the United States, making the long journey across the western prairies to Virginia City, Nevada, in the days before the railroads. By the time that he was ready for high school his parents had moved to San Francisco, and on leaving high school in that city, at the age of seventeen, he went alone to Washington and persuaded President Grant to give him a presidential appointment to the Naval Academy at Annapolis, from which he graduated in 1873. After his graduation he served two years as a midshipman and then became instructor in physics and chemistry at Annapolis under Admiral Sampson, continuing this work until 1879. At that time he was not particularly interested in science but, determined to do a good job, he soon began experimenting and devising his own apparatus. Because there was no appropriation for material, Ensign Michelson spent ten dollars of his own money to construct an apparatus to measure the speed of light in a class-room demonstration. To his great surprise, he was able to make the measurement with greater accuracy than had ever before been achieved.

With his interest in science thus aroused, after a year in the Nautical Almanac Office at Washington Michelson went abroad for further study at the universities of Berlin and Heidelberg, and at the Collège de France and the École Polytechnique in Paris. Upon his return to the United States in 1883 he became professor of physics in the Case School of Applied Science, Cleveland, Ohio.

It was while he was at Cleveland that Professor Michelson collaborated with Professor Morley in their joint experiment, which was first carried out in 1883, and it was for the purpose of that experiment that Michelson invented his particular form of interferometer, with the to-and-fro beams at right angles. Later, he applied it in Paris to the determination of the metre in wave-lengths of light. The way this should be done had been first described in a paper by Morley and himself in 1887[4], though an actual attempt to make the wave-length of sodium light a standard of length was made by Peirce in 1879[5].

After six years at Cleveland, he was called in 1889 to Clark University, where he remained as professor until 1892. Then President Rainey Harper, on the look-out for promising men, brought him to the newly founded University of Chicago as Professor of Physics and Head of the Department. In June 1925 he was appointed to the first of the Distinguished Service Professorships made possible by the new development programme of the university.

During the world war Professor Michelson re-entered the naval service with the rank of lieutenant-commander, giving his whole time to seeking new devices for naval use, especially a range-finder which became part of the U.S. Navy equipment.

In 1924 he undertook a very accurate determination of the velocity of light, using Mount Wilson and San Antonio, 22 miles apart, as the stations. He remained in active work at the University of Chicago, engaging in both research and teaching, until he became seriously ill in the autumn of 1929, and underwent two operations. Upon his recovery, he asked to be relieved of active duties and took the status of Professor-emeritus on July 1, 1930. That summer he went to Pasadena, California, to repeat for the last time his famous experiment on the velocity of light. Finding the climate agreeable, he decided to establish a permanent residence in California. The university, however, continued to support his researches, and provided him with a technical assistant who built and installed equipment for the experiment.

Sir Oliver Lodge's article gives a lucid account of Michelson's achievements, but for members of the Physical Society a short reminder of the most important of them will suffice. In presenting the Duddell Medal for 1929 to Michelson, the President said:

"The interferometers invented by Professor Michelson, of which the first was used for carrying out in 1887 the famous Michelson and Morley experiment, have been applied by him, always with complete adequacy of design, to other important and difficult problems, most of them of audacious novelty. These problems included the measurement in 1892 and 1893 of the metre in wave-lengths of light[6]: the measurement of the diameters of stars: the re-measurement of the earth tides: and the testing of the effect of the earth's rotation on the velocity of light[7]. These measurements have had great consequences, of which the following examples may be mentioned:

"The difficulties of reconciling the result of the Michelson-Morley experiment with the then prevailing physical conception of the nature of the universe were the direct cause of the enquiry of Albert Einstein[8], which resulted in the theory of relativity. The measurement of the metre in wave-lengths of light[9] resulted in establishing a standard of length free from the uncertainty concerning possible variation which attaches to all material standards. The interferometer for the measurement of the diameter of stars, suggested by Michelson in 1890[10] and first applied by him to Betelgeuse[11], has not only confirmed the correctness of the previously almost incredible dimensions yielded by indirect means of calculation, but has detected fresh stellar phenomena in the variable diameter of Mira Ceti, and effected the separation of double stars too close for resolution by the telescope.

"The invention by Professor Michelson of the échelon diffraction grating[12]

provided physicists with a potent tool for the investigation of the fine structure of spectral lines, knowledge concerning which has become of such great importance in modern physics. Professor Michelson has also designed a ruling-engine with which very large gratings have been ruled.

"As a final example of Professor Michelson's work on scientific instruments for the advancement of knowledge, mention may be made of the completion by him in 1926, with apparatus designed by himself, of a re-determination of the velocity of light[13]. The elaborate precautions taken to secure freedom from error included means whereby the distance of 82 miles* traversed by the light was measured to a higher degree of accuracy than had ever been reached in triangulation."

An account by Michelson himself of his work in the years immediately preceding 1921 will be found in the sixth Guthrie Lecture[14].

In 1921, at the suggestion of Dr L. Silberstein, Michelson arranged an "ether-drift" experiment, in which the effect looked for was that which would be due to the rotation of the earth. The result, published in 1925, was in Michelson's opinion a positive one[15], confirming the existence of a stagnant ether. The writer of this notice has not seen any informed comment on this interesting result.

Professor Michelson before resigning his position at the University of Chicago had already worked at Mount Wilson Observatory, and a brief account of repetitions of the famous Michelson-Morley experiment, with a diagram of the apparatus, was contributed to *Nature* of January 19, 1929, by him and his collaborators. The results then obtained showed no displacement of the interferometer fringes so great as one-fifteenth of that to be expected on the supposition of an effect due to a motion of the solar system of three hundred kilometres per second through the ether.

His last work was concerned with a further determination of the velocity of light. This was described by the President of the Société française de Physique, Professor M. A. Cotton, in announcing Michelson's death to that Society, in the following words:

"Parmi les constantes physiques importantes, il en est une à laquelle la théorie de la relativité, comme le faisait déjà la théorie électromagnétique, assigne une place à part: c'est la valeur de la vitesse de la lumière. Michelson a consacré à plusieurs reprises à la mesure précise de cette vitesse toute son ingéniosité et son extrême habileté expérimentale. C'est cette mesure qu'il s'est appliqué encore à perfectionner durant les dernières années de sa vie, pendant lesquelles il a conservé toute son activité. Michelson avait décidé hardiment cette fois de supprimer la cause des petites erreurs qui peuvent persister toujours dans les mesures les plus soignées de la vitesse de la lumière, lorsqu'on les fait à la surface de la terre: celles qui proviennent du défaut d'homogénéité optique de la colonne gazeuse traversée par le faisceau lumineux, de ces variations de l'indice de réfraction de l'air qui font déjà le tourment des astronomes. Il avait donc décidé de faire le mesure dans le vide. Aussitôt remis d'une grave maladie qu'il avait éprouvée au début de l'hiver de 1929 il était retourné en Californie pour mettre en œuvre les puissants moyens qu'il avait su réunir. Mettant à profit une subvention de 750,000 francs provenant à la fois de la

* Subsequently reduced to 22 miles in view of effects of smoke.

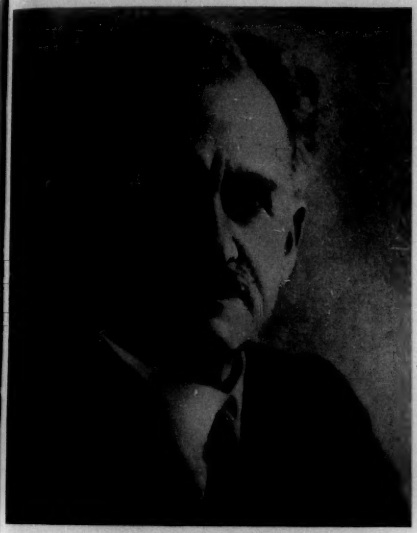
Fondation Rockefeller et de la Carnegie Corporation, utilisant les ressources de l'Observatoire du Mont Wilson, il installa sur un terrain approprié, dans la propriété de James Irvine, à 6 milles de Santa Ana, un tube d'acier de 1,610 mètres de long, de 91 centimètres de largeur, formé de morceaux de 20 mètres, avec des joints étanches: c'est dans l'intérieur que le faisceau lumineux poursuit sa route, parcourant plusieurs fois, grâce à des réflexions auxiliaires, la longueur de la base, avant de revenir sur le miroir tournant. Le dernier Rapport de l'Observatoire du Mont Wilson indique qu'on avait réussi à faire dans le tube un vide déjà satisfaisant et que les mesures allaient être commencées. Elles étaient terminées, assure-t-on, et Michelson faisait les calculs qui s'y rapportaient, lorsque la mort est venue le surprendre, en plein travail, à soixante-dix-huit ans."

In the Polish paper *Illustracja Polska*, No. 38, there appears the illustration here reproduced of this latest apparatus, showing the evacuated tube traversed by the beam of light. The control room of the apparatus is shown also. While actually working on these instruments Michelson had a sudden fainting fit. Immediately he regained consciousness, he asked what were the results of the experiment, and those were his last words.

Of Michelson's personality something may be added. "What he set out to do he accomplished with a minimum of effort because added to rare ability he had remarkable powers of concentration. Anything with which he was occupied so completely absorbed him that he thought of nothing else. His unusual concentration enabled him to drop whatever he was doing, engage in some other activity, and return to his original task. After several hours in his laboratory, he might play a game of chess or a few sets of tennis, forgetting entirely his experiments. But his companion in the recreation, walking toward his laboratory with Professor Michelson, would find that the game had absolutely passed from his mind, and that he was again absorbed in planning his investigation. Because of his concentration, he always had leisure to do everything that interested him, but he enjoyed only those activities in which he could participate.

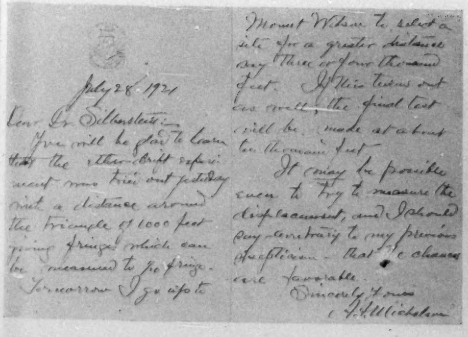
"He preferred to play the violin rather than to attend concerts: he expressed his interest in art by sketching and painting rather than by attending exhibitions. With no patience for mediocrity, whatever he decided to do he wanted to do well, and he would set himself to acquire the necessary technique. When he first came to the University of Chicago he was an indifferent billiard player, but because that game was one of the few diversions at the Quadrangle Club in which he could join his associates, he practised assiduously until he was proficient. More than thirty years ago, when he was nearly fifty years of age, he decided to improve his tennis game, and with the same determination with which he worked on an experiment, he practised strokes.

"In bridge he was not proficient, because he never set himself to master it. One reason, perhaps, was that his memory was of the 'long type'—he could remember every move of chessmen in a game played a year before, but he could not remember cards he had just played. When he was in the navy he was ordered to participate in a topographic survey for planning coast defences. In the mechanics of warfare he



Prof. A. A. MICHELSON.

Exterior of apparatus for determining the velocity of light.



July 28 1924

Dear Dr Silberstein:—

You will be glad to learn that the other Britt experiment was tried out yesterday with a distance around the triangle of 1000 feet giving fringes which can be measured to 1/10 fringe.

To-morrow I go up to Mount Wilson to select a site for a greater distance say three or four thousand feet. If this turns out as well the final test will be made at about the maximum limit.

It may be possible even to try to measure the displacement, and I should pay attention to my previous scepticism—that the chances are favorable.

Sincerely Yours
A. A. Michelson

Letter to Dr Silberstein.

Control room of apparatus for determining the velocity of light.

[*To face p.* 628

was not interested, and though his maps were beautifully accurate, he incurred the censure of his superiors because the borders were illustrated with pictures he had sketched of scenes that had caught his eye.

"His intense interests were satisfied without the development of any eccentricities. Something of the navy tradition remained with him throughout his life, reflected in his meticulous regard for his appearance and his punctilious observance of social form. In Ryerson Laboratory, associates recall, he was a little the admiral on the quarterdeck, courteously formal, the officer in command. No one ever saw him hurried or anxious, nor heard him raise his voice.

"Absolute honesty, the kindliest of courtesy, simplicity, and rigid self-control were his predominating personal qualities. Cordial and kindly to everyone, he had few intimate friends. Even these he did not call by their first names, for the discipline of his early career had given him a formality that was tempered only by his serenity and kindliness. He had, however, a deep feeling for those upon whom he bestowed affection, and would drop any task to spend a few minutes with his grandchildren.

"Whatever of vanity he had was early suppressed: whatever realization he had of his greatness and secure position in the history of civilization was never apparent. He could be brought to talk of his experiments only by direct questioning, and then he told the facts simply and impersonally. He had, though few even of his associates ever knew so, opinions and views on many questions and personalities, but he did not make them public property. He was a lifelong sufferer from insomnia, averaging in the last decade of his life less than five hours' sleep a night, but it never occurred to him to talk of the fact nor to discuss any other phases of his private life.

"He was self-contained to the point of being self-effacing, not through pride nor desire to be secretive, but wholly because he could not conceive of any interest in himself. When he came into the Quadrangle Club for lunch, he would take a seat by himself, but should any of his colleagues care to join the scientist he was quietly welcomed. He was one of the most inconspicuous figures on the campus. To those who asked, he would show his sketches and paintings, but visitors learned not to admire them openly, for Professor Michelson would insist on his guest accepting as a gift any work that was praised.

"He had a highly developed aesthetic sense, which found expression not only in his painting and composition of music, but in other more unusual forms. Twenty years ago, on a trip to South America, he found a species of beetle with bright and beautiful colours. Thinking they might please his children, he collected a group of them. On shipboard, his active mind began considering the physical causes of the brilliant coloration, and the speculation opened up a minor scientific problem that amused him for nearly ten years. He examined beetles, humming birds, moths and butterflies, figuring angles of incidence and other formulae, until he reached a definite conclusion. Incidentally, that diversion produced a spirited scientific debate with a celebrated English physicist, who did not agree with his conclusions. Professor Michelson had a deep philosophical interest and was a member of the American Philosophical Society, the organization founded by Benjamin Franklin. For years he attended its annual meetings in Philadelphia, greatly enjoying the

contact with thinkers in fields other than his. He knew many modern languages, and one of his milder diversions took the form of wide reading in Spanish and Italian literature[16]."

The following further notes on his personality are derived from two who knew him[17].

"Michelson dealt with facts without regard to whether they corresponded to his preconceptions or not, and, in fact, invariably attempted to avoid preconceptions of any kind. While he had a good knowledge of analysis, he was not interested particularly to follow the physical implications of the work he did. In his apparatus he frequently designed very simple and ingenious ways of doing things in which the temptation might be to do them in a more complicated manner. He was a good athlete, and was interested in sports, particularly tennis, throughout his life. Millikan mentioned that in playing tennis with him it was interesting to note that his calls were absolutely precise: he never gave either his opponent or himself the advantage of a doubt—indeed, he never seemed to have a doubt. When a ball was out, he said 'out,' and when it was in, he said 'in.' He was very positive in his views, and upheld them vigorously. He was not a teacher, except to those who were doing research with him: and indeed did not enjoy having students to work with him, considering that one of two things happened. If the student was poor, he was in the way: if he was good, he thought that the work was his and not Michelson's. He was a spasmodic rather than a steady worker, working at high speed when he was interested, and his output of work might seem to one near him not to be great." How impressive that work becomes when viewed from afar we all know, and this is doubtless because his achievements, though small in number, were all singularly complete and perfect in conception and execution, and all of them were of the very first importance. Perhaps only the most difficult problems fully aroused his genius.

"He was excellent company, bright and jovial, taking a lively part in the conversation, delighted to hear and to tell a good joke. He rather avoided scientific conversation, except in general enthusiastic terms, or concerning some particularly intricate technical difficulties of an experiment which he had just succeeded in conquering. He did not sympathize very much with relativity, and, publicly, scarcely ever alluded to that theory: yet he was not diffident of it: on the contrary, he seemed to attribute to relativity some uncanny power of predicting phenomena which were all verified. He liked a good vigorous walk in mild weather or in a biting frost, though he was equally alert in the scorching heat which is often experienced in Chicago, and he delighted in touring excursions, and spoke with enthusiasm of the beauty of the desert of Arizona where he toured extensively. He was, even at 70, one of the best tennis players in the university circles of Chicago, and an excellent billiard-player. Once, on a very hot afternoon, he played tennis with an opponent under the very hard self-imposed condition that his points should not count unless forming a suite of 5, and he won after some two hours. Before the play was ended a fellow professor came to remind him of a promise to play with him. "Well,' said Michelson, 'let me finish this, then I shall gladly play with you': which he did, vigorously and victoriously, and all this at some 90 or 95° F."

Of honours there is a long list. The chief was the Nobel Prize in 1907, which he was the first American to receive for science. In the same year he received the Copley Medal of the Royal Society, of which he was a foreign member. He was a corresponding member of the British Association, a Fellow and Gold Medallist of the Royal Astronomical Society, an honorary member of the Royal Institution and the Cambridge Philosophical Society, and an honorary or corresponding member of various other learned societies in America and other countries. He was a past president of the American Physical Society and of the American Association for the Advancement of Science, and Exchange Professor at Göttingen in 1911 and at the University of Paris in 1920. Honorary degrees were conferred on him by Cambridge, Yale, Pennsylvania, Leipzig, Göttingen, Christiania, Paris and McGill Universities. He married, in 1899, Edna Stanton of Lake Forest, Illinois, and had one son and three daughters.

As portrayed in the sources of information which I have consulted I have found the character of Michelson rather enigmatical. With all his varied activities, wide circle of acquaintances, and ready acceptance of good company, the man himself in his inmost being seems to have held aloof from any real intimacies. On this account I think it worth while concluding this notice by Michelson's statement of how he himself felt about his work.

"It has happened more frequently than might have been expected that research in pure science has resulted in tremendous industrial developments, as in the case of the researches of Faraday, Maxwell, and Hertz, which laid the foundation for all of our immense organization of electric light and power: telegraph, telephone, wireless, and other applications of electricity and magnetism.

"But quite apart from such direct benefits to humanity, it seems to me that scientific research should be regarded as a painter regards his art, a poet his poems, a composer his music. It would be quite as unfair to ask of these an apology for their efforts; and the kind of benefit which I should most appreciate from research in pure science is much more allied to such non-material results—results which help to increase the pleasure to us all of matter-of-fact existence, and which help to teach man his true relation to his surroundings—his place in Nature."

So the man whose experiments had such unforeseen and impressive results prized them rather as works of art in themselves. The passage reveals, I think, rare serenity and balance of mind.

F. TWYMAN

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(10) *Phil Mag.* **30**, 1 (1890).
(11) *Astrophys. J.* **51**, 257 (1920).
(12) *Ibid.* **8**, 37 (1898).
(13) *Ibid.* **65**, 1 (1927).
(14) *Proc. Phys. Soc.* **33**, 275 (1921).
(15) *Astrophys. J.* (1925).
(16) *Personality sketch from the Department of Public Relations, University of Chicago.*
(17) Professor Millikan (communicated to the writer of the notice by one of the audience who heard his obituary address on Michelson in Pasadena) and Dr L. Silberstein, who came in touch with Michelson by suggesting to him the experiment carried out in 1921 on the possible effect of the rotation of the earth in producing an effect on the velocity of light.

MR HERBERT TOMLINSON, F.R.S.

The death of Mr Herbert Tomlinson on June 12, 1931, will recall to the minds of many of our older physicists the nature of the research work done in the later Victorian years. He was born in 1845 and went to St Peter's School, York, from which he gained a scholarship at Christ Church, Oxford. There he studied mathematics and physics, receiving the B.A. degree in 1868.

Shortly after graduation he went as demonstrator and lecturer in physics to King's College, London, under Professor Grylls Adams. It will be remembered that Clerk Maxwell, the centenary of whose birth is being commemorated in October, was professor of physics at King's College from 1860–65, and he had made the laboratory a centre of research. When he resigned, Professor Grylls Adams, brother of the discoverer of the planet Neptune, was appointed, and Mr Tomlinson on coming into this genial and intellectual atmosphere soon had his mind turned to research. The present writer, in looking over the subjects of Mr Tomlinson's papers, feels that he can trace the influence of Maxwell in the problems and methods used in them. The viscosity of air is one of the subjects that attracted Tomlinson. Maxwell had made measurements with oscillating plates; Tomlinson used spheres attached to a rod which was arranged to oscillate at the end of a torsion wire. Sir George Stokes, then president of the Royal Society, was interested in the experiment and calculated a correction for the rotational effect of the spheres, and indeed communicated the paper to the Society. It is interesting to compare the final result, 0·00017711 at 11°·79 C., with the recent determination by the capillary-tube method made by Professor Rankine who found 0·0001770 at 11°·2 C. and 0·0001803 at 15°·5 C.

Many of Tomlinson's other papers, most of which were communicated to the Royal Society by Professor Grylls Adams, dealt with the internal friction of metals and the effects of temperature and magnetism on this friction, and with the velocity of sound in wires determined directly and indirectly. These subjects formed the matter of many extensive papers in the Royal Society's *Transactions*, and for his work he was elected a Fellow on June 6, 1889. Among the others elected at the same meeting were John Aitken, Horace T. Brown, Latimer Clark, Professor McKenny Hughes and Professor Sollas.

Tomlinson joined the Physical Society in its first session, and the 1926 list contains the name of only one other who joined that session. He was a life member, and served on the Council.

In 1894 Tomlinson gave up the work at King's College for the post of Principal of the newly built South-western Polytechnic at Chelsea, and this work in technical education absorbed his activity until 1904. Among the many classes that he instituted was a Saturday morning class for team-work in research. He and a number of the more advanced students met for the purpose of a united attack on

some problem in elasticity or magnetism. The results were difficult to utilize, for although he was an experienced research-worker himself, his assistants could claim but little facility at practical work.

In 1904 he resigned the principalship and retired to Bexhill, where he devoted much time to wireless telegraphy, then in its infancy. It was characteristic of him that when he left London he told the present writer that he would never revisit it; and so far as the writer knows he kept his word. Unfortunate circumstances deprived him of his small fortune, and as he was ineligible for any educational pension he received a Civil List pension for his scientific work. During the War he served for some time as a science teacher at Lancing College.

Tomlinson was a man of intense devotion to a task that interested him, but he had also the capacity of putting aside a subject and taking up another with equal intensity. He was a most pleasant and cordial principal and received the devotion of his staff and students. His intimate friends included the late Professor Reinold, but he was not known to many of the younger school of physicists. His work was of importance in tracing changes in the properties of matter under varying conditions, and he brought to general notice many of these properties which in his time were little appreciated.

S. SKINNER

REVIEWS OF BOOKS

An Introduction to Applied Optics, Vol. 1, *General and Physiological*, by L. C. MARTIN, D.Sc., A.R.C.S., D.I.C. Pp. ix + 324. (London: Sir Isaac Pitman and Sons, Ltd., 1930.) 21*s*.

In general our text-books can easily be divided into two categories. The material of a new text-book of one class can be practically all found in two or three of its predecessors, and very often the errors of an earlier text are repeated in the new. Unfortunately this class is by far the more numerous, since the writing of such text-books involves a minimum expenditure of time and effort. It is this class of writing that is responsible for the common complaint that new and important material does not "get into the text-books" for ten or more years. Occasionally, however, we are rewarded with a book of the other class, which presents the subject from a fresh viewpoint. It is usually written by one who has specialized in the subject for many years, and therefore contains much material that has never found its way into the stereotyped class book.

Dr Martin's book is clearly of this latter class and he is to be congratulated on achieving so much in the almost superhuman task of giving a fair *résumé* of present day applied optics. The elementary theory given in the first chapter includes Dowell's new graphical construction for tracing a ray through an optical system, and a numerical example of trigonometric tracing as used in lens designing. Chapter 2 deals with the general theory of optical systems both graphically and analytically. In the third chapter, just enough physical optics is introduced to enable the reader to understand the nature of the diffractional effects obtained with telescopes, and the chief methods of investigating the properties of radiation are briefly summarized. The fourth chapter is a masterly account of the defects of optical images. This is probably unequalled in any other book, and no serious student of optics should miss the clear account of aberration that is given here. Chapter 5 deals with the eye and physiological optics and is followed by further chapters on physical optics, optical glass (including lens-polishing) and spectacles.

The book is singularly free from misprints, but one occurs on p. 184. The phase-change on reflection at a denser medium is an *advance* of π, so that the minus sign should be used in the expression for the total path-difference. The text for striae given on p. 241 is very much improved when the eye is placed behind an obstacle slightly larger than the image of the small light-source that falls on it. These are very minor points, and the book can be whole-heartedly recommended to all who are interested in practical optics and to those who wish to make the most of the optical instruments that they use as tools. The appearance of the second volume, dealing with the telescope and the microscope, will be awaited with interest.

W. E. W.

The Scientific Journal of the Royal College of Science, Vol. 1. Pp. 158. (London: Imperial College Union, 1931.) 3*s*.

With the rapid growth of the sciences comes a growing need for accounts which, while sufficiently elementary to appeal to students not specially versed in the details of the science dealt with, are yet technical enough to be of interest to the expert. It is not easy to find such accounts; the difficulties are patent and well-known—they are in fact the difficulties that face every one whose lot it has been to contribute an article to an encyclo-

pædia. But addresses given before student societies are as likely as not to hit the mark, and, to whomsoever be the credit, it was a happy thought to induce three of the student societies of the Imperial College to join forces. The result has been the production of a volume of permanent interest. The three societies concerned are the Imperial College Chemical Society, the R.C.S. Natural History Society, and the R.C.S. Mathematical and Physical Society. The first-named society is not new to the business—it has for nine years past produced an annual journal. The present volume continues worthily the traditions of its predecessors and contains five authoritative addresses by W. H. Perkin, on high-pressure chemical plant, on the mechanism of the combination of hydrogen and oxygen, and on the polymerizing capacities of unsaturated compounds. This in itself is good enough fare, but the volume is greatly strengthened by the addition of three natural history lectures on the Alps, on plant-breeding, and on tree crops, and of four lectures—on moving magnetic fields, time, the determation of gases and the nature of vowel and consonant sounds—which were delivered before the Physical and Mathematical Societies.

The societies concerned are to be heartily congratulated on their venture; their first volume is pleasant to look at and to handle and good to read, and we trust that it will be the ancestor of a long and healthy line. A. F.

A Survey of Physics (*for College Students*), by Prof. F. A. SAUNDERS. Pp. x + 635. (London: G. Bell and Sons, Ltd.) 14*s*.

The chief virtues of this book lie in its modern viewpoint and its thoroughly practical outlook. Among other things, it gives brief accounts of or mentions series spectra, the Schottky effect, the Raman effect, wave mechanics, electron-diffraction and relativity. While no very obvious immediate purpose is served by brief non-mathematical accounts of some of these, there is much to be said for introducing some mention of them at an early stage in physics courses.

The book is good on the practical side, and it freely mentions modern technical applications. The exercises and problems are generally so devised as to give sound ideas on common physical magnitudes rather than mere practice in arithmetical manipulation. The practical outlook and general style may be illustrated by quotation from a passage (p. 21) which accompanies a highly moral tale of the discomfiture of an inventor who lacked instruction in the elements of theoretical physics: "It makes an instructive illustration of the magnitude of the air pressure to exhaust the air from the inside of an old tin can, provided that it is no longer needed." This might have been less ambiguously expressed, but it would be difficult to devise a mode of exposition more likely to drive the lesson home.

On the systematic and theoretical side the treatment is less adequate—less so than that of many established texts of similar range. It is at times even a little misleading. For example, the electronic change is deduced (p. 363) by dividing $6{\cdot}06 \times 10^{23}$ (Avogadro's number per gramme-molecule) into Faraday's constant, and stress is laid upon the exactitude of the agreement of this result with Millikan's oil-drop value. It is surely a little disingenuous to suggest that this particular agreement proves anything beyond the reliability of the ordinary rules of arithmetic.

In spite of defects of this kind, the book is clearly and vigorously written, and most junior students of physics could read it with both profit and pleasure. It contains many things which are not to be found in the more conventional elementary texts, and some excellent photographs.

It should be pointed out (after the announcement on the dust-cover) that the book provides only a *preliminary* "survey of all the general physics demanded... by a pass degree examination." The text and the many excellent problems and exercises are in the main of a standard not very different from those associated with work for intermediate science, pre-medical and school-certificate examinations in this country. H. R. R.

Thermodynamics, by ALFRED W. PORTER, D.Sc., F.R.S. Fcap. 8vo, pp. vii + 96. (London: Methuen, 1931.) *2s 6d.*

The number of possible solutions of the problem of selection of material for a small book on thermodynamics must be very large. The one embodied in this work seems thoroughly suited for a third-year course for students reading for an honours degree. After an historical introduction of 20 pages, which commences with the foundations of mechanics, it deals in succession with reversible changes and their applications, with irreversible changes and with equilibrium. The treatment of gas equations is particularly good and will give a student a much clearer idea of their limitations than he can obtain from many larger works. There are few misprints (pp. 36, 37), but the absence of numbering from the equations is an inconvenience when "it has been shewn that" is encountered.

C. H. L.

Krieselräder als Pumpen und Turbinen, Vol. I, by WILHELM SPANNHAKE, Professor at the Karlsruhe Technical High School. 8vo, pp. viii + 320. (Berlin: Springer, 1931.) 29 marks.

In this volume, which is the first of what promises to be an extensive treatise, the author establishes the hydrodynamical theorems applicable to machines and plant which deal with the flow of fluids. Turbulent motion is not included, but in cases where the one-dimensional stream-line theory, which is the principal feature of the book, is found insufficient, corrections, such as experience has shewn to be necessary, are applied to the velocities and energies. This enables an outline of the subject to be traced, which the second volume will fill in and render more exact, while the third volume will deal with the mechanical details of the machines. The volume is well printed and illustrated.

C. H. L

Meteorological Office, Professional Notes. No. 52: *Bumpiness on the Cairo-Basra Air Route*, by J. DURWARD, M.A. 8vo, pp. 6. (London: H.M. Stationery Office, 1929.) *3d.*

The rises and falls of an aircraft due to vertical air currents, which are brought about generally by irregular country, are least in the early morning, but may at other times be experienced at altitudes of 12,000 feet.

C. H. L.

Meteorological Office, Professional Notes. No. 53: *The Relation between the Duration of Bright Sunshine and the Amount of Cloud*, by C. E. P. BROOKS, D.Sc. 8vo, pp. 15. (London: H.M. Stationery Office, 1929.) *9d.* net.

By comparing the records of Campbell-Stokes sunshine-recorders with those of cloudiness, the author has deduced a formula giving the duration of bright sunshine in terms of cloudiness which allows the sunshine to be calculated from observations of clouds, made at three or more periods of the day, at places where sunshine recorders are not available.

C. H. L.

Meteorological Office Réseau Mondial 1924. 4to, pp. xv + 115. (London: H.M. Stationery Office, 1931.) 25*s*. net.

No change has been made in the form of publication of this volume as compared with that of the previous year.* The number of stations utilized was 479 for the year, as against 463 for the previous year. C. H. L

Meteorological Office, Geophysical Memoirs. No. 48: *The Meteorological Results of Journeys in the Southern Sahara,* 1922 and 1927. Made by F. R. Rodd, F.R.G.S. Discussed by C. E. P. Brooks, D.Sc. and S. T. A. Mirrlees, M.A. 4to, pp. 40. (London: H.M. Stationery Office, 1929.) 3*s*. 6*d*. net.

The region north of Kano (12° N. to 20° N.) has a winter dry season with north-easterly winds and a summer wet season with south-westerly winds and a rainfall of 34 inches at Kano, considerably less to the north, and in the northern plains probably less than 5 inches. In summer the nights are warm and oppressive. C. H. L.

Meteorological Office, Geophysical Memoirs. No. 49: *Two Notes on the Operation of Galitzin Seismographs,* by F. J. Scrase, M.A., B.Sc. Pp. 9. (London: H.M. Stationery Office, 1930.) 1*s*. net.

A diagram is given enabling the damping-constant and the free period of a Galitzin seismograph to be readily evaluated. The variation of the period of the pendulum with position, brought about by the single control spring, can be overcome only by the use of additional springs. C. H. L.

Meteorological Office, Geophysical Memoirs. No. 52: *Some Characteristics of Eddy Motion in the Atmosphere,* by F. J. Scrase, M.A., B.Sc. Pp. 16. (London: H.M. Stationery Office, 1930.) 1*s*. 6*d*. net.

The eddy motions in the wind over Salisbury Plain have been studied by means of kinematograph records of the position of a wind-vane free to move both horizontally and vertically, and of a Dines anemometer. For average eddy motion the eddy speeds across and down the wind are proportional to the mean wind speed, the horizontal cross-wind speed being greater than the down-wind speed, and that greater than the vertical speed. The ratios of these speeds vary with height above the ground. C. H. L.

Novius Organum, by James Clark McKerrow, M.B. Pp. viii + 277. (London: Longmans Green and Co. Ltd., 1931.) 9*s*.

Mr McKerrow's title is well chosen; his philosophy is in the classical tradition, even if his exposition of it is not in the classical style. For he will explain the universe to us, whether under the aspect of cosmogony or of social economics, in terms of a single principle—the principle of "habit," which makes things repeat their activity.

The only question that need be asked here is whether Mr McKerrow's arguments, often acute and entertaining, are of particular interest to physicists. The answer is that they are not. The "limited, second-hand, and popular knowledge" to which he confesses, does not necessarily make his "liberty of thought border on licence"; but a philosopher who appeals to scientists must take the scientific attitude towards science and refrain from criticizing theories on the ground that they fail to solve problems lying entirely outside their range. N. R. C.

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